Charmed Bottom Mesons from Lattice QCD

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Why Charmed-bottom mesons ?

$n\,^{2s+1}\ell_J$ J^{PC}	I = 0 $c\bar{c}$	I = 0 $b\bar{b}$	I = $\frac{1}{2}$ $c\bar{u}$, $c\bar{d}$; $\bar{c}u$, $\bar{c}d$	I = 0 $c\bar{s}$; $\bar{c}s$	I = $\frac{1}{2}$ $b\bar{u}$, $b\bar{d}$; $\bar{b}u$, $\bar{b}d$	I = 0 $b\bar{s}$; $\bar{b}s$	I = 0 $b\bar{c}$; $\bar{b}c$
$1\,^1S_0$ 0^{-+}	$\eta_c(1S)$	$\eta_b(1S)$	D	$D_s^{\pm}$	B	B_s^0	$B_c^{\pm}$
$1\,^3S_1$ 1^{--}	$J/\psi(1S)$	$\Upsilon(1S)$	D^*	$D_s^{*\pm}$	B^*	B_s^*	
$1\,^1P_1$ 1^{+-}	$h_c(1P)$	$h_b(1P)$	$D_1(2420)$	$D_{s1}(2536)^{\pm}$	$B_1(5721)$	$B_{s1}(5830)^0$	
$1\,^3P_0$ 0^{++}	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$D_0^*(2400)$	$D_{s0}^*(2317)^{\pm\dagger}$			
$1\,^3P_1$ 1^{++}	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$	$D_1(2430)$	$D_{s1}(2460)^{\pm\dagger}$			
$1\,^3P_2$ 2^{++}	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$	$D_2^*(2460)$	$D_{s2}^*(2573)^{\pm}$	$B_2^*(5747)$	$B_{s2}^*(5840)^0$	
$1\,^3D_1$ 1^{--}	$\psi(3770)$			$D_{s1}^*(2860)^{\pm\ddagger}$			
$1\,^3D_3$ 3^{--}				$D_{s3}^*(2860)^{\pm}$			
$2\,^1S_0$ 0^{-+}	$\eta_c(2S)$	$\eta_b(2S)$	$D(2550)$				
$2\,^3S_1$ 1^{--}	$\psi(2S)$	$\Upsilon(2S)$		$D_{s1}^*(2700)^{\pm\ddagger}$			
$2\,^1P_1$ 1^{+-}		$h_b(2P)$					
$2\,^3P_{0,1,2}$ $0^{++}, 1^{++}, 2^{++}$	$\chi_{c0,2}(2P)$	$\chi_{b0,1,2}(2P)$				**@PDG**	
$3\,^3P_{0,1,2}$ $0^{++}, 1^{++}, 2^{++}$		$\chi_b(3P)$					

Set up

– Gauge Configurations: HISQ 2+1+1

– Valence quark propagators :

-- Light to charm : Overlap

-- Bottom : NRQCD with improved coefficients

Overlap Fermions

- Some desirable features:
  - No $O(a)$ error.

$$(1-\tfrac{1}{2}D)D(m)^{-1} = (D_c + ma)^{-1}$$

  - The effective propagator :

$D_c = D/(1 - D/2)$ is chirally symmetric, i.e., $\{\gamma_5, D_c\} = 0$.

  - D_c + m is like in the continuum formalism.
  - Multi-mass algorithm (more than 20 masses
    -10-15% overhead
  - Renormalization may be relatively simple (e.g. with chiral Ward identity).

- Undesirable feature:
  -- *Cost*

Overlap fermions on 2+1+1 Flavors HISQ Configurations

- Lattices used for this study :

  HISQ gauge configurations from MILC

  24^3 x 64 , a = 0.12 fm, m_l/m_s = 1/5, $m_\pi L$ = 4.54, m_π = 305 MeV

  32^3 x 96 , a = 0.089 fm, m_l/m_s = 1/5, $m_\pi L$ = 4.5, m_π = 312 MeV

  48^3 x 144 , a = 0.058 fm, m_l/m_s = 1/5, $m_\pi L$ = 4.51, m_π = 319 MeV

  PHYSICAL REVIEW D 87, 054505 (2013) (MILC)

- **HYP smearing on gauge fields**
- **Both point source and coulomb gauge fixed wall source are used**
- **No of eigenvectors projected : 350 (a = 0.012 fm)**

  : 350 (a = 0.09 fm)

  : 75 (a = 0.058 fm)

Rest mass Vs Kinetic mass

Charm mass is tuned by meson kinetic mass and not from rest mass ……a la FermiLab formulation

El-khadra et al, PRD55, 3933(1997)

