Light-cone distribution amplitudes of the rho meson

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with the help of
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intuitive picture of a hadron in the infinite momentum frame: superposition of Fock states with different numbers of quarks and gluons

(corresponding hadron wave functions at small transverse distances of the constituents (valence quarks in the simplest case) in the Fock state decomposition: (light cone) distribution amplitudes (DAs)

leading twist (twist 2): probability amplitude for finding valence quarks with fixed momentum fractions in the hadron

DAs complementary to parton distribution functions (PDFs)

PDFs: “directly” measurable

DAs: not directly measurable (appear always in convolutions)

single-particle probabilities (or densities)

amplitudes (or wave functions)

DAs: nonperturbative input for the theoretical description of hard exclusive processes
Distribution amplitudes of vector mesons in the continuum

Theoretical description: matrix elements of a nonlocal light-cone operator

For a positively charged vector meson, e.g., $\rho^+$ (similarly for $K^*$, ...)
at leading twist ($=2$)

\[ \langle 0 | \bar{d}(z_{2n}) \gamma \sigma(z_{2n}, z_{1n}) u(z_{1n}) | \rho^+(p, \lambda) \rangle = mf_L(e(\lambda) \cdot n) \int_0^1 dx e^{-i(z_1 x + z_2 (1-x)p \cdot n)} \phi_L(x, \mu^2) \]

\[ n^\nu \langle 0 | \bar{d}(z_{2n}) \sigma_{\mu\nu} [z_{2n}, z_{1n}] u(z_{1n}) | \rho^+(p, \lambda) \rangle = i f_T [e^\mu (p \cdot n) - p^\mu (e(\lambda) \cdot n)] \]

\[ \times \int_0^1 dx e^{-i(z_1 x + z_2 (1-x)p \cdot n)} \phi_T(x, \mu^2) \]

$\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ $n$: light-cone vector ($n^2 = 0$) $z_{1,2}$: real numbers $[z_{2n}, z_{1n}]$: Wilson line

$p_\mu$: momentum of the meson ($p^2 = m^2$) $e^{(\lambda)}$: polarisation vector of the meson

$\phi^L(x, \mu^2)$ $\phi^T(x, \mu^2)$ twist-2 DAs of the longitudinally transversely polarised vector meson at renormalisation scale $\mu$
decay constants defined by
\[ \langle 0 | \bar{d}(0) \gamma_{\mu} u(0) | \rho^+(p, \lambda) \rangle = f^L \epsilon_{\mu}^{(\lambda)} \]
\[ \langle 0 | \bar{d}(0) \sigma_{\mu\nu} u(0) | \rho^+(p, \lambda) \rangle = i f^T (\epsilon_{\mu}^{(\lambda)} p_\nu - \epsilon_{\nu}^{(\lambda)} p_\mu) \]

normalisation: \[ \int_0^1 dx \phi^Q(x, \mu^2) = 1 \] where Q is either L or T

\[ x \quad \text{fraction of the meson momentum carried by the } u\text{-quark} \]
\[ 1 - x \quad \text{fraction of the meson momentum carried by the } \bar{d}\text{-antiquark} \]
\[ \xi = x - (1 - x) = 2x - 1 \quad \text{difference of the momentum fractions} \]

expansion in terms of Gegenbauer polynomials
\[ \phi^Q(x, \mu^2) = 6x(1 - x) \left[ 1 + \sum_{n=1}^{\infty} a_n^Q(\mu^2) C_n^{3/2}(\xi) \right] \]

Gegenbauer moments \( a_n^Q \) parameterise the deviation of the DA from its asymptotic form related to moments of the DAs

\[ \langle \xi^n \rangle^Q(\mu^2) = \int_0^1 dx \ (2x - 1)^n \phi^Q(x, \mu^2) \]

\( e.g. \), \( a_2^Q = \frac{7}{12} (5 \langle \xi^2 \rangle^Q - 1) \)

odd moments vanish in the equal-mass case
moments related to matrix elements of the local renormalised operators (of twist 2)

\[
\mathcal{M}_{\mu_0 \ldots \mu_{k+l}}^{L(k,l)}(t) = \bar{d}(0) \gamma(\mu_0 D_{\mu_1} \ldots D_{\mu_k} D_{\mu_{k+1}} \ldots D_{\mu_{k+l}}) u(0)
\]

\[
\mathcal{M}_{\nu \mu_0 \ldots \mu_{k+l}}^{T(k,l)}(t) = \bar{d}(0) \sigma(\nu_0 D_{\mu_1} \ldots D_{\mu_k} D_{\mu_{k+1}} \ldots D_{\mu_{k+l}}) u(0)
\]

