Running coupling $\alpha_s(\mu)$ from Wilson flow for three quark flavors

G. Schierholz

Deutsches Elektronen-Synchrotron DESY

With

R. Horsley, Y. Nakamura, H. Perlt, P.E.L. Rakow, A. Schiller, J. Zanotti

QCDSF Collaboration

Synopsis

$$t^2 E(t) \;=\; \frac{3}{4\pi}\,\alpha_{WF}(\mu) \quad + \quad \Sigma^2\, t$$

$\uparrow$ perturbative $\quad\quad$ $\uparrow$ nonperturbative

$$\mu = \frac{1}{\sqrt{8\,t}}$$

$$\alpha_{WF} = \alpha_s + k_1\,\alpha_s^2 + k_2\,\alpha_s^3 + O(\alpha_s^4)$$

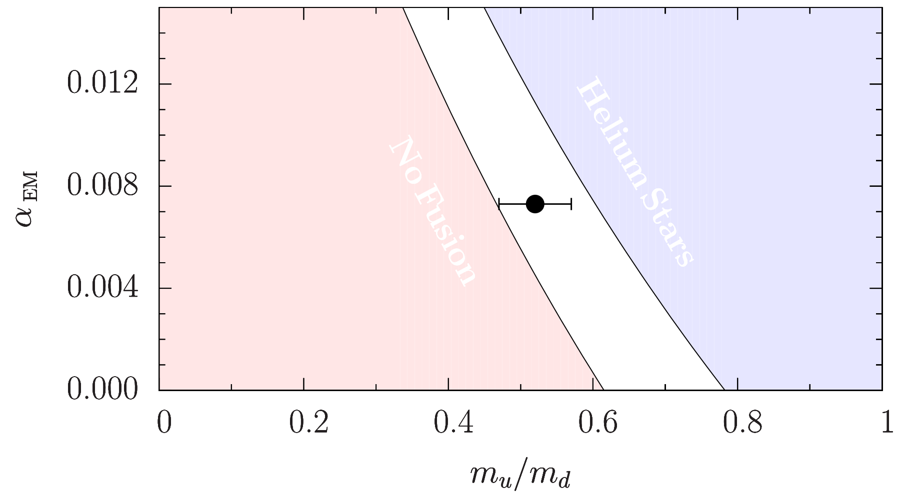
Focus on

- $O(a^2)$ corrections
- Determination of $\alpha_s(\mu)$ at medium to small scales μ
- Infrared fixed point?

Simulations done at 5 lattice spacings a

Of central importance to low-energy phenomenology

The long-distance attributes of QCD, that are independent of the short-distance structure of the theory, such as the LECs (quark masses, condensates, etc.) of the chiral Lagrangian, are ultimately related to the existence of infrared fixed points



If $\beta(\alpha_s)$ vanishes when α_s runs to an infrared fixed point $\alpha_s(0)$, QCD becomes a conformal theory at zero quark mass (i.e. $\Theta_{\mu\mu} = 0$), with scale invariance being spontaneously broken, giving rise to a dilaton σ. This results in a mass gap at the fixed point, and $\chi PT_3 \rightarrow \chi PT_\sigma$

Conformal behavior of QCD is the basis for commensurate scale relations, which relate observables to each other without renormalization scale or scheme ambiguities, e.g. the Crewther relation

Data

Action

$$S_G = \frac{6}{g^2} \sum_{x,\mu<\nu} \frac{1}{3} \mathrm{Tr} \left\{ c_0 \left[1 - U_{\mu\nu}(x)\right] + c_1 \left[1 - R_{\mu\nu}(x)\right] \right\}$$

$$\begin{aligned} S_F = \sum_{q=u,d,s} \sum_x \Big\{ \sum_\mu \Big[& \overline{q}(x) \frac{\gamma_\mu - 1}{2} \tilde{U}_\mu(x) q(x+\hat{\mu}) \\ & - \overline{q}(x) \frac{\gamma_\mu + 1}{2} \tilde{U}^\dagger_\mu(x-\hat{\mu}) q(x-\hat{\mu}) \Big] \\ & + \frac{1}{2\kappa_q} \overline{q}(x) q(x) - \frac{1}{4} c_{SW} \sum_{\mu\nu} \overline{q}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) q(x) \Big\} \end{aligned}$$