Expanding the energy momentum relation in powers of pa

$$E(p)^2 = M_1^2 + \frac{M_1}{M_2}\mathbf{p}^2 + O(\mathbf{p}^4)$$

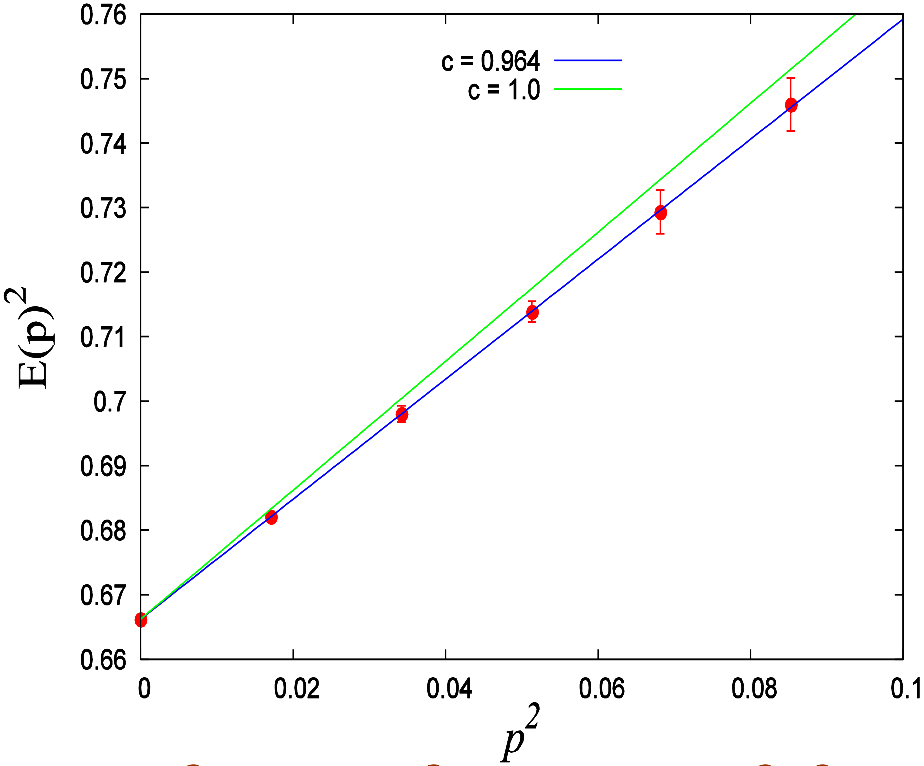
$$= M_1^2 + c^2 p^2$$

$$|\mathbf{p}| << m_0, 1/a$$

Rest mass : $M_1 = E(\mathbf{0})$

Kinetic mass : $M_2 = M_1/c^2$

Dispersion relation (at charm mass)



$$E^2(p) = E^2(p=0) + p^2c^2$$

Finite momentum wall source is used to project to particular momentum state which reduce errorbars substantially.

Lattice spacings and tuning of charm and strange masses

Lattice spacings are calculated by Omega(sss) mass = 1672 GeV

48^3 x 144 : 0.0582(5) fm
32^3 x 96 : 0.0877(10) fm
24^3 x 64 : 0.1192(14) fm
which are quite consistent with lattice spacings determined by MILC

- **Strange mass is tuned by setting pseudoscalar $\underline{s}s$ mass at 685 MeV**

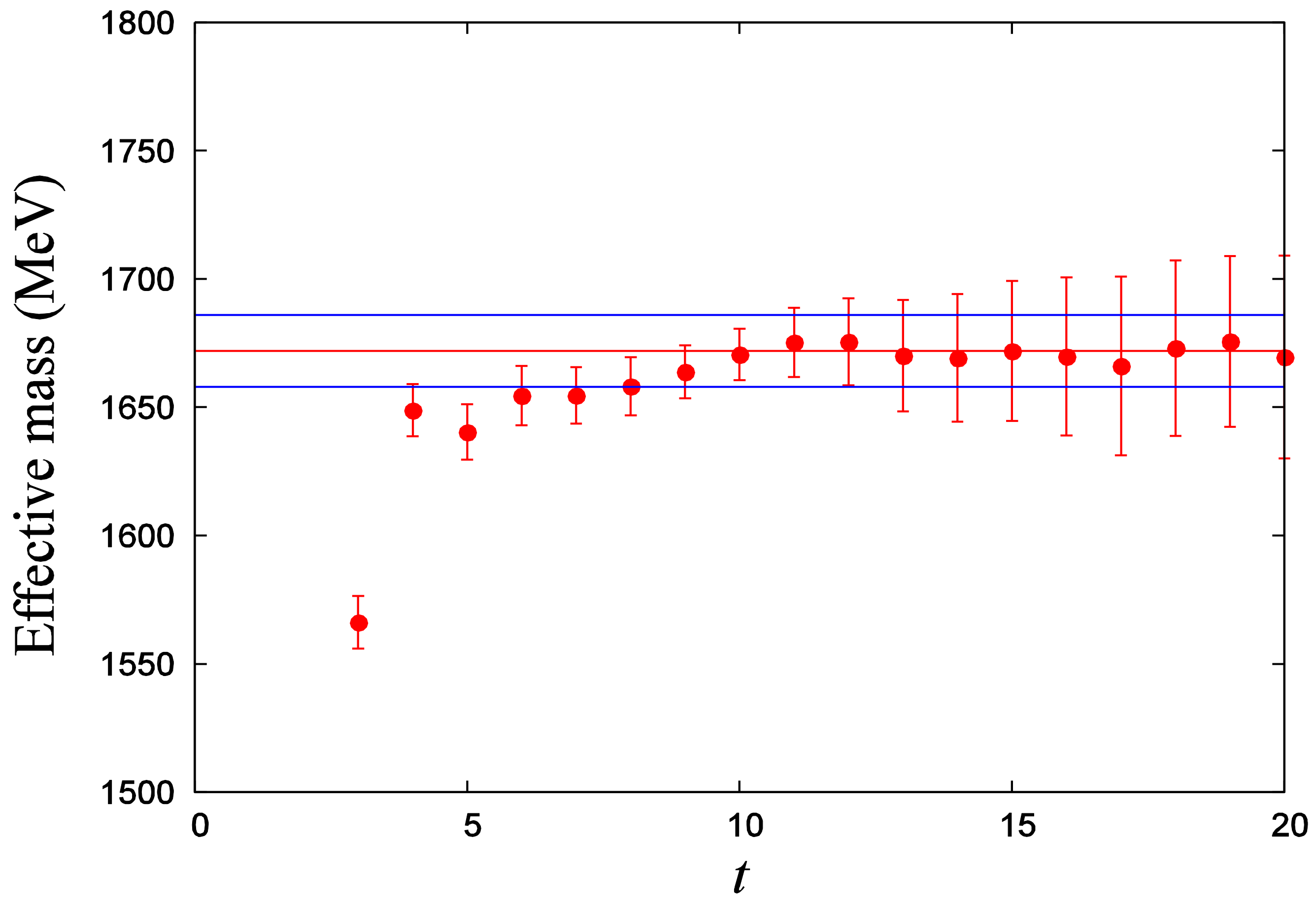
$$m_s a = 0.0738 \ (a = 0.0118\text{fm})$$
$$= 0.048 \ (a = 0.0888 \text{ fm})$$
$$= 0.028 \ (a = 0.0582\text{fm})$$