\(\ldots\): symmetrization of all enclosed Lorentz indices and subtraction of traces

\[
i^{k+l} \langle 0 | \mathcal{M}_{\mu_0 \ldots \mu_{k+l}}^{L(k,l)} | \rho^+(p, \lambda) \rangle = f T m e^{(\lambda)}_\mu p_\mu_1 \cdots p_\mu_{k+l} \langle (1 - x)^k x^l \rangle^L
\]

\[
i^{k+l} \langle 0 | \mathcal{M}_{\nu \mu_0 \ldots \mu_{k+l}}^{T(k,l)} | \rho^+(p, \lambda) \rangle = i f T \left( e^{(\lambda)}_\nu p_\nu - p_\nu e^{(\lambda)}_\nu \right) p_{\mu_1} \cdots p_{\mu_{k+l}} \langle (1 - x)^k x^l \rangle^T
\]

DA moments to be determined from ratios written schematically as

\[
\frac{\langle 0 | \mathcal{M}_{\mu_0 \ldots \mu_{k+l}}^{L(k,l)} | \rho^+(p, \lambda) \rangle}{\langle 0 | d \gamma_\mu u | \rho^+(p, \lambda) \rangle} \quad \text{or} \quad \frac{\langle 0 | \mathcal{M}_{\nu \mu_0 \ldots \mu_{k+l}}^{T(k,l)} | \rho^+(p, \lambda) \rangle}{\langle 0 | d \sigma_{\mu \nu} u | \rho^+(p, \lambda) \rangle}
\]

isospin symmetry \(\Rightarrow\) \(\langle (1 - x)^k x^l \rangle^Q = \langle (1 - x)^l x^k \rangle^Q\)

product rule for derivatives \(\partial_{\mu_{k+l+1}} \mathcal{M}_{\mu_0 \ldots \mu_{k+l+1}}^{Q(k,l)}(t) = \mathcal{M}_{\mu_0 \ldots \mu_{k+l+1}}^{Q(k+1,l)}(t) + \mathcal{M}_{\mu_0 \ldots \mu_{k+l+1}}^{Q(k,l+1)}(t)\)

\(\Rightarrow\) \(\langle (1 - x)^k x^l \rangle^Q = \langle (1 - x)^{k+1} x^l \rangle^Q + \langle (1 - x)^k x^{l+1} \rangle^Q\) (momentum-conservation constraint)
Operators in the Euclidean continuum

consider bare operators in Euclidean space (continuum)

\[ \mathcal{O}_{\mu \nu \rho}^{L\pm}(x) = \bar{d}(x)\gamma_{\mu} \left( \overleftarrow{D}_{\nu} \overleftarrow{D}_{\rho} + \overrightarrow{D}_{\nu} \overrightarrow{D}_{\rho} \pm 2\overleftarrow{D}_{\nu} \overrightarrow{D}_{\rho} \right) u(x) \]

\[ \mathcal{O}_{\mu \nu \rho \sigma}^{T\pm}(x) = \bar{d}(x)\sigma_{\mu \nu} \left( \overleftarrow{D}_{\rho} \overleftarrow{D}_{\sigma} + \overrightarrow{D}_{\rho} \overrightarrow{D}_{\sigma} \pm 2\overleftarrow{D}_{\rho} \overrightarrow{D}_{\sigma} \right) u(x) \]

matrix elements \[ \langle 0 | \mathcal{O}^{Q-} | \rho^+(p, \lambda) \rangle \]
proportional to the bare value of \[ \langle (1 - x)^2 \rangle^Q + \langle x^2 \rangle^Q - 2\langle x(1 - x) \rangle^Q = \langle \xi^2 \rangle^Q \]

matrix elements \[ \langle 0 | \mathcal{O}^{Q+} | \rho^+(p, \lambda) \rangle \]
proportional to the bare value of \[ \langle (1 - x)^2 \rangle^Q + \langle x^2 \rangle^Q + 2\langle x(1 - x) \rangle^Q = \langle (x + 1 - x)^2 \rangle^Q = \langle 1^2 \rangle^Q \]

note:

\[ \mathcal{O}_{(\mu \nu \rho)}^{L-}(x) = \bar{d}(x)\gamma_{(\mu} \overleftarrow{D}_{\nu} \overleftarrow{D}_{\rho)} u(x) , \quad \mathcal{O}_{\mu(\nu \rho \sigma)}^{T-}(x) = \bar{d}(x)\sigma_{\mu(\nu} \overleftarrow{D}_{\rho} \overleftarrow{D}_{\sigma)} u(x) \]

