Anderson localisation of Dirac eigenmodes in high temperature QCD

Analysis of background gauge fields

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Outline & motivation

- Anderson localisation basics (disorder)
- QCD spectrum at high temperature
    - Anderson localisation of eigenmodes above T_c
- What is the source of disorder in QCD? (*)
- Dirac low-modes: analysis of background gauge fields
- Conclusions

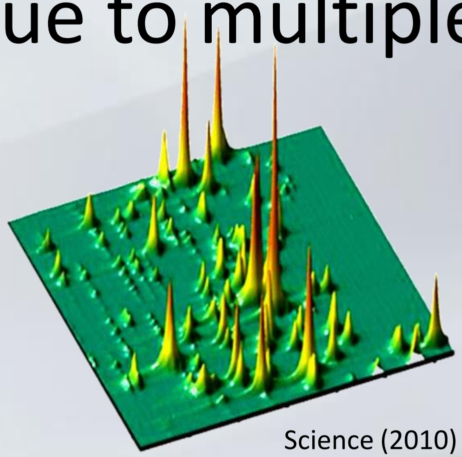
(*) *Anderson Localization in high temperature QCD: background configuration properties and Dirac eigenmodes*, **G. C.** and **S. Hashimoto**, JHEP 1606(2016) 056

Anderson localization (AL)

Random Schrödinger operator

$$\Big(-\Delta + V(x)\Big)\psi = E\psi$$

- AL: Spatial **localization** of the states of a system due to multiple quantum interference caused by **disorder**
- **Huge** number of experimental evidences



Science (2010) 327, 1352-1355

- $V(x)$ random potential, **disorder**
- Anderson tight-binding model (1958) is the discretised version
- Exponential spectral localisation above a critical disorder

Anderson localization (AL)

Random Schrödinger operator

$$\Big(-\Delta + V(x)\Big)\psi = E\psi$$

- Above a critical energy E_c, mobility edge → delocalisation
- (Second order) *quantum* phase transition at E_c

Two regions

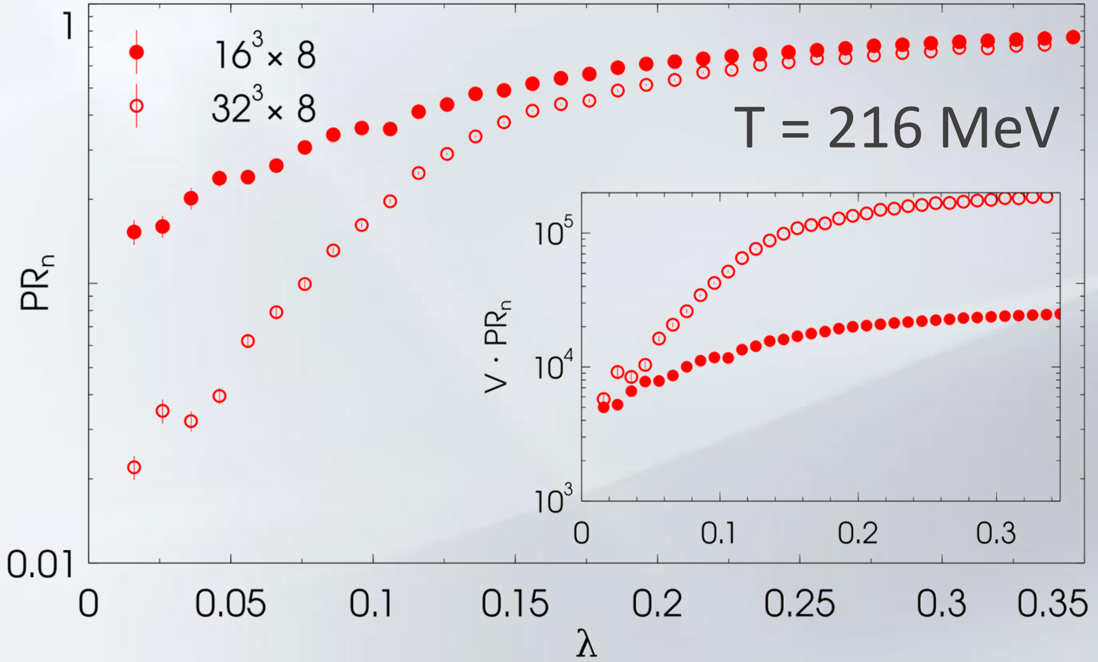
- Localised states: Poisson distributed (classical dynamics)
- Delocalised states: Random Matrix Theory (chaotic dynamics)

Bohigas, Giannoni, Schmit conjecture 1984

Anderson localization (AL) in QCD

QCD Dirac operator

$$D\psi = \lambda\psi$$

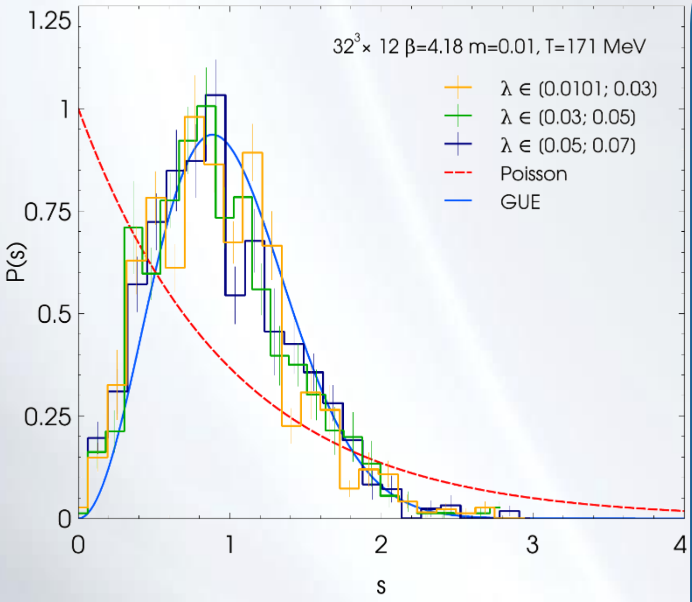


- At $T > T_c$: low modes localisation
- Second order phase transition at some $\lambda_c(T)$
- Critical exponents of the 3D Anderson model (Kovacs et al.)
- Multifractal scaling of eigenmodes at $\lambda_c(T)$ (Kovacs et al.)