Taking m_s = 100 MeV
$$m_s a = 0.0450 \ (a = 0.0888\text{fm}),$$
$$= 0.0295 (a = 0.0582\text{fm})$$

- **Charm mass is tuned by** $\frac{1}{4}(m_{\eta_c} + 3 m_{J/\psi})$

$$m_c a = 0.527 \ (a = 0.1192\text{fm})$$
$$= 0.428 \ (a = 0.0888\text{fm}),$$
$$= 0.29 \ (a = 0.0582 \text{ fm})$$

Considering kinetic masses of mesons (a la Fermilab formulation)



Omega(sss) effective mass

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NRQCD Action

$$aH = aH_0 + a\delta H; \quad aH_0 = -\frac{\Delta^{(2)}}{2am_b},$$

$$a\delta H = -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2}(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla)$$
$$- c_3 \frac{1}{8(am_b)^2} \sigma \cdot (\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla})$$
$$- c_4 \frac{1}{2am_b} \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24am_b} - c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2}.$$

∇ is the symmetric lattice derivative

$\Delta^{(2)}$ and $\Delta^{(4)}$ ➡ $\sum_i D_i^2$ and $\sum_i D_i^4$

Tadpole improvement.

H_0 ➡ $\mathcal{O}(v^2)$

Coefficients are improved $\mathcal{O}(\alpha_s^2 a^2 v^4)$

δH ➡ $\mathcal{O}(v^4)$

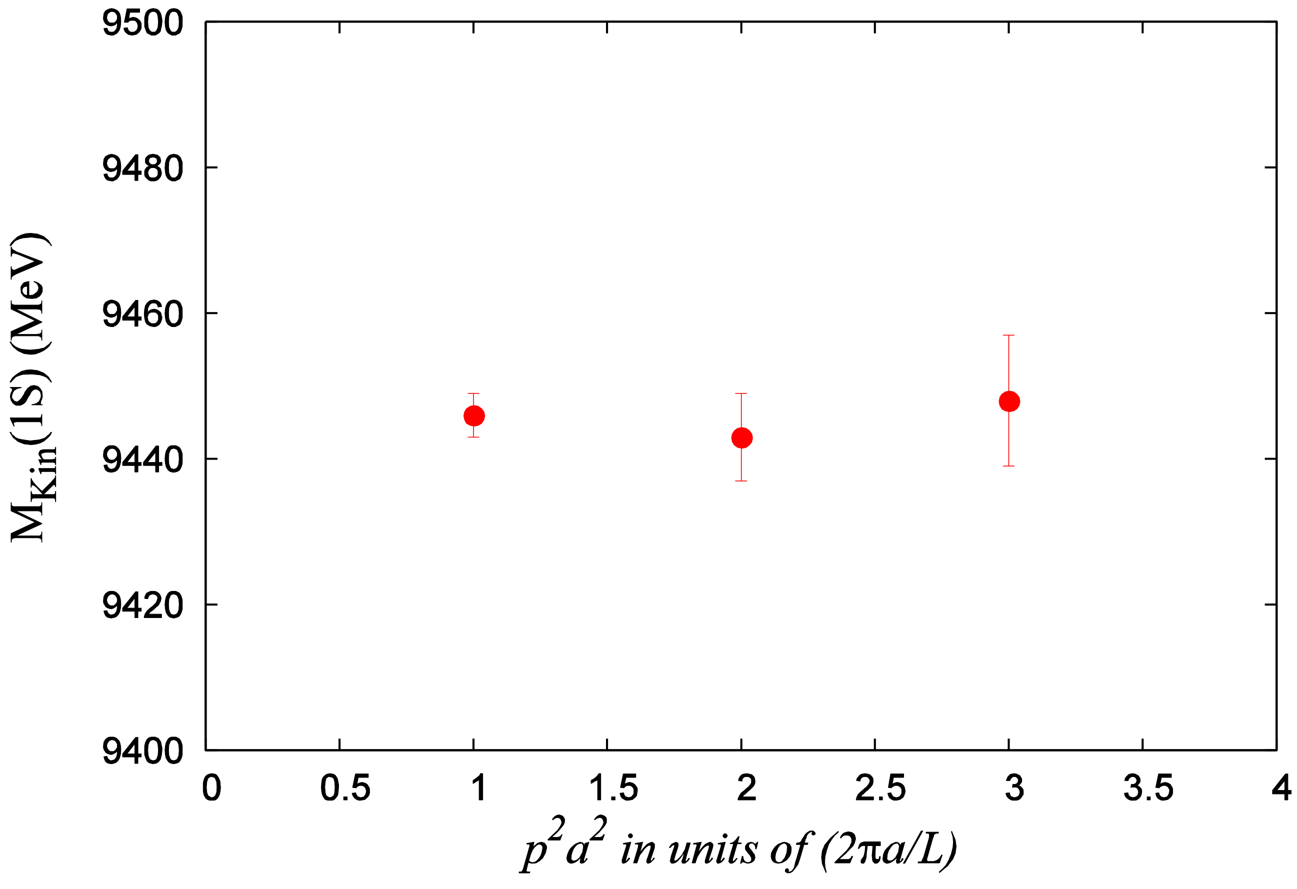
Dowdall et all, PRD 85,054509,2012
A. Hart et all, PRD 79, 074008 (2009)

