Complex spectrum of spin models for finite-density QCD

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Talk@Lattice2016

27 July 2016

<HN, M. Ogilvie, and K. Pangeni, PRD93, 094501>

Outline

- Introduction

  - Spin models for QCD at finite μ

  - *CK* symmetry

- Main Results

  - Complex mass spectrum

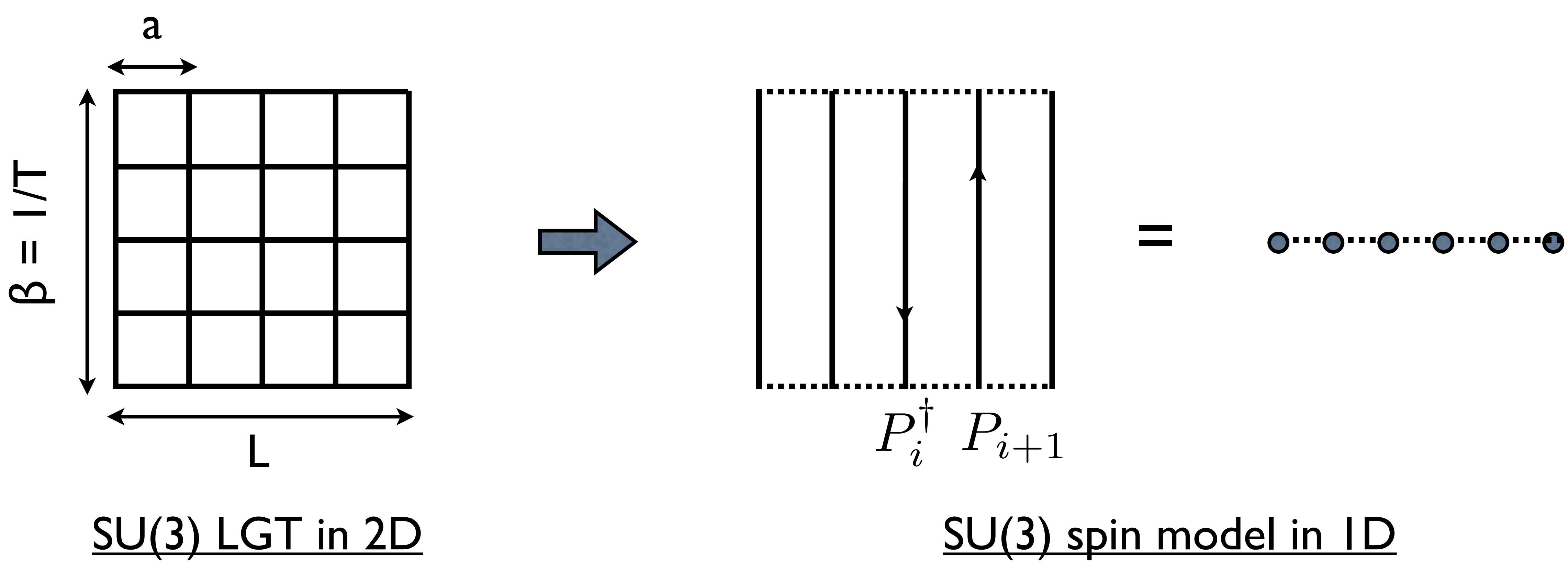
  - Sinusoidal oscillation for correlation functions

- Conclusions

Introduction

Yang-Mills in (1+1) dimensions

- Integrate out the spatial links using the character expansion



- Construct the transfer matrix with the heat kernel action

$$T_0 = \langle P_{i+1} | \, e^{-aH_0} \, | P_i \rangle \qquad \text{where} \qquad H_0 = \frac{g^2 \beta}{2} C$$

<P. Menotti and E. Onofri, 1981> etc

$$\to \langle r' | \, e^{-aH_0} \, | r \rangle = \mathrm{drag}(1, e^{-4a/3}, e^{-4a/3}, e^{-3a} \cdots)$$

Static quarks

- Inserting static quarks in the transfer matrix

$$T = \langle r' |\, e^{-aH_0/2} \det(1 + z_1 P) \det(1 + z_2 P^\dagger)\, e^{-aH_0/2}\, | r \rangle$$

$$\text{where} \;\; z_1 = e^{(\mu - M)/T} \;\; \text{and} \;\; z_2 = e^{(-\mu - M)/T}$$

- Raising and lowering operators

$$\det(1 + z_1 P) = 1 \; + z_1 \; \square \; + z_1^2 \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \; + z_1^3$$

- Transfer matrix is non-Hermitian: A manifestation of the sign problem.