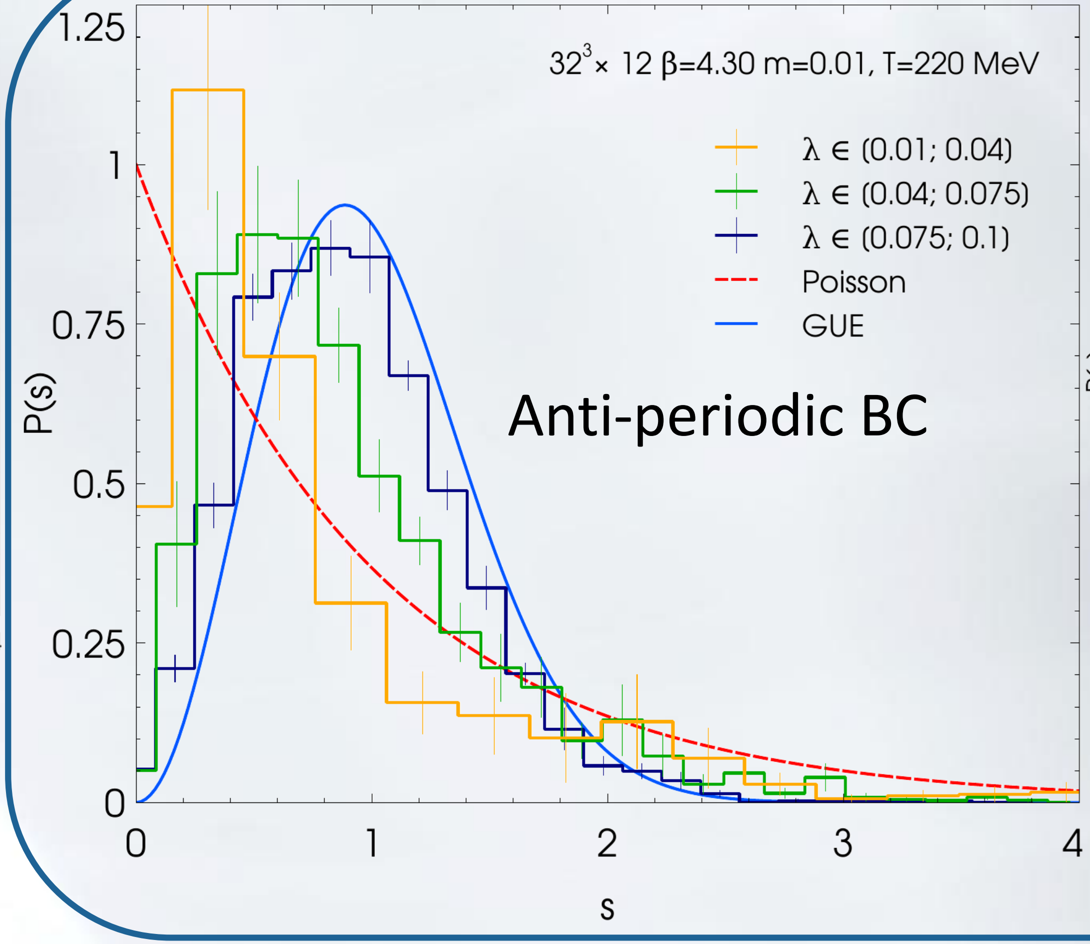
QCD with chiral fermions

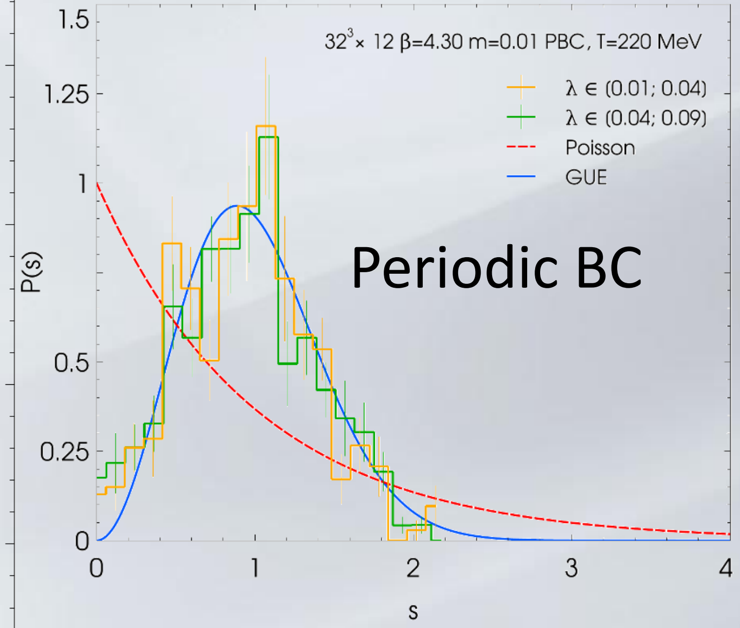
- Chiral fermions, domain-wall (JHEP 1606(2016) 056)
- Main question: what is the source of disorder?
- Let’s investigate the gauge field background configurations
- Gauge invariant observables (after configuration smearing):
  - Polyakov loop
  - Local action
  - Local topology
- Main conclusion: Disorder → monopole-instantons (dyons)
- Chiral properties (left- and right-handed projections)
  - Is localisation triggering the chiral symmetry restoration?