$$aE(P) = aE(0) + \sqrt{a^2P^2 + a^2M_{\mathrm{Kin}}^2}$$

$$aM_{\mathrm{Kin}} = \frac{a^2P^2 - (a\Delta E)^2}{2a\Delta E}$$

$$\bar{M}_{\mathrm{Kin}}(1S) = \frac{(3M_{\mathrm{Kin}}(\Upsilon) + M_{\mathrm{Kin}}(\eta_b))}{4}$$

b-quark mass tunning

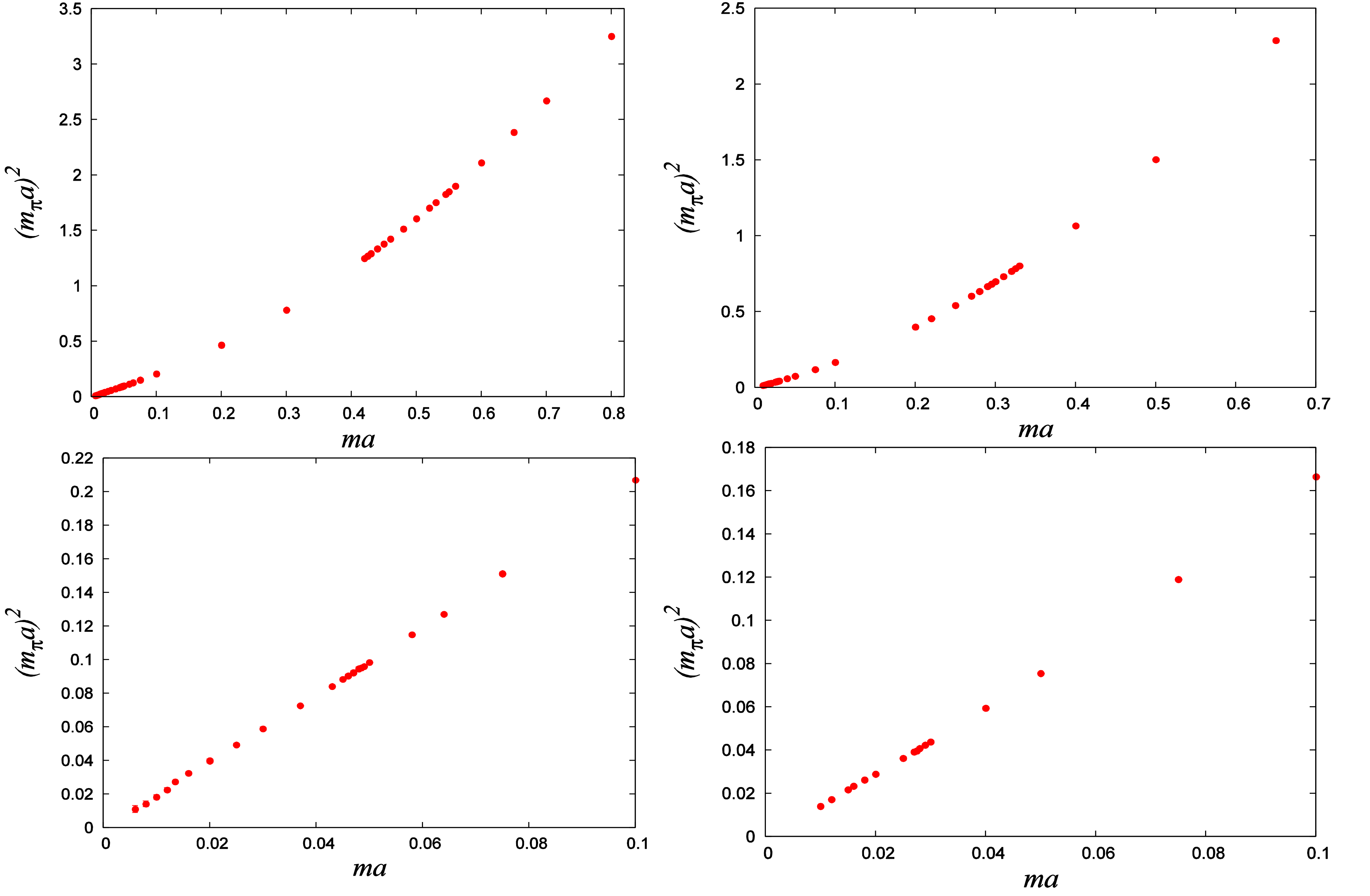


- We have calculated ground state energy spectra of mesons and baryons for all possible quantum numbers with ***l*, *s*, *c*** and ***b*** quarks