\[ \mathcal{O}_{(\mu \nu \rho)}^{L+}(x) = \partial_{(\mu} \partial_{\nu} \bar{d}(x)\gamma_{\rho)} u(x) \]

\[ \mathcal{O}_{\mu(\nu \rho \sigma)}^{T+}(x) = \bar{d}(x)\sigma_{\mu(\nu} \overleftarrow{D}_{\rho} \overleftarrow{D}_{\sigma)} \]

with \( \overleftarrow{D}_{\mu} = \overrightarrow{D}_{\mu} - \overleftarrow{D}_{\mu} \)

total derivatives

similarly for \( \mathcal{O}_{\mu(\nu \rho \sigma)}^{T+}(x) \)
Lattice formulation

bare operators on the lattice: discretize the covariant derivative in

\[
O_{\mu\nu\rho}(x) = \bar{d}(x)\gamma_{\mu} \left( \overleftarrow{D_\nu D_\rho} + \overrightarrow{D_\nu D_\rho} \pm \overleftarrow{2D_\nu D_\rho} \right) u(x)
\]

\[
O_{\mu\nu\rho\sigma}(x) = \bar{d}(x)\sigma_{\mu\nu} \left( \overleftarrow{D_\rho D_\sigma} + \overrightarrow{D_\rho D_\sigma} \pm \overleftarrow{2D_\rho D_\sigma} \right) u(x)
\]

\[O_{(\mu\nu\rho)}^{-}\] and \[O_{(\mu\nu\rho)}^{+}\]

\[O_{\mu(\nu\rho\sigma)}^{-}\] and \[O_{\mu(\nu\rho\sigma)}^{+}\]

mix under renormalisation (even in the continuum)

additional mixing (with operators of lower dimension) avoided by a suitable choice of operators

transforming irreducibly under H(4), the symmetry group of the hypercubic lattice

### Operator Multiplets:

<table>
<thead>
<tr>
<th>Longitudinal</th>
<th>Transverse</th>
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<tbody>
<tr>
<td>(O_{L\pm}^{12(24)})</td>
<td>(O_{T\pm}^{12(24)})</td>
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<td>(O_{T\pm}^{12(23)})</td>
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<tr>
<td>(O_{L\pm}^{12(123)})</td>
<td>(O_{T\pm}^{12(123)})</td>
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<td>(O_{L\pm}^{412})</td>
<td>(O_{T\pm}^{412})</td>
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<tr>
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<td>(O_{T\pm}^{21(13)})</td>
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<td>(O_{T\pm}^{(32)})</td>
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<tr>
<td>(O_{T\pm}^{34(42)})</td>
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</table>

etc.

in addition:

\[V_\mu(x) = \bar{d}(x)\gamma_\mu u(x),\]

\[T_{\mu\nu}(x) = \bar{d}(x)\sigma_{\mu\nu} u(x)\] for determining \(f_L, f_T\)
bare matrix elements extracted from two-point correlation functions of the operators \( \mathcal{O}^{Q\pm}, V_\mu, T_{\mu\nu} \) with suitable interpolating fields \( J \) for the mesons \((V^{\text{smeared}}_\mu, T^{\text{smeared}}_{\mu\nu})\)

for moments of DAs consider ratios at large \( t \)

\[
\frac{\sum_x e^{-ip\cdot x} \langle \mathcal{O}^{L\pm}(x, t) J(0) \rangle}{\sum_x e^{-ip\cdot x} \langle V(x, t) J(0) \rangle} = F^{L\pm}(p) R^{L\pm}, \quad \frac{\sum_x e^{-ip\cdot x} \langle \mathcal{O}^{T\pm}(x, t) J(0) \rangle}{\sum_x e^{-ip\cdot x} \langle T(x, t) J(0) \rangle} = F^{T\pm}(p) R^{T\pm}
\]

with kinematical factors \( F^{Q\pm} \) such that

\[
\langle \xi^2 \rangle^Q_{\text{bare}} = R^Q, \quad \left( a^Q_2 \right)_{\text{bare}} = \frac{7}{12} \left( 5R^Q - R^Q_+ \right)
\]

\[
\text{note: } R^{Q_+} \neq 1 \Rightarrow \left( a^Q_2 \right)_{\text{bare}} \neq \frac{7}{12} \left( 5\langle \xi^2 \rangle^Q_{\text{bare}} - 1 \right)
\]

renormalised operators given by

\[
\mathcal{O}_{\text{MS}}^{Q_-} = Z_{11} \mathcal{O}^{Q_-} + Z_{12} \mathcal{O}^{Q_+} \\
\mathcal{O}_{\text{MS}}^{Q_+} = Z_{22} \mathcal{O}^{Q_+}
\]

