Infrared features of dynamical QED+QCD simulations

Ross Young
CSSM & CoEPP
University of Adelaide

Lattice 2016
34th International Symposium on Lattice Field Theory, 24-30 July 2016
University of Southhampton

Thanks to my collaborators

- QCDSF-UKQCD-CSSM
    - J. Crilly, R. Horsley, Z. Koumi, Y. Nakamura,
    - H. Perlt, D. Pleiter, P. Rakow, G. Schierholz,
    - A. Schiller, R. Stokes, H. Stüben, R. Young,
    - J. Zanotti

Thanks to my collaborators

- QCDSF-UKQCD-CSSM
  - J. Crilly, R. Horsley, Z. Koumi, Y. Nakamura,
  - H. Perlt, D. Pleiter, P. Rakow, G. Schierholz
  - A. Schiller, R. Stokes, H. Stüben, R. Young,
  - J. Zanotti

QCDSF

Thanks to my collaborators

- QCDSF-UKQCD-CSSM
  - J. Crilly, R. Horsley, Z. Koumi, Y. Nakamura,
  - H. Perlt, D. Pleiter, P. Rakow, G. Schierholz
  - A. Schiller, R. Stokes, H. Stüben, R. Young,
  - J. Zanotti

QCDSF

UKQCD

Thanks to my collaborators

- QCDSF-UKQCD-CSSM
  - J. Crilly, R. Horsley, Z. Koumi, Y. Nakamura,
  - H. Perlt, D. Pleiter, P. Rakow, G. Schierholz,
  - A. Schiller, R. Stokes, H. Stüben, R. Young,
  - J. Zanotti

QCDSF
UKQCD
Adelaide

Thanks to my collaborators

- QCDSF-UKQCD-CSSM
    - J. Crilly, R. Horsley, Z. Koumi, Y. Nakamura,
    - H. Perlt, D. Pleiter, P. Rakow, G. Schierholz,
    - A. Schiller, R. Stokes, H. Stüben, R. Young,
    - J. Zanotti

QCDSF
UKQCD
Adelaide
RIKEN-Kobe

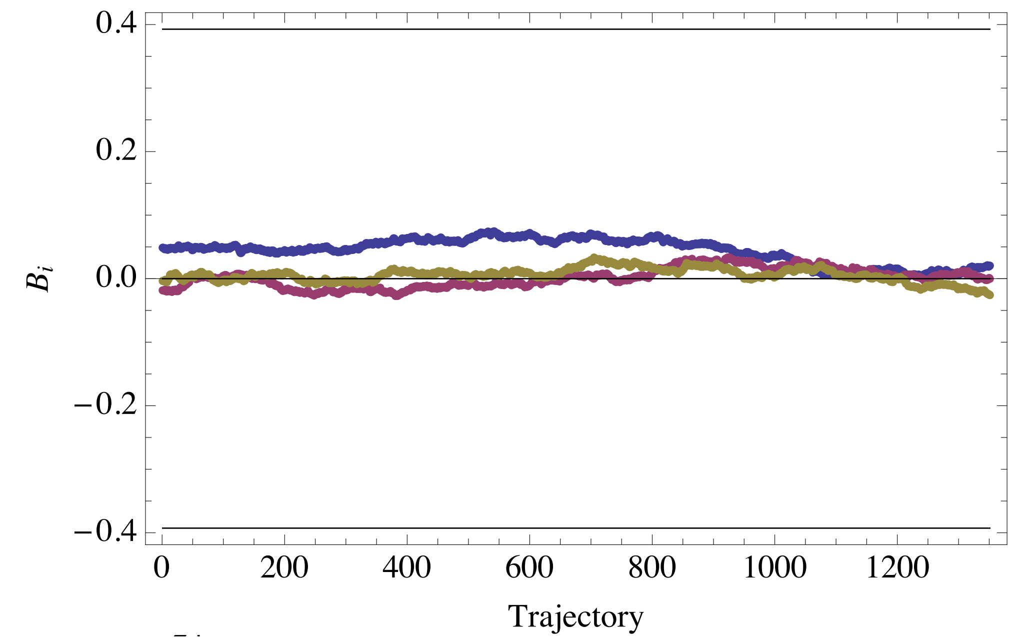
Thanks to my collaborators

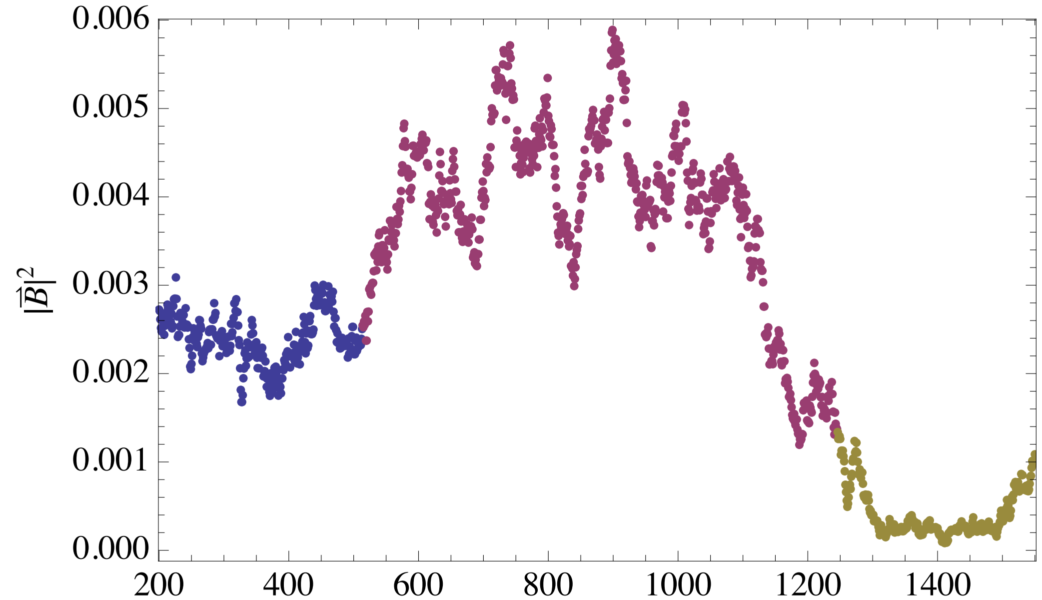
- QCDSF-UKQCD-CSSM
  - J. Crilly, R. Horsley, Z. Koumi, Y. Nakamura,
  - H. Perlt, D. Pleiter, P. Rakow, G. Schierholz
  - A. Schiller, R. Stokes, H. Stüben, R. Young,
  - J. Zanotti

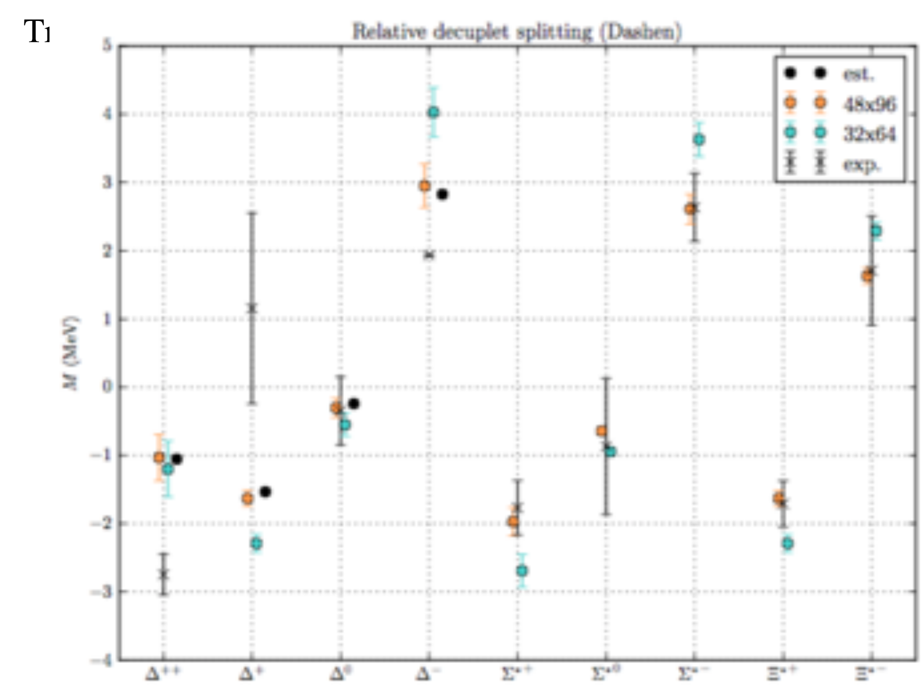
QCDSF
UKQCD
Adelaide
RIKEN-Kobe
Collaboration

