Infrared features of dynamical QED+QCD simulations

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  - J. Crilly, R. Horsley, Z. Koumi, Y. Nakamura,
  - H. Perlł, D. Pleiter, P. Rakow, G. Schierholz,
  - A. Schiller, R. Stokes, H. Stüben, R. Young,
  - J. Zanotti
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Collaboration
Outline

- Photon zero modes
  - Charged particle correlators and mass extraction
  - Comparison with Uno & Hayakawa gauge fixing
- Very briefly:
  - Charge renormalisation
  - Decuplet baryon isospin splittings
"Isospin splittings of meson and baryon masses from three-flavor lattice QCD + QED"
R. Horsley et al., to appear JPG(2016), arXiv:1508.06401

"QED effects in the pseudoscalar meson sector"
R. Horsley et al., JHEP1604,093(2016), arXiv:1509.00799
Photon zero mode
Lattice QCD+QED

\[ S = S_G + S_{QED} + S_F^u + S_F^d + S_F^s. \]

\[ S_{QED} = \frac{1}{2e^2} \sum_{x, \mu < \nu} (A_\mu(x) + A_\nu(x + \mu) - A_\mu(x + \nu) - A_\nu(x))^2 \]

\[ S_F^q = \sum_x \left\{ \sum_\mu \left[ \bar{q}(x) \frac{\gamma_\mu - 1}{2} e^{-iQqA_\mu(x)} \tilde{U}_\mu(x) q(x + \mu) \right. \right. \]

\[ \left. \left. - \bar{q}(x) \frac{\gamma_\mu + 1}{2} e^{iQqA_\mu(x)} \tilde{U}_\mu^\dagger(x - \mu) q(x - \mu) \right] \right. \]

\[ + \frac{1}{2\kappa_q} \bar{q}(x)q(x) - \frac{1}{4} c_{SW} \sum_{\mu\nu} \bar{q}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) q(x) \right\} \]

\[ Q_u = \frac{2}{3}, \quad Q_d = Q_s = -\frac{1}{3} \]

We work with a gauge coupling corresponding to \( \alpha_{QED} = 0.1 \)
Photon zero mode

• Consider zero mode of EM gauge field

\[ \overline{A}_\mu \equiv A_\mu(k = 0) = \frac{1}{L^3T} \sum_x A_\mu(x) \]

• Photon action invariant:

\[ \frac{\partial S_{QED}}{\partial \overline{A}_\mu} = 0 \]

• Not constrained by Lorenz gauge-fixing condition:

\[ S_{GF} = \frac{1}{2e^2 \lambda} \sum_x \left( \sum_\mu \overline{A}_\mu A_\mu \right)^2 \quad \Rightarrow \quad \frac{\partial S_{GF}}{\partial \overline{A}_\mu} = 0 \]

Landau gauge: \( \lambda = 0 \)

• Couplings to quarks:

\[ \frac{\partial S^q_F}{\partial \overline{A}_\mu} = L^3T \sum_x \frac{\partial S^q_F}{\partial A_\mu(x)} = L^3T \sum_x J^q_\mu(x) \]

action not invariant under continuous transformation of zero mode
Gauge transformations on a box

- Consider U(1) field transformation:
  \[ A_\mu(x) \rightarrow A_\mu(x) + \Delta_\mu \alpha(x) \]

- Shift of zero mode:
  \[ \Delta_\mu \alpha(x) = c_\mu \text{ [indep. of spacetime } x]\]

- Gauge-field action invariant

- Simple closed Wilson loops invariant

- Polyakov loops:
  \[ L^q_\mu(x) = \exp \left[ iQ_q \sum_{x_\mu} A_\mu(x) \right] \]
  \[ \rightarrow \exp \left[ iQ_q N_\mu c_\mu + iQ_q \sum_{x_\mu} A_\mu(x) \right] \text{ no summation} \]

  Quantisation condition: Polyakov loops invariant if
  \[ c_\mu = n_\mu \frac{2\pi}{Q_q N_\mu}, \quad n_\mu \in \mathbb{Z} \]
Gauge-fixing in a box

- On each configuration we map the zero mode into the interval

\[-\frac{\pi}{|Q_d|N_\mu} < A_\mu \leq \frac{\pi}{|Q_d|N_\mu}\]

- Preserves importance sampling of path integral

![Graph showing periodicity and subset of full ensemble](image-url)
Gauge-fixing in a box

• On each configuration we map the zero mode into the interval

$$-\frac{\pi}{|Q_d| N_\mu} < \bar{A}_\mu \leq \frac{\pi}{|Q_d| N_\mu}$$

• Preserves importance sampling of path integral

Impact on spectrum?
Correlators in uniform U(1) external field

• Consider quarks propagating in uniform background field $B_\mu$

• Absorb U(1) phase into fermion field transformation:
  • Twisted boundary conditions for charged hadrons
    $$k_\mu \rightarrow k_\mu + QB_\mu$$