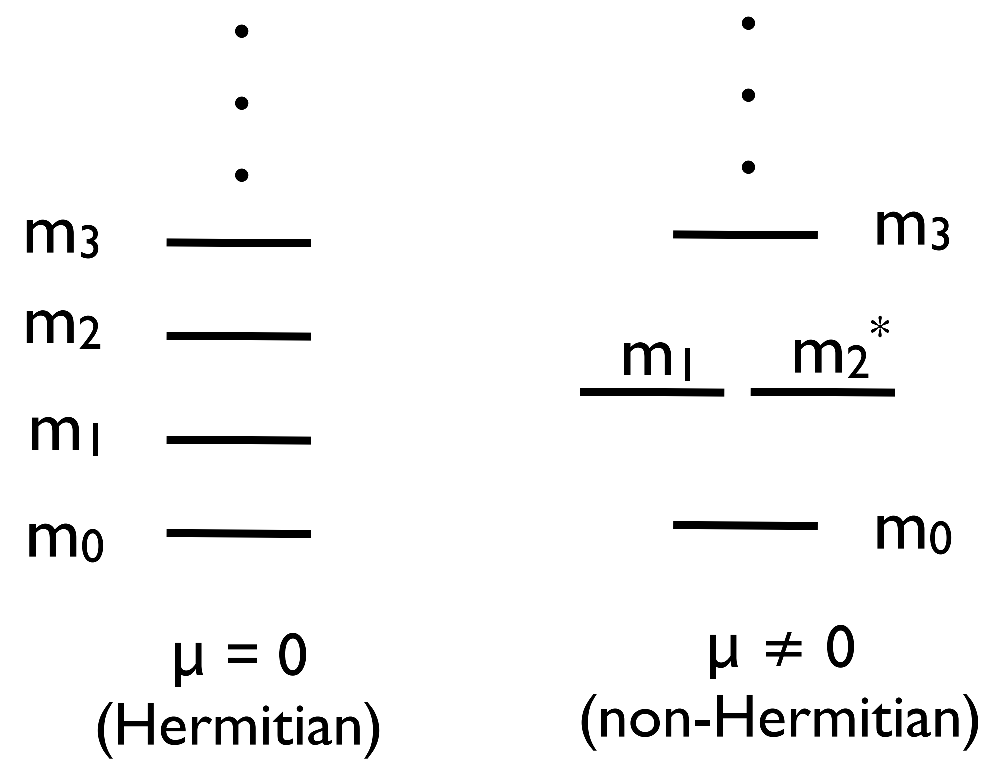
Pure SU(3) → With quarks ($z_2 = 0$)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{e^{4/3}} & 0 & 0 \\ 0 & 0 & \frac{1}{e^{4/3}} & 0 \\ 0 & 0 & 0 & \frac{1}{e^3} \end{pmatrix} \rightarrow \begin{pmatrix} 1 + z_1^3 & \frac{z_1}{e^{2/3}} & \frac{z_1^2}{e^{2/3}} & 0 \\ \frac{z_1^2}{e^{2/3}} & \frac{1+z_1^3}{e^{4/3}} & \frac{z_1}{e^{4/3}} & \frac{z_1^2}{e^{13/6}} \\ \frac{z_1}{e^{2/3}} & \frac{z_1^2}{e^{4/3}} & \frac{1+z_1^3}{e^{4/3}} & \frac{z_1}{e^{13/6}} \\ 0 & \frac{z_1}{e^{13/6}} & \frac{z_1^2}{e^{13/6}} & \frac{1+z_1^3}{e^3} \end{pmatrix}$$

CK-symmetric systems

- Eigenvalues are real or form a complex conjugate pair due to *CK*-symmetry

$$T = \mathrm{diag}(e^{-m_0 a}, e^{-m_1 a}, \ldots)$$



- Charge conjugation (*C*)
- Complex conjugation (*K*)

$$\rightarrow \quad \det M(\mu) = [\det M(-\mu)]^*$$

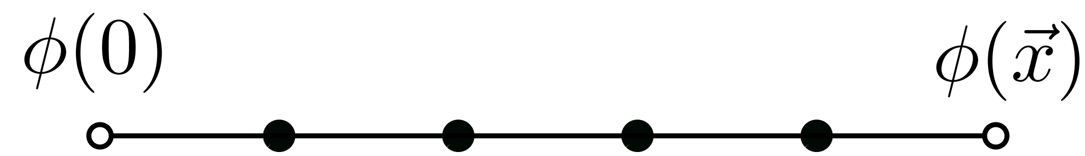
<HN, M. Ogilvie, and K. Pangeni, 2014>

- Correlation function: sinusoidal exponential decay if $m_2 = m_1^*$

$$\left\langle \mathrm{tr}_F P^\dagger(x)\, \mathrm{tr}_F P(0) \right\rangle_C \sim \exp\left(-\mathrm{Re}[m_1 - m_0]\, x\right) \cos(\mathrm{Im}[m_1 - m_0]\, x) \qquad (x, L \to \infty)$$

For higher dimensions

- In (1+1)-dim with static quarks, the results of mass spectrum are exact.