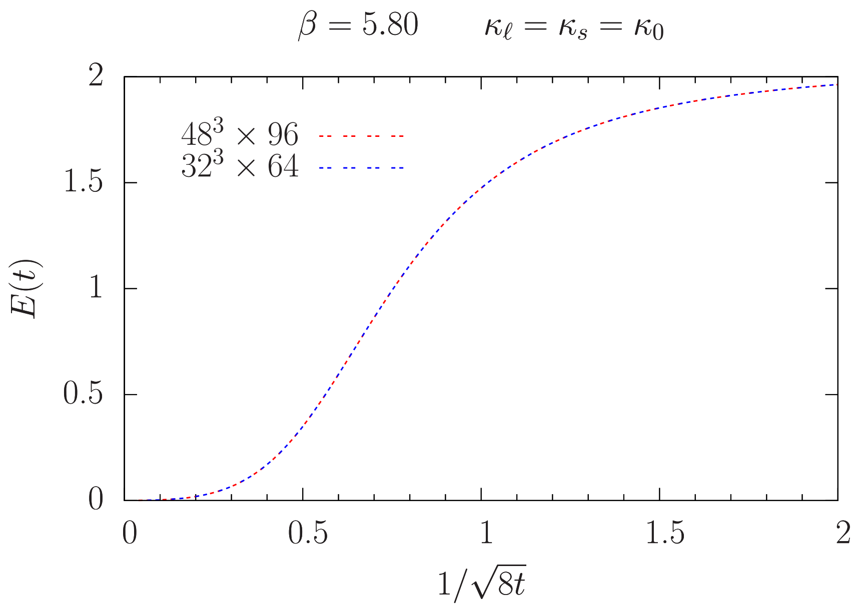
We keep the average quark mass $\bar{m} = (m_u + m_d + m_s)/3$ constant, with $\bar{m}$ tuned to the physical value of the average pseudoscalar meson mass $(M^2_{K^0} + M^2_{K^+} + 2M^2_{\pi^0} - M^2_{\pi^+})/3$

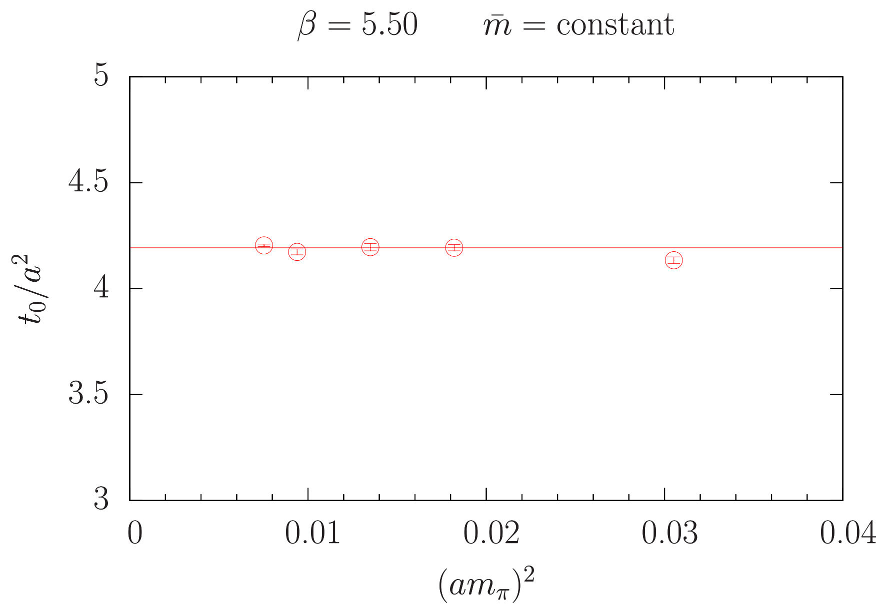
WF Ensemble

β	V	κ_0	κ_l	κ_s
5.95	$48^3 \times 96$	0.123460	0.123460	0.123460
5.80	$48^3 \times 96$	0.122810	0.122810	0.122810
5.80	$32^3 \times 64$	0.122810	0.122810	0.122810
5.80	$48^3 \times 96$	0.122810	0.122880	0.122670
5.80	$48^3 \times 96$	0.122810	0.122940	0.122551
5.65	$32^3 \times 64$	0.122030	0.122030	0.122030
5.50	$32^3 \times 64$	0.120900	0.120900	0.120900
5.50	$32^3 \times 64$	0.120900	0.121040	0.120620
5.50	$32^3 \times 64$	0.120900	0.121095	0.120512
5.50	$32^3 \times 64$	0.120900	0.121145	0.120413
5.50	$48^3 \times 96$	0.120900	0.121166	0.120371
5.40	$24^3 \times 48$	0.119930	0.119930	0.119930
5.40	$24^3 \times 48$	0.119930	0.120048	0.119695

$N_f = 2 + 1 : \ \kappa_u = \kappa_d \equiv \kappa_\ell \qquad 2/\kappa_\ell + 1/\kappa_s = 3/\kappa_0$

Volume & mass dependence ?

