Approaching the Bottom Using Fine Lattices With Domain-Wall Fermions

Brendan Fahy

JLQCD Collaboration

Lattice 2016 Southampton UK

Getting to the Bottom

- Lattice discretization effects are significant at large quark masses as some cutoff effects go as am.
- The JLQCD collaboration has recently produced very fine Domain Wall Lattices $a = 0.080$ to 0.044fm.
- We look at the charmed mesons and find that the cutoff effects are only a few percent.
- How far can we push the limits beyond charm and extrapolate to the bottom?

JLQCD Lattices

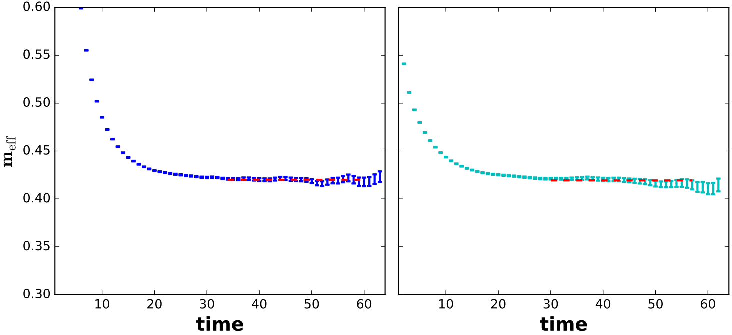
- $N_f = 2 + 1$ simulations on 15 Ensembles with 10,000 MD times for each.
- Simulations at three lattice spacing $a^{-1} \approx 2.4, 3.6$ and 4.5GeV
- $m_\pi \approx 230, 300, 400, 500$ MeV
- Domain-Wall (Möbius) fermions
- Stout link-smearing
- $m_{\text{res}} \approx 1MeV$ on our coarsest lattice;
- $m_{\text{res}} \approx 0$ on the finer lattices.

JLQCD Lattices

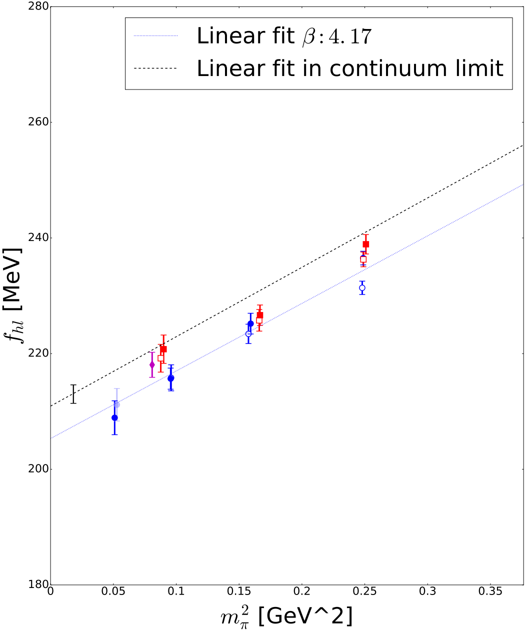
Lattice Spacing	$L^3 \times T$	L_5	am_{ud}	am_s	m_π [MeV]	$m_\pi L$
$\beta = 4.17, a = 0.080$fm $a^{-1} = 2.453(4)$ GeV	$32^3 \times 64$	12	0.0035	0.040	230	3.0
			0.0070	0.030	310	4.0
			0.0070	0.040	310	4.0
			0.0120	0.030	400	5.2
			0.0120	0.040	400	5.2
			0.0190	0.030	500	6.5
			0.0190	0.040	500	6.5
	$48^3 \times 96$	12	0.0035	0.040	230	4.4
$\beta = 4.35, a = 0.055$fm $a^{-1} = 3.610(9)$ GeV	$48^3 \times 96$	8	0.0042	0.018	300	3.9
			0.0042	0.025	300	3.9
			0.0080	0.018	410	5.4
			0.0080	0.025	410	5.4
			0.0120	0.018	500	6.6
			0.0120	0.025	500	6.6
$\beta = 4.47, a = 0.044$fm $a^{-1} = 4.496(9)$ GeV	$64^3 \times 128$	8	0.0030	0.015	280	4.0

Measurements

- Correlators calculated on each lattice for both smeared and unsmeared Z_2 sources
- Measurements were produced on 100 configurations with $6-8$ source points each.
- Combined fit to Axial and Pseudoscalar correlators

