Towards extracting the timelike pion form factor on CLS 2-flavour ensembles

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Motivation

- It has been shown by Meyer [1] that one can extract the pion form factor in the timelike region $2m_\pi \leq \sqrt{s} \leq 4m_\pi$ using lattice QCD.
- Knowing this quantity is crucial to reduce the uncertainty in theoretical calculations of the muon $g-2$.
- In order to obtain it, we need to study the ρ resonance on the lattice, whose principal decay is $\rho \to \pi\pi$
- Challenge with $\pi\pi$-interpolators on the lattice: Also sink-to-sink quark propagators have to be calculated.
- One approach which is able to handle those is (stochastic) distillation [2,3] using Laplacian-Heavyside (LapH) smearing, which is used in this work.

Distillation approach

- The inversion of the Dirac matrix K is computationally very expensive.
- Distillation uses a special kind of hermitian smearing matrix $\mathcal{S} = V_S V_S^\dagger$. $\Rightarrow$ Just $V_S^\dagger K^{-1} V_S$, a much smaller matrix has to be calculated and stored on disk.
- Quark lines $\mathcal{Q}$ (a smeare-to-smeared quark propagator) can be expressed as an expectation value of Distillation-sink vectors φ and Distillation-source vectors ϱ:

$$\mathcal{Q} = \sum_b E(\varphi^{[b]}(\rho)(\varrho^{[b]}(\rho))^\dagger).$$

$$\varphi^{[b]}(\rho) = \mathcal{S}\,\mathrm{K}^{-1} V_S P^{(b)} \rho$$
$$\varrho^{[b]}(\rho) = V_S P^{(b)} \rho.$$

- The noise vectors ρ are defined in the Distillation subspace, obeying $E(\rho) = 0$ and $E(\rho\rho^\dagger) = 1$.
- $P^{(b)}$ are the Dilution projectors in the Distillation subspace.
- We use full distillation for the quark lines connected to the source timeslice, and stochastic distillation for the sink-to-sink lines.

Lattice ensemble

- This work is an initial exploratory study on a $\beta = 5.3$ CLS 2-flavour lattice
- $\mathcal{O}(a)$ improved Wilson fermions
- $\mathrm{T} \times \mathrm{L}^3 = 64 \times 32^3$ at a lattice spacing of $a = 0.0658\,\mathrm{fm}$ [4]
- pion mass of $437\,\mathrm{MeV}$ [5]

Interpolator setup

- We analyse 4 different frames:
- Centre-of-mass frame (CMF), $\mathbf{P} = \frac{2\pi}{L}\mathbf{d} = 0$
- Moving frames: $\mathbf{d}^2 \in 1, 2, 3$, averaged over all possible directions on the lattice.
- We analyse different lattice irreps in the moving frames.
- We use a matrix (size depending on the frame) which contains a ρ interpolator and $\pi\pi$ interpolators with $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}$:

$$\rho^0(\mathbf{P}, t) = \frac{1}{2L^{3/2}} \sum_{\mathbf{x}} e^{-i\mathbf{P}\cdot\mathbf{x}} \Big(\bar{u}(\mathbf{a}\cdot\gamma)u - \bar{d}(\mathbf{a}\cdot\gamma)d\Big)(t)$$

$$(\pi\pi)(t) = \pi^+(\mathbf{p}_1, t)\pi^-(\mathbf{p}_2, t) - \pi^-(\mathbf{p}_1, t)\pi^+(\mathbf{p}_2, t),$$

$$\pi^+(\mathbf{q}, t) = \frac{1}{2L^{3/2}} \sum_{\mathbf{x}} e^{-i\mathbf{q}\cdot\mathbf{x}} \big(\bar{u}\gamma_5 d\big)(\mathbf{x}, t)$$

$$\pi^-(\mathbf{q}, t) = \frac{1}{2L^{3/2}} \sum_{\mathbf{x}} e^{-i\mathbf{q}\cdot\mathbf{x}} \big(\bar{d}\gamma_5 u\big)(\mathbf{x}, t)$$

- We use isospin symmetry $m_u = m_d$

GEVP

- We get a correlator matrix:

$$C(t) = \begin{pmatrix} \langle \rho(t)\rho^\dagger(0)\rangle & \langle \rho(t)(\pi\pi)^\dagger(0)\rangle \\ \langle (\pi\pi)(t)\rho^\dagger(0)\rangle & \langle (\pi\pi)(t)(\pi\pi)^\dagger(0)\rangle \end{pmatrix}.$$

- In each frame and in each lattice irreducible representation (irrep) we solve a generalized eigenvalue problem (GEVP) [6,7] of this matrix:

$$C(t)\mathbf{v} = \lambda(t) C(t_0)\mathbf{v}$$

- Window method [8]: $t_0 = t - t_w$; $t_w = 3$ gives the best results for our data and is used in this work.
- The eigenvalues λ can be used to extract effective energies because their leading order behaves for large times as:

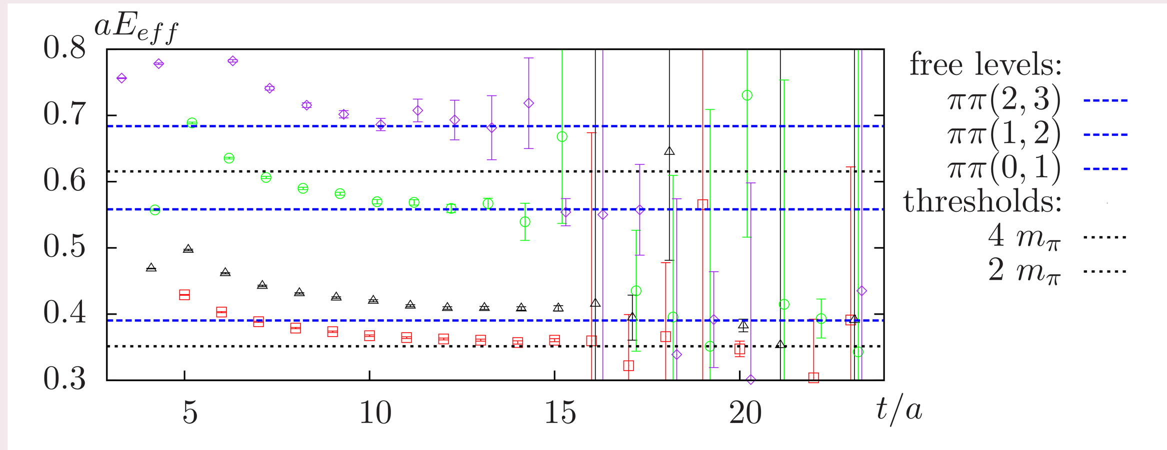
$$\lambda^{(k)}(t) \to e^{-(t-t_0)\,E_k}$$

- By taking the logarithm of these eigenvalues, effective energies (effective masses) can be extracted:

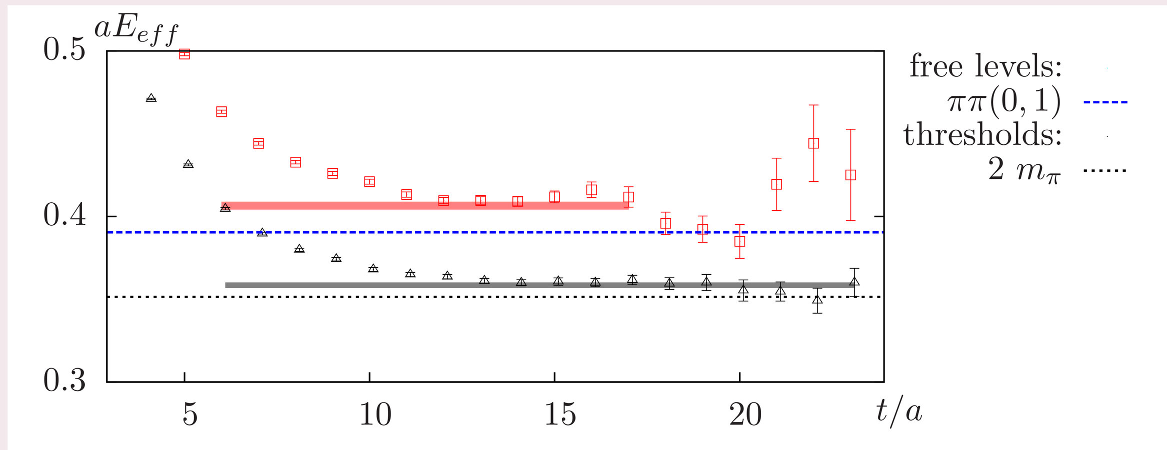
$$E_{eff}^{(k)}(t) \equiv \frac{-1}{t_w} \ln \lambda^{(k)}(t)$$

Extracting energy levels

- We plot the effective energies in each frame

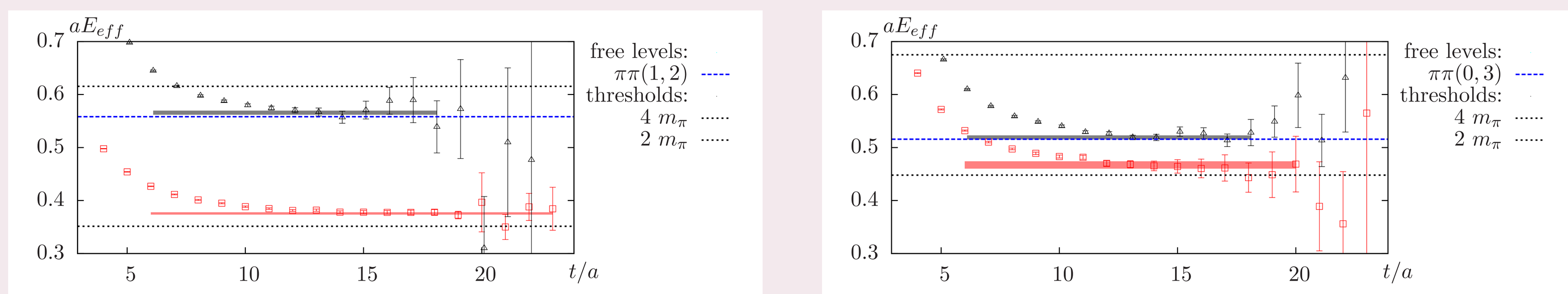


Effective energies of the 4-matrix states in the A_1 irrep in the $\mathbf{d}^2 = 1$ moving frame. Each colour depicts an effective energy from a different chain of eigenvalues.