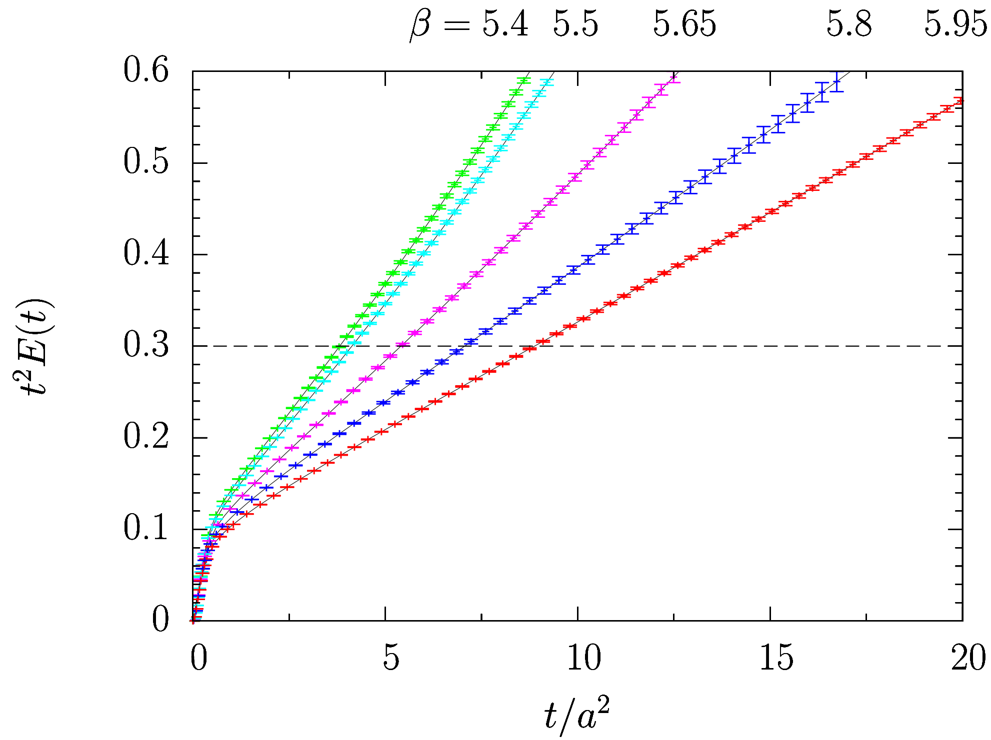




Can limit ourselves to SU(3) symmetric point: $\kappa_\ell = \kappa_s = \kappa_0$