Level spacing distribution, *P(s)*



$T < T_c$

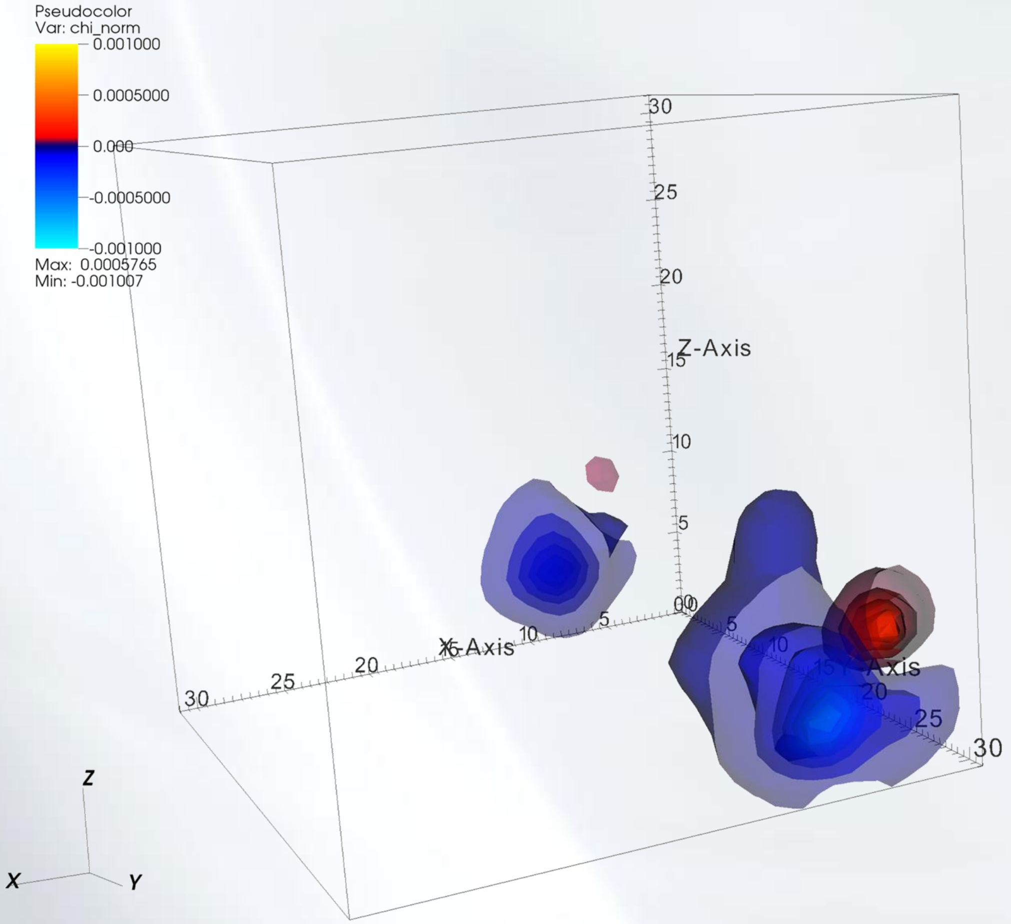




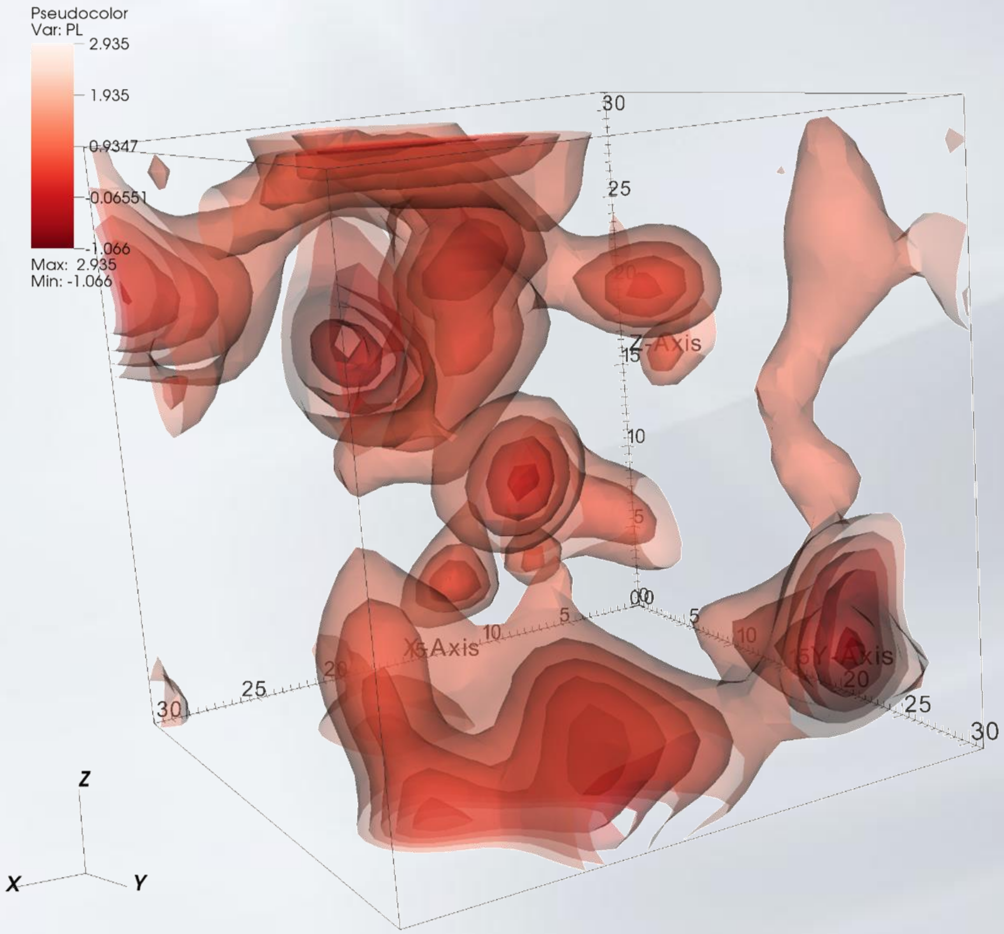
$T > T_c$

Investigating the disorder – Polyakov loop

Chirality



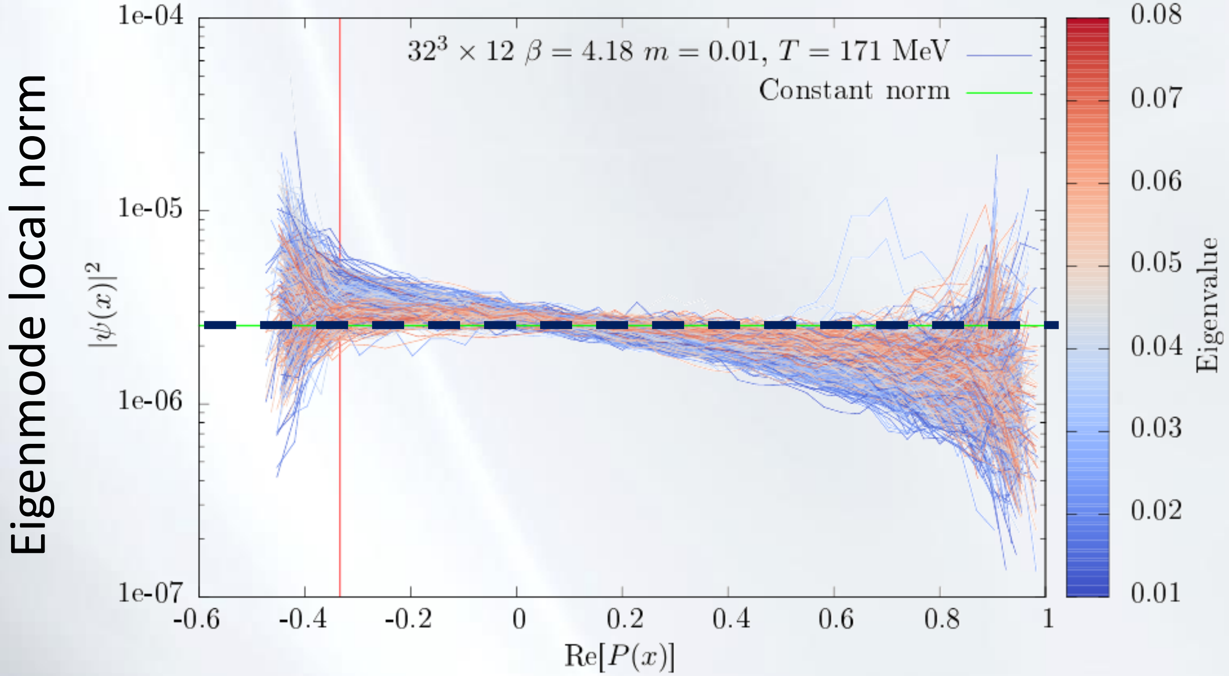
Re(Polyakov-loop)



