Light-cone distribution amplitudes of the rho meson

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with the help of

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Introduction

intuitive picture of a hadron in the infinite momentum frame:
superposition of Fock states with different numbers of quarks and gluons

corresponding hadron wave functions at small transverse distances of the constituents (valence quarks in the simplest case) in the Fock state decomposition:

(light cone) distribution amplitudes (DAs)

leading twist (twist 2):
probability amplitude for finding valence quarks with fixed momentum fractions in the hadron

DAs complementary to parton distribution functions (PDFs)

PDFs:	"directly" measurable	DAs:	not directly measurable (appear always in convolutions)
	single-particle probabilities (or densities)		amplitudes (or wave functions)

DAs: nonperturbative input for the theoretical description of hard exclusive processes

Distribution amplitudes of vector mesons in the continuum

theoretical description: matrix elements of a nonlocal light-cone operator

for a positively charged vector meson, e.g., ρ^+ (similarly for $K^*, \ldots$)
at leading twist (=2)

$$\langle 0|\bar{d}(z_2 n)\; \not{n}\; [z_2 n, z_1 n]\, u(z_1 n)|\rho^+(p,\lambda)\rangle = m f^{\mathrm{L}}(e^{(\lambda)}\cdot n)\int_0^1 \mathrm{d}x\, \mathrm{e}^{-\mathrm{i}(z_1 x + z_2(1-x))p\cdot n}\phi^{\mathrm{L}}(x,\mu^2)$$

$$n^\nu\langle 0|\bar{d}(z_2 n)\,\sigma_{\mu\nu}\,[z_2 n, z_1 n]\, u(z_1 n)|\rho^+(p,\lambda)\rangle = \mathrm{i} f^{\mathrm{T}}[e_\mu^{(\lambda)}(p\cdot n) - p_\mu(e^{(\lambda)}\cdot n)] \times \int_0^1 \mathrm{d}x\, \mathrm{e}^{-\mathrm{i}(z_1 x + z_2(1-x))p\cdot n}\phi^{\mathrm{T}}(x,\mu^2)$$

$\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma_\mu, \gamma_\nu]$ n: light-cone vector ($n^2 = 0$) $z_{1,2}$: real numbers $[z_2 n, z_1 n]$: Wilson line

p_μ: momentum of the meson ($p^2 = m^2$) $e_\mu^{(\lambda)}$: polarisation vector of the meson

$\phi^{\mathrm{L}}(x,\mu^2)$ / $\phi^{\mathrm{T}}(x,\mu^2)$ twist-2 DAs of the longitudinally / transversely polarised vector meson at renormalisation scale μ

decay constants defined by

$$\langle 0|\bar{d}(0)\gamma_\mu u(0)|\rho^+(p,\lambda)\rangle = f^{\mathrm{L}} m e_\mu^{(\lambda)}$$

$$\langle 0|\bar{d}(0)\sigma_{\mu\nu} u(0)|\rho^+(p,\lambda)\rangle = \mathrm{i} f^{\mathrm{T}} (e_\mu^{(\lambda)} p_\nu - e_\nu^{(\lambda)} p_\mu)$$

normalisation: $\int_0^1 \mathrm{d}x\, \phi^{\mathrm{Q}}(x,\mu^2) = 1$ where Q is either L or T

x	fraction of the meson momentum carried by the u-quark
$1-x$	fraction of the meson momentum carried by the $\bar{d}$-antiquark
$\xi = x-(1-x) = 2x-1$	difference of the momentum fractions

expansion in terms of Gegenbauer polynomials

$$\phi^{\mathrm{Q}}(x,\mu^2) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n^{\mathrm{Q}}(\mu^2) C_n^{3/2}(\xi)\right]$$

Gegenbauer moments a_n^{Q} parameterise the deviation of the DA from its asymptotic form

related to moments of the DAs

$$\langle \xi^n \rangle^{\mathrm{Q}}(\mu^2) = \int_0^1 \mathrm{d}x\,(2x-1)^n \phi^{\mathrm{Q}}(x,\mu^2)$$

e.g., $a_2^{\mathrm{Q}} = \frac{7}{12}\left(5\langle\xi^2\rangle^{\mathrm{Q}} - 1\right)$

odd moments vanish in the equal-mass case moments related to matrix elements of the local renormalised operators (of twist 2)

$$\mathcal{M}^{\mathrm{L}(k,l)}_{\mu_0\cdots\mu_{k+l}} = \bar{d}(0)\gamma_{(\mu_0}\overleftarrow{D}_{\mu_1}\ldots\overleftarrow{D}_{\mu_k}\overrightarrow{D}_{\mu_{k+1}}\ldots\overrightarrow{D}_{\mu_{k+l})}u(0)$$
$$\mathcal{M}^{\mathrm{T}(k,l)}_{\nu\mu_0\cdots\mu_{k+l}} = \bar{d}(0)\sigma_{\nu(\mu_0}\overleftarrow{D}_{\mu_1}\ldots\overleftarrow{D}_{\mu_k}\overrightarrow{D}_{\mu_{k+1}}\ldots\overrightarrow{D}_{\mu_{k+l})}u(0)$$

D_μ: covariant derivative

$(\cdots)$: symmetrization of all enclosed Lorentz indices and subtraction of traces

$$\mathrm{i}^{k+l}\langle 0|\mathcal{M}^{\mathrm{L}(k,l)}_{\mu_0\cdots\mu_{k+l}}|\rho^+(p,\lambda)\rangle = f^{\mathrm{L}} m e^{(\lambda)}_{(\mu_0} p_{\mu_1}\cdots p_{\mu_{k+l})}\langle(1-x)^k x^l\rangle^{\mathrm{L}}$$
$$\mathrm{i}^{k+l}\langle 0|\mathcal{M}^{\mathrm{T}(k,l)}_{\nu\mu_0\cdots\mu_{k+l}}|\rho^+(p,\lambda)\rangle = \mathrm{i} f^{\mathrm{T}}\left(e^{(\lambda)}_{\nu} p_{(\mu_0} - p_\nu e^{(\lambda)}_{(\mu_0}\right) p_{\mu_1}\cdots p_{\mu_{k+l})}\langle(1-x)^k x^l\rangle^{\mathrm{T}}$$