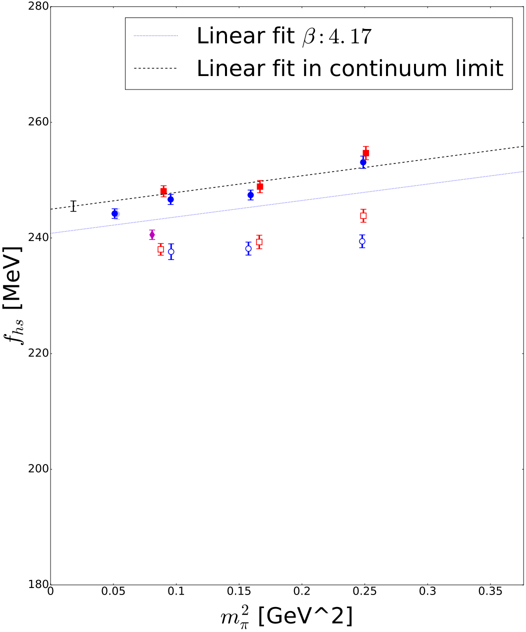


D decay constant



- Chiral and Continuum extrapolation of f_D
- The lattice spacing dependence is small
- $f_D = 212.8 \pm 1.7 \pm 3.6$ MeV

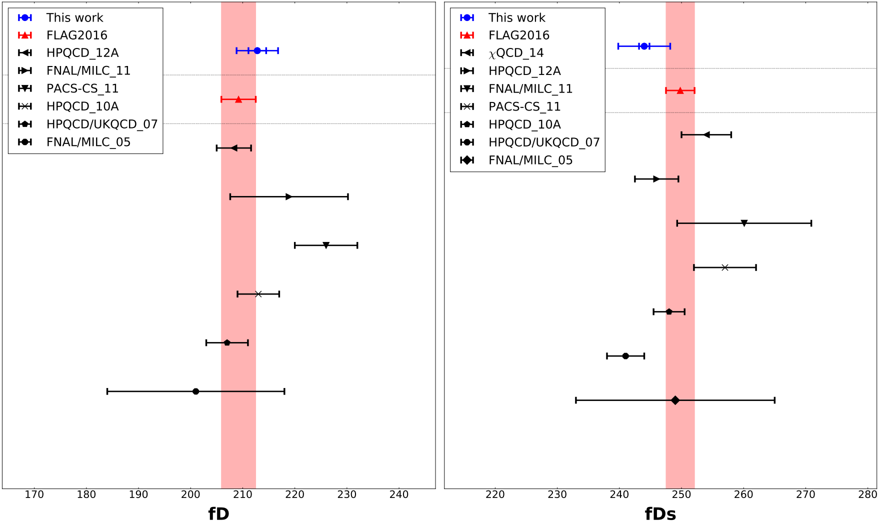
D_s decay constant



- Chiral and Continuum extrapolation of f_{D_s}
- Fit does not go through the lines due to miss tuning of m_s
- Interpolated using $2m_K^2 - m_\pi^2$
- $f_{D_s} = 244.0 \pm 0.84 \pm 4.1$ MeV

Comparison of $f_{D_{(s)}}$ to existing results

(PRELIMINARY)

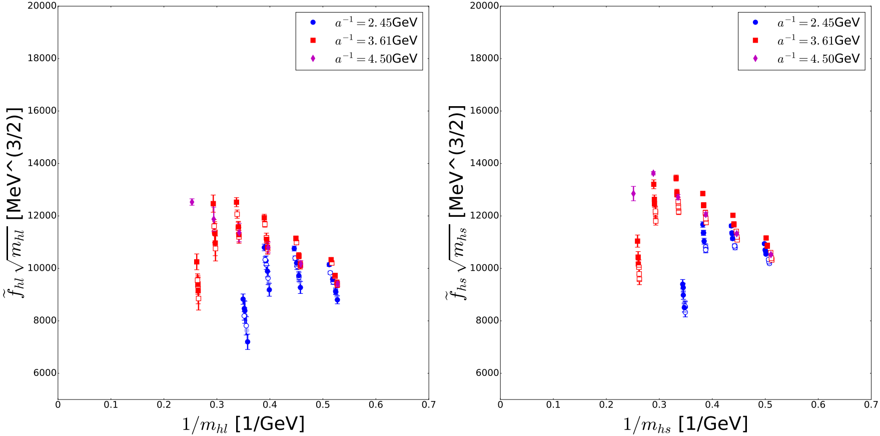


Beyond Charm

- Since cutoff effects at the charm are reasonably controlled, how far above the charm mass can we go?
- Bare quark masses chosen $m_i = (1.25)^i m_c$:
- All heavy quarks treated with DW