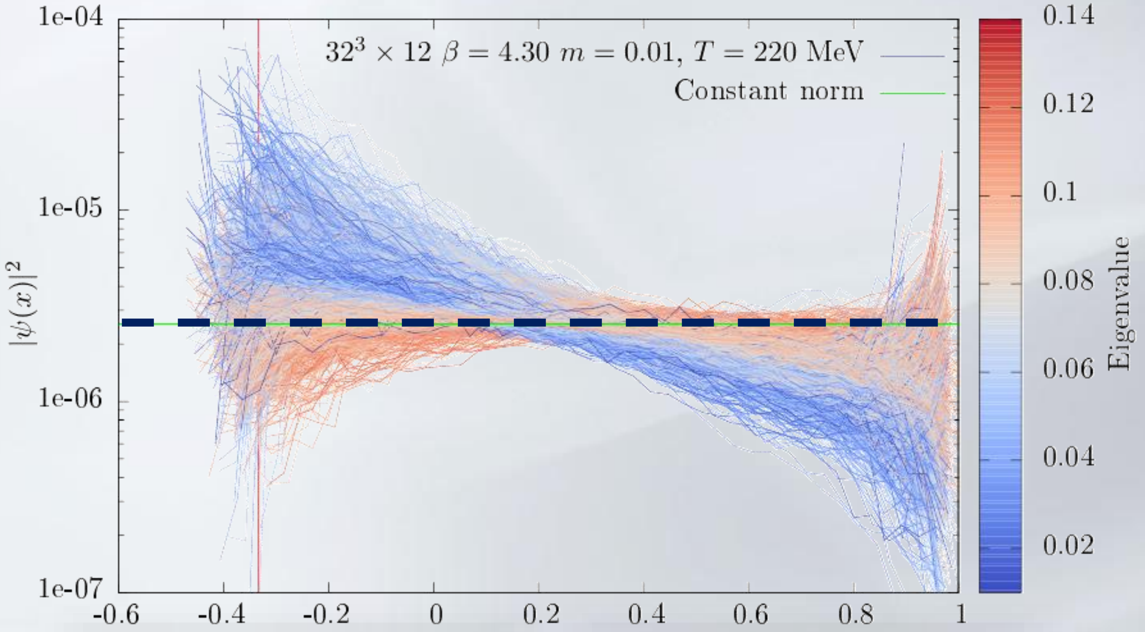
Low-mode

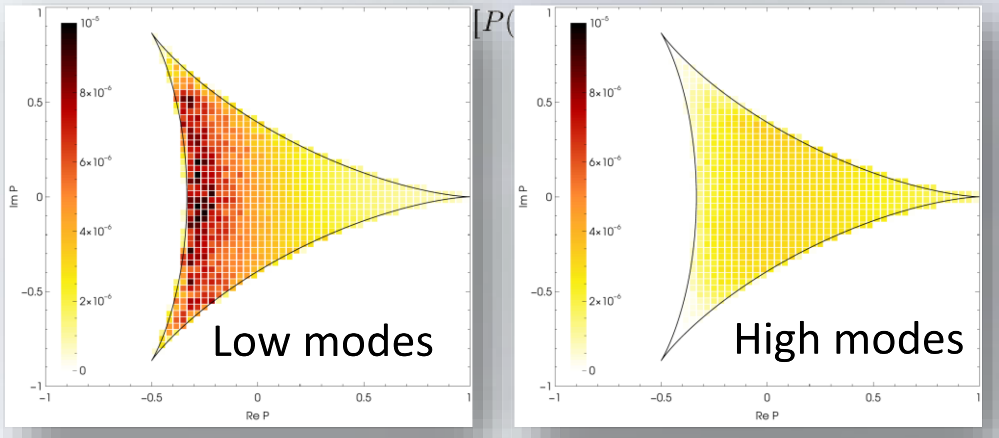
Correlations with the Polyakov loop

Below the phase transition



Above the phase transition



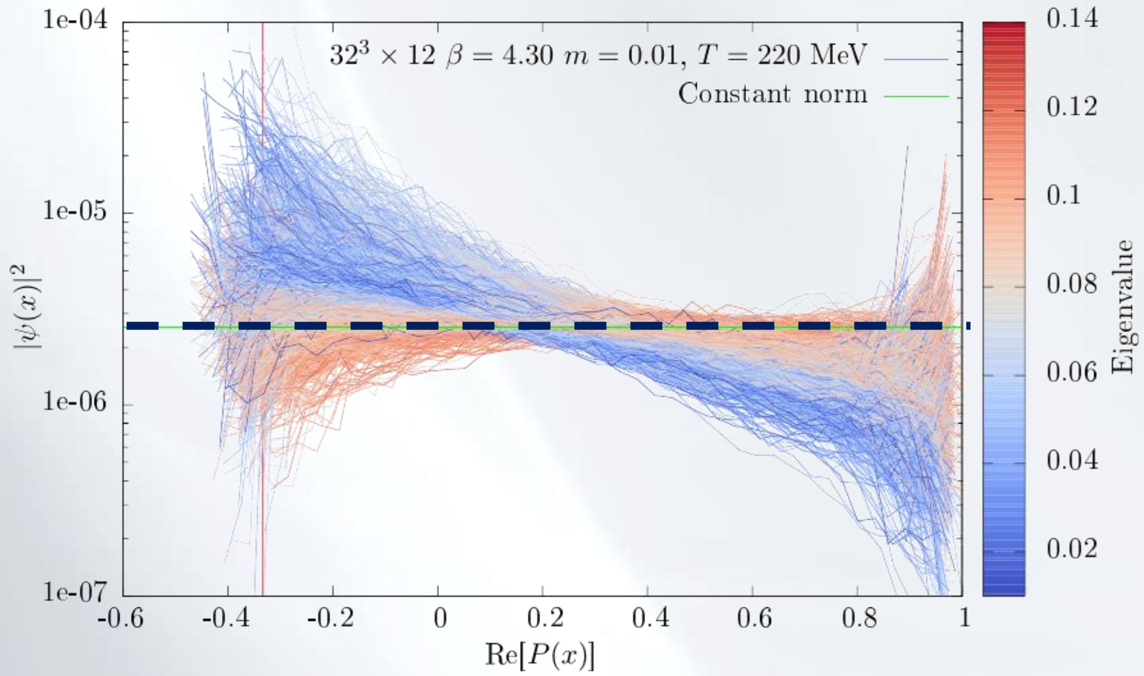


Correlations with the Polyakov loop

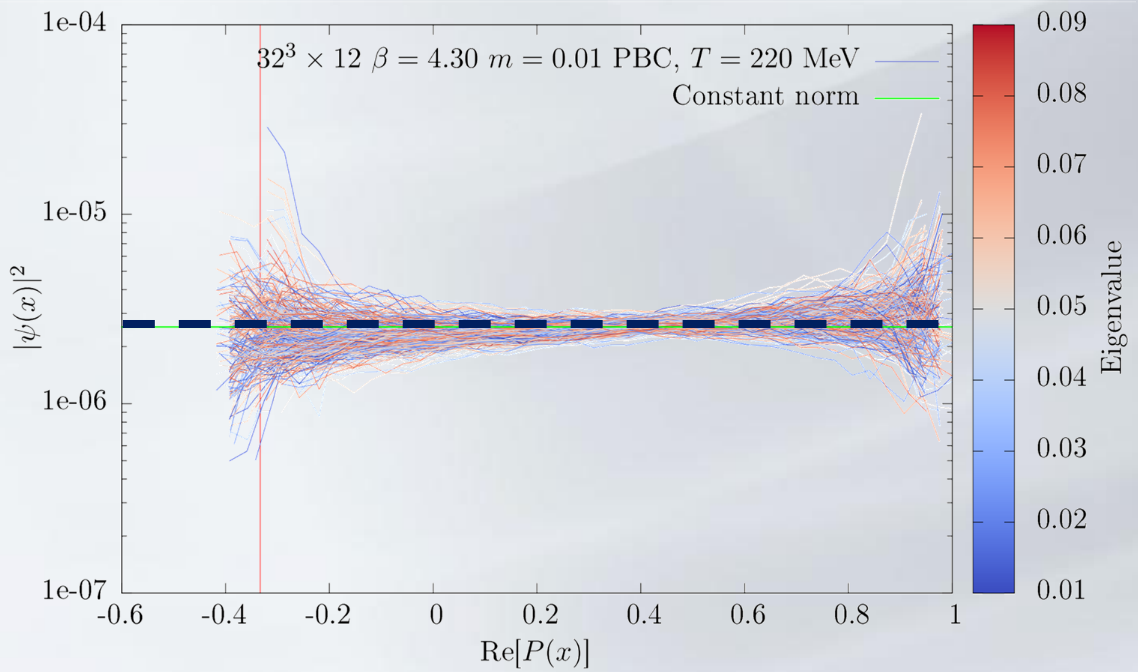
Now change the boundary conditions (BC) of the Dirac operator for the measurements