DA moments to be determined from ratios written schematically as

$$\frac{\langle 0|\mathcal{M}^{\mathrm{L}(k,l)}_{\mu_0\cdots\mu_{k+l}}|\rho^+(p,\lambda)\rangle}{\langle 0|\bar{d}\gamma_\mu u|\rho^+(p,\lambda)\rangle} \quad \text{or} \quad \frac{\langle 0|\mathcal{M}^{\mathrm{T}(k,l)}_{\mu_0\cdots\mu_{k+l}}|\rho^+(p,\lambda)\rangle}{\langle 0|\bar{d}\sigma_{\mu\nu} u|\rho^+(p,\lambda)\rangle}$$

isospin symmetry $\Rightarrow \langle(1-x)^k x^l\rangle^{\mathrm{Q}} = \langle(1-x)^l x^k\rangle^{\mathrm{Q}}$

product rule for derivatives $\partial_{(\mu_{k+l+1}}\mathcal{M}^{\mathrm{Q}(k,l)}_{\mu_0\cdots\mu_{k+l})} = \mathcal{M}^{\mathrm{Q}(k+1,l)}_{\mu_0\cdots\mu_{k+l+1}} + \mathcal{M}^{\mathrm{Q}(k,l+1)}_{\mu_0\cdots\mu_{k+l+1}}$

$\Rightarrow \langle(1-x)^k x^l\rangle^{\mathrm{Q}} = \langle(1-x)^{k+1} x^l\rangle^{\mathrm{Q}} + \langle(1-x)^k x^{l+1}\rangle^{\mathrm{Q}}$ (momentum-conservation constraint)

Operators in the Euclidean continuum

consider bare operators in Euclidean space (continuum)

$$\mathcal{O}^{\mathrm{L}\pm}_{\mu\nu\rho}(x) = \bar{d}(x)\gamma_\mu \left(\overleftarrow{D}_\nu \overleftarrow{D}_\rho + \overrightarrow{D}_\nu \overrightarrow{D}_\rho \pm 2\overleftarrow{D}_\nu \overrightarrow{D}_\rho\right) u(x)$$

$\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$

$$\mathcal{O}^{\mathrm{T}\pm}_{\mu\nu\rho\sigma}(x) = \bar{d}(x)\sigma_{\mu\nu} \left(\overleftarrow{D}_\rho \overleftarrow{D}_\sigma + \overrightarrow{D}_\rho \overrightarrow{D}_\sigma \pm 2\overleftarrow{D}_\rho \overrightarrow{D}_\sigma\right) u(x)$$

matrix elements $\langle 0|\mathcal{O}^{\mathrm{Q}-}|\rho^+(p,\lambda)\rangle$
proportional to the bare value of $\langle (1-x)^2\rangle^{\mathrm{Q}} + \langle x^2\rangle^{\mathrm{Q}} - 2\langle x(1-x)\rangle^{\mathrm{Q}} = \langle \xi^2\rangle^{\mathrm{Q}}$

matrix elements $\langle 0|\mathcal{O}^{\mathrm{Q}+}|\rho^+(p,\lambda)\rangle$
proportional to the bare value of $\langle (1-x)^2\rangle^{\mathrm{Q}} + \langle x^2\rangle^{\mathrm{Q}} + 2\langle x(1-x)\rangle^{\mathrm{Q}} = \langle (x+1-x)^2\rangle^{\mathrm{Q}} = \langle 1^2\rangle^{\mathrm{Q}}$

note:

$$\mathcal{O}^{\mathrm{L}-}_{(\mu\nu\rho)}(x) = \bar{d}(x)\gamma_{(\mu}\overleftrightarrow{D}_\nu \overleftrightarrow{D}_{\rho)} u(x)\ ,\ \mathcal{O}^{\mathrm{T}-}_{\mu(\nu\rho\sigma)}(x) = \bar{d}(x)\sigma_{\mu(\nu}\overleftrightarrow{D}_\rho \overleftrightarrow{D}_{\sigma)} u(x)$$

with $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$

$$\mathcal{O}^{\mathrm{L}+}_{(\mu\nu\rho)}(x) = \partial_{(\mu}\partial_\nu \bar{d}(x)\gamma_{\rho)} u(x)$$

total derivatives

similarly for $\mathcal{O}^{\mathrm{T}+}_{\mu(\nu\rho\sigma)}(x)$

Lattice formulation

bare operators on the lattice: discretize the covariant derivative in

$$\mathcal{O}^{\mathrm{L}\pm}_{\mu\nu\rho}(x) = \bar{d}(x)\gamma_\mu \left(\overleftarrow{D}_\nu\overleftarrow{D}_\rho + \overrightarrow{D}_\nu\overrightarrow{D}_\rho \pm 2\overleftarrow{D}_\nu\overrightarrow{D}_\rho\right) u(x)$$

$$\mathcal{O}^{\mathrm{T}\pm}_{\mu\nu\rho\sigma}(x) = \bar{d}(x)\sigma_{\mu\nu} \left(\overleftarrow{D}_\rho\overleftarrow{D}_\sigma + \overrightarrow{D}_\rho\overrightarrow{D}_\sigma \pm 2\overleftarrow{D}_\rho\overrightarrow{D}_\sigma\right) u(x)$$

$\mathcal{O}^{\mathrm{L}-}_{(\mu\nu\rho)}$ and $\mathcal{O}^{\mathrm{L}+}_{(\mu\nu\rho)}$
$\mathcal{O}^{\mathrm{T}-}_{\mu(\nu\rho\sigma)}$ and $\mathcal{O}^{\mathrm{T}+}_{\mu(\nu\rho\sigma)}$ mix under renormalisation (even in the continuum)

additional mixing (with operators of lower dimension) avoided by a suitable choice of operators transforming irreducibly under H(4), the symmetry group of the hypercubic lattice

operator multiplets:

longitudinal	transverse
$\mathcal{O}^{\mathrm{L}\pm}_{(423)}$	$\mathcal{O}^{\mathrm{T}\pm}_{13(32)} + \mathcal{O}^{\mathrm{T}\pm}_{23(31)} - \mathcal{O}^{\mathrm{T}\pm}_{14(42)} - \mathcal{O}^{\mathrm{T}\pm}_{24(41)}$
$\mathcal{O}^{\mathrm{L}\pm}_{(413)}$	$\mathcal{O}^{\mathrm{T}\pm}_{12(23)} + \mathcal{O}^{\mathrm{T}\pm}_{32(21)} - \mathcal{O}^{\mathrm{T}\pm}_{14(43)} - \mathcal{O}^{\mathrm{T}\pm}_{34(41)}$
$\mathcal{O}^{\mathrm{L}\pm}_{(412)}$	$\mathcal{O}^{\mathrm{T}\pm}_{12(24)} + \mathcal{O}^{\mathrm{T}\pm}_{42(21)} - \mathcal{O}^{\mathrm{T}\pm}_{13(34)} - \mathcal{O}^{\mathrm{T}\pm}_{43(31)}$
$\mathcal{O}^{\mathrm{L}\pm}_{(123)}$	$\mathcal{O}^{\mathrm{T}\pm}_{21(13)} + \mathcal{O}^{\mathrm{T}\pm}_{31(12)} - \mathcal{O}^{\mathrm{T}\pm}_{24(43)} - \mathcal{O}^{\mathrm{T}\pm}_{34(42)}$

etc.