renormalised moments in the \( \overline{\text{MS}} \) scheme:

\[
\langle \xi^2 \rangle_{\text{MS}}^Q = \zeta_{11} R^Q - \zeta_{12} R^Q_+ \\
\left( a^Q_2 \right)_{\text{MS}} = \frac{7}{12} \left[ 5\zeta_{11} R^Q - (5\zeta_{12} - \zeta_{22}) R^Q_+ \right]
\]

with \( \zeta_{11} = \frac{Z_{11}}{Z_V}, \zeta_{12} = \frac{Z_{12}}{Z_V}, \zeta_{22} = \frac{Z_{22}}{Z_V} \) for \( Q = L \) (for \( Q = T \): \( Z_V \rightarrow Z_T \))
in particular:

**continuum limit,** \( t \) large

\[
Z_{22} \sum_x e^{-ip \cdot x} \langle O^L(x, t) J(0) \rangle = Z_V F^{L+}(p) \sum_x e^{-ip \cdot x} \langle V(x, t) J(0) \rangle
\]

**remember:** \( O^L_{(\mu\nu\rho)}(x) = \partial_{(\mu} \partial_{\nu} \bar{d}(x) \gamma_{\rho}) u(x) \)

hence for large \( t \)

\[
\langle 1^2 \rangle_{\text{MS}}^{L} = \frac{Z_{22} \sum_x e^{-ip \cdot x} \langle O^L(x, t) J(0) \rangle}{Z_V F^{L+}(p) \sum_x e^{-ip \cdot x} \langle V(x, t) J(0) \rangle} = \zeta_{22} R^{L+}
\]

should approach unity in the continuum limit

similarly in the transverse case

then

\[
\left( a_2^Q \right)_{\text{MS}} = \frac{7}{12} \left( 5 \langle \xi^2 \rangle_{\text{MS}}^Q - \langle 1^2 \rangle_{\text{MS}}^Q \right) \xrightarrow{a \to 0} \frac{7}{12} \left( 5 \langle \xi^2 \rangle_{\text{MS}}^Q - 1 \right)
\]
Renormalisation

renormalised operators:

\[ \mathcal{O}^{Q-}_{\text{MS}} = Z_{11} \mathcal{O}^{Q-} + Z_{12} \mathcal{O}^{Q+} \]
\[ \mathcal{O}^{Q+}_{\text{MS}} = Z_{22} \mathcal{O}^{Q+} \]

\[ \mathcal{O}^{L-} \] : lattice version of \( \bar{d}(x)\gamma(\mu)\overleftrightarrow{D}_\nu\overleftrightarrow{D}_\rho u(x) \)
\[ \mathcal{O}^{L+} \] : lattice version of \( \partial(\mu)\partial(\nu)\bar{d}(x)\gamma(\rho)u(x) \)

similarly for \( \mathcal{O}^{T\pm} \)

forward matrix elements of \( \mathcal{O}^{Q+} \) vanish

\( \rightarrow \) use RI'–SMOM variant of the Rome-Southampton method instead of RI'–MOM
to compute the mixing coefficient \( Z_{12} \) nonperturbatively

follow the same strategy as in the case of the pion DA:

for the conversion to the \( \overline{\text{MS}} \) scheme:
renormalised vertex functions in the \( \overline{\text{MS}} \) scheme at two loops for SMOM kinematics

taken from the work of John Gracey:
\( \mathcal{O}^{L\pm} \): Phys. Rev. D84 (2011) 016002 (arXiv:1105.2138)
\( \mathcal{O}^{T\pm} \): private communication (unpublished)
Bare data

two degenerate flavours of clover fermions

<table>
<thead>
<tr>
<th>β</th>
<th>κ</th>
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<th>$m_\pi$/MeV</th>
<th>$m_\rho$/MeV</th>
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<td>48$^3$ x 64</td>
<td>260</td>
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2-point correlation functions with open spinor indices already used for the pion DA

scale setting: Sommer parameter $r_0 = 0.5$ fm

$m_\rho$ determined “naively” from smeared-smeared 2-point functions
although on most of the ensembles, $\rho$ could “in principle” decay . . .
but dispersion relation (for one “unit” of momentum) well satisfied

a: generated on QPACE
b: generated within the QCDSF collaboration
red points (right): energies calculated for one “unit” of momentum ($2\pi/L$)

blue points (left): energies calculated from mass and momentum

<table>
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<tr>
<th>size</th>
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<th>$m_\rho$</th>
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<tr>
<td>9</td>
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<td>795</td>
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Results