Effective energies of the states in the A_1 irrep in the $\mathbf{d}^2 = 1$ moving frame from a reduced 2 matrix with just the lowest $\pi\pi$ interpolator. The bands are fit values from a fit of a constant plus one exponent, fit window depicted by the length of the bands.

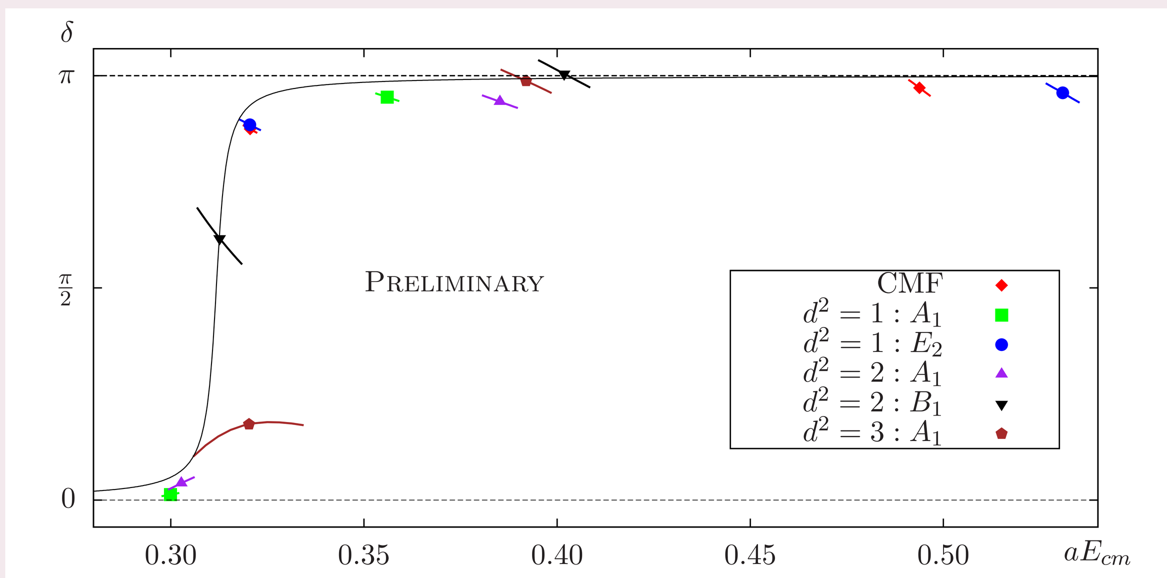
Other irreps and frames



Effective energies of the lowest 2×2 matrices in the E_2 irrep ($\mathbf{d}^2 = 1$) on the left panel and the A_1 irrep ($\mathbf{d}^2 = 3$) on the right panel.

Phase shift diagram

- Using Lüscher's method [9] we can match the finite-box lattice data to the continuum theory.



Phase shift δ plotted as a function of the centre-of-mass energy E_{cm}. The different colours correspond to the different irreps in the respective frames. The lowest two energy levels in each irrep have been used for this plot. Because δ is actually a function of E_{cm} the errorbars in this plot are curvy lines.

- We can also plot δ into a K-matrix plot and perform a linear fit in the function:

$$\frac{p_{cm}^3}{E_{cm}} \cot(\delta) = \frac{6\pi}{g_{\rho\pi\pi}^2}(m_\rho^2 - E_{cm}^2)$$

- The results of this fit are:

$$am_\rho = 0.3118(42) \qquad g_{\rho\pi\pi} = 6.87(1.41)$$

- The naive rho mass in comparison: $am_{\rho,\text{naive}} = 0.3208(29)$

Future work

- The statistics used in this study so far are about 35% of the configurations on our exploratory ensemble. We want to improve our results by increasing statistics to the full ensemble.
- We want to continue this study on more ensembles with $2+1$ flavours and more chiral ones.
- We have all the tools needed to calculate correlators such as $\langle V_i(t) O_j^\dagger(0)\rangle$ with pointlike and point-split currents at the sink.
- Using those we will be able to obtain the matrix elements $\langle \Omega | V_i | k\rangle$ which can be used to better understand the large-t behaviour of the vector-vector correlator $G(t) = \langle V_i(t) V_i(0)\rangle$ that is used for a_μ^{HVP}. [10]
- These matrix elements are also necessary to obtain the pion form factor in the timelike region [1] from our data, which we can use to reduce finite-volume effects in G(t). [10]

References

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