in addition: $V_\mu(x) = \bar{d}(x)\gamma_\mu u(x)$, $T_{\mu\nu}(x) = \bar{d}(x)\sigma_{\mu\nu}u(x)$ for determining f^{L}, f^{T}

bare matrix elements extracted from two-point correlation functions of the operators $\mathcal{O}^{\mathrm{Q}\pm}$, V_μ, $T_{\mu\nu}$ with suitable interpolating fields J for the mesons (V_μ^{smeared}, $T_{\mu\nu}^{\mathrm{smeared}}$)

for moments of DAs consider ratios at large t

$$\frac{\sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle \mathcal{O}^{\mathrm{L}\pm}(\mathbf{x},t) J(0)\rangle}{\sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle V(\mathbf{x},t) J(0)\rangle} = F^{\mathrm{L}\pm}(\mathbf{p}) R^{\mathrm{L}\pm} \,, \quad \frac{\sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle \mathcal{O}^{\mathrm{T}\pm}(\mathbf{x},t) J(0)\rangle}{\sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle T(\mathbf{x},t) J(0)\rangle} = F^{\mathrm{T}\pm}(\mathbf{p}) R^{\mathrm{T}\pm}$$

with kinematical factors $F^{\mathrm{Q}\pm}$ such that $$\langle \xi^2 \rangle^{\mathrm{Q}}_{\mathrm{bare}} = R^{\mathrm{Q}-} \,, \quad \left(a_2^{\mathrm{Q}}\right)_{\mathrm{bare}} = \frac{7}{12}\left(5R^{\mathrm{Q}-} - R^{\mathrm{Q}+}\right)$$

note: $$R^{\mathrm{Q}+} \neq 1 \quad \Rightarrow \quad \left(a_2^{\mathrm{Q}}\right)_{\mathrm{bare}} \neq \frac{7}{12}\left(5\langle \xi^2 \rangle^{\mathrm{Q}}_{\mathrm{bare}} - 1\right)$$

renormalised operators given by
$$\mathcal{O}^{\mathrm{Q}-}_{\overline{\mathrm{MS}}} = Z_{11}\mathcal{O}^{\mathrm{Q}-} + Z_{12}\mathcal{O}^{\mathrm{Q}+}$$
$$\mathcal{O}^{\mathrm{Q}+}_{\overline{\mathrm{MS}}} = Z_{22}\mathcal{O}^{\mathrm{Q}+}$$

renormalised moments in the $\overline{\mathrm{MS}}$ scheme:
$$\langle \xi^2 \rangle^{\mathrm{Q}}_{\overline{\mathrm{MS}}} = \zeta_{11} R^{\mathrm{Q}-} + \zeta_{12} R^{\mathrm{Q}+}$$
$$\left(a_2^{\mathrm{Q}}\right)_{\overline{\mathrm{MS}}} = \frac{7}{12}\left[5\zeta_{11} R^{\mathrm{Q}-} + (5\zeta_{12} - \zeta_{22}) R^{\mathrm{Q}+}\right]$$

with $\zeta_{11} = \dfrac{Z_{11}}{Z_V} \,, \ \zeta_{12} = \dfrac{Z_{12}}{Z_V} \,, \ \zeta_{22} = \dfrac{Z_{22}}{Z_V}$ for $\mathrm{Q} = \mathrm{L}$ (for $\mathrm{Q} = \mathrm{T}$: $Z_V \to Z_T$)

in particular:

continuum limit, t large

$$Z_{22} \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle \mathcal{O}^{\mathrm{L}+}(\mathbf{x},t) J(0) \rangle = Z_V F^{\mathrm{L}+}(\mathbf{p}) \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle V(\mathbf{x},t) J(0) \rangle$$

remember: $\mathcal{O}^{\mathrm{L}+}_{(\mu\nu\rho)}(x) = \partial_{(\mu}\partial_\nu \bar{d}(x)\gamma_{\rho)} u(x)$

hence for large t

$$\langle 1^2 \rangle^{\mathrm{L}}_{\overline{\mathrm{MS}}} = \frac{Z_{22} \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle \mathcal{O}^{\mathrm{L}+}(\mathbf{x},t) J(0) \rangle}{Z_V F^{\mathrm{L}+}(\mathbf{p}) \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle V(\mathbf{x},t) J(0) \rangle} = \zeta_{22} R^{\mathrm{L}+}$$

should approach unity in the continuum limit

similarly in the transverse case

then $$\left(a_2^{\mathrm{Q}}\right)_{\overline{\mathrm{MS}}} = \frac{7}{12}\left(5\langle \xi^2 \rangle^{\mathrm{Q}}_{\overline{\mathrm{MS}}} - \langle 1^2 \rangle^{\mathrm{Q}}_{\overline{\mathrm{MS}}}\right) \xrightarrow{a \to 0} \frac{7}{12}\left(5\langle \xi^2 \rangle^{\mathrm{Q}}_{\overline{\mathrm{MS}}} - 1\right)$$

Renormalisation

renormalised operators:

$$\mathcal{O}^{\mathrm{Q}-}_{\overline{\mathrm{MS}}} = Z_{11}\mathcal{O}^{\mathrm{Q}-} + Z_{12}\mathcal{O}^{\mathrm{Q}+}$$

$$\mathcal{O}^{\mathrm{Q}+}_{\overline{\mathrm{MS}}} = Z_{22}\mathcal{O}^{\mathrm{Q}+}$$

$\mathcal{O}^{\mathrm{L}-}$: lattice version of $\bar{d}(x)\gamma_{(\mu}\overleftrightarrow{D}_{\nu}\overleftrightarrow{D}_{\rho)}u(x)$