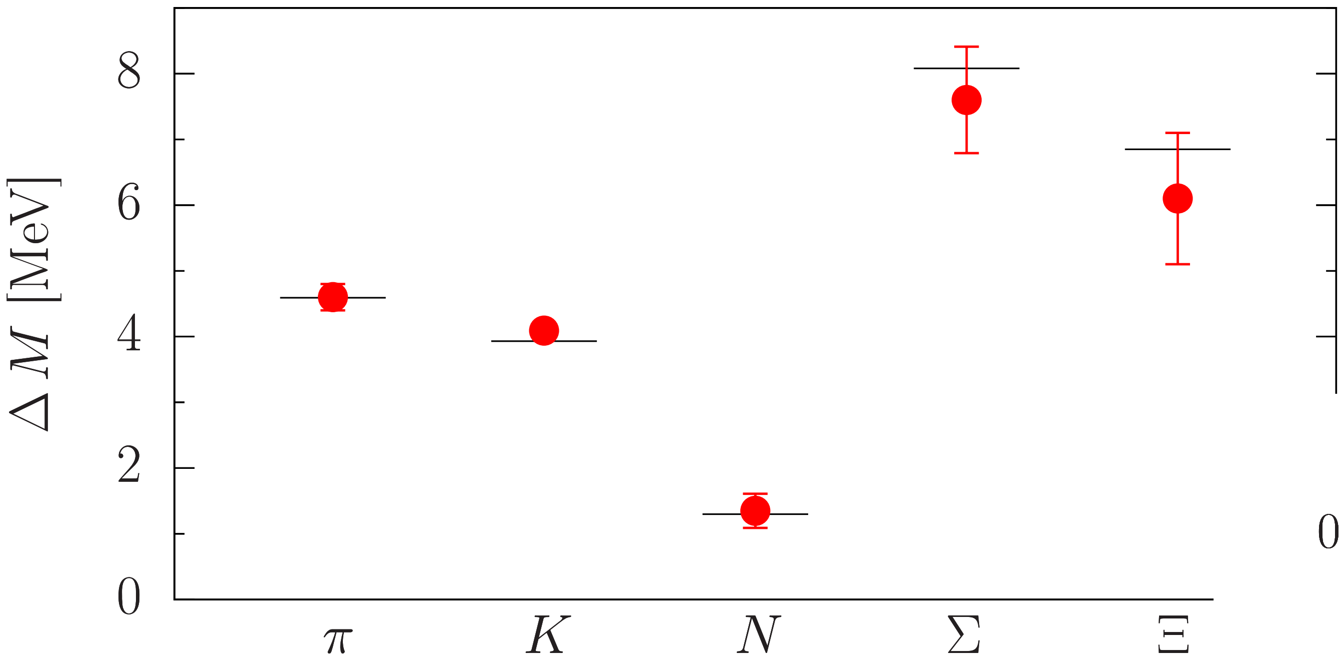
Outline

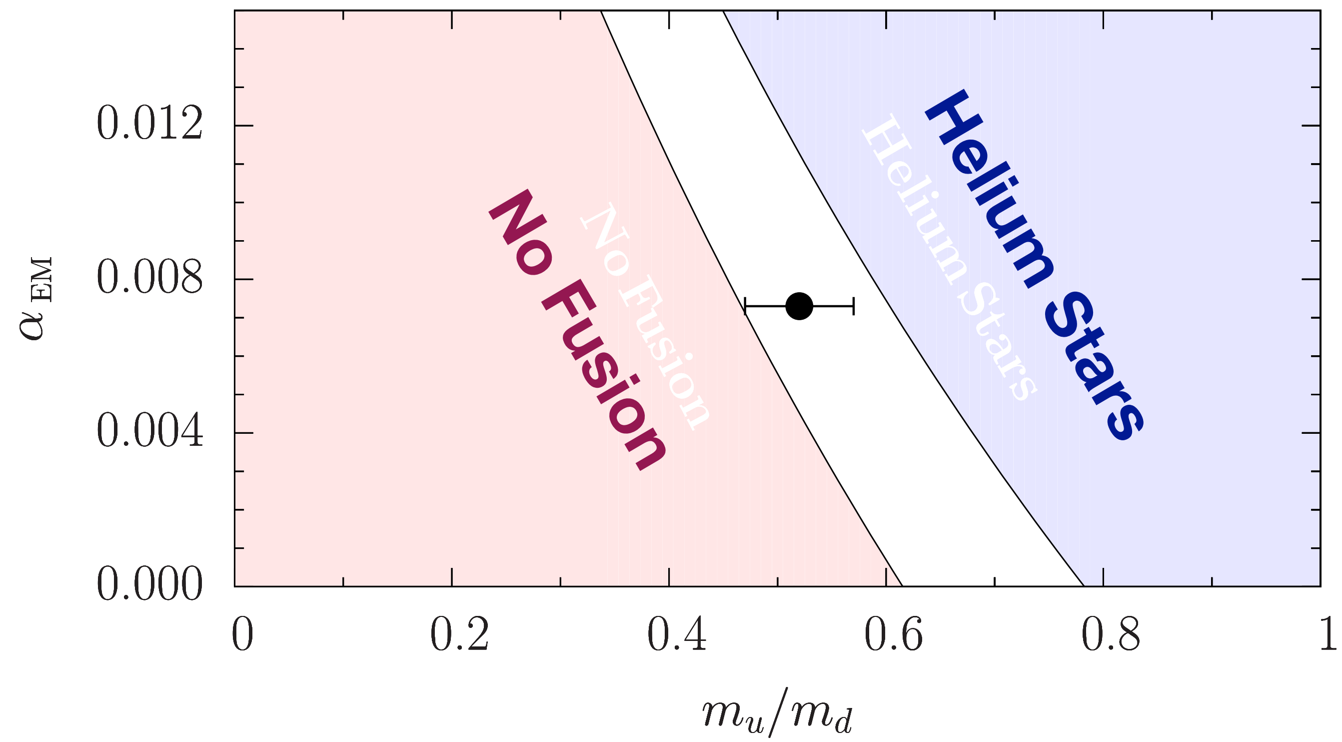
- **Photon zero modes**
  - Charged particle correlators and mass extraction
  - Comparison with Uno & Hayakawa gauge fixing
- Very briefly:
  - Charge renormalisation
  - Decuplet baryon isospin splittings











"Isospin splittings of meson and baryon masses from three-flavor lattice QCD + QED"
R. Horsley *et al.*, *to appear* JPG(2016), `arXiv:1508.06401`

"QED effects in the pseudoscalar meson sector"
R. Horsley *et al.*, JHEP1604,093(2016), `arXiv:1509.00799`

Photon zero mode

Lattice QCD+QED

$$S = S_G + S_{QED} + S_F^u + S_F^d + S_F^s\,.$$

$$S_{QED} = \frac{1}{2e^2}\sum_{x,\mu<\nu}\left(A_\mu(x) + A_\nu(x+\mu) - A_\mu(x+\nu) - A_\nu(x)\right)^2$$

noncompact

$$S_F^q = \sum_x \Big\{ \sum_\mu \Big[\overline{q}(x)\frac{\gamma_\mu - 1}{2}e^{-iQ_q A_\mu(x)}\tilde{U}_\mu(x)q(x+\hat{\mu})$$
$$-\overline{q}(x)\frac{\gamma_\mu + 1}{2}e^{iQ_q A_\mu(x)}\tilde{U}_\mu^\dagger(x-\hat{\mu})q(x-\hat{\mu})\Big]$$
$$+\frac{1}{2\kappa_q}\overline{q}(x)q(x) - \frac{1}{4}c_{SW}\sum_{\mu\nu}\overline{q}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)q(x)\Big\}$$

$$Q_u = \frac{2}{3}\,,\; Q_d = Q_s = -\frac{1}{3}$$

We work with a gauge coupling corresponding to $\alpha_{QED} = 0.1$

Photon zero mode

- Consider zero mode of EM gauge field

$$\overline{A}_\mu \equiv A_\mu(k=0) = \frac{1}{L^3 T} \sum_x A_\mu(x)$$

- Photon action invariant: $$\frac{\partial S_{QED}}{\partial \overline{A}_\mu} = 0$$

- Not constrained by Lorenz gauge-fixing condition:

$$S_{GF} = \frac{1}{2e^2\lambda} \sum_x \left(\sum_\mu \overline{\Delta}_\mu A_\mu \right)^2 \quad \Rightarrow \quad \frac{\partial S_{GF}}{\partial \overline{A}_\mu} = 0$$

Landau gauge: $\lambda = 0$

- Couplings to quarks:

$$\frac{\partial S_F^q}{\partial \overline{A}_\mu} = L^3 T \sum_x \frac{\partial S_F^q}{\partial A_\mu(x)} = L^3 T \sum_x J_\mu^q(x)$$

action *not* invariant under continuous transformation of zero mode

Gauge transformations on a box

- Consider U(1) field transformation:

$$A_\mu(x) \to A_\mu(x) + \Delta_\mu \alpha(x)$$

- Shift of zero mode:

$$\Delta_\mu \alpha(x) = c_\mu \ [\text{indep. of spacetime } x]$$

  - Gauge-field action invariant
  - Simple closed Wilson loops invariant
  - Polyakov loops: $L_\mu^q(x) = \exp\left[iQ_q \sum_{x_\mu} A_\mu(x)\right]$

$$\to \exp\left[iQ_q N_\mu c_\mu + iQ_q \sum_{x_\mu} A_\mu(x)\right]$$

$N = (L, L, L, T)$

no summation

Quantisation condition: Polyakov loops invariant if
[& action]