SAME background configurations on both panels

Anti-periodic BC



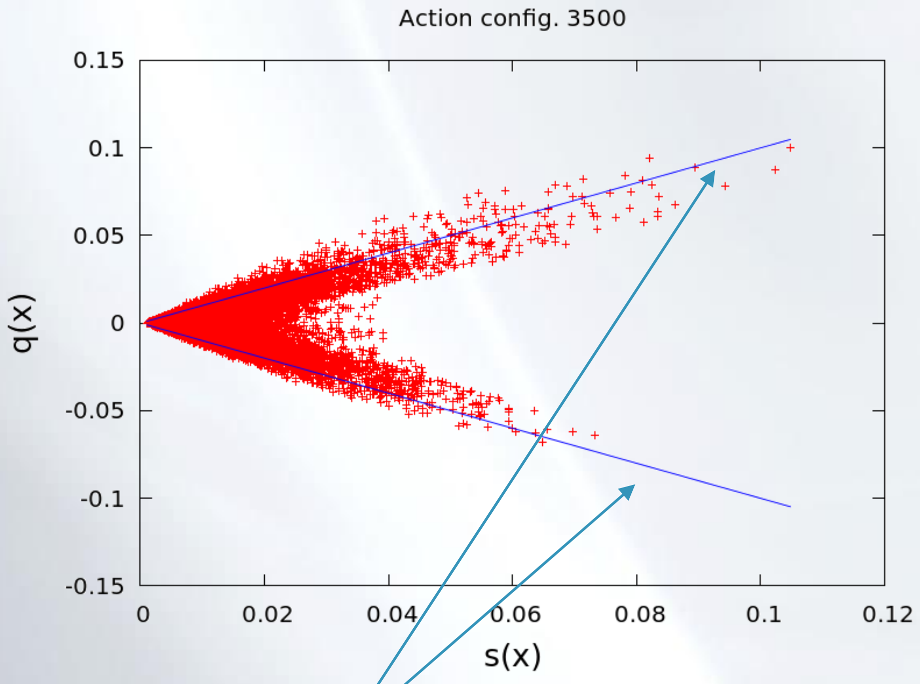
Periodic BC



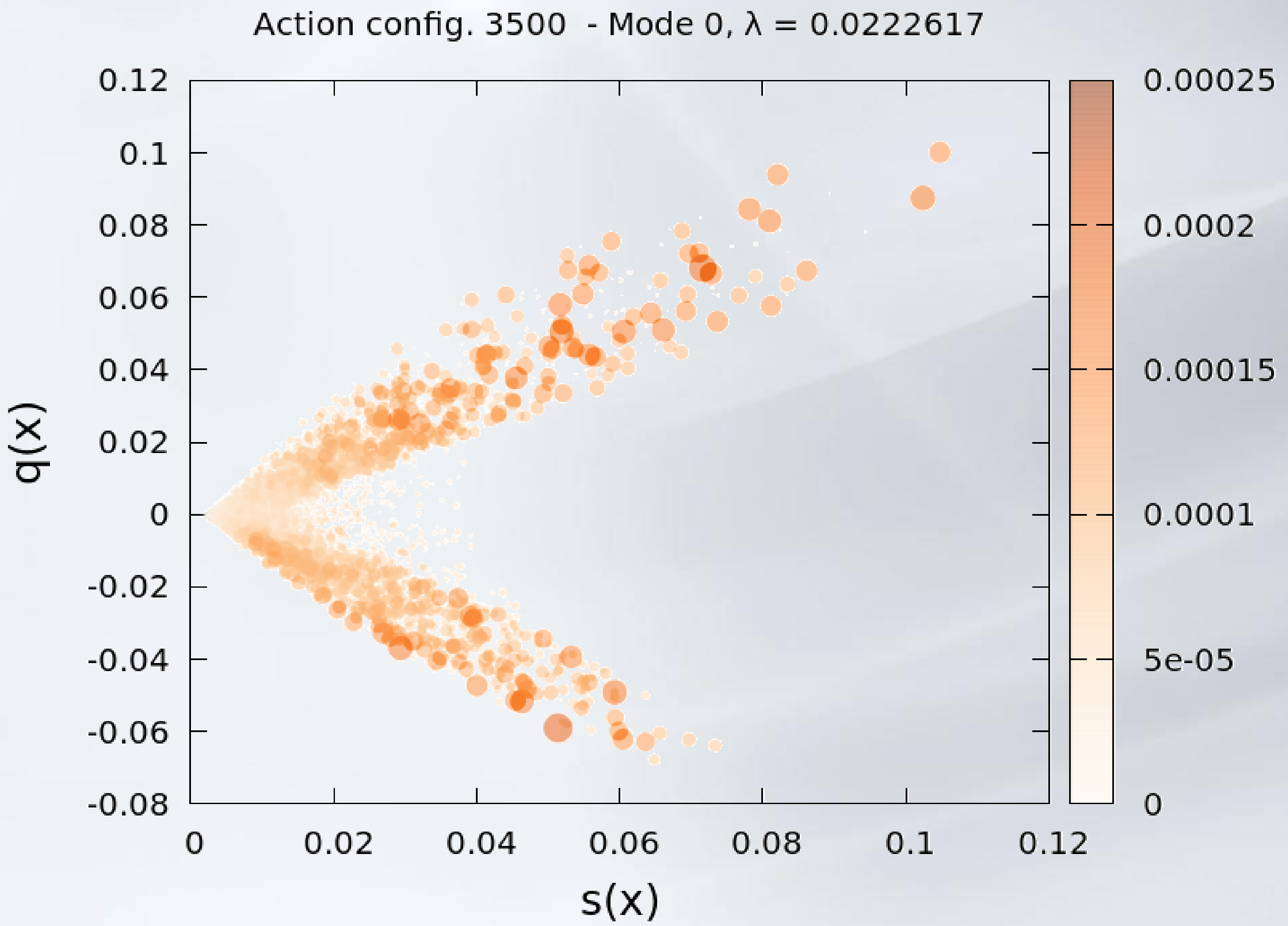