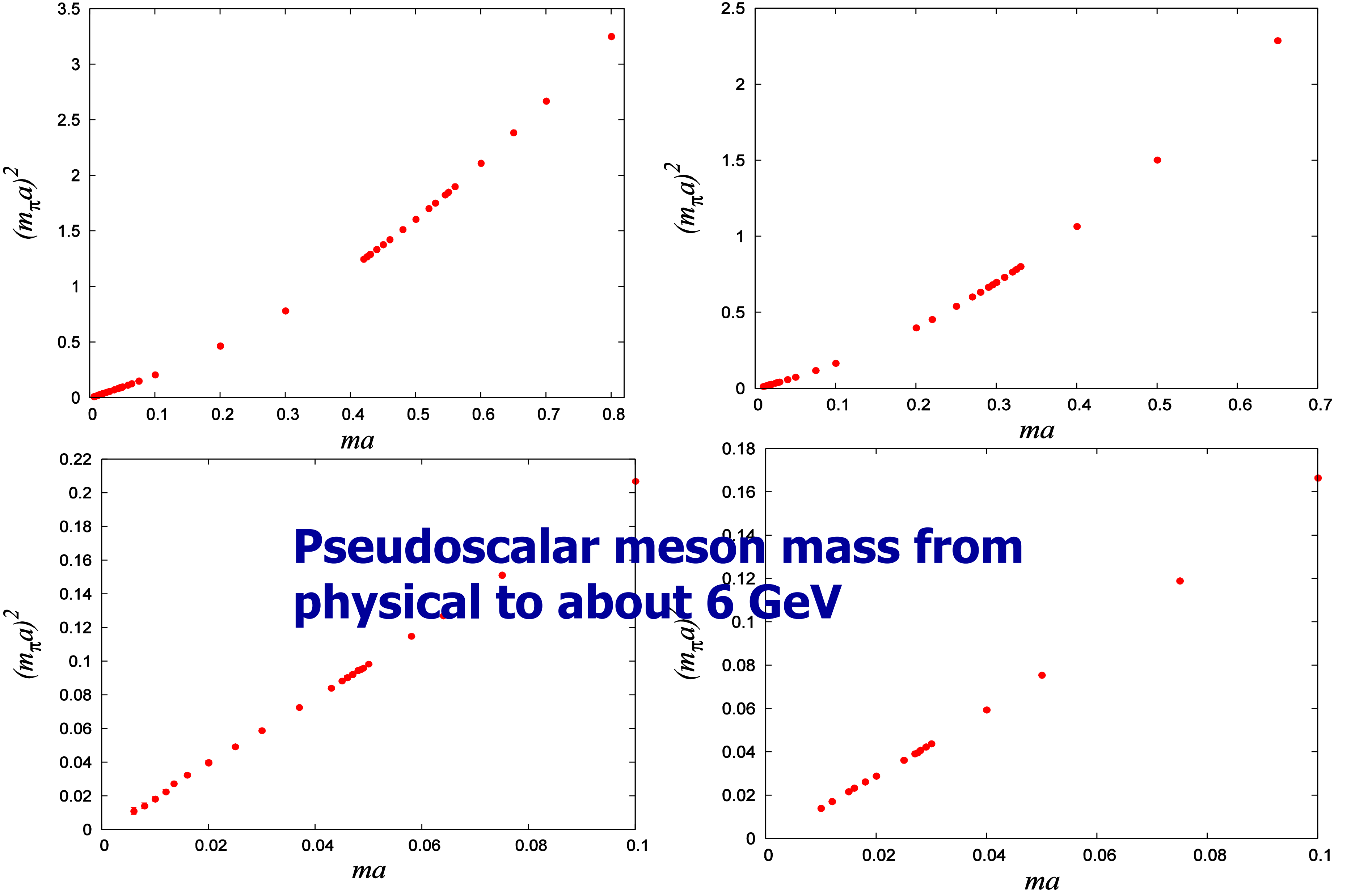
- Decay constants from two points functions are also being evaluated