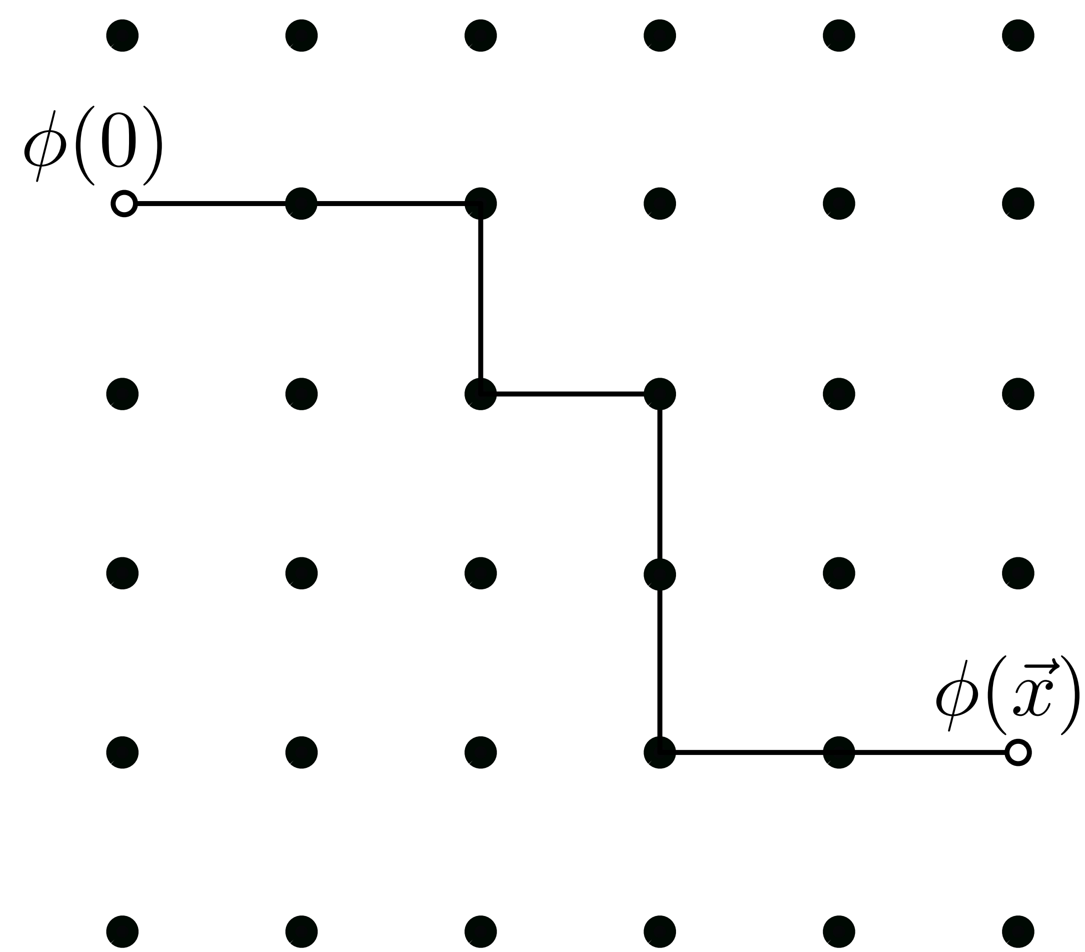


- They are also the results for higher dimensions at leading order in strong coupling.

The leading diagrams for $\langle \phi(\vec{x}) \phi(0) \rangle$

are the shortest possible paths.

<J. Kogut, D. Sinclair, R. Pearson, J. Richardson, and J. Shigemitsu 1981>



Results

1. Static quark: $T = \langle r' |\, e^{-aH_0/2} \det(1 + z_1 P) \det(1 + z_2 P^\dagger)\, e^{-aH_0/2}\, | r \rangle$

$$\text{where} \quad z_1 = e^{(\mu - M)/T} \quad \text{and} \quad z_2 = e^{(-\mu - M)/T}$$

- Particle-Antiparticle (*C*): $(z_1, z_2) \rightarrow (z_2, z_1)$

- Particle-Hole (*K*): $(z_1, z_2) \rightarrow (1/z_1, 1/z_2)$

PH: $\det(1 + z_1 P) \longrightarrow \det(1 + P/z_1)$

K: $\det(1 + z_1 P) \longrightarrow \det(1 + z_1 P^\dagger)$

$$= z_1^N \det(1 + P/z_1)$$

- Combined transformation (*CK*): $(z_1, z_2) \rightarrow (1/z_2, 1/z_1)$

1. Static quark: $T = \langle r'|\, e^{-aH_0/2} \det(1+z_1 P) \det(1+z_2 P^\dagger)\, e^{-aH_0/2}\, |r\rangle$

- Invariant under $z_1 \to 1/z_1$
- Hermitian point at $z_1 = 1$.
- The Polyakov loop goes to zero for μ>>M

  Also observed in other methods:

  - Strong-coupling

    <J. Langelage, M. Neuman, and O. Philipsen, 2014>

    <T. Rindlisbacher and P. de Forcrand, 2015>

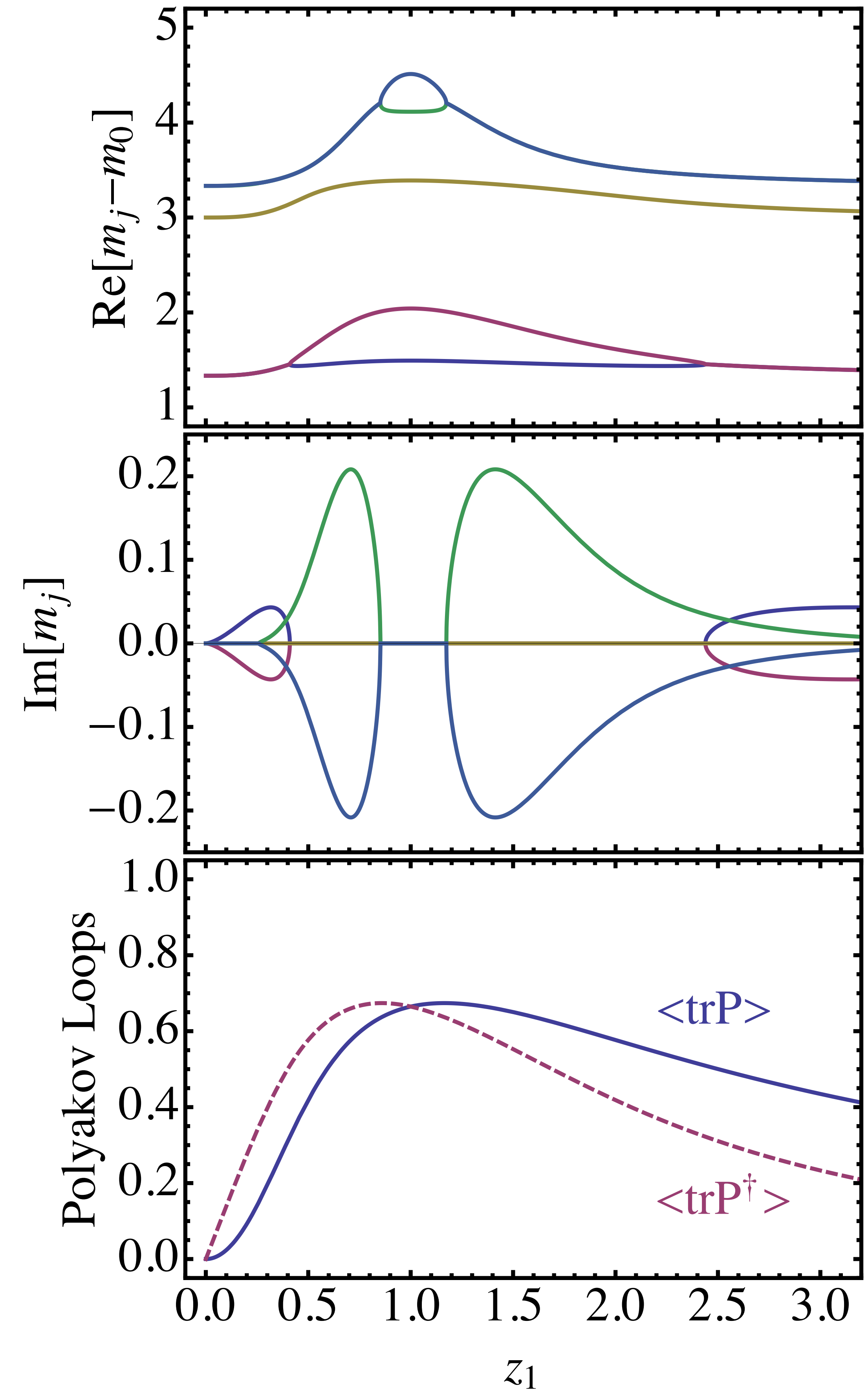
    and more

  - Complex Langevin