Wilson Flow

Observable E: clover



$$\left.\frac{t}{a^2}\right|_{t^2E(t)=0.3} \equiv \frac{t_0}{a^2}$$

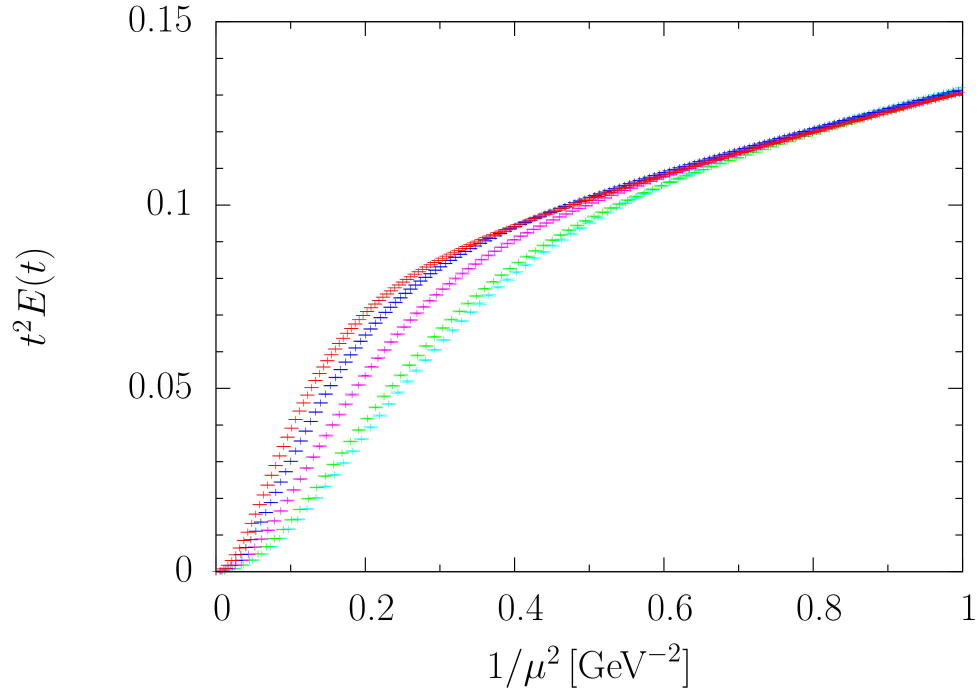
Lattice spacings

β	5.40	5.50	5.65	5.80	5.95
a [fm]	0.0818(09)	0.0740(04)	0.0684(04)	0.0588(03)	0.0507(05)

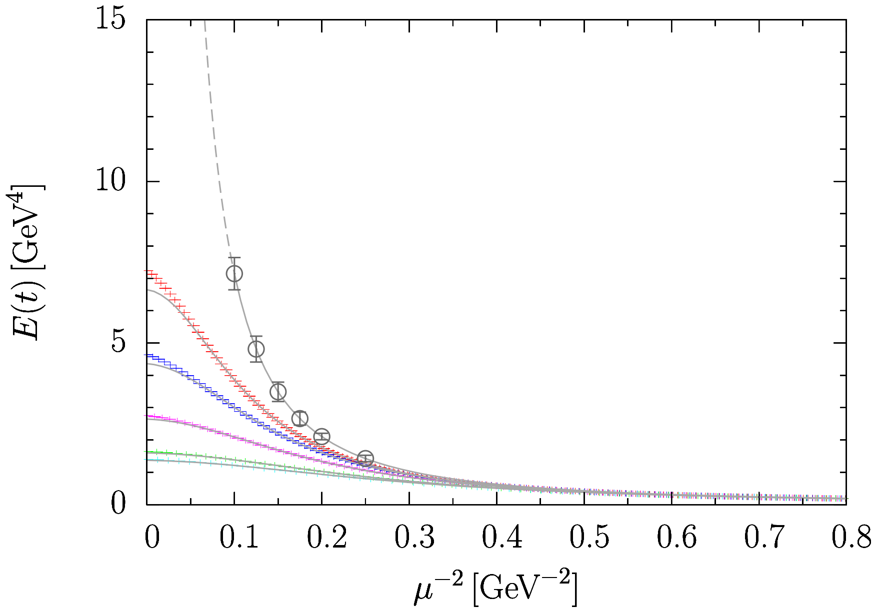
from $\sqrt{t_0} = 0.151$ [fm]