• Correlator
  $$\sim e^{-t\sqrt{\left(k + QB\right)^2 + m^2}} e^{iQB_4 t}$$

Just the same as twisted BCs that have been studied extensively in the literature.
eg. Sachrajda & Villadoro, Tiburzi et al., Bijnens et al.
Extracting a mass: Complex phase (toy model)

- Zero modes move through simulation time
  - Not a simple uniform phase

\[
\langle B_4 \rangle = \langle A_4 \rangle = -0.00993
\]
\[
\langle B_4^2 \rangle = \langle A_4^2 \rangle = 0.000255
\]

Blue: “Perfect” eff. mass
Purple: Ensemble average complex phase

Fit real part of correlator:
\[
m^{\text{“exact”}} = 0.160
\]
\[
m^{\text{“fit”}} = 0.162
\]
Extracting a mass: Kinetic shift

- Rest energies get a spread in kinetic energy from spatial zero mode
  \[ E = \sqrt{m^2 + Q^2 \overline{B}^2} \]

- Mass estimate on ensemble average
  \[ m = \sqrt{E^2 - Q^2 \langle \overline{B}^2 \rangle} \]

- Charge 1 meson ("u-dbar")
  \[ E = 0.1708(12) \]
  \[ \rightarrow m = 0.1626(12) \]

\[ \langle \overline{B}^2 \rangle = 0.0027 \]
Bin ensembles by $B^2$

- Compute energies on binned subensembles
Bin ensembles by $B^2$

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Spatial zero mode subtraction

Estimate complex phase correction
Uno & Hayakawa gauge fixing

- To eliminate zero mode complications (preserve reflection positivity), Uno & Hayakawa (2008) proposed additional gauge-fixing condition where all spatial zero modes of the U(1) field are eliminated

\[ \tilde{A}_\mu(\bar{k} = 0, t) = 0, \quad \forall t \]

- In continuum, any gauge field can be transformed to this condition by a simple gauge transformation

- Because of box quantisation condition, we cannot satisfy this condition by a gauge transformation
  - In practice, U(1) field updates must be performed in Fourier space

- For our ensembles, we can compare our results by forcing this zero mode condition
  - Amounts to a “partial-quenching” of twists in the sea
  - Potential to be corrected by appropriate reweighting
New results with spatial zero modes removed

- Recompute spectra on modified gauge fields
  - Consider same binning of trajectories as above

![Graph showing energy levels $E$ versus $B^2$. The graph indicates a trend with zero modes eliminated.]
Larger volume comparison

24$^3$x48

32$^3$x64

\[ E \]

\[ B^2 \]
Charge renormalisation
Photon propagators

- Fourier transform $U(1)$ field to compute propagators:
  \[
  \frac{\beta}{V} \langle \tilde{A}_\mu(k) \tilde{A}_\nu^*(k) \rangle = \frac{Z_3}{\hat{k}^2} \left[ \delta_{\mu\nu} - \frac{\hat{k}_\mu \hat{k}_\nu^*}{\hat{k}^2} \right] + \ldots
  \]

- Define
  \[
  D_{\mu\nu}(k) = \frac{\beta}{V} \hat{k}^2 \langle \tilde{A}_\mu(k) \tilde{A}_\nu^*(k) \rangle
  = D(k^2) \left[ \delta_{\mu\nu} - \frac{\hat{k}_\mu \hat{k}_\nu^*}{\hat{k}^2} \right] + \ldots
  \]

- Charge renormalisation
  \[
  Z_3 = \lim_{k^2 \to 0} D(k^2)
  \]

- Vacuum polarisation
  \[
  \Pi(k^2) = \frac{1}{e^2} \left[ 1 - \frac{1}{D(k^2)} \right]
  \]
Lattice symmetries

- Lattice irreps for asymmetric (L/T) lattices [see Aubin et al. PRD(2016)]

\[ A_1^{(\text{spatial})} : \sum_i D_{ii}(k) = \left( 3 - \frac{\hat{k} \cdot \hat{k}^*}{\hat{k}^2} \right) D_{A1}(k^2) \]

\[ A_1^{(\text{temporal})} : \quad D_{44}(k) = \left( 1 - \frac{\hat{k}_4 \hat{k}_4^*}{\hat{k}^2} \right) D_{A1}(k^2) \]
Charge renormalisation

$Z_3 \sim 0.9$
Decuplet isospin splittings
Decuplet splittings in vicinity of SU(3) symmetric point

Phenomenological “data”: Cutkosky PRC(1993)

Good agreement with phenomenology
Final remarks

- Zero modes disrupt simple extraction of charged hadron masses
- Numerical evidence suggests these can be reasonably controlled by accounting for systematics [further investigations warranted]
- Potential for using Uno & Hayakawa with reweighting
- Charge renormalisation ~0.9 [more soon]
- First look at decuplet isospin splittings