renormalisation scale $\mu^2 = 4 \text{GeV}^2$

$$\langle 1^2 \rangle_{\text{MS}}^L$$

$$\langle 1^2 \rangle_{\text{MS}}^T$$

difficult to disentangle the possible sources for $\langle 1^2 \rangle_{\text{MS}}^T \neq 1$:

lattice artefacts, finite size effects, …
linear fits of the mass dependence
data point at $m_\pi = 150\,\text{MeV}$ included
bands indicate the one- and two-sigma statistical error

$$(a_2^L)_{\overline{\text{MS}}} > (a_2^T)_{\overline{\text{MS}}}$$  at the physical point?
linear fits of the mass dependence
data point at $m_\pi = 150 \text{ MeV}$ excluded
bands indicate the one- and two-sigma statistical error

$\left(a_2^L\right)_{\overline{\text{MS}}} > \left(a_2^T\right)_{\overline{\text{MS}}}$ at the physical point??
pion mass dependence of $f^L$, $f^T$, $f^T/f^L$ from chiral perturbation theory:

\[
f_{\text{MS}}^L = f_{\text{L}}^{(0)} \left( 1 - \frac{m_\pi^2}{16\pi^2 F_\pi^2} \ln \left( \frac{m_\pi^2}{M^2} \right) \right) + f_{\text{L}}^{(2)} m_\pi^2 + f_{\text{L}}^{(3)} m_\pi^3 + O(m_\pi^4)
\]

\[
f_{\text{MS}}^T = f_{\text{T}}^{(0)} \left( 1 - \frac{m_\pi^2}{32\pi^2 F_\pi^2} \ln \left( \frac{m_\pi^2}{M^2} \right) \right) + f_{\text{T}}^{(2)} m_\pi^2 + f_{\text{T}}^{(3)} m_\pi^3 + O(m_\pi^4)
\]

\[
\frac{f_{\text{MS}}^T}{f_{\text{MS}}^L} = \left( \frac{f_{\text{T}}}{f_{\text{L}}} \right)^{(0)} \left( 1 + \frac{m_\pi^2}{32\pi^2 F_\pi^2} \ln \left( \frac{m_\pi^2}{M^2} \right) \right) + f_{\text{R}}^{(2)} m_\pi^2 + f_{\text{R}}^{(3)} m_\pi^3 + O(m_\pi^4)
\]

chiral renormalisation scale $M = 775$ MeV, $F_\pi = 92.4$ MeV fixed
Comparison with previous results

all results (at the physical pion mass) renormalised in the $\overline{\text{MS}}$ scheme

<table>
<thead>
<tr>
<th></th>
<th>$\mu$ [GeV]</th>
<th>$f^L$ [MeV]</th>
<th>$f^T$ [MeV]</th>
<th>$f^T/f^L$</th>
<th>$a^L_2$</th>
<th>$a^T_2$</th>
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<tbody>
<tr>
<td>this work (preliminary) (all data points included)</td>
<td>2</td>
<td>196(4)</td>
<td>121(3)</td>
<td>0.625(7)</td>
<td>0.132(13)</td>
<td>0.101(18)</td>
</tr>
<tr>
<td>QCD sum rules$^{(1)}$</td>
<td>1</td>
<td>206(7)</td>
<td>165(9)</td>
<td>0.79(5)</td>
<td>0.14(6)</td>
<td>0.15(7)</td>
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<tr>
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<td>155(8)</td>
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<td>0.11(5)</td>
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<td>experiment$^{(1)}$</td>
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<td>209(2)</td>
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</table>


$^{(2)}$ obtained by LO evolution with $N_f = 2$

some previous lattice results:

$N_f = 2 + 1$ (domain wall), $a = 0.1141(18)$ fm, $\mu = 2$ GeV  \( \langle \xi^2 \rangle^L = 0.27(1)(2) \rightarrow a^L_2 = 0.20(6) \)

$N_f = 0$ (chirally improved fermions), $a = 0.10$ fm, $\mu = 2$ GeV  \( f^T/f^L = 0.742(14) \)

our lattice spacings in this work: $a = 0.081$ fm, 0.071 fm, 0.060 fm
Summary and outlook

• 2-point correlation functions with open spinor indices
  on a set of ensembles with two dynamical flavours of clover fermions


• $\rho$ DA (with caveats): in progress, preliminary results shown
  instability effects in the matrix elements?

• $\omega$: isosinglet analogue of $\rho$, much smaller width!
  additional disconnected contributions for the DA to be calculated

• CLS ensembles with 2+1 flavours
  – also K and K* can be studied
  – reliable continuum limit?