$\mathcal{O}^{\mathrm{L}+}$: lattice version of $\partial_{(\mu}\partial_{\nu}\bar{d}(x)\gamma_{\rho)}u(x)$

similarly for $\mathcal{O}^{\mathrm{T}\pm}$

forward matrix elements of $\mathcal{O}^{\mathrm{Q}+}$ vanish

$\rightarrow$ use $\mathrm{RI}' - \mathrm{SMOM}$ variant of the Rome-Southampton method instead of $\mathrm{RI}' - \mathrm{MOM}$

to compute the mixing coefficient Z_{12} nonperturbatively

follow the same strategy as in the case of the pion DA:

V.M. Braun et al., Phys. Rev. D92 (2015) 014504, arXiv:1503.03656

for the conversion to the $\overline{\mathrm{MS}}$ scheme:

renormalised vertex functions in the $\overline{\mathrm{MS}}$ scheme at two loops for SMOM kinematics

taken from the work of John Gracey:

$\mathcal{O}^{\mathrm{L}\pm}$: Phys. Rev. D84 (2011) 016002 (arXiv:1105.2138)

$\mathcal{O}^{\mathrm{T}\pm}$: private communication (unpublished)

vector and tensor currents: Eur. Phys. J. C71 (2011) 1567 (arXiv:1101.5266)

Bare data

two degenerate flavours of clover fermions

	β	κ	size	m_π/MeV	m_ρ/MeV
a	5.20	0.13596	$32^3 \times 64$	280	835
a	5.29	0.13620	$32^3 \times 64$	422	916
b	5.29	0.13632	$32^3 \times 64$	294	849
b	5.29	0.13632	$40^3 \times 64$	289	825
a	5.29	0.13632	$64^3 \times 64$	285	796
a	5.29	0.13640	$64^3 \times 64$	150	734
b	5.40	0.13640	$32^3 \times 64$	491	958
a	5.40	0.13647	$32^3 \times 64$	430	930
b	5.40	0.13660	$48^3 \times 64$	260	795

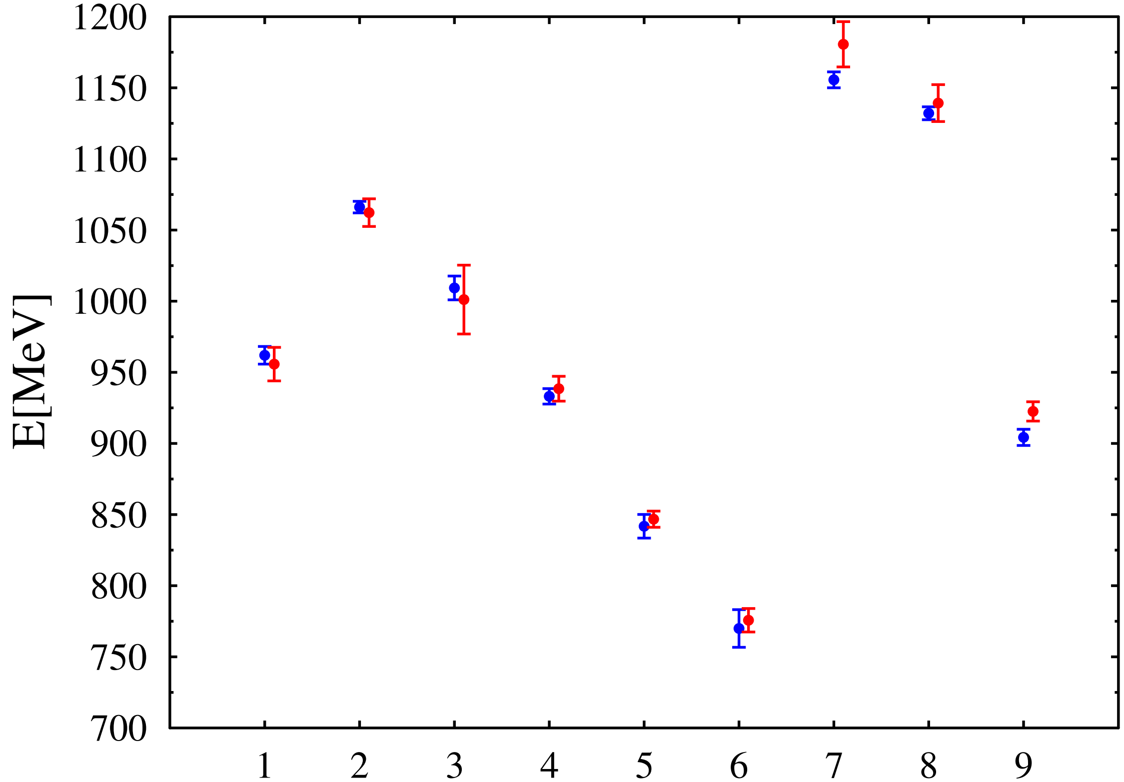
a: generated on QPACE

b: generated within the QCDSF collaboration

2-point correlation functions with open spinor indices already used for the pion DA
V.M. Braun et al., Phys. Rev. D92 (2015) 014504, arXiv:1503.03656

scale setting: Sommer parameter $r_0 = 0.5\,\mathrm{fm}$

m_ρ determined “naively” from smeared-smeared 2-point functions
although on most of the ensembles, ρ could “in principle” decay . . .
but dispersion relation (for one “unit” of momentum) well satisfied



	size	m_π	m_ρ
1	32	280	835
2	32	422	916
3	32	294	849
4	40	289	825
5	64	285	796
6	64	150	734
7	32	491	958
8	32	430	930
9	48	260	795