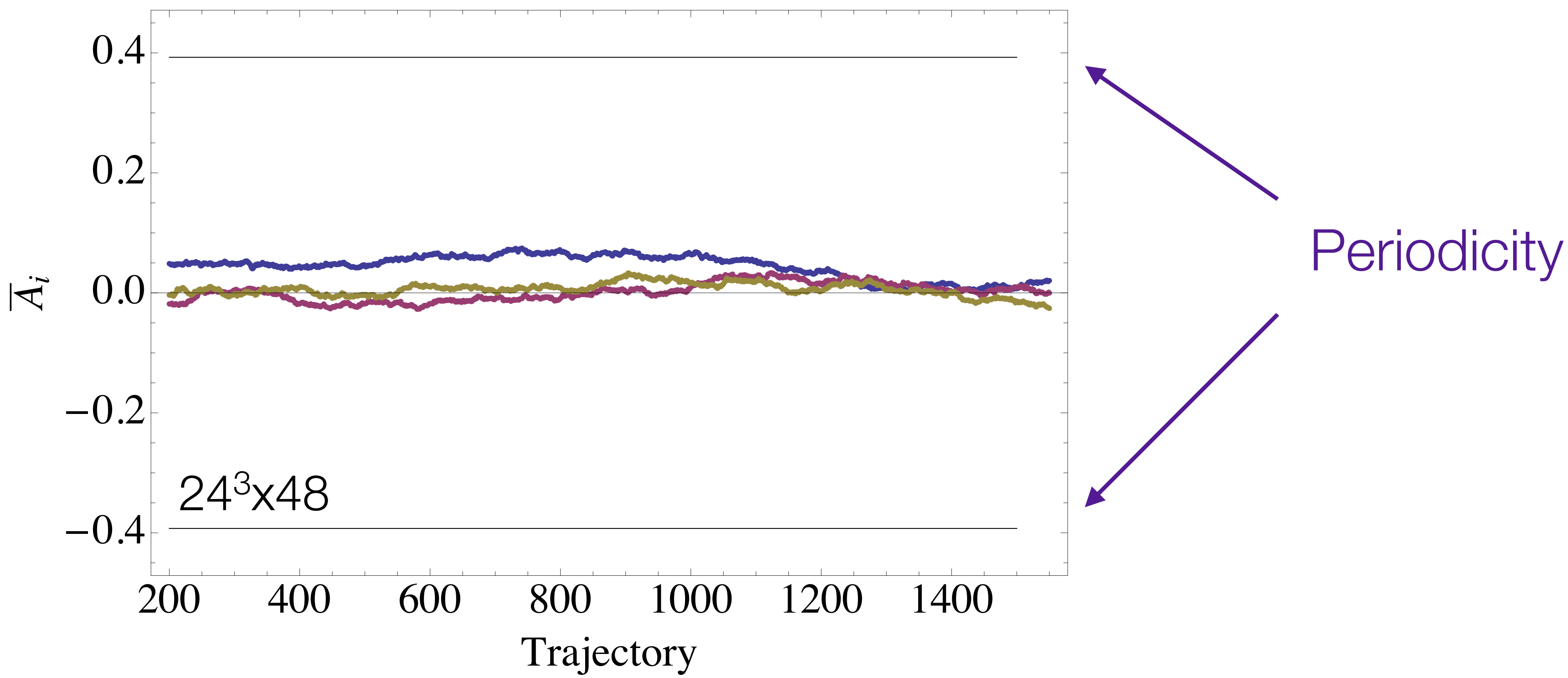
$$c_\mu = n_\mu \frac{2\pi}{Q_q N_\mu}, \quad n_\mu \in \mathbb{Z}$$

Gauge-fixing in a box

- On each configuration we map the zero mode into the interval

$$-\frac{\pi}{|Q_d|N_\mu} < \overline{A}_\mu \leq \frac{\pi}{|Q_d|N_\mu}$$

  - Preserves importance sampling of path integral



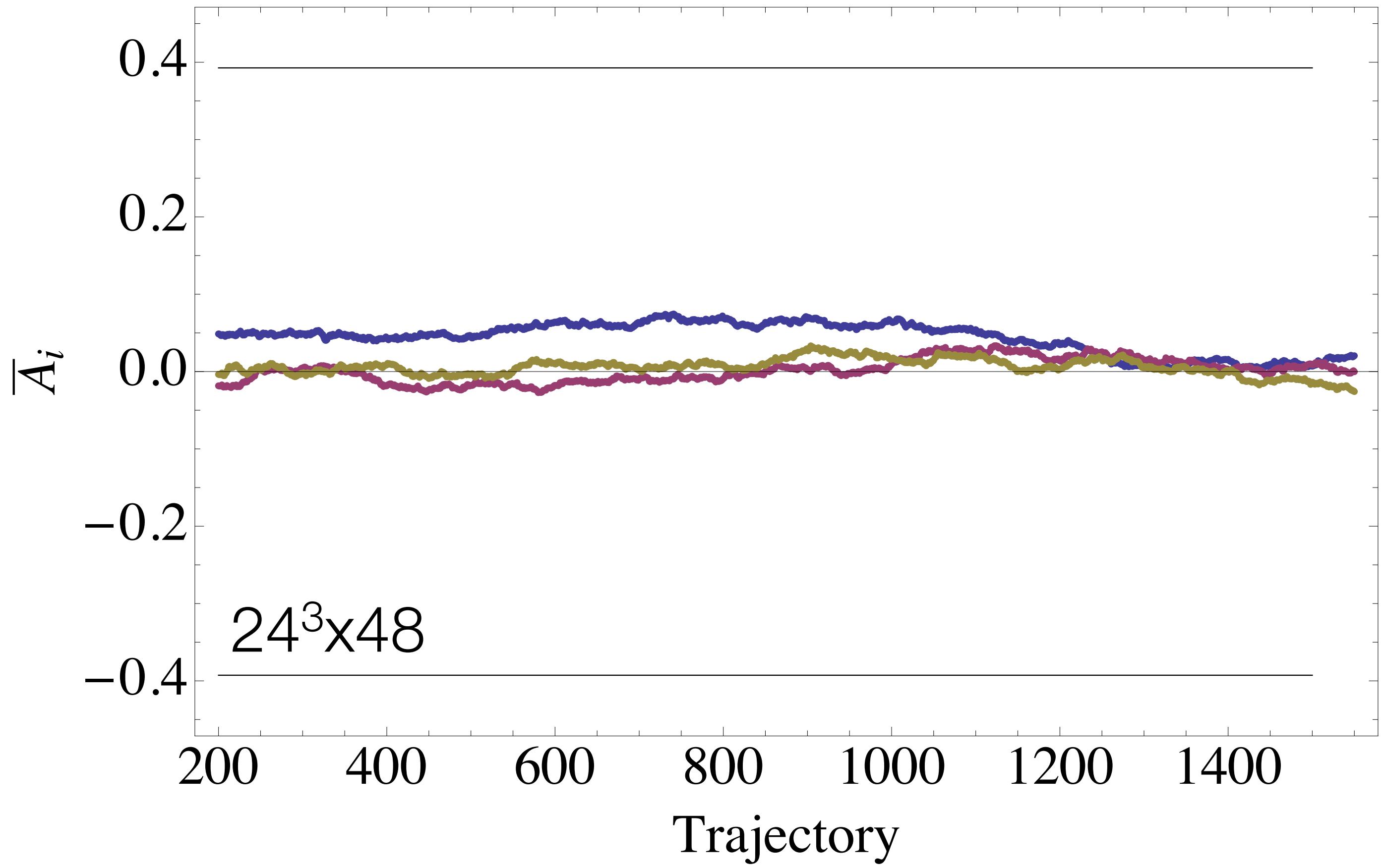
subset of full ensemble

Gauge-fixing in a box

- On each configuration we map the zero mode into the interval

$$-\frac{\pi}{|Q_d|N_\mu} < \overline{A}_\mu \leq \frac{\pi}{|Q_d|N_\mu}$$

  - Preserves importance sampling of path integral



Periodicity

subset of full ensemble

Impact on spectrum?

Correlators in uniform U(1) external field

- Consider quarks propagating in uniform background field B_μ

- Absorb U(1) phase into fermion field transformation:

    - Twisted boundary conditions for charged hadrons

$$k_\mu \to k_\mu + QB_\mu$$

- Correlator

$$\sim e^{-t\sqrt{(\vec{k}+Q\vec{B})^2+m^2}}e^{iQB_4 t}$$

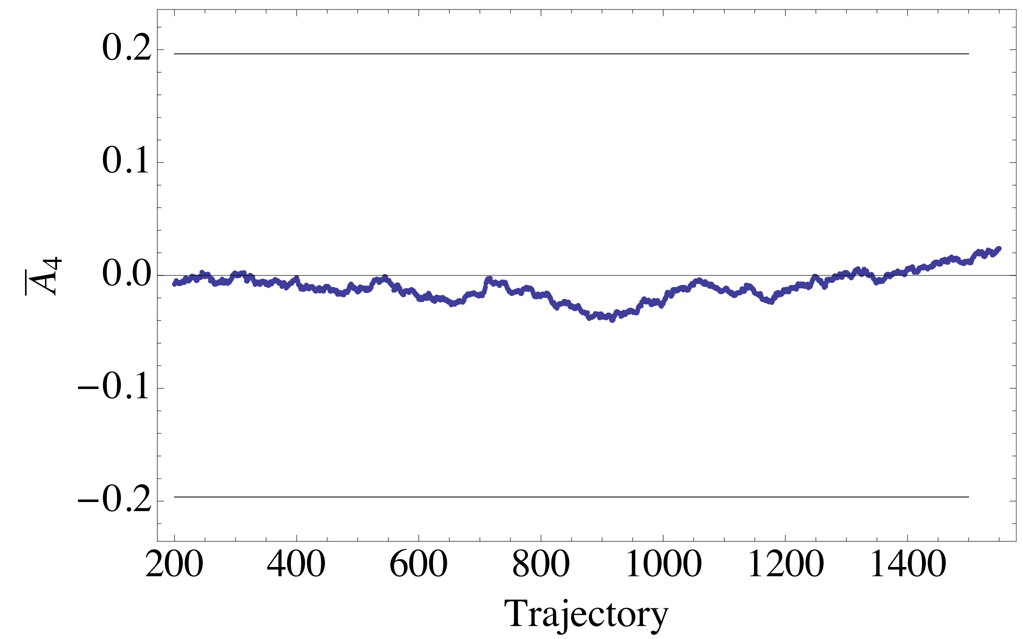
Just the same as twisted BCs that have been studied extensively in the literature.
eg. Sachrajda & Villadoro, Tiburzi *et al*., Bijnens *et al*.