    <G. Aarts, E. Seiler, D. Sexty, and I. Stamatescu, 2014>

    <G. Aarts, F. Attanasio, B. Jäger, and D. Sexty, 2016>

    and more



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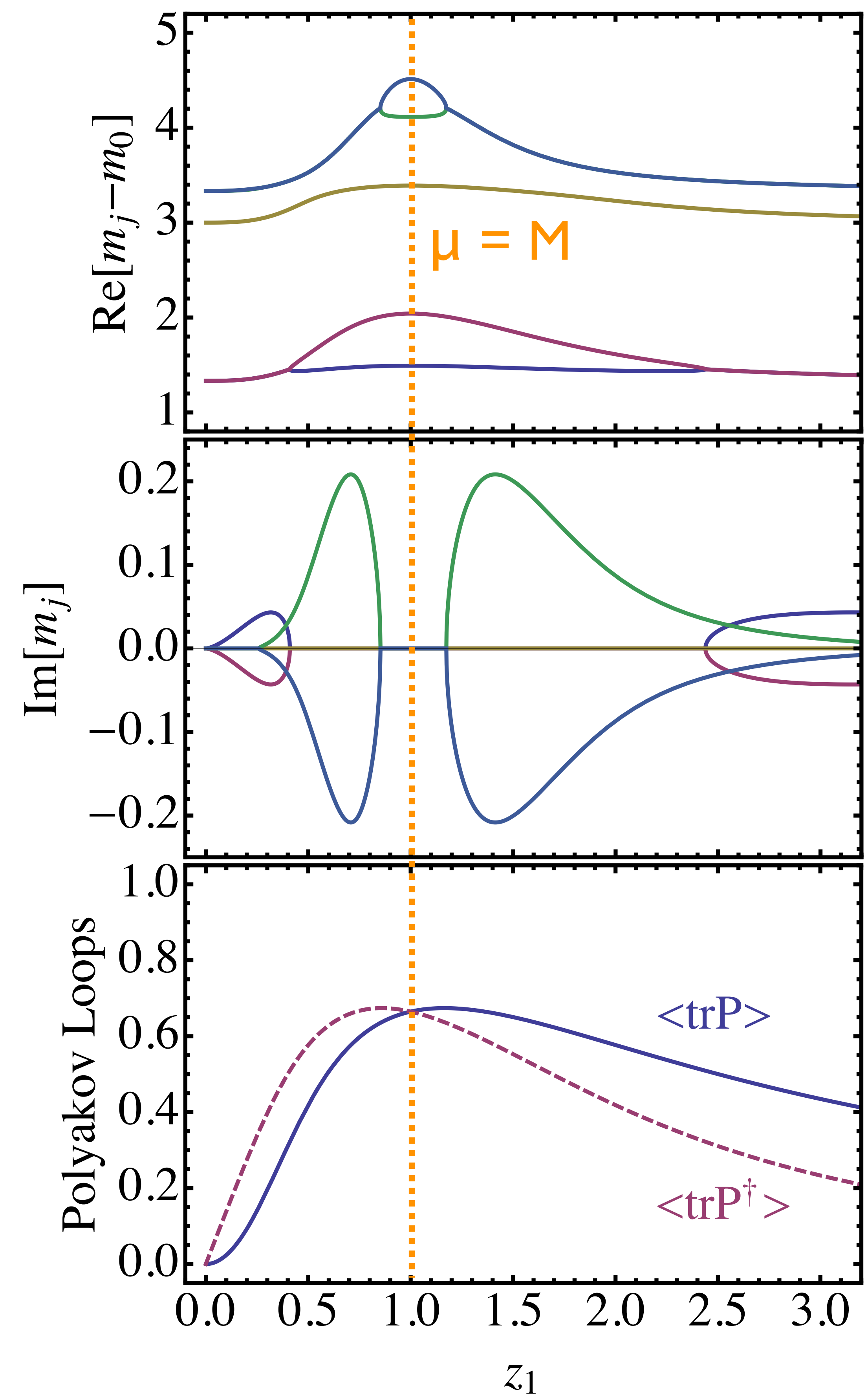