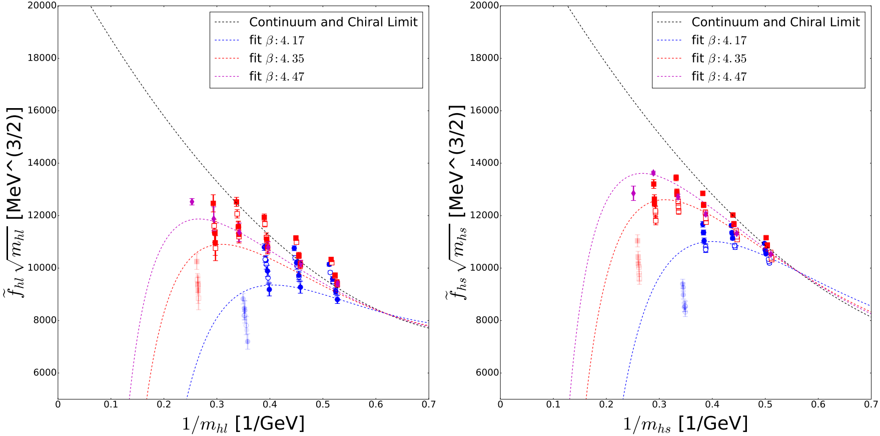
Beta	$m_0 = m_c$	m_1	m_2	m_3	m_4	m_5
4.17	0.4404	0.5505	0.6881	0.8600		
4.35	0.2729	0.3411	0.4264	0.5330	0.6661	0.8327
4.45	0.2105	0.2631	0.3289	0.4111	0.5139	0.6423

Heavy-light and heavy-strange results



$F\sqrt{m}$ for both h-l and h-s for each of our heavy quark masses.
Contains large discretization effects.

Heavy-light and heavy-strange results



Global fit to $(1 + C_1/m + C_2/m^2)$ excluding $m_q > 0.7$
with $\gamma_1(a^2m^2),\ \gamma_2(a^2))$ and linear chiral and m_s corrections.

Account for the leading discretization effects

- Adjust the meson masses using m_1 and m_2 from $E = m_1 + \frac{p^2}{2m_2} + \dots$

- In the Continuum

$$S(p) = \frac{1}{\not{p} + m} \quad \rightarrow C(t, \vec{p} = 0) = \int \frac{dp_0}{2\pi} S(p) e^{ip_0 t} = \frac{1+\gamma^0}{2} e^{-mt}$$

- On the lattice this is not a simple exponential due to the non-locality of 4D effective Dirac operator of DW.

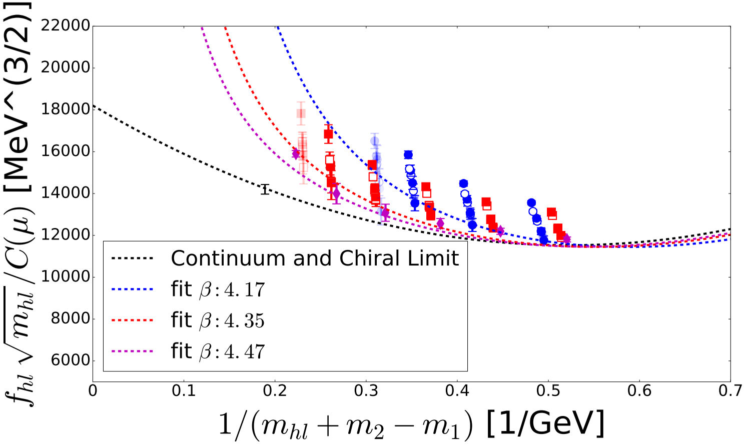
- In order to eliminate the leading discretization effects, we divide the correlator by the tree-level heavy quark propagator of DW and multiply back the corresponding continuum exponential. This is an extension of the Fermilab approach for DW.