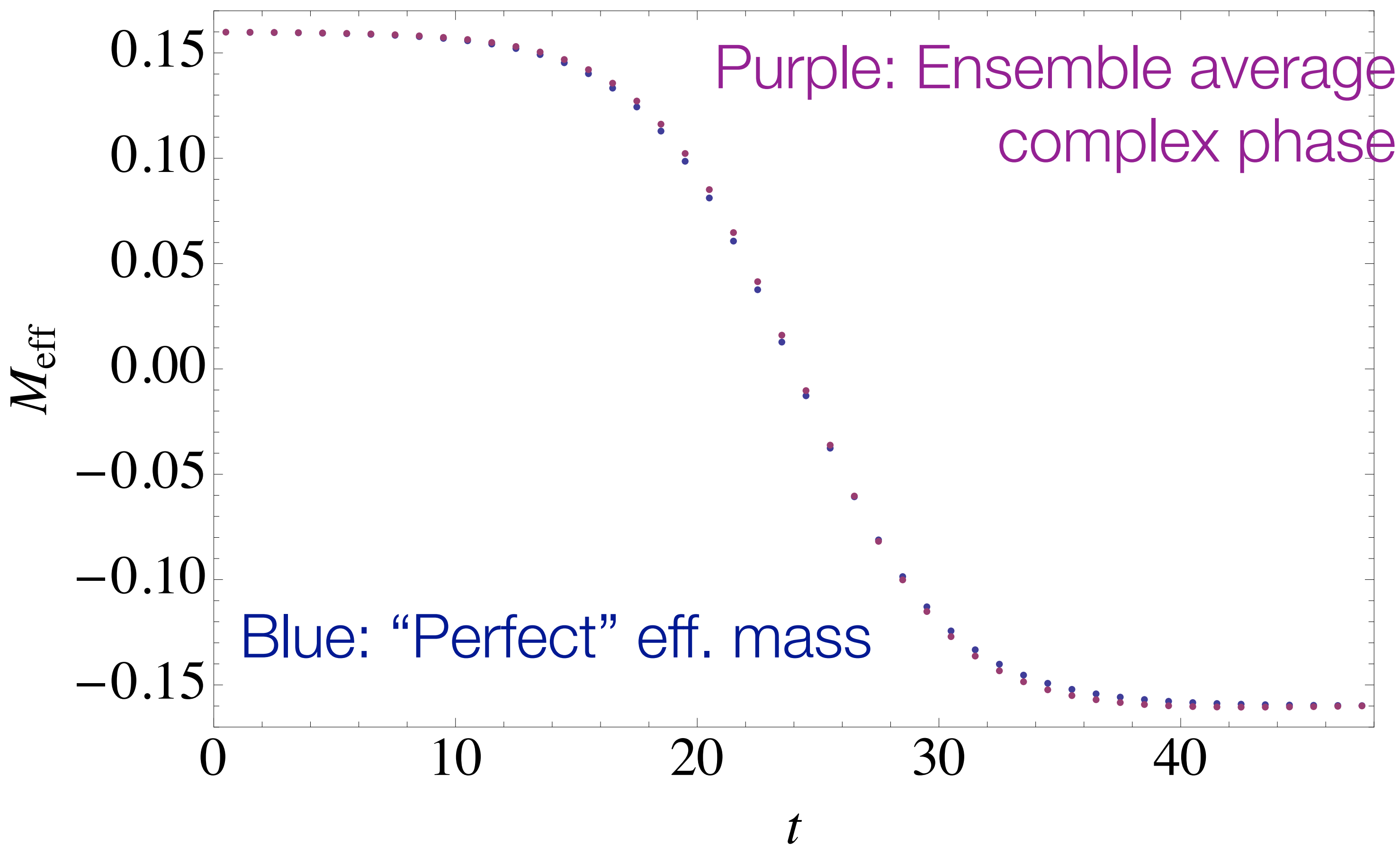
Extracting a mass: Complex phase (toy model)

- Zero modes move through simulation time
  - Not a simple uniform phase

$$\langle B_4 \rangle = \langle \overline{A}_4 \rangle = -0.00993$$
$$\langle B_4^2 \rangle = \langle \overline{A}_4^2 \rangle = 0.000255$$

24^3x48





$$m_{\text{“exact”}} = 0.160$$

Fit real part of correlator:

$$m_{\text{“fit”}} = 0.162$$

Extracting a mass: Kinetic shift

- Rest energies get a spread in kinetic energy from spatial zero mode

$$E = \sqrt{m^2 + Q^2\vec{B}^2}$$

  - Mass estimate on ensemble average

$$m = \sqrt{E^2 - Q^2\langle\vec{B}^2\rangle}$$

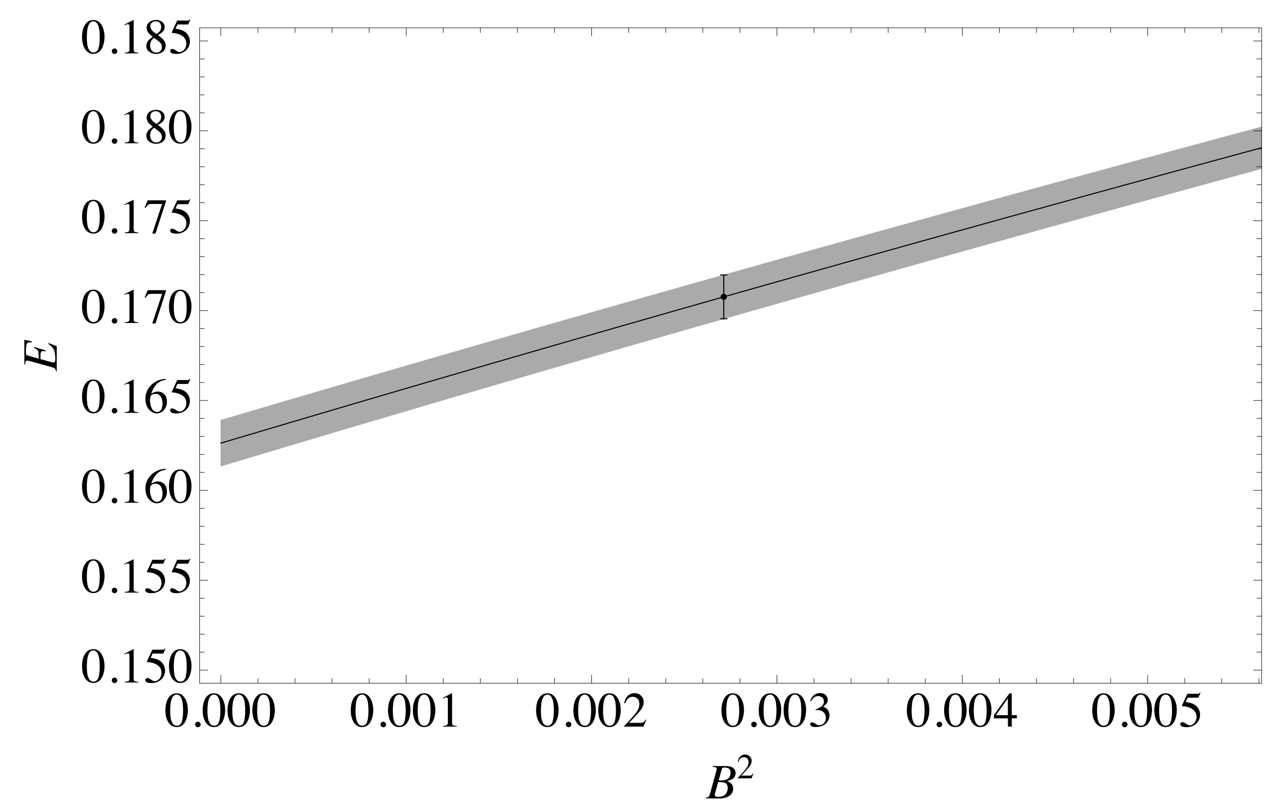
24^3x48 (subset)

$$\langle\vec{B}^2\rangle = 0.0027$$

- Charge 1 meson ("u-dbar")