Action and topology, single configuration

$$s(x) = F_{\mu\nu}F_{\mu\nu}(x)$$

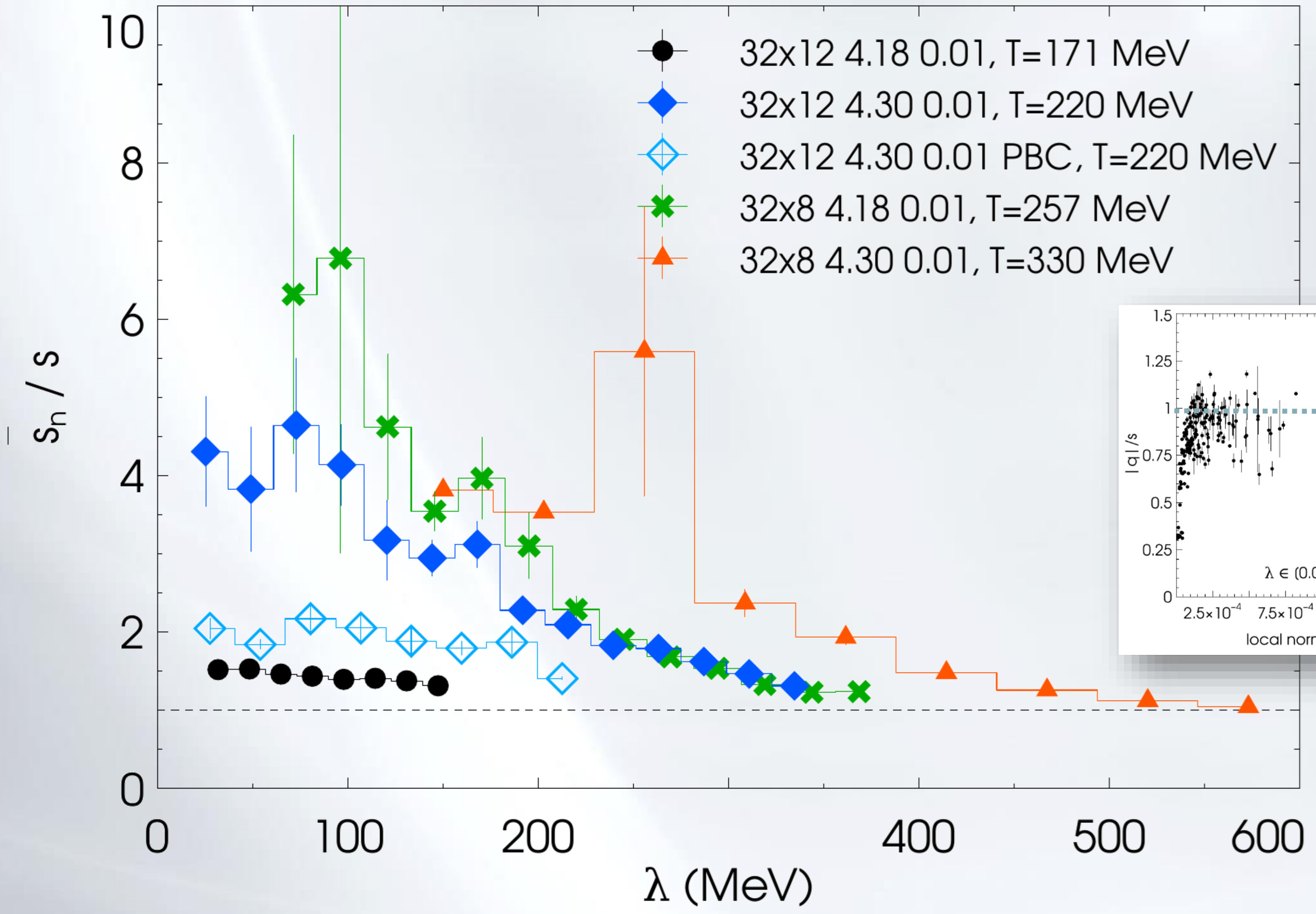
$$q(x) = F_{\mu\nu}\tilde{F}_{\mu\nu}(x)$$