- Results for charmed-bottom mesons and baryons will be presented

Pseudoscalar meson mass

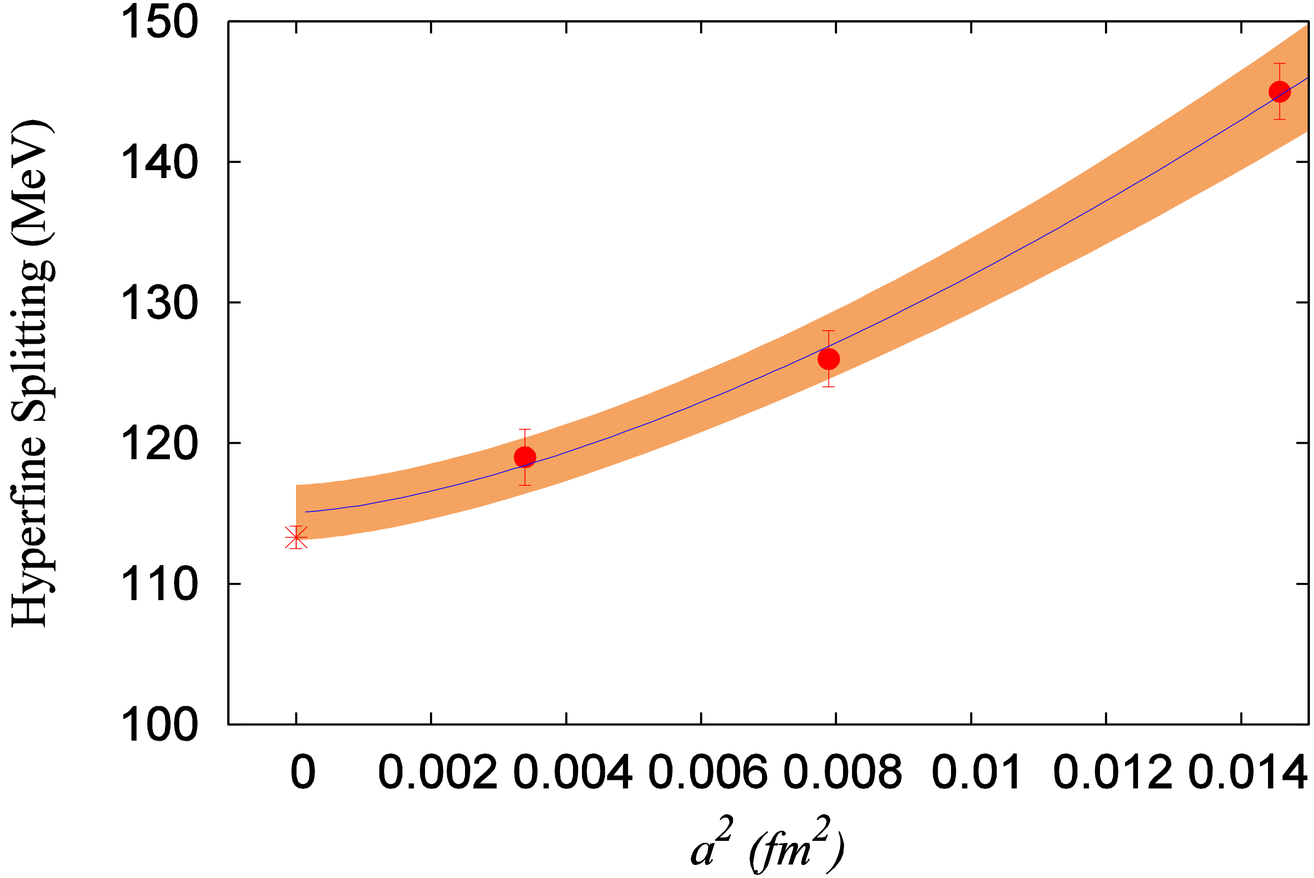


Pseudoscalar meson mass

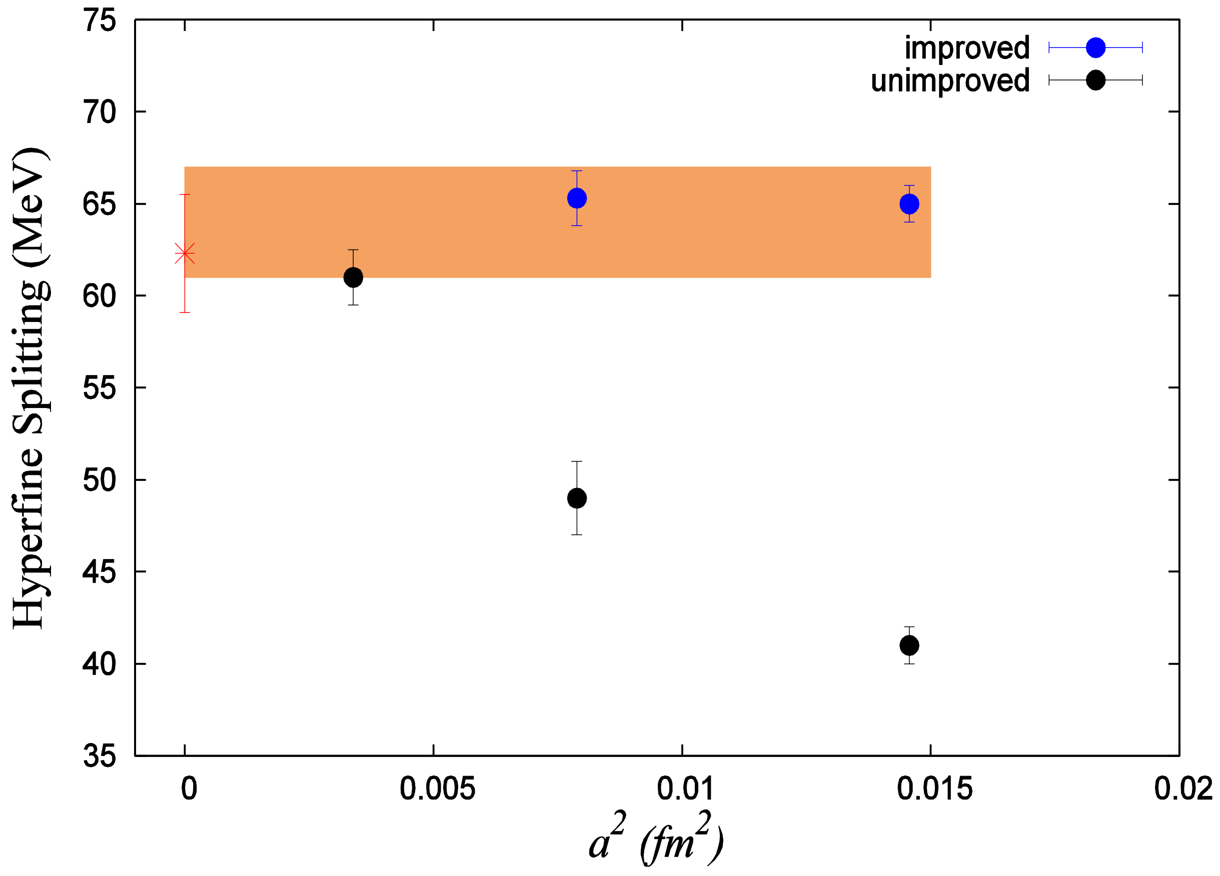