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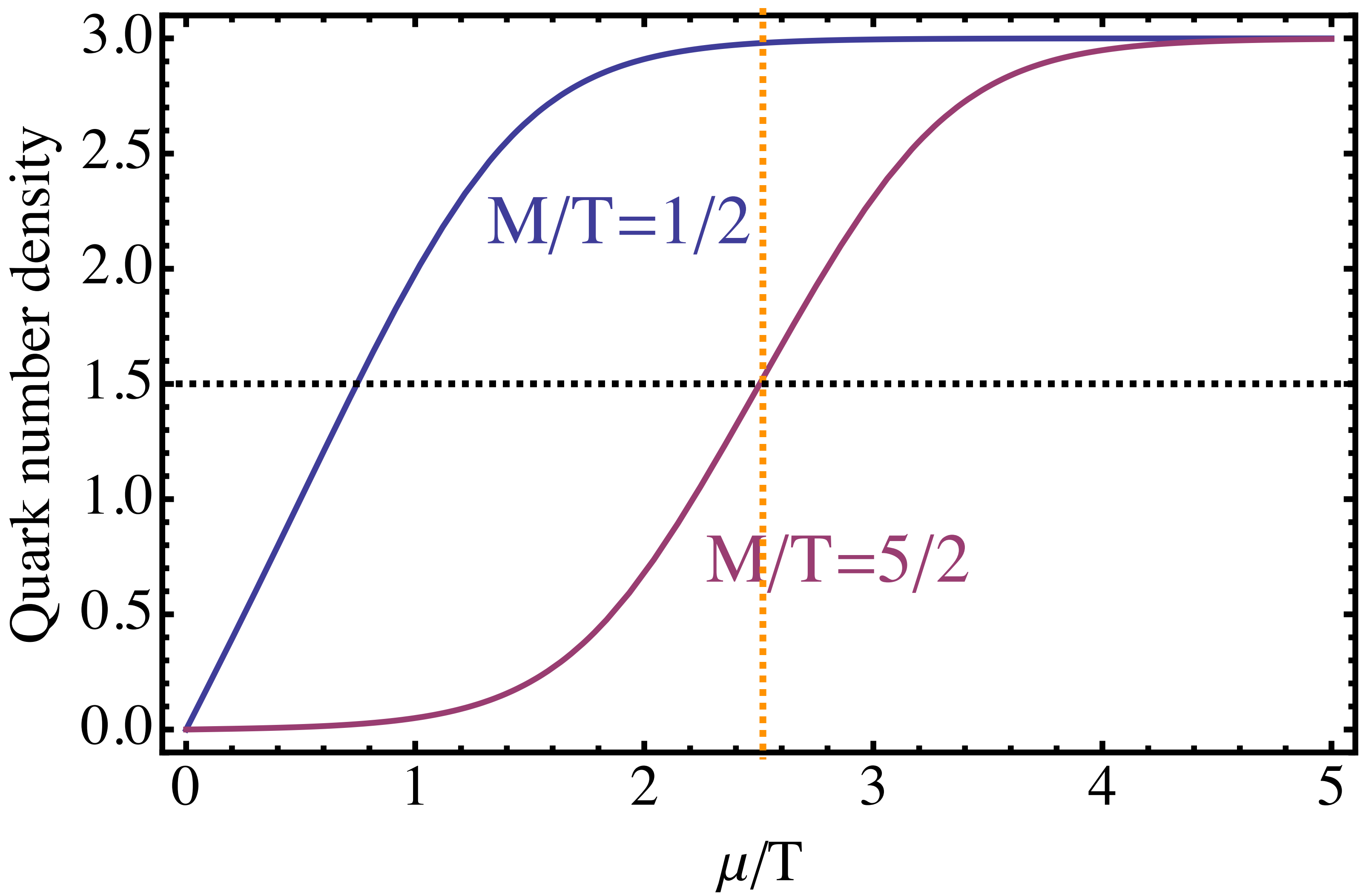
- The number density saturates ($n_q \to 3$) for $\mu >> M$