Self-dual fields

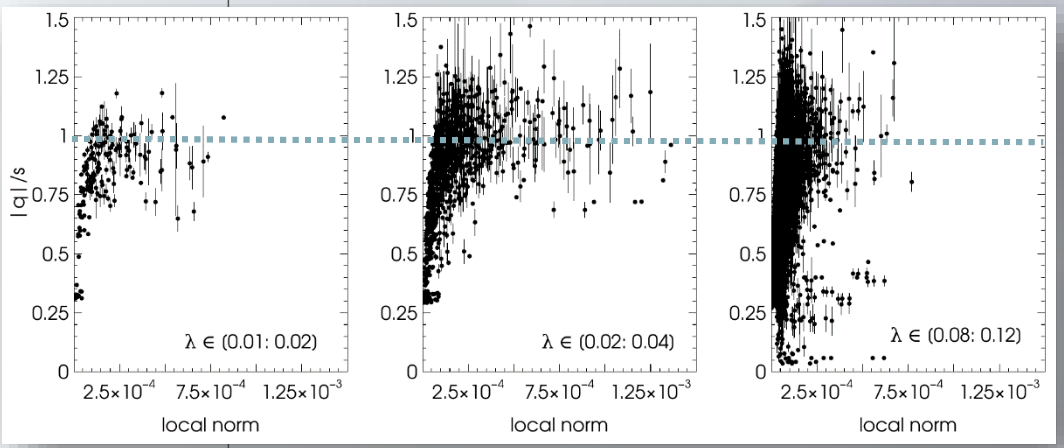


Action and topology, systematic analysis



$$\bar{s}_n = \frac{\int \rho_n(x)^2 s(x) d^4x}{\int \rho_n(x)^2 d^4x}$$

$$\rho_n(x) = |\psi_n(x)|^2$$



Interpretation in terms of topological fluctuations

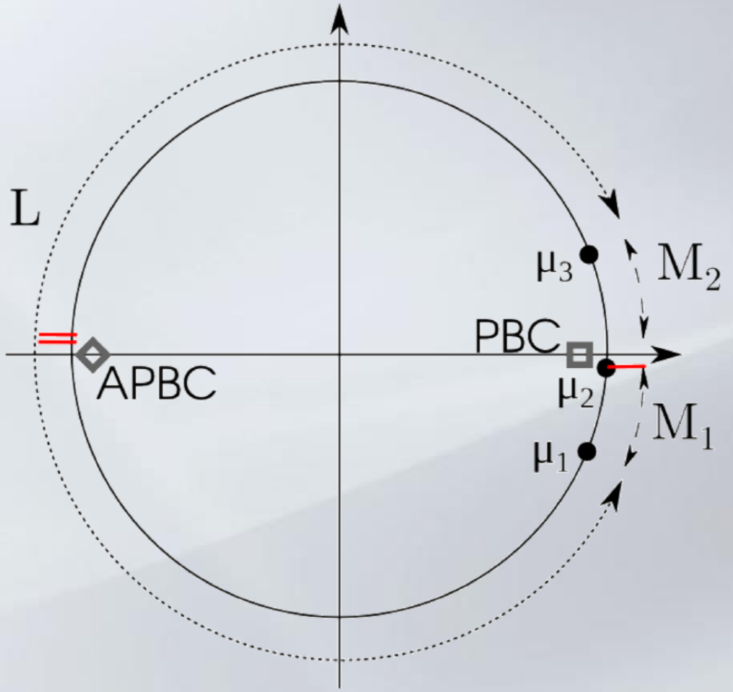
Class of solutions of the YM equation of motion in a non trivial Polyakov loop background (Van Baal et al.): monopole-instantons

$$P(\infty) = \exp[2\pi i \ \mathrm{diag}(\mu_1, \mu_2, \mu_3)], \qquad \nu_m \equiv \mu_{m+1} - \mu_m$$

- ✓ Self dual solutions, charged in each Cartan subgroup
- ✓ SU(N) N-1 species of BPS monopoles, 1 Kaluza-Klein (KK) from the compact dimension
- ✓ Finite temperature calorons are composite objects
  - ✓ N-1 BPS + 1 KK, electrically and magnetically neutral