Account for the leading discretization effects

- Matching between QCD and HQET. This allows $1/m$ expansion.
- $A_\mu^{\text{QCD}} = C(\mu) A_\mu^{\text{HQET}}(\mu)$
- Perturbative calculation available[1] up to three loops (α_s^3)
- Global fit to with continuum limit $(A + B/m + C/m^2)$ excluding $m_q > 0.7$
- Fit function accounts for $\gamma_1 \alpha_s (a^2 m^2)$, $\gamma_2 (a^2)$ and linear chiral and m_s corrections. Note tree level $(am)^2$ is already removed.

[1] Bekavac et al. arXiv:0911.3356

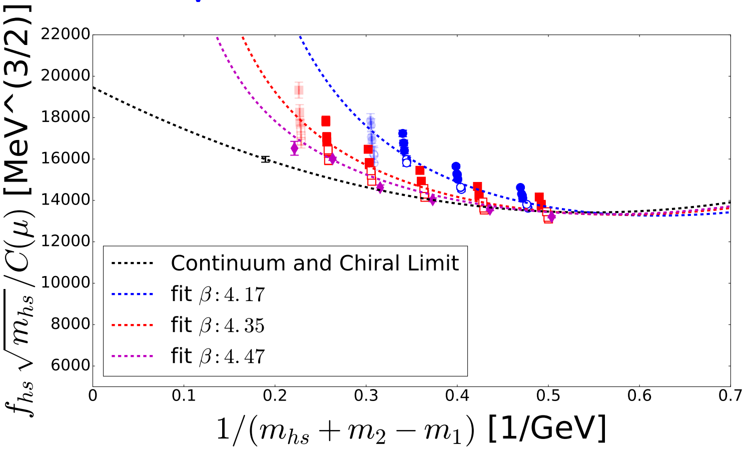
Corrected $f_{hl}\sqrt{m_{hl}}$



$f_B : 195.5 \pm 3.2 \pm 3.3$ MeV

Check: at the charm this gives $f_D : 215.5 \pm 2.0$ MeV consistent with the charm only analysis

Corrected $f_{hs}\sqrt{m_{hs}}$

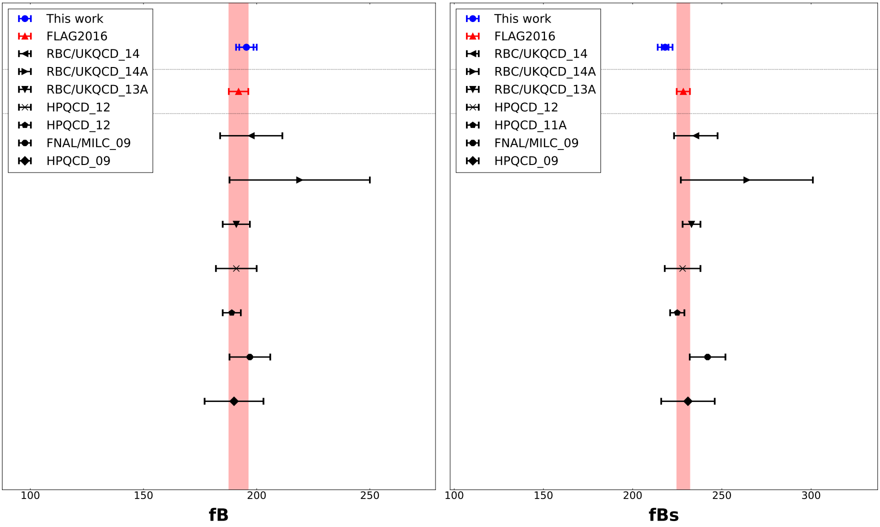


$f_{B_s}: 218.2 \pm 1.9 \pm 3.7$ MeV

Check: at the charm $f_{D_s}: 244.7 \pm 1.0$ MeV consistent with the charm only analysis

Comparison of $f_{B_{(s)}}$ to existing results

(PRELIMINARY)



Conclusions and Future work

- Results of heavy mesons seem promising and the cutoff effects for heavy domain wall fermions can be partially understood
- Leading a^2 effects seem to be identifiable and corrected for
- Extrapolation to the B using standard DW fermions seems somewhat reasonable

---

- Investigate f_{B_s}/f_B
- Further explore the “ratio method” using ratios of successive heavy masses to constrain the extrapolation

Thank You.