Pseudoscalar meson mass from physical to about 6 GeV

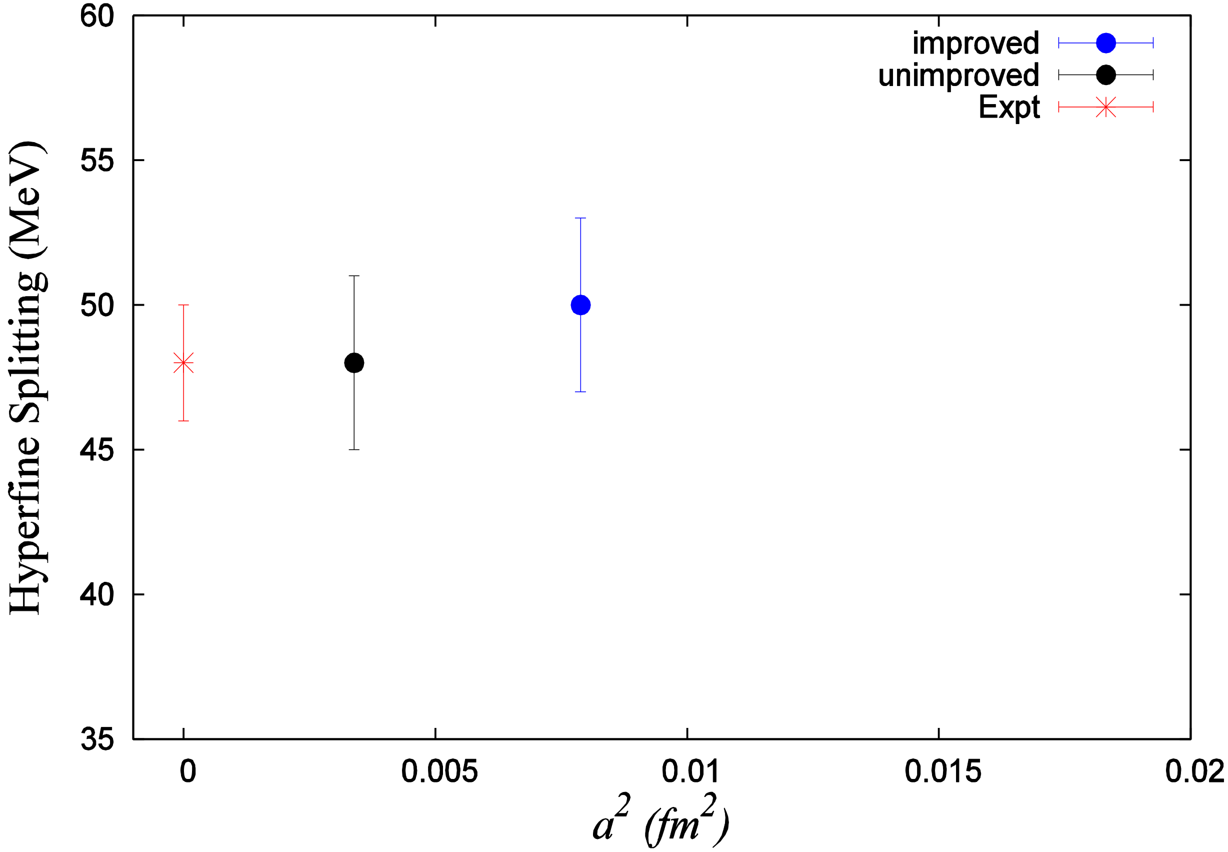
Hyperfine splitting in charmonia



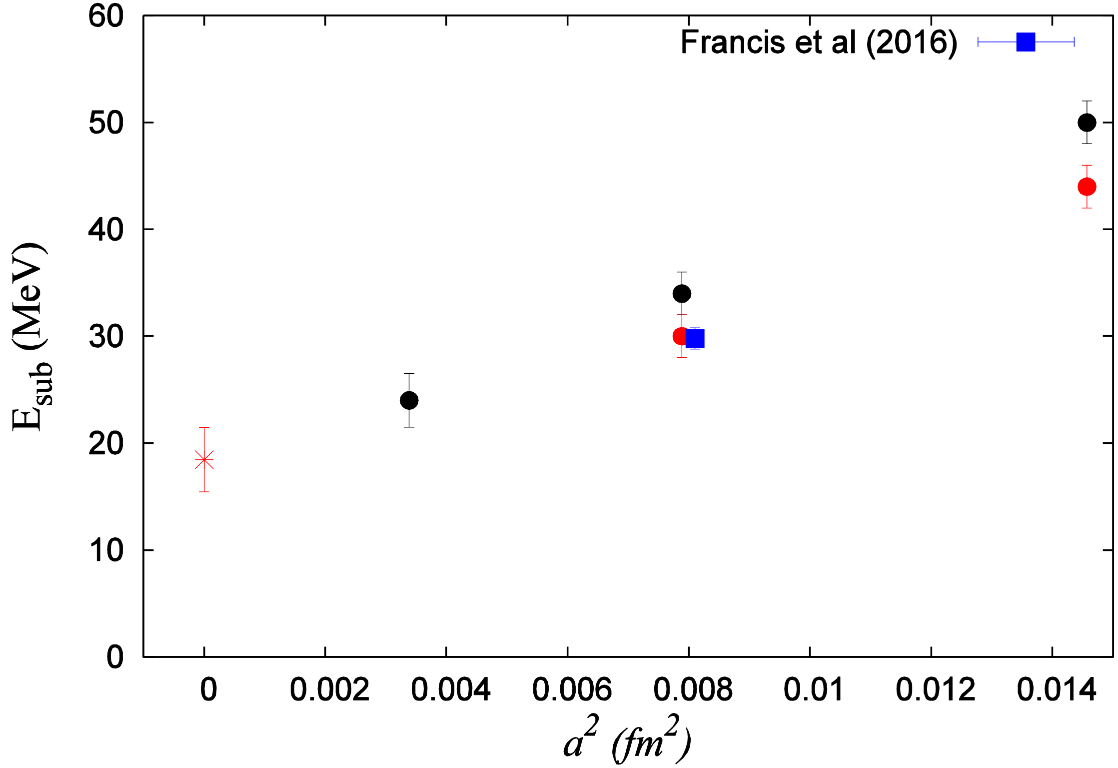
Hyperfine splitting in bottomonia