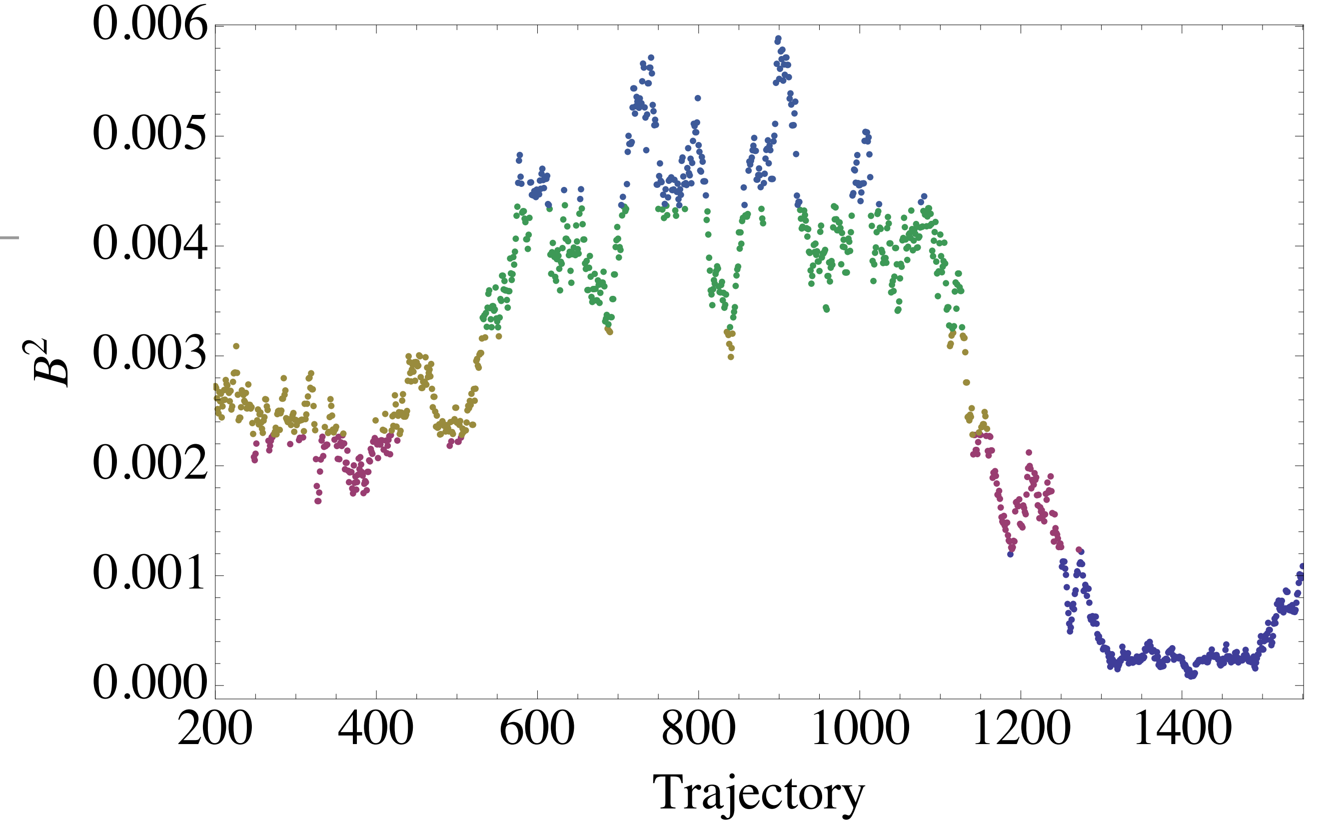
$$E = 0.1708(12)$$

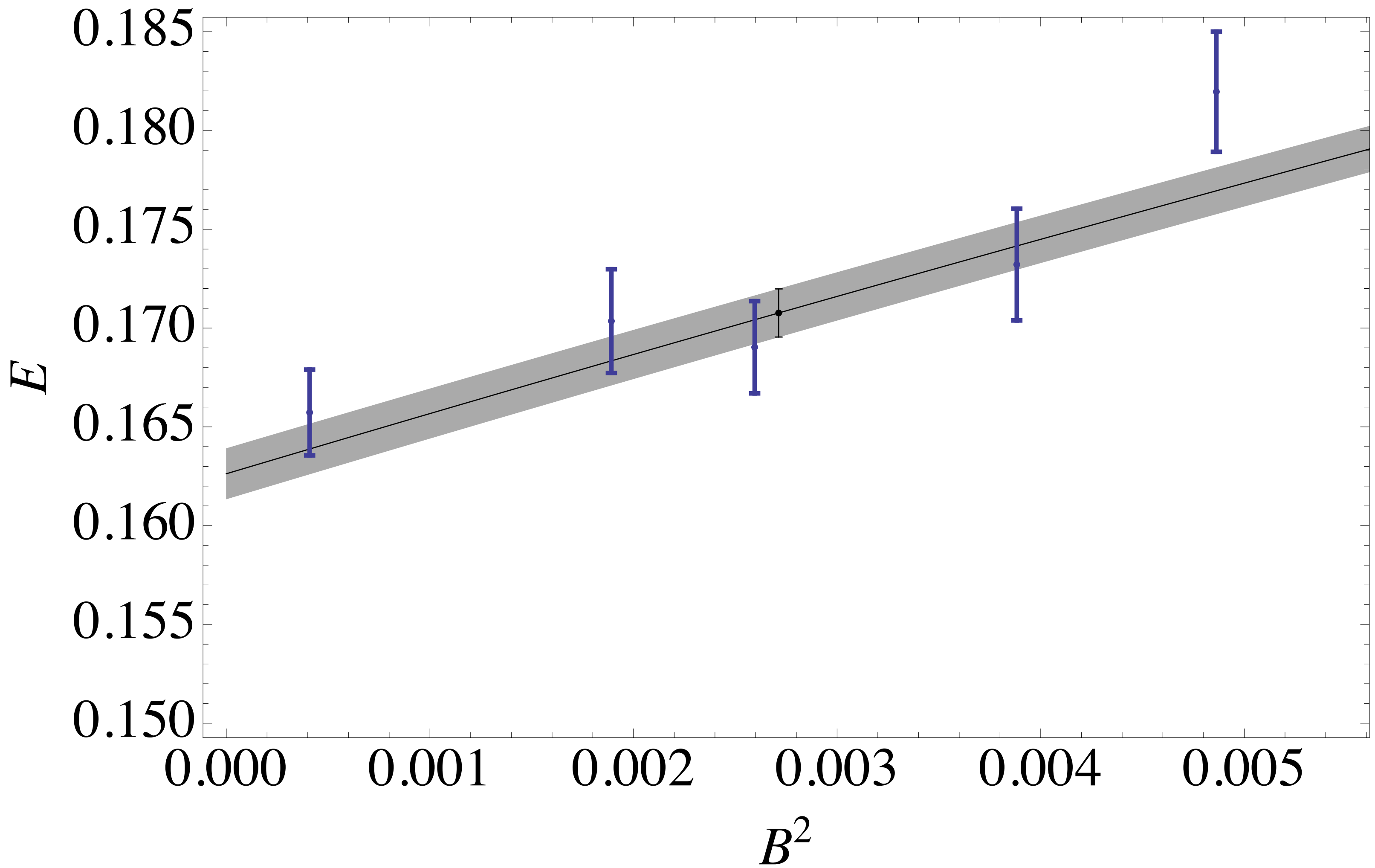
$$\rightarrow m = 0.1626(12)$$



Bin ensembles by B^2

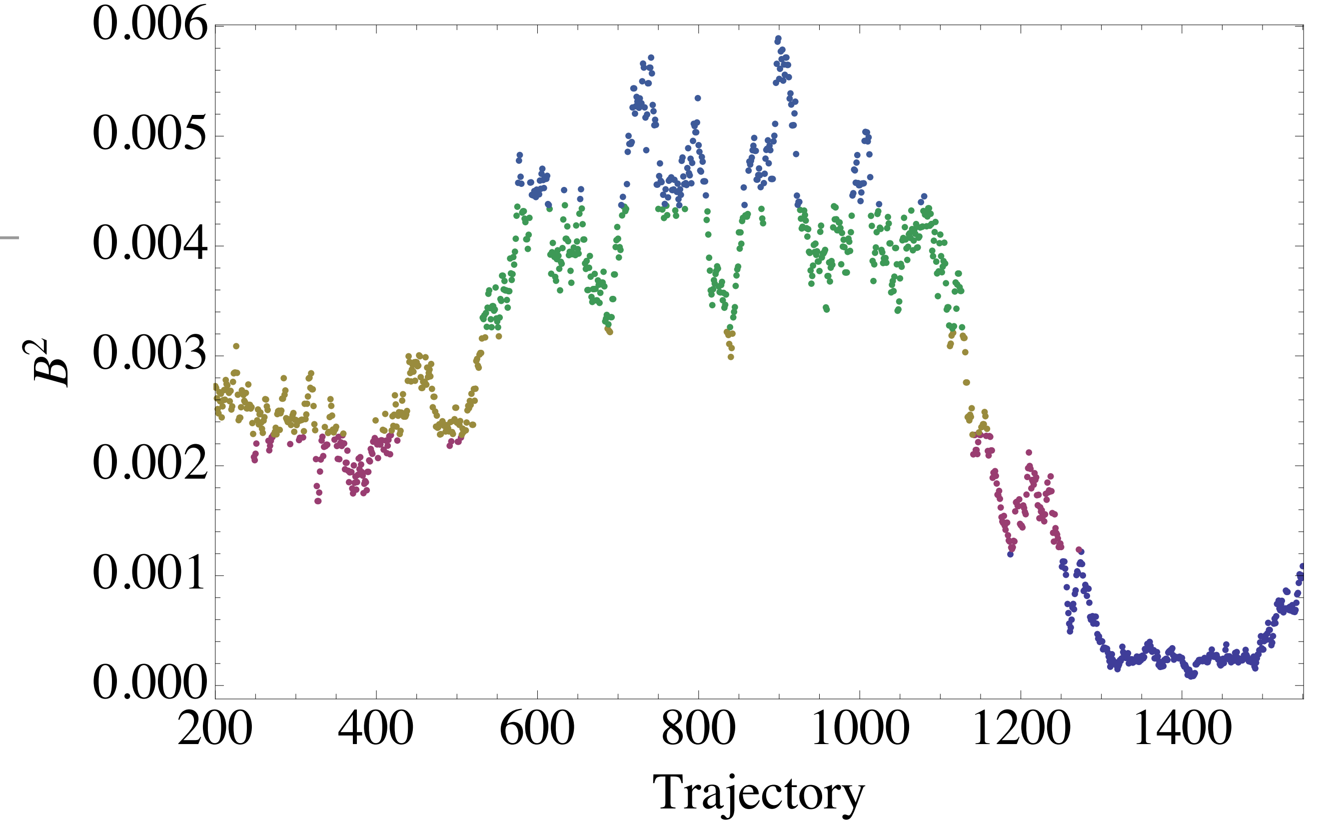
- Compute energies on binned subensembles