Interpretation in terms of topological fluctuations

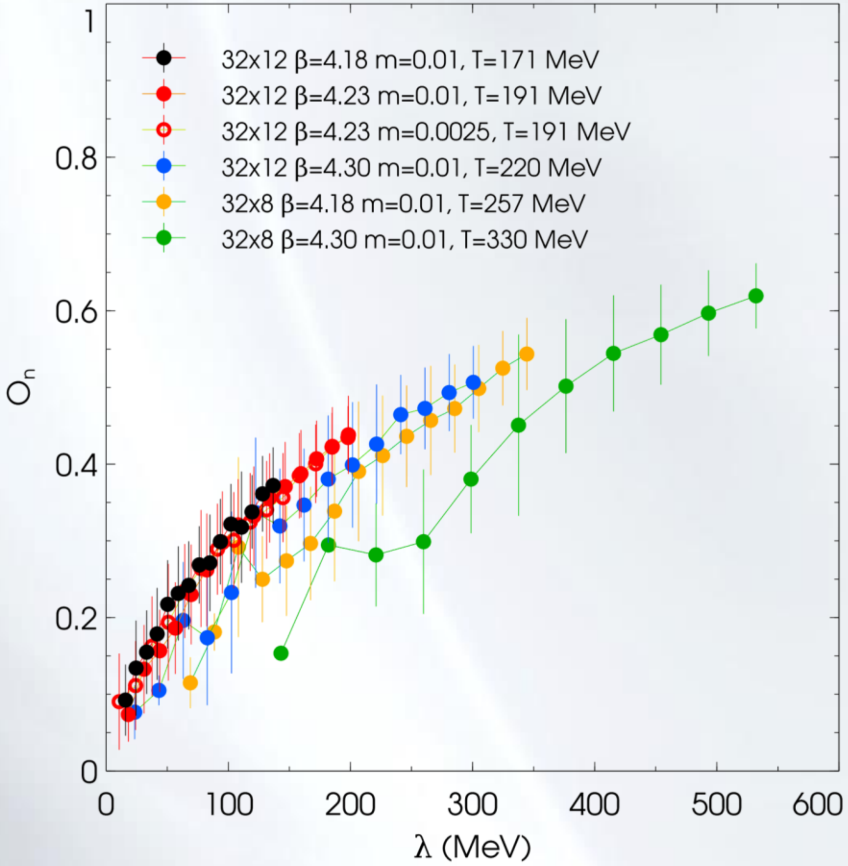
$$P(\infty) = \exp[2\pi i \,\mathrm{diag}(\mu_1, \mu_2, \mu_3)], \qquad \nu_m \equiv \mu_{m+1} - \mu_m$$



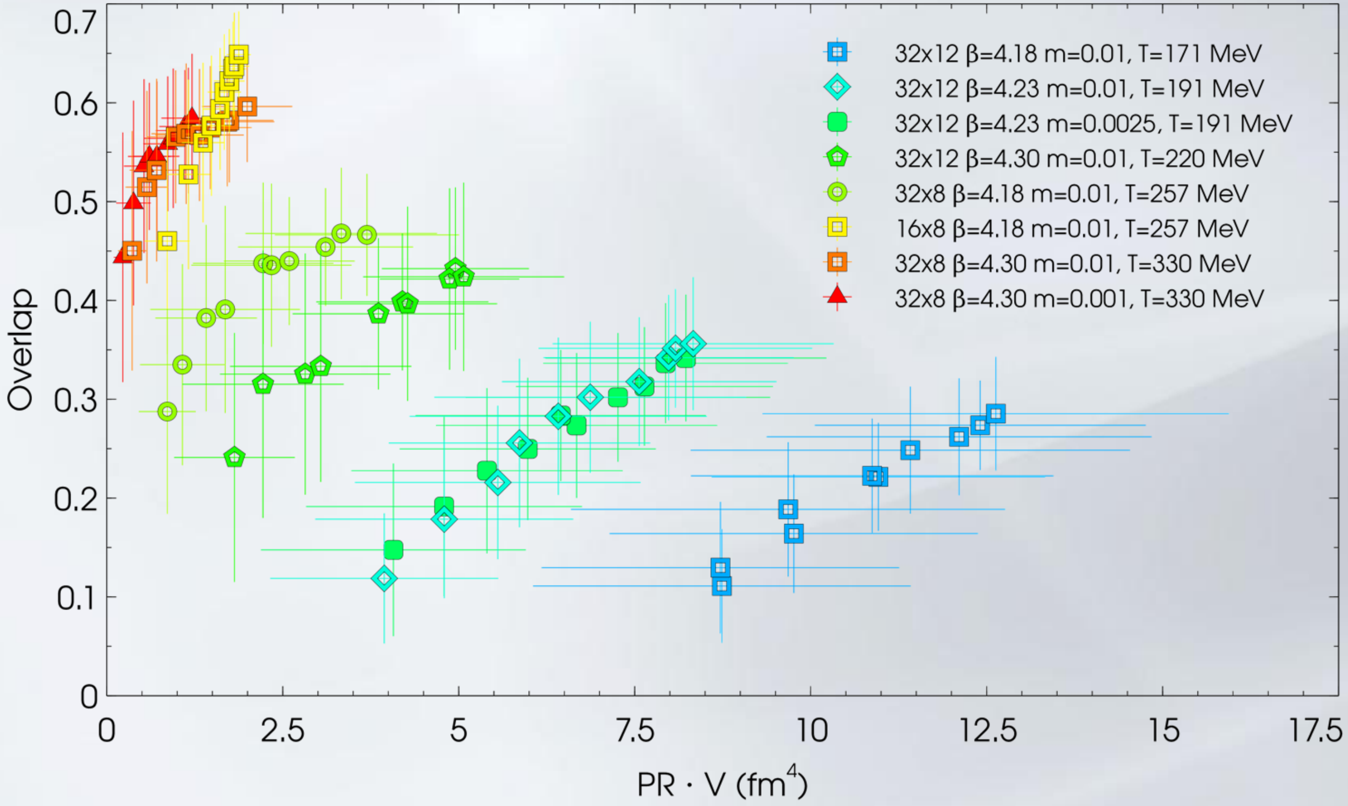
- ✓ Supported action: $S = \frac{8\pi^2}{g^2}\nu_m$
- ✓ Topological charge fractional in general
- ✓ KK monopoles at high T:
  - ✓ Large action support (“heavy”), suppressed
  - ✓ Polyakov loop at their centre = -1/3
- ✓ Boundary condition dependence of zero modes localisation
- ✓ Low temperature all monopoles have the same action on average
  - ✓ No change of the spectrum with the boundary conditions below T_c

The properties measured on the lattice agree with the characteristics of molecules (pairs) of Kaluza-Klein monopole-instantons in SU(N)

Overlap of left-right eigenmode projections



Overlap monotonic with the eigenvalue



Overlap increases for more localized states

Conjecture: localisation triggers chiral symmetry restoration.

Conclusions

- ✓We measured the properties of gauge invariant observables in correlation with each one of the Dirac low eigenmodes
- ✓Boundary condition dependence of the localisation mechanism
- ✓Properties agree with the characteristics of KK monopole-instantons pairs in *SU(N)*

- ✓Conjectured mechanism relating restoration of chiral symmetry at high temperature to localisation
  - ✓Increased localisation → larger overlap → larger eigenvalues → chiral condensate suppressed
    - ✓Polyakov loop transition → localisation → chiral transition?

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Thank you!

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