- Particle-Hole symmetry if $\mu/T = M/T >> 1$ → Half-filling

See also <T. Rindlisbacher and P. de Forcrand, 2015>



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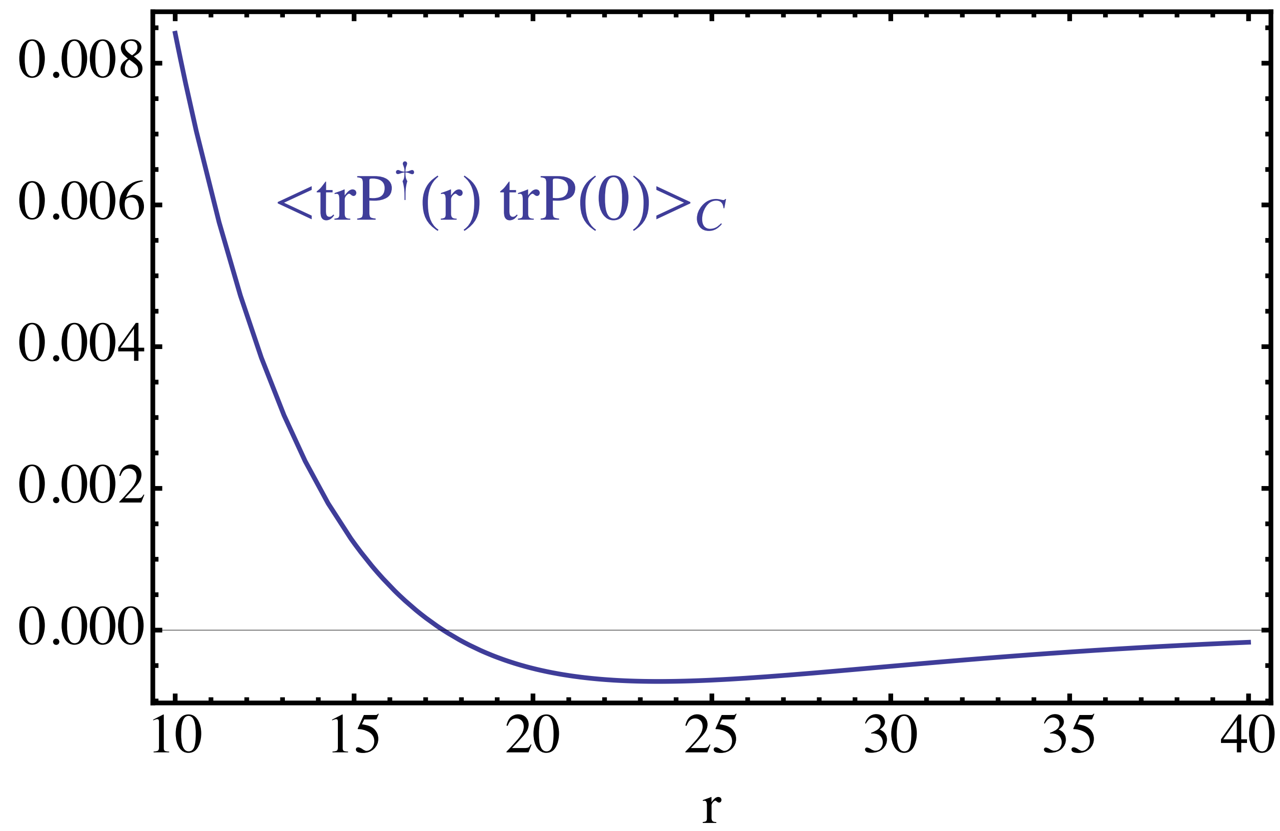
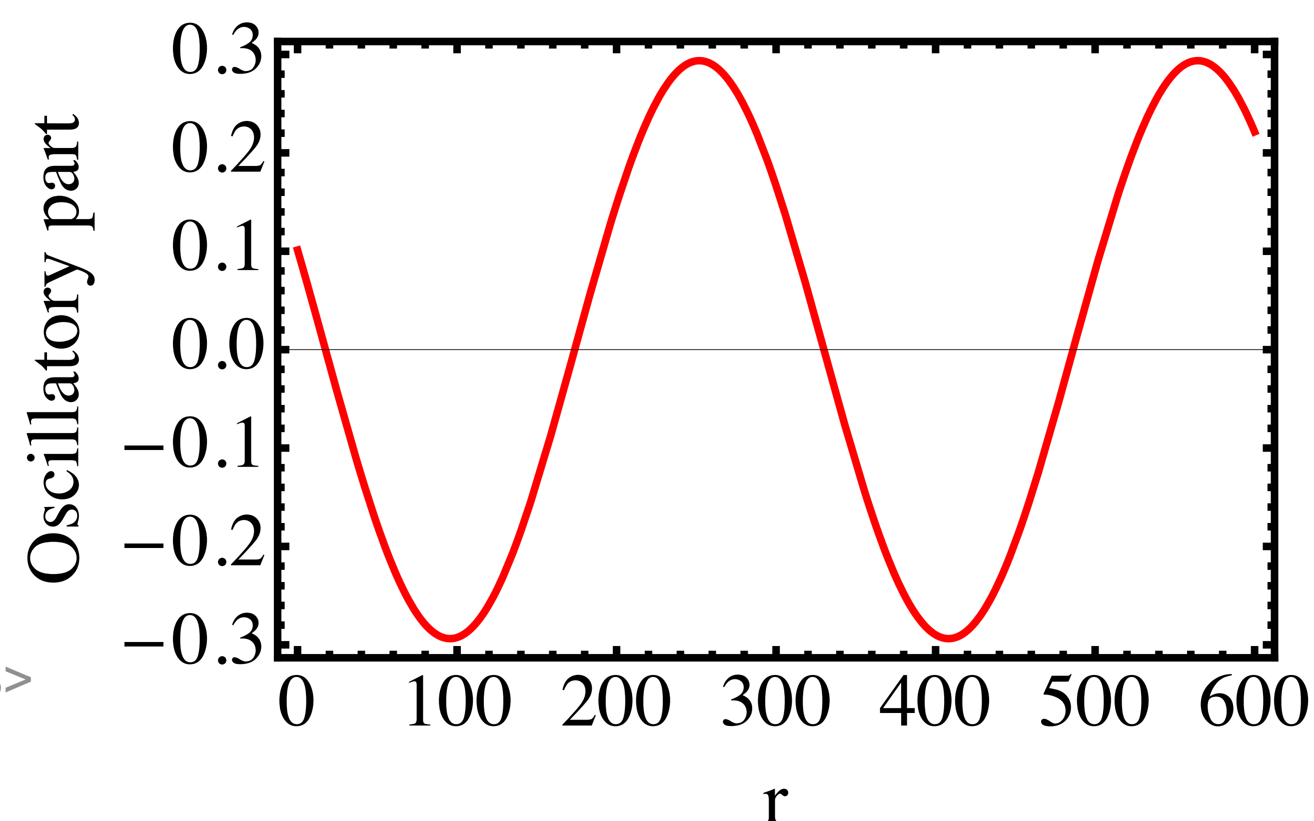


- Drops below zero.
- Sinusoidal modulation.
- Could check with other methods:

  Complex Langevin and Reweighting?