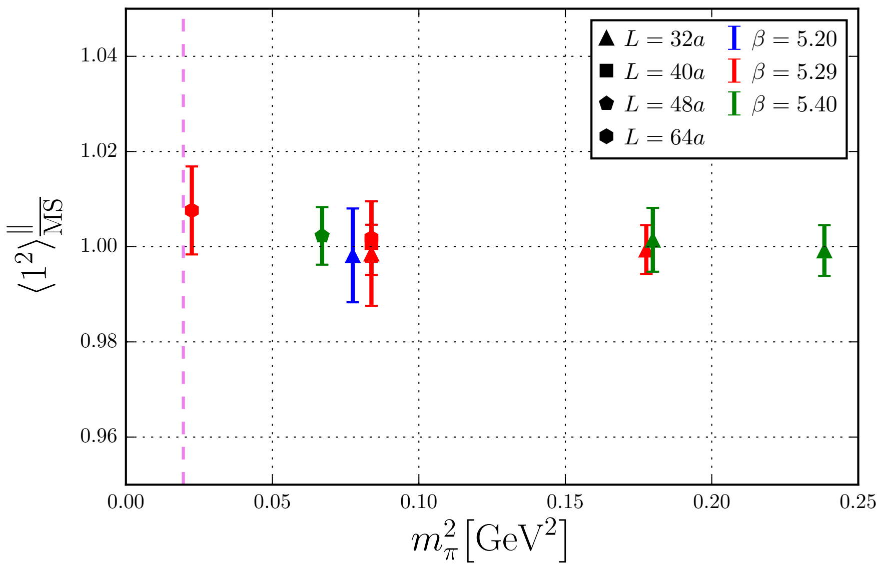
red points (right): energies calculated for one “unit” of momentum ($2\pi/L$)

blue points (left): energies calculated from mass and momentum

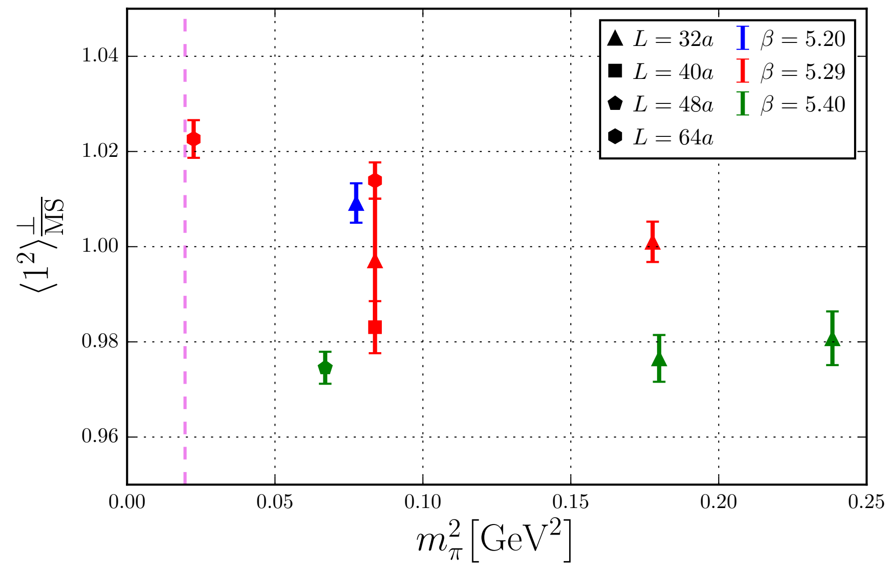
Results

renormalisation scale $\mu^2 = 4\,\mathrm{GeV}^2$

$\langle 1^2 \rangle^{\mathrm{L}}_{\overline{\mathrm{MS}}}$



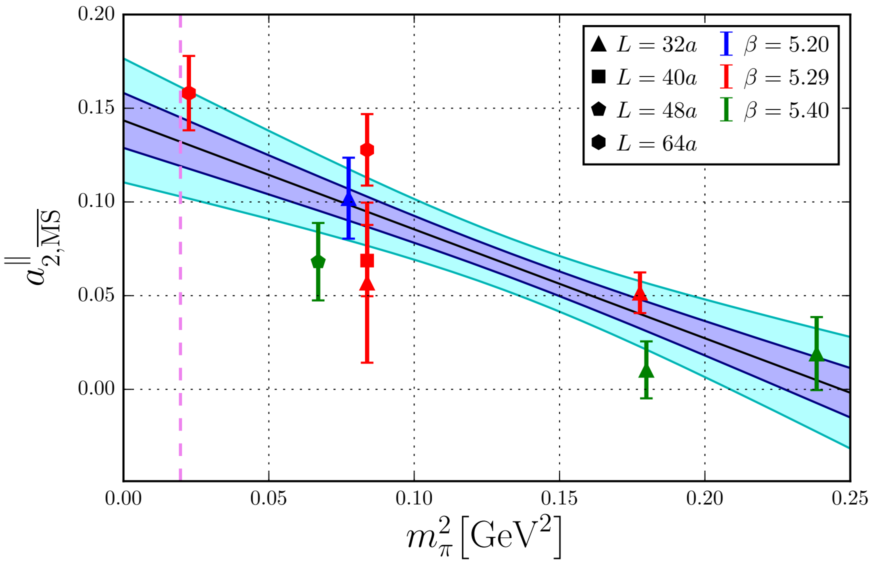
$\langle 1^2 \rangle^{\mathrm{T}}_{\overline{\mathrm{MS}}}$



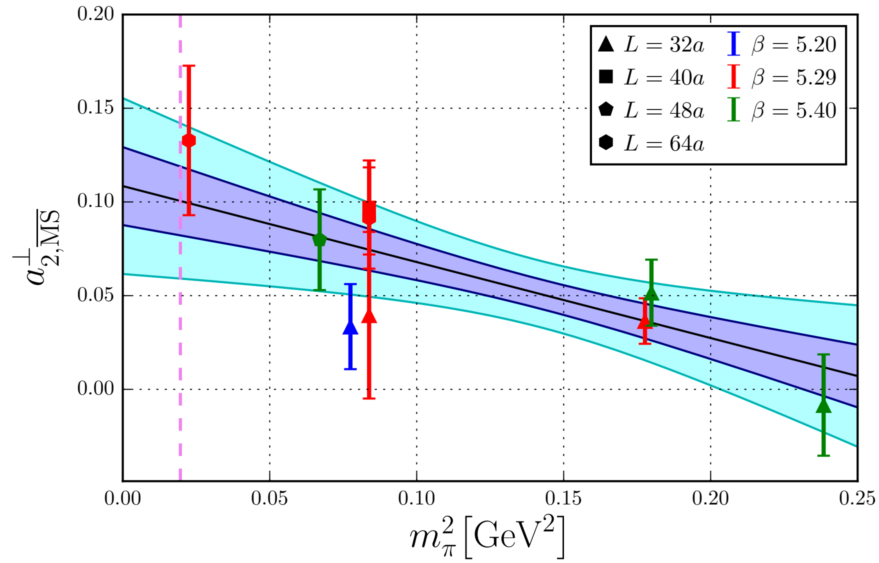
difficult to disentangle the possible sources for $\langle 1^2 \rangle^{\mathrm{T}}_{\overline{\mathrm{MS}}} \neq 1$:

lattice artefacts, finite size effects, ...