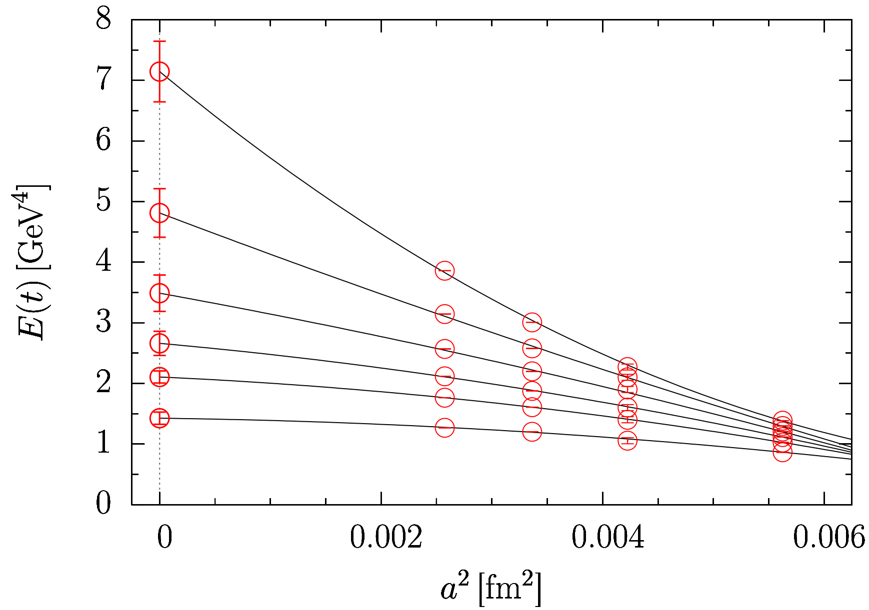
arXiv:1508.05916

In physical units

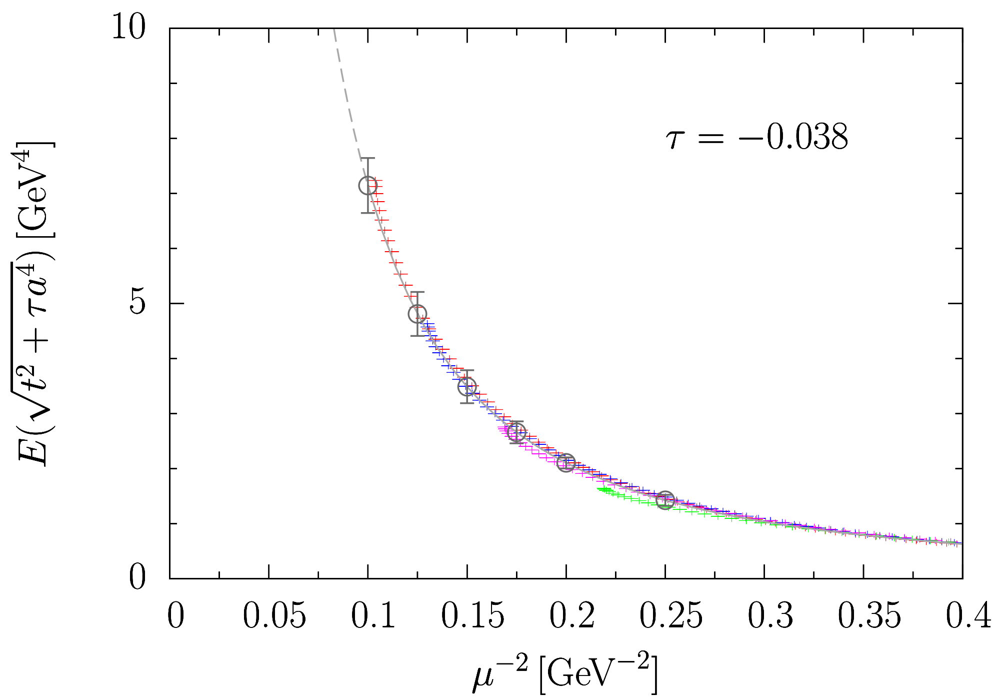


$O(a^2)$ Corrections





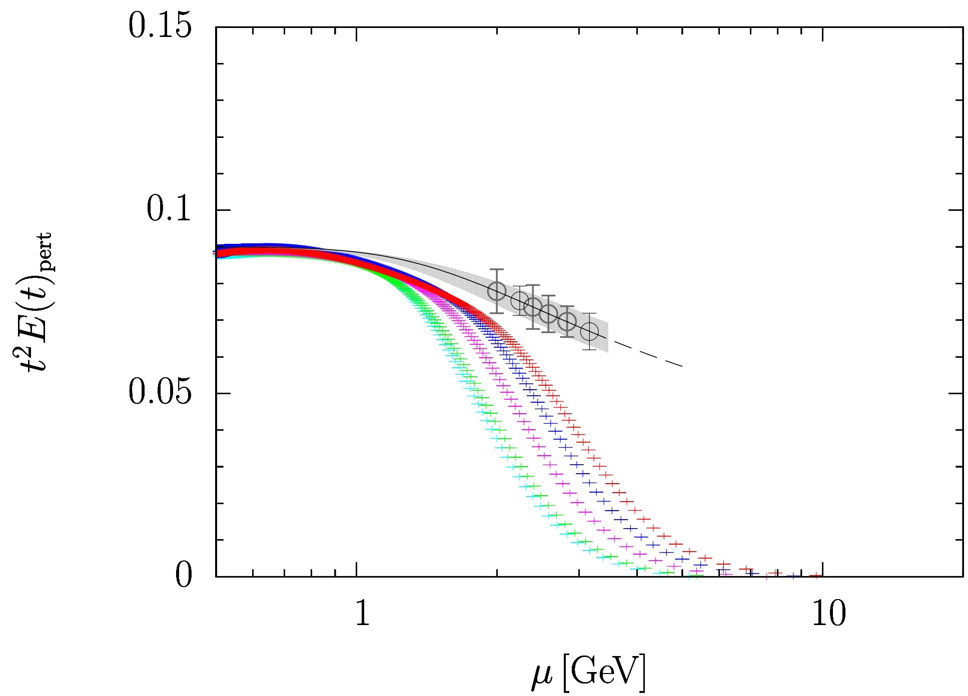
Alternatively

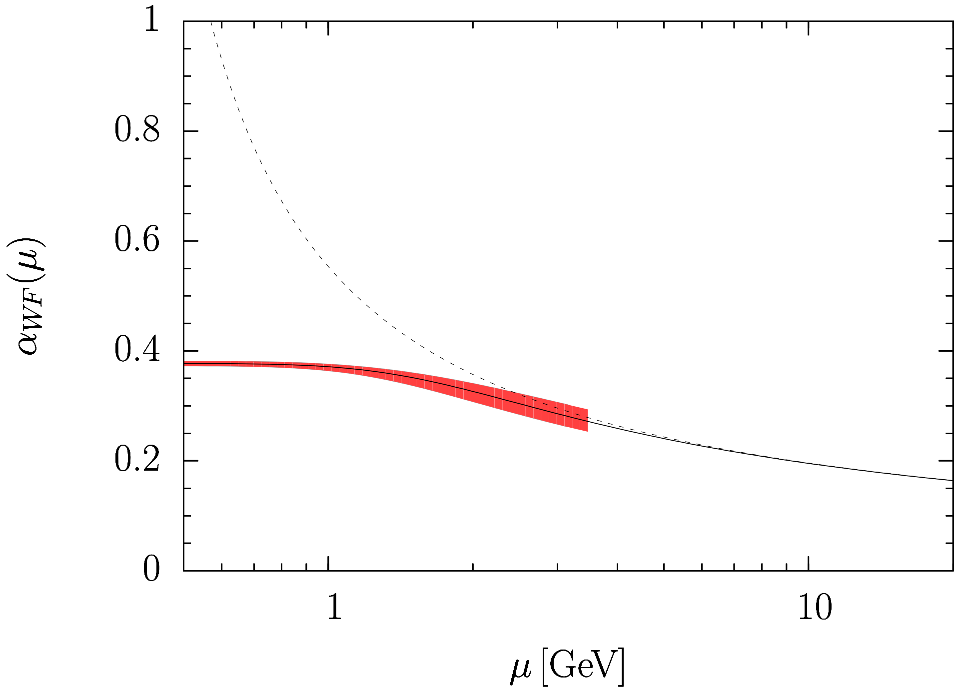


$$t^2 E(t) + O(a^2) \simeq t^2 E(\sqrt{t^2 + \tau a^4})$$

$\alpha_s(\mu)$

Perturbative part of $t^2 E(t)$



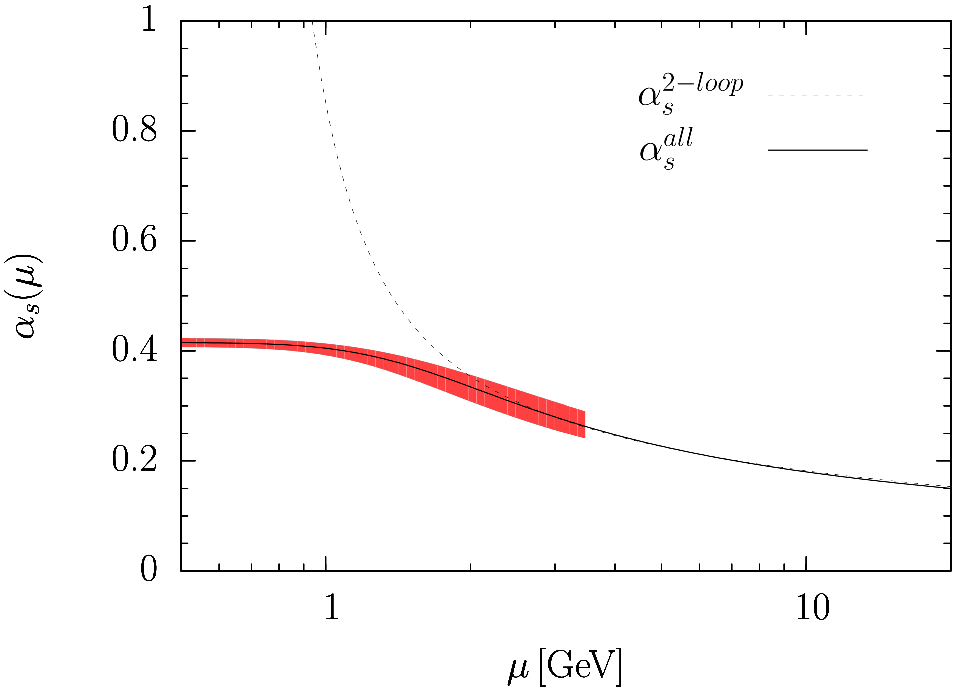


$$\alpha_{WF} = \alpha_s + k_1\,\alpha_s^2 + k_2\,\alpha_s^3$$