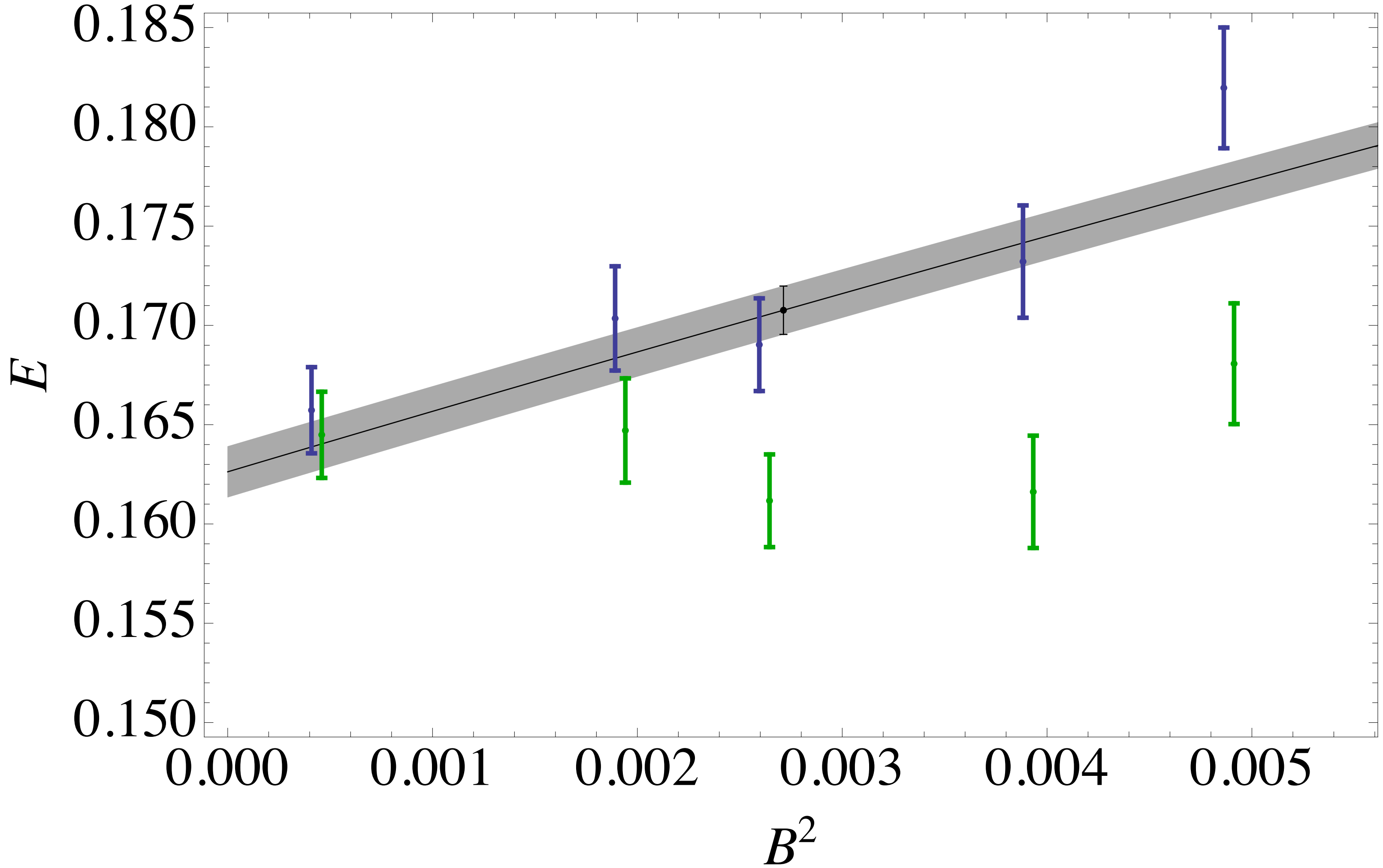




Bin ensembles by B^2

- Compute energies on binned subensembles

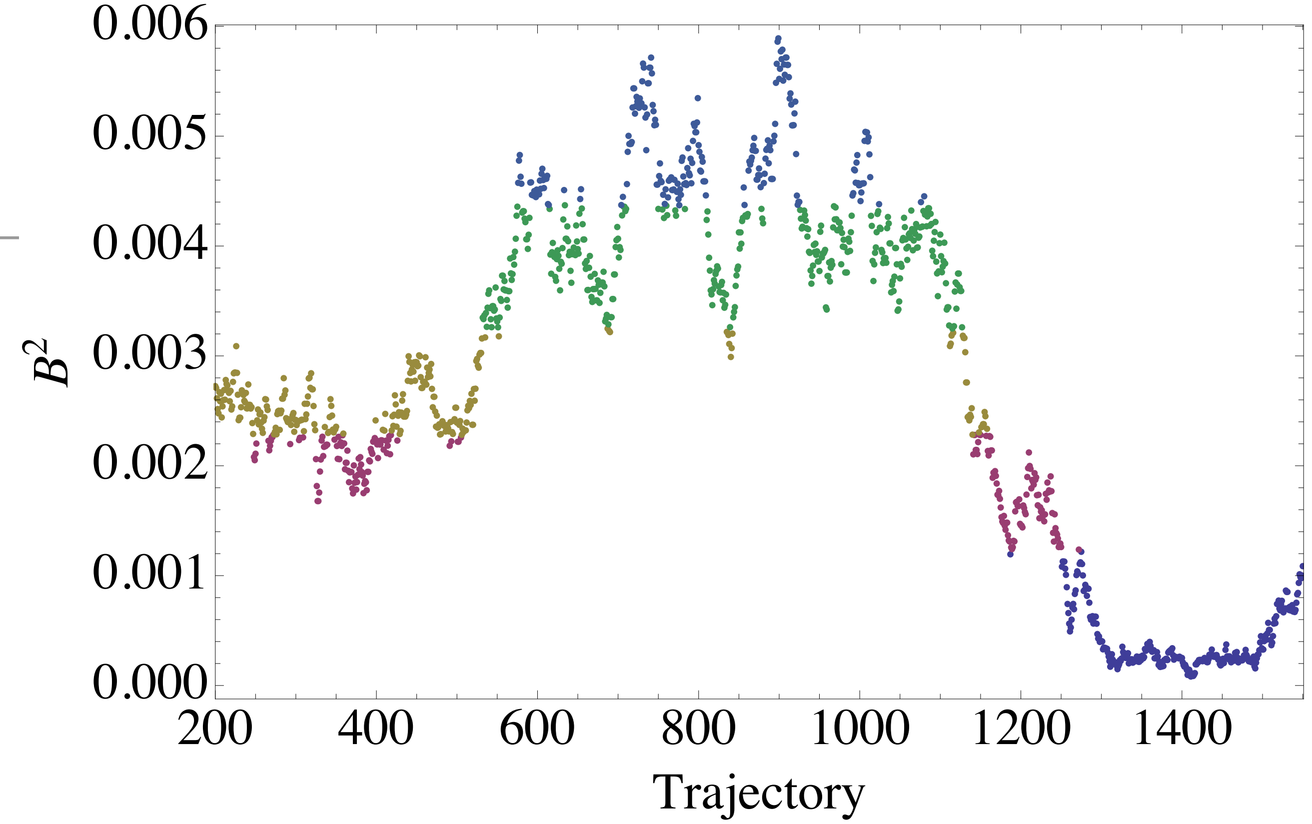


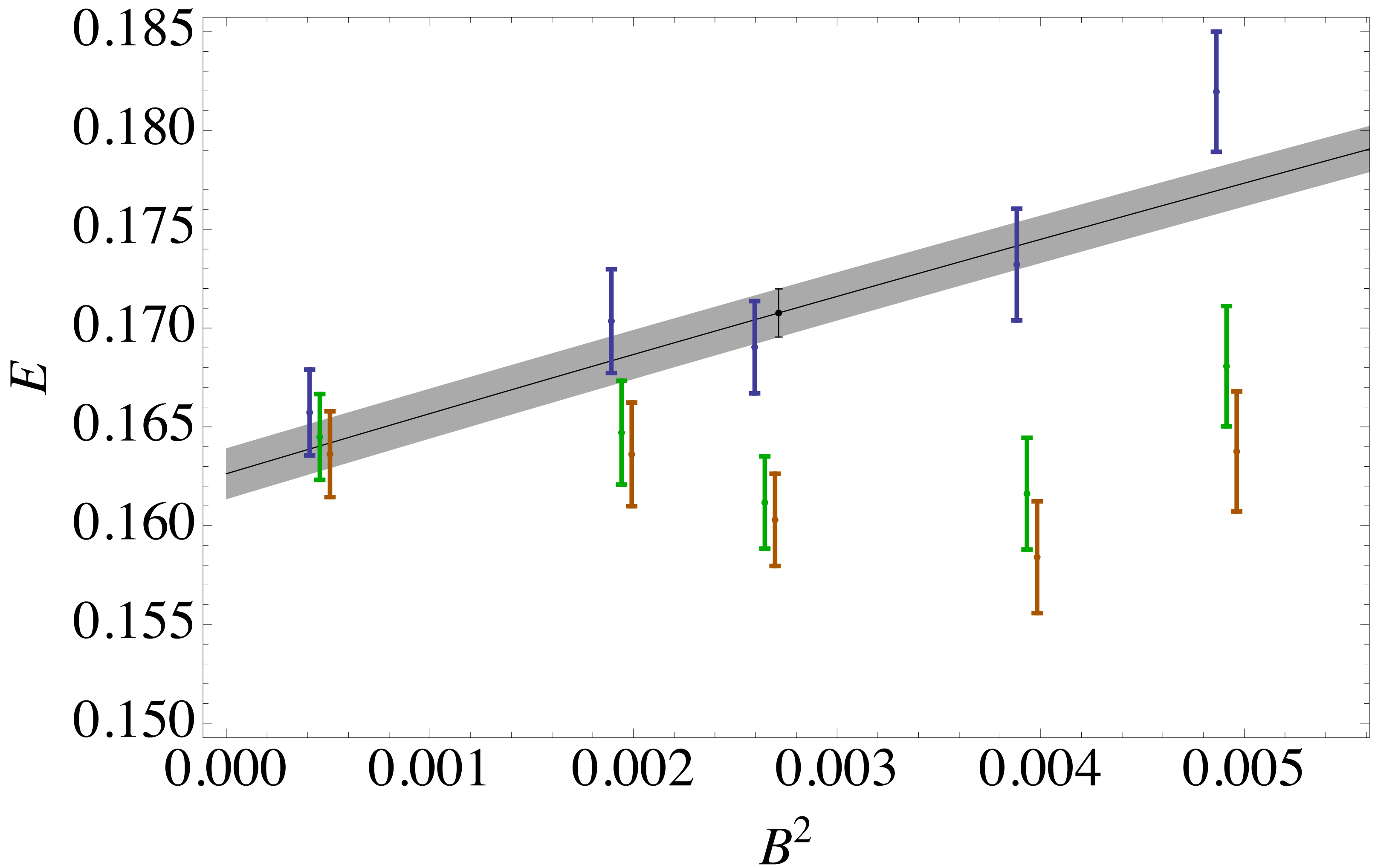


Spatial zero mode subtraction

Bin ensembles by B^2

- Compute energies on binned subensembles





Spatial zero mode subtraction

Estimate complex phase correction

Uno & Hayakawa gauge fixing

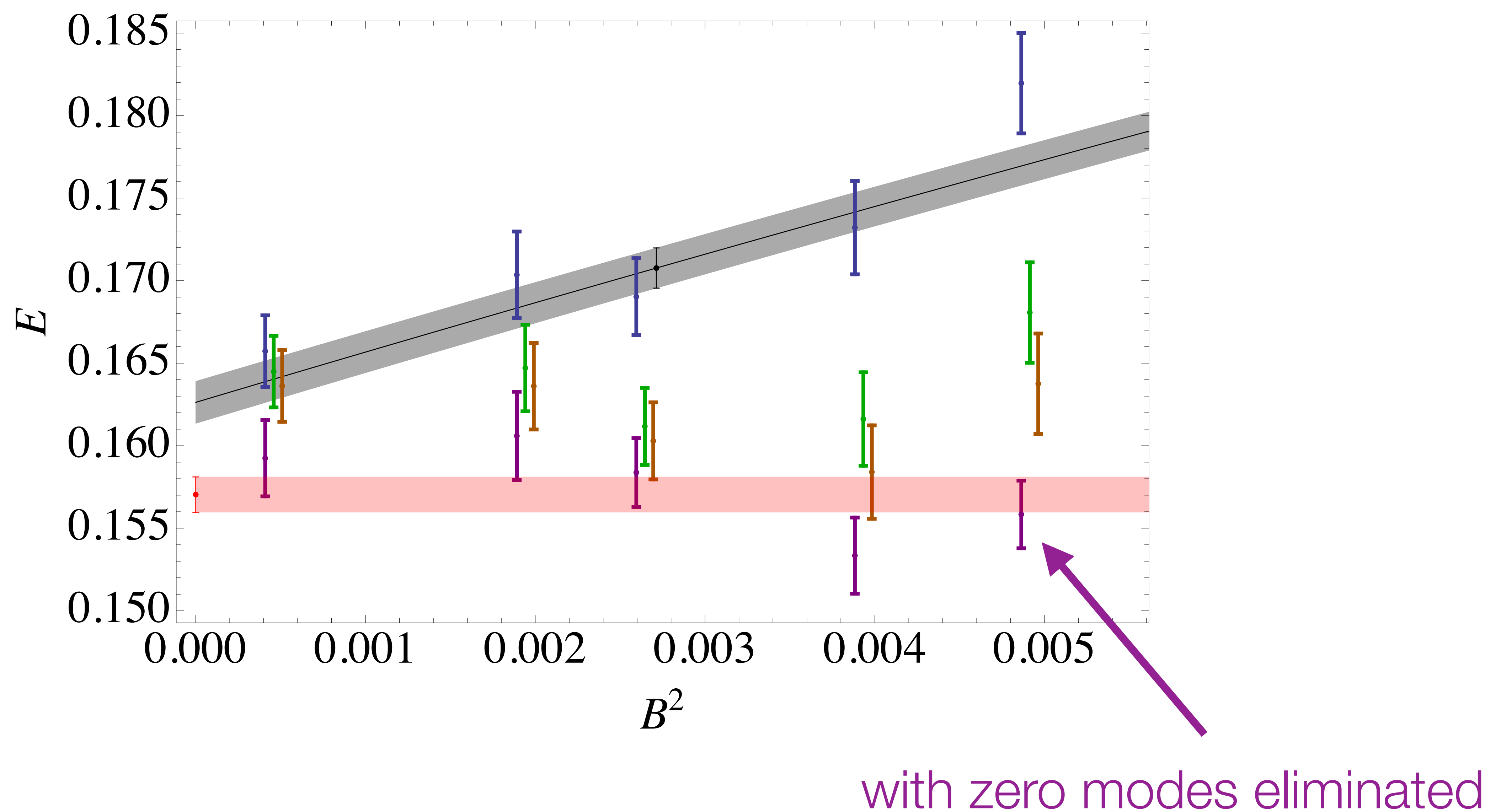
- To eliminate zero mode complications (preserve reflection positivity), Uno & Hayakawa (2008) proposed additional gauge-fixing condition where all spatial zero modes of the U(1) field are eliminated

$$\tilde{A}_\mu(\vec{k}=0,t)=0, \quad \forall t$$