$\left(a_2^{\mathrm{L}}\right)_{\overline{\mathrm{MS}}}$



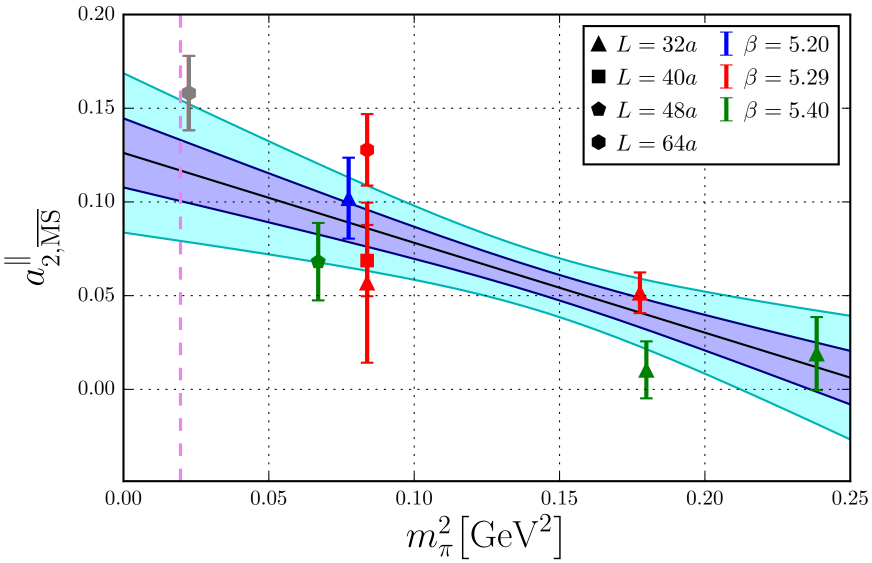
$\left(a_2^{\mathrm{T}}\right)_{\overline{\mathrm{MS}}}$



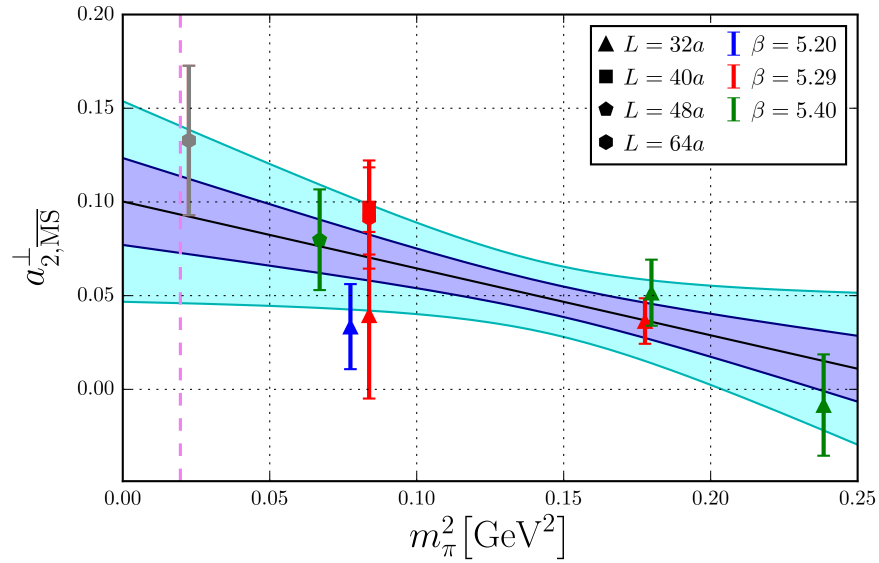
linear fits of the mass dependence
data point at $m_\pi = 150\,\mathrm{MeV}$ included
bands indicate the one- and two-sigma statistical error

$\left(a_2^{\mathrm{L}}\right)_{\overline{\mathrm{MS}}} > \left(a_2^{\mathrm{T}}\right)_{\overline{\mathrm{MS}}}$ at the physical point?

$\left(a_2^{\mathrm{L}}\right)_{\overline{\mathrm{MS}}}$



$\left(a_2^{\mathrm{T}}\right)_{\overline{\mathrm{MS}}}$



linear fits of the mass dependence
data point at $m_\pi = 150\,\mathrm{MeV}$ excluded
bands indicate the one- and two-sigma statistical error

$\left(a_2^{\mathrm{L}}\right)_{\overline{\mathrm{MS}}} > \left(a_2^{\mathrm{T}}\right)_{\overline{\mathrm{MS}}}$ at the physical point??

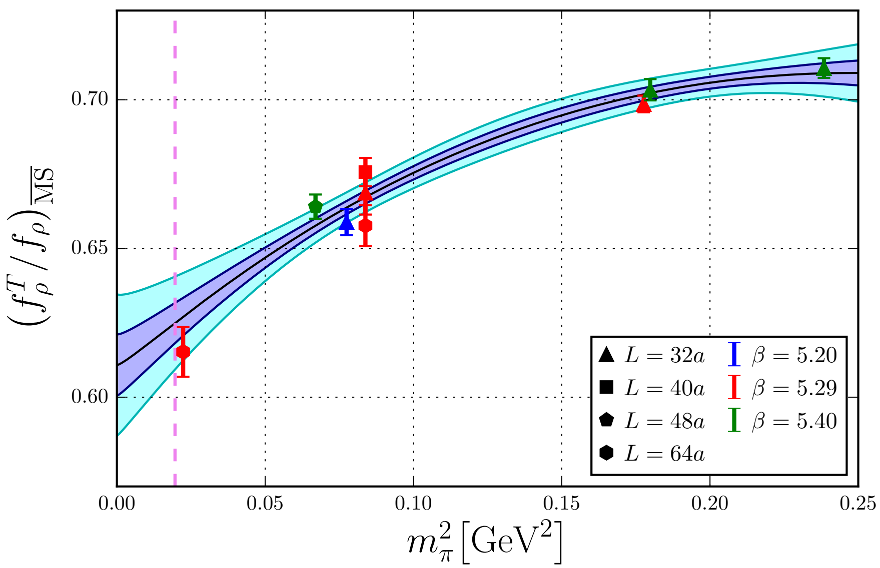
pion mass dependence of f^{L}, f^{T}, $f^{\mathrm{T}}/f^{\mathrm{L}}$ from chiral perturbation theory:

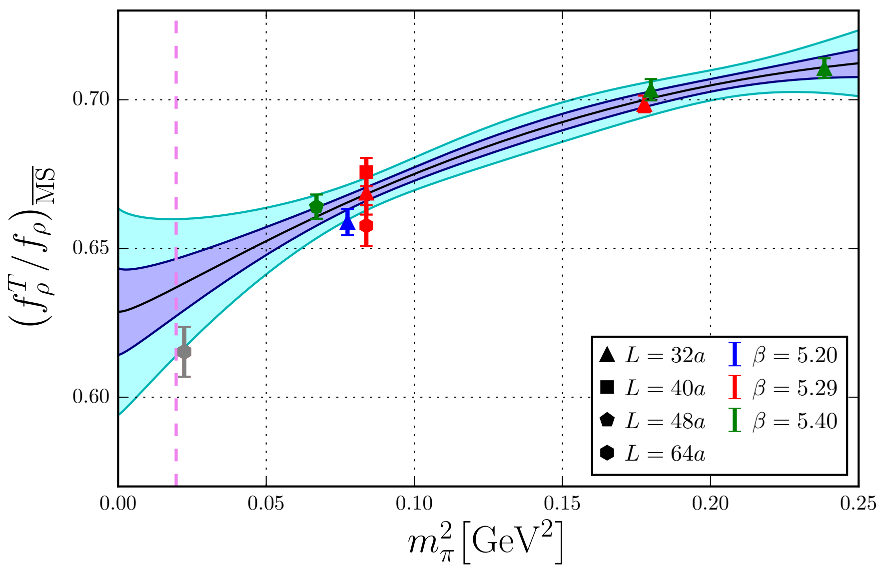
$$f^{\mathrm{L}}_{\overline{\mathrm{MS}}} = f^{\mathrm{L}(0)}\left(1 - \frac{m_\pi^2}{16\pi^2 F_\pi^2}\ln\left(\frac{m_\pi^2}{M^2}\right)\right) + f^{\mathrm{L}(2)} m_\pi^2 + f^{\mathrm{L}(3)} m_\pi^3 + O(m_\pi^4)$$

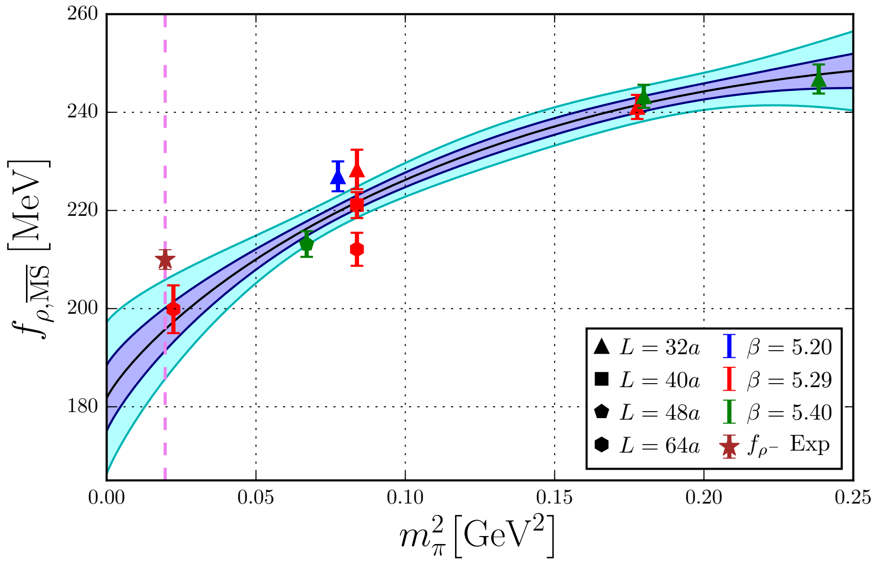
$$f^{\mathrm{T}}_{\overline{\mathrm{MS}}} = f^{\mathrm{T}(0)}\left(1 - \frac{m_\pi^2}{32\pi^2 F_\pi^2}\ln\left(\frac{m_\pi^2}{M^2}\right)\right) + f^{\mathrm{T}(2)} m_\pi^2 + f^{\mathrm{T}(3)} m_\pi^3 + O(m_\pi^4)$$

$$\frac{f^{\mathrm{T}}_{\overline{\mathrm{MS}}}}{f^{\mathrm{L}}_{\overline{\mathrm{MS}}}} = \left(\frac{f^{\mathrm{T}}}{f^{\mathrm{L}}}\right)^{(0)}\left(1 + \frac{m_\pi^2}{32\pi^2 F_\pi^2}\ln\left(\frac{m_\pi^2}{M^2}\right)\right) + f^{\mathrm{R}(2)} m_\pi^2 + f^{\mathrm{R}(3)} m_\pi^3 + O(m_\pi^4)$$

chiral renormalisation scale $M = 775\,\mathrm{MeV}$, $F_\pi = 92.4\,\mathrm{MeV}$ fixed



