Anderson localisation of Dirac eigenmodes in high temperature QCD

Analysis of background gauge fields

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Outline & motivation

• Anderson localisation basics (disorder)
• QCD spectrum at high temperature
  • Anderson localisation of eigenmodes above $T_c$
• What is the source of disorder in QCD? (*)
• Dirac low-modes: analysis of background gauge fields
• Conclusions

(*) Anderson Localization in high temperature QCD: background configuration properties and Dirac eigenmodes, G. C. and S. Hashimoto, JHEP 1606(2016) 056
Anderson localization (AL)

Random Schrödinger operator

\[
\left( -\Delta + V(x) \right) \psi = E \psi
\]

- AL: Spatial localization of the states of a system due to multiple quantum interference caused by disorder
- Huge number of experimental evidences

\( V(x) \) random potential, disorder

- Anderson tight-binding model (1958) is the discretised version
- Exponential spectral localisation above a critical disorder
Anderson localization (AL)

Random Schrödinger operator

\[
\left( -\Delta + V(x) \right) \psi = E\psi
\]

• Above a critical energy \( E_c \), mobility edge \( \rightarrow \) delocalisation
• (Second order) quantum phase transition at \( E_c \)

Two regions
• Localised states: Poisson distributed (classical dynamics)
• Delocalised states: Random Matrix Theory (chaotic dynamics)

Bohigas, Giannoni, Schmit conjecture 1984
Anderson localization (AL) in QCD

QCD Dirac operator

\[ D\psi = \lambda \psi \]

- At \( T > T_c \): low modes localisation
- Second order phase transition at some \( \lambda_c(T) \)
- Critical exponents of the 3D Anderson model (Kovacs et al.)
- Multifractal scaling of eigenmodes at \( \lambda_c(T) \) (Kovacs et al.)

\( T = 216 \text{ MeV} \)
QCD with chiral fermions

• Chiral fermions, domain-wall (JHEP 1606(2016) 056)
• Main question: what is the source of disorder?
• Let’s investigate the gauge field background configurations
• Gauge invariant observables (after configuration smearing):
  • Polyakov loop
  • Local action
  • Local topology
• Main conclusion: Disorder → monopole-instantons (dyons)
• Chiral properties (left- and right-handed projections)
  • Is localisation triggering the chiral symmetry restoration?
Level spacing distribution, $P(s)$

$T < T_c$

$T > T_c$
Investigating the disorder – Polyakov loop

Chirality

Re(Polyakov-loop)

Low-mode
Correlations with the Polyakov loop

Below the phase transition

Above the phase transition

Eigenmode local norm

Eigenvalue

Low modes

High modes
Correlations with the Polyakov loop

Now change the boundary conditions (BC) of the Dirac operator for the measurements
SAME background configurations on both panels

Anti-periodic BC

Periodic BC
Action and topology, single configuration

\[ s(x) = F_{\mu \nu} F_{\mu \nu}(x) \]

\[ q(x) = F_{\mu \nu} \tilde{F}_{\mu \nu}(x) \]

Self-dual fields
Action and topology, systematic analysis

$$\bar{s}_n = \frac{\int \rho_n(x)^2 s(x) d^4x}{\int \rho_n(x)^2 d^4x}$$

$$\rho_n(x) = |\psi_n(x)|^2$$

32x12 4.18 0.01, T=171 MeV
32x12 4.30 0.01, T=220 MeV
32x12 4.30 0.01 PBC, T=220 MeV
32x8 4.18 0.01, T=257 MeV
32x8 4.30 0.01, T=330 MeV
Interpretation in terms of topological fluctuations

Class of solutions of the YM equation of motion in a non trivial Polyakov loop background (Van Baal et al.): monopole-instantons

\[ P(\infty) = \exp[2\pi i \, \text{diag}(\mu_1, \mu_2, \mu_3)], \quad \nu_m \equiv \mu_{m+1} - \mu_m \]

✓ Self dual solutions, charged in each Cartan subgroup
✓ SU(N) N-1 species of BPS monopoles, 1 Kaluza-Klein (KK) from the compact dimension
✓ Finite temperature calorons are composite objects
  ✓ N-1 BPS + 1 KK, electrically and magnetically neutral
Interpretation in terms of topological fluctuations

\[ P(\infty) = \exp[2\pi i \text{ diag}(\mu_1, \mu_2, \mu_3)], \quad \nu_m \equiv \mu_{m+1} - \mu_m \]

- Supported action: \( S = \frac{8\pi^2}{g^2} \nu_m \)
- Topological charge fractional in general
- KK monopoles at high T:
  - Large action support ("heavy"), suppressed
  - Polyakov loop at their centre = -1/3
- Boundary condition dependence of zero modes localisation
- Low temperature all monopoles have the same action on average
  - No change of the spectrum with the boundary conditions below \( T_c \)

The properties measured on the lattice agree with the characteristics of molecules (pairs) of Kaluza-Klein monopole-instantons in SU(N)
Overlap of left-right eigenmode projections

Conjecture: localisation triggers chiral symmetry restoration.

Overlap monotonic with the eigenvalue

Overlap increases for more localized states
Conclusions

✓ We measured the properties of gauge invariant observables in correlation with each one of the Dirac low eigenmodes

✓ Boundary condition dependence of the localisation mechanism

✓ Properties agree with the characteristics of KK monopole-instantons pairs in SU(N)

✓ Conjectured mechanism relating restoration of chiral symmetry at high temperature to localisation
  ✓ Increased localisation → larger overlap → larger eigenvalues → chiral condensate suppressed
    ✓ Polyakov loop transition → localisation → chiral transition?
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Thank you!

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