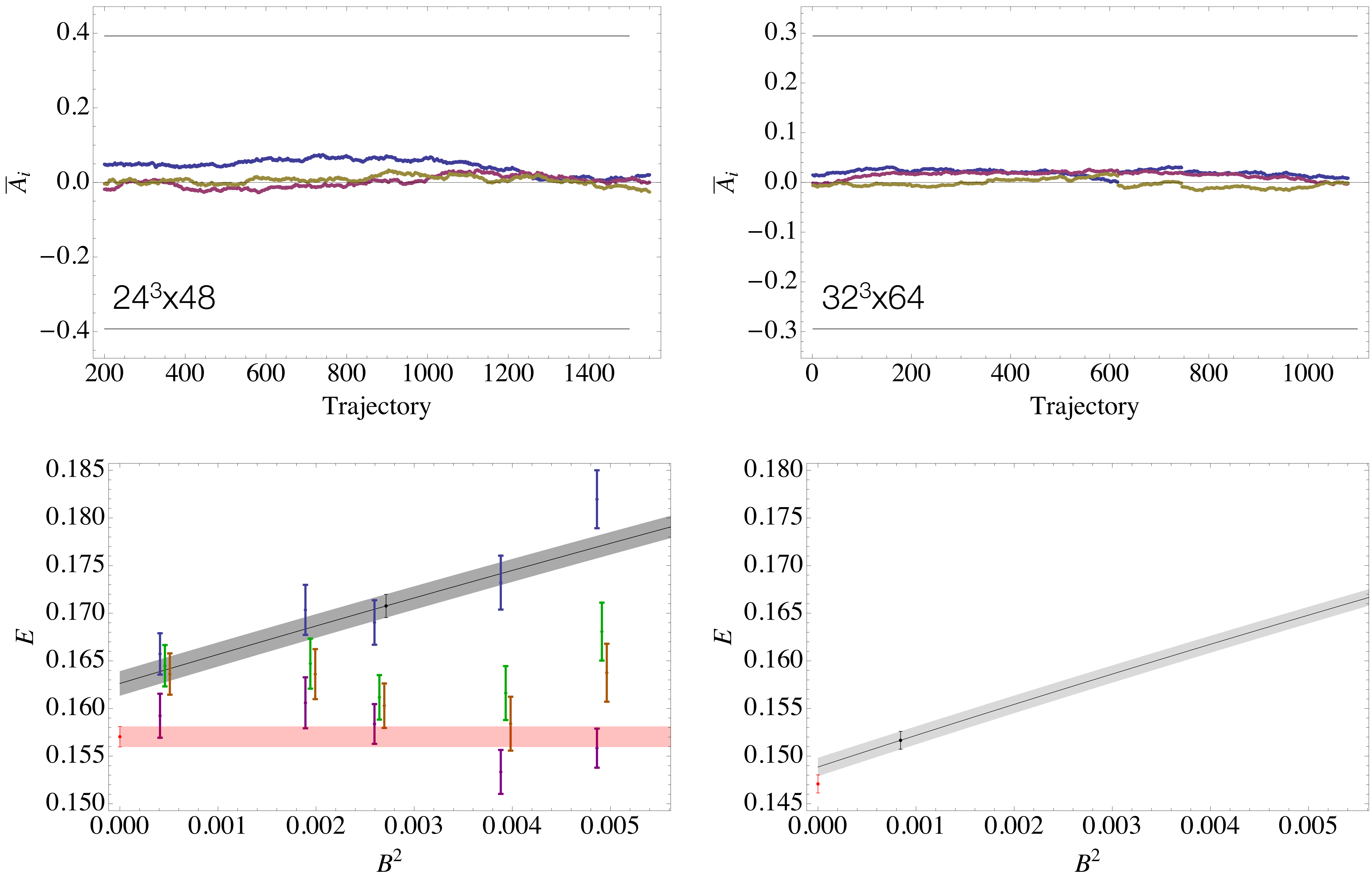
- In continuum, any gauge field can be transformed to this condition by a simple gauge transformation
- Because of box quantisation condition, we cannot satisfy this condition by a gauge transformation
    - In practice, U(1) field updates must be performed in Fourier space
- For our ensembles, we can compare our results by forcing this zero mode condition
    - Amounts to a “partial-quenching” of twists in the sea
    - Potential to be corrected by appropriate reweighting

New results with spatial zero modes removed

- Recompute spectra on modified gauge fields
  - Consider same binning of trajectories as above