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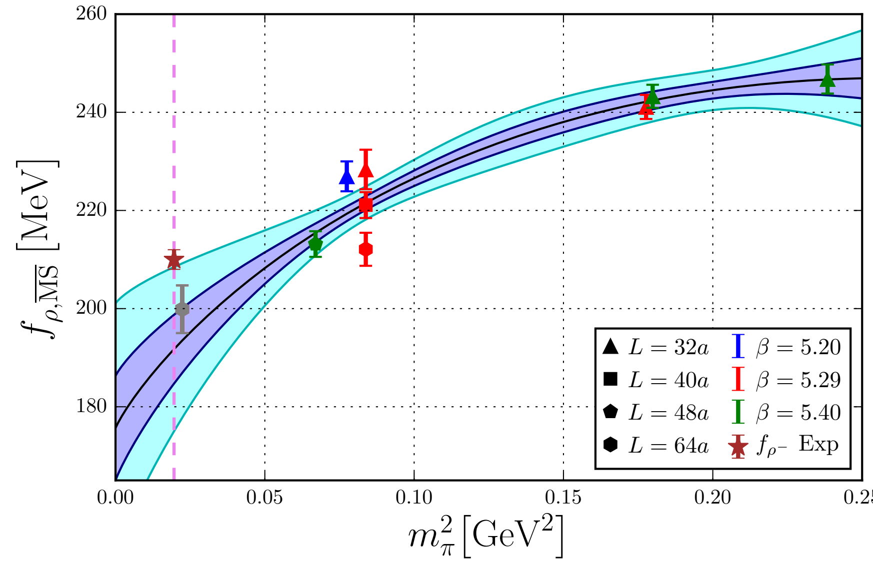
$f_{\rho,\overline{\mathrm{MS}}}$ [MeV]
$m_\pi^2\,[\mathrm{GeV}^2]$
$L = 32a$
$L = 40a$
$L = 48a$
$L = 64a$
$\beta = 5.20$
$\beta = 5.29$
$\beta = 5.40$
f_{ρ^-} Exp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$f_{\rho,\overline{\mathrm{MS}}}$ [MeV]
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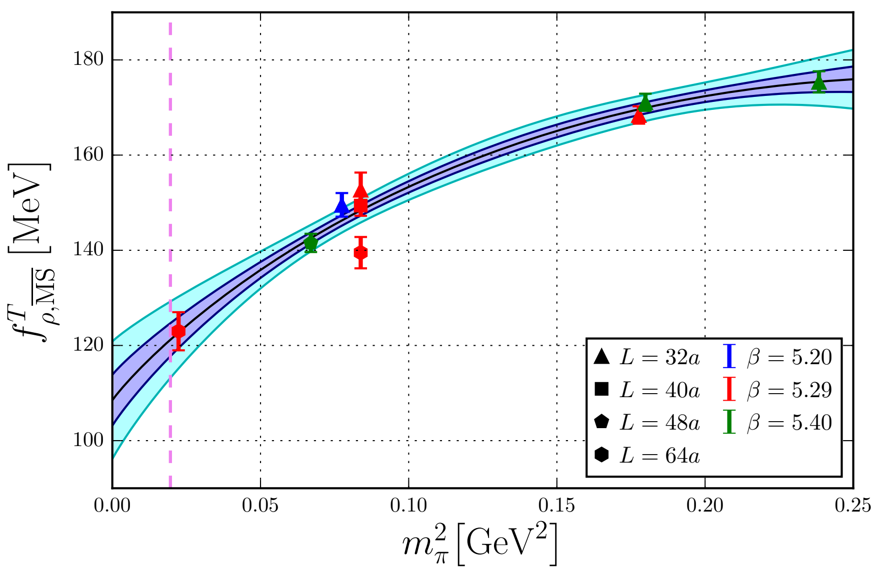
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$f^T_{\rho,\overline{\mathrm{MS}}}$ [MeV]
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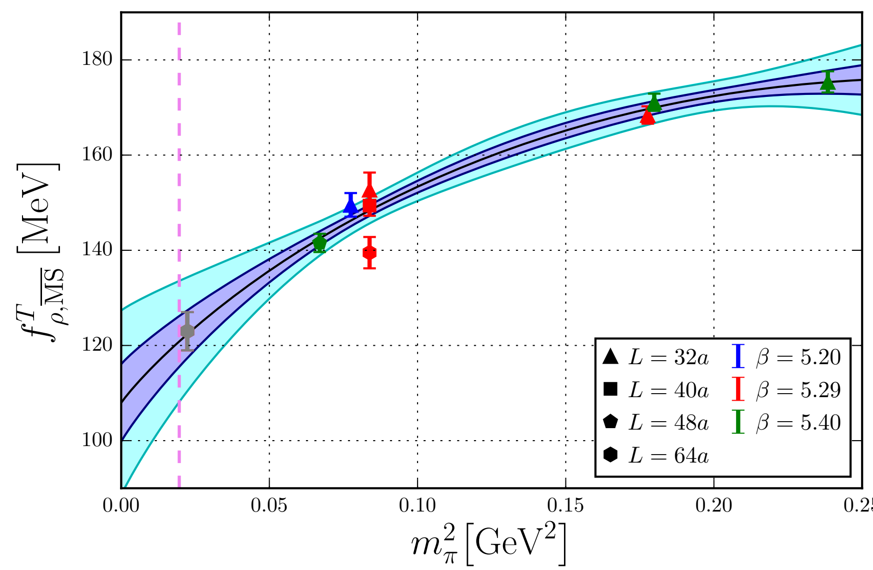
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$\beta = 5.20$
$\beta = 5.29$
$\beta = 5.40$

Comparison with previous results

all results (at the physical pion mass) renormalised in the $\overline{\text{MS}}$ scheme

	μ[GeV]	f^{L}[MeV]	f^{T}[MeV]	$f^{\text{T}}/f^{\text{L}}$	a_2^{L}	a_2^{T}
this work (preliminary)	2	196(4)	121(3)	0.625(7)	0.132(13)	0.101(18)
(all data points included)						
QCD sum rules(1)	1	206(7)	165(9)	0.79(5)	0.14(6)	0.15(7)
QCD sum rules(2)	2	206(7)	155(8)	0.74(5)	0.11(5)	0.11(5)
experiment(1)		209(2)				

(1) P. Ball, R. Zwicky, JHEP 04 (2006) 046, hep-ph/0603232
(2) obtained by LO evolution with $N_f = 2$

some previous lattice results:

RBC-UKQCD: R. Arthur et al., Phys. Rev. D83 (2011) 074505, arXiv:1011.5906
$N_f = 2+1$ (domain wall), $a = 0.1141(18)\,\text{fm}$, $\mu = 2\,\text{GeV}$ $\langle\xi^2\rangle^{\text{L}} = 0.27(1)(2) \to a_2^{\text{L}} = 0.20(6)$

BGR: V.M. Braun et al., Phys. Rev. D68 (2003) 054501, hep-lat/0306006
$N_f = 0$ (chirally improved fermions), $a = 0.10\,\text{fm}$, $\mu = 2\,\text{GeV}$ $f^{\text{T}}/f^{\text{L}} = 0.742(14)$

our lattice spacings in this work: $a = 0.081\,\text{fm}$, $0.071\,\text{fm}$, $0.060\,\text{fm}$

Summary and outlook

- 2-point correlation functions with open spinor indices
  on a set of ensembles with two dynamical flavours of clover fermions

- pion DA: V.M. Braun et al., Phys. Rev. D92 (2015) 014504, arXiv:1503.03656

- ρ DA (with caveats): in progress, preliminary results shown
  instability effects in the matrix elements?

- ω: isosinglet analogue of ρ, much smaller width!
  additional disconnected contributions for the DA to be calculated

- CLS ensembles with 2+1 flavours
  - also K and K^* can be studied
  - reliable continuum limit?