$$k_1 = 1.122\,,\ \ k_2 = -1.165$$

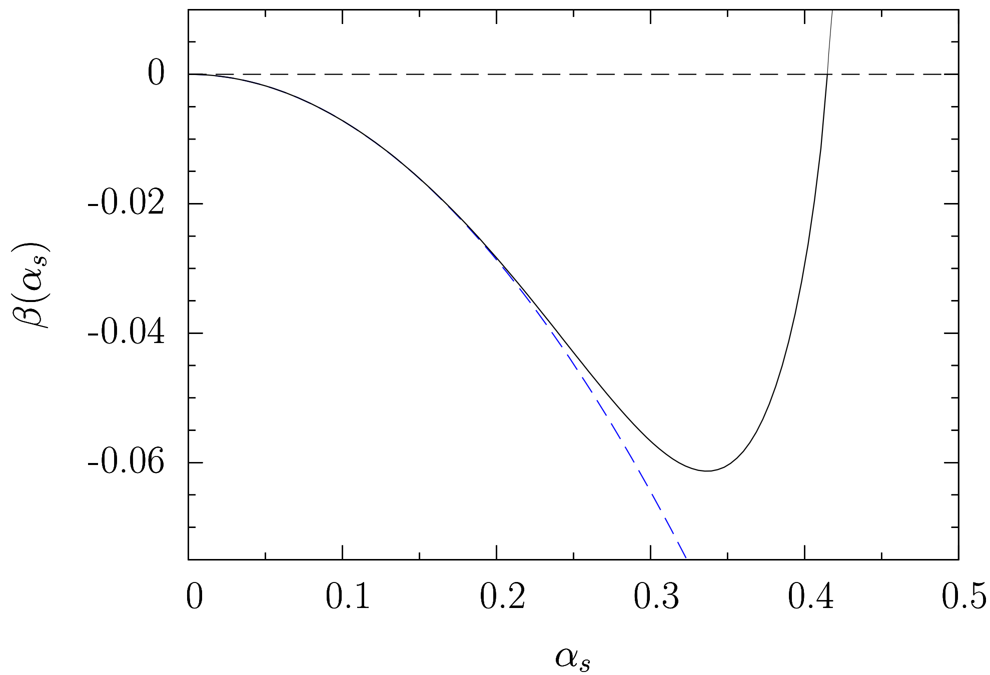
Lüscher

Harlander & Neumann



$$\alpha_s^{all}(\mu) = \left[\left(\frac{1}{\alpha_s^{2-loop}(\mu)} \right)^n + \left(\frac{1}{\alpha_s(0)} \right)^n \right]^{-\frac{1}{n}} \qquad \Lambda_{\overline{MS}} \approx 300\ \text{MeV}$$

β Function



Other schemes: Choi & Shrock

Conclusions

- Find large $O(a^2)$ corrections for lattice spacings $a \gtrsim 0.05$ fm

  Can be removed reasonably well at scales below $\mu \lesssim 4$ GeV

- Infrared behavior appears to be little affected by $O(a^2)$ corrections though

- The running coupling $\alpha_s(\mu)$ freezes at $\alpha_s(0) \approx 0.4$

- This results in an infrared attractive fixed point of the beta function $\beta(\alpha_s)$

  Possibly $\exists$ a deconfined phase above $\alpha_s \approx 0.4$