Larger volume comparison



Charge renormalisation

Photon propagators

- Fourier transform U(1) field to compute propagators:

$$\frac{\beta}{V}\langle \tilde{A}_\mu(k)\tilde{A}^*_\nu(k)\rangle = \frac{Z_3}{\hat{k}^2}\left[\delta_{\mu\nu} - \frac{\hat{k}_\mu \hat{k}^*_\nu}{\hat{k}^2}\right] + \ldots$$

- Define

$$\begin{aligned} D_{\mu\nu}(k) &= \frac{\beta}{V}\hat{k}^2\langle \tilde{A}_\mu(k)\tilde{A}^*_\nu(k)\rangle \\ &= D(k^2)\left[\delta_{\mu\nu} - \frac{\hat{k}_\mu \hat{k}^*_\nu}{\hat{k}^2}\right] + \ldots \end{aligned}$$

- Charge renormalisation

$$Z_3 = \lim_{k^2 \to 0} D(k^2)$$

- Vacuum polarisation

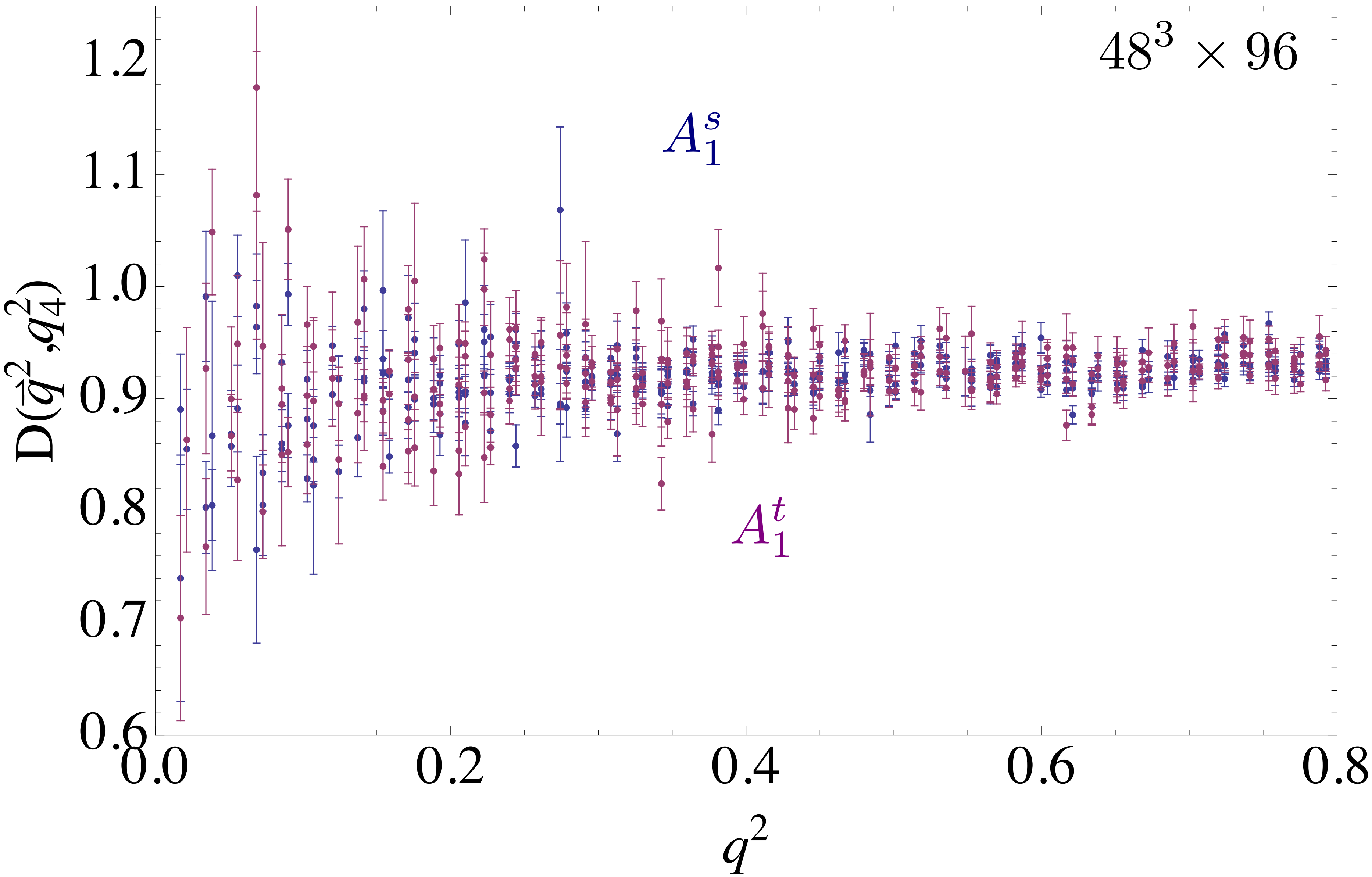
$$\Pi(k^2) = \frac{1}{e^2}\left[1 - \frac{1}{D(k^2)}\right]$$

Lattice symmetries

- Lattice irreps for asymmetric (L/T) lattices [see Aubin *et al*. PRD(2016)]

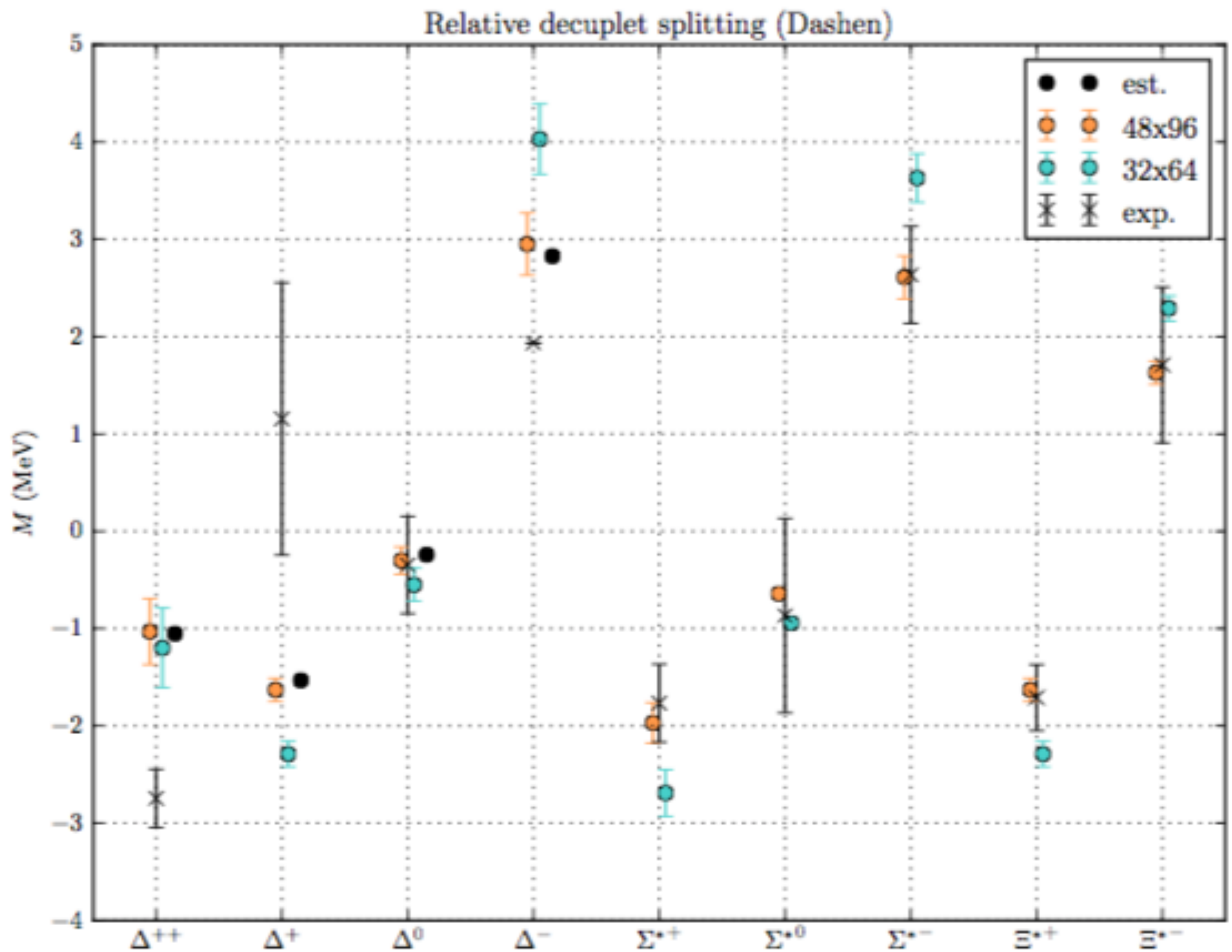
$$A_1^{(\text{spatial})} : \quad \sum_i D_{ii}(k) = \left(3 - \frac{\hat{\vec{k}}.\hat{\vec{k}}^*}{\hat{k}^2}\right) D_{A_1^s}(k^2)$$

$$A_1^{(\text{temporal})} : \quad D_{44}(k) = \left(1 - \frac{\hat{k}_4 \hat{k}_4^*}{\hat{k}^2}\right) D_{A_1^t}(k^2)$$



Charge renormalisation | $Z_3 \sim 0.9$

Decuplet isospin splittings



Phenomenological "data": Cutkosky PRC(1993)

Decuplet splittings in vicinity of SU(3) symmetric point

Good agreement with phenomenology

Final remarks

- Zero modes disrupt simple extraction of charged hadron masses
  - Numerical evidence suggests these can be reasonably controlled by accounting for systematics [further investigations warranted]
  - Potential for using Uno & Hayakawa with reweighting
- Charge renormalisation ~0.9 [more soon]
- First look at decuplet isospin splittings