See also:
<P. Meisinger and M. Ogilvie, 2014>
<O. Akerlund, P. de Forcrand, and T. Rindlisbacher, 2016>

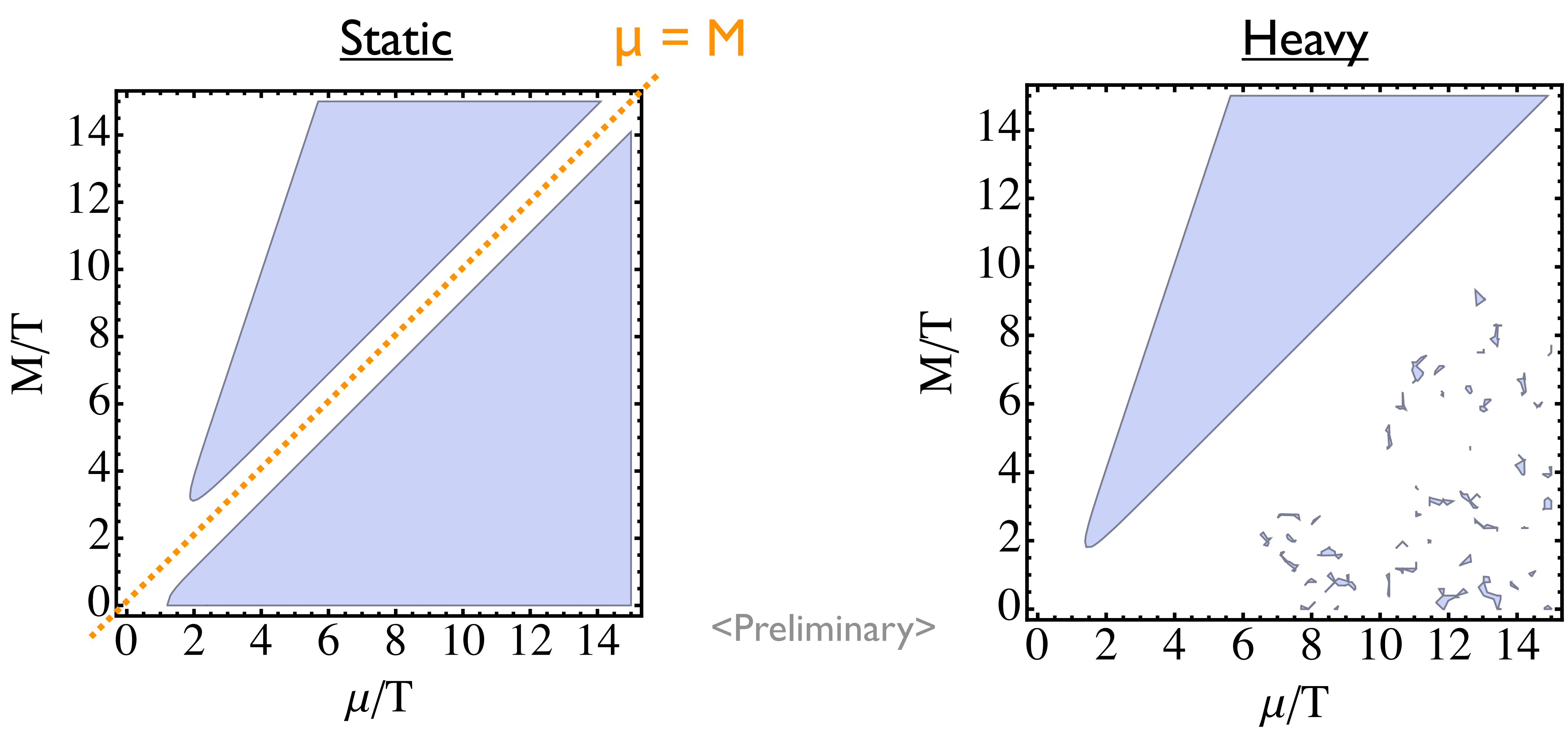
2. Heavy quark: $T = \langle r' | \, e^{-aH_0/2} \exp\left[z_1 \mathrm{tr}_F P\right] \exp\left[z_2 \mathrm{tr}_F P^\dagger\right] e^{-aH_0/2} \, |r\rangle$

- Spin model with "complex" magnetic field

<T. DeGrand and C. DeTar, 1983>

$$H \sim -J \sum_{\langle i,j \rangle} \mathrm{Re}\left[\mathrm{tr} P_i \mathrm{tr} P_j^\dagger\right] - \underbrace{h_R}_{z_1 + z_2} \sum_i \mathrm{Re}\left[\mathrm{tr} P_i\right] - \underbrace{i h_I}_{z_1 - z_2} \sum_i \mathrm{Im}\left[\mathrm{tr} P_i\right]$$

- Region plots where $m_1 = m_2^* \in \mathbb{C}$ for SU(3)



<Preliminary>

Conclusions

- We constructed the transfer matrix of strong-coupling lattice QCD with static quarks and heavy quarks.
- The transfer matrix is not Hermitian, but *CK*-symmetric.
- Mass spectrum of spin models for finite-density QCD becomes complex when chemical potential is non-zero.
- Oscillation of correlation functions of the Polyakov loops.