Hyperfine splitting : B_s



B_c (0^-)

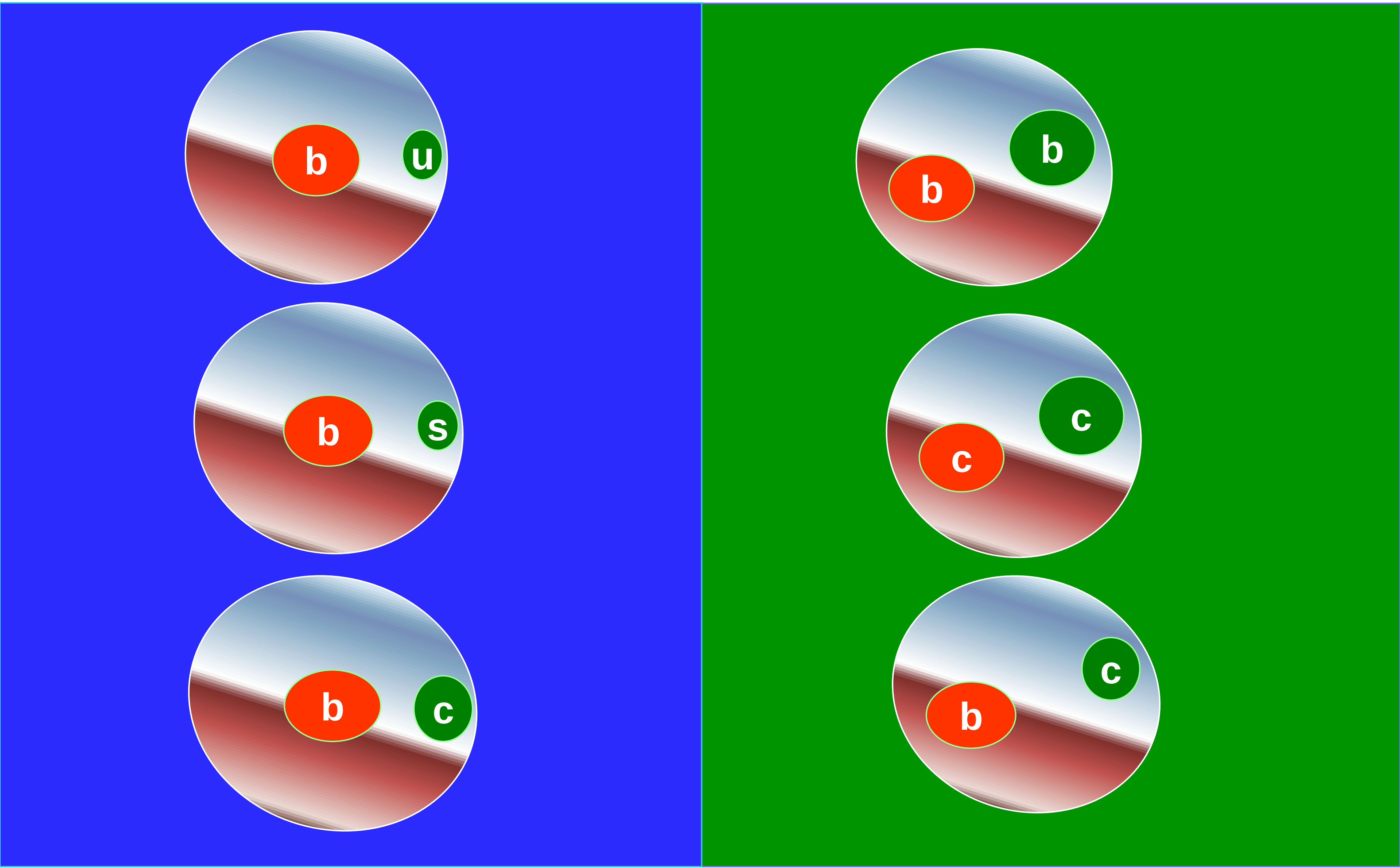


$$E_X^{(\mathrm{sub})} = E_X - \frac{n_c}{2}\overline{E}_{c\bar{c}} - \frac{n_b}{2}\overline{E}_{b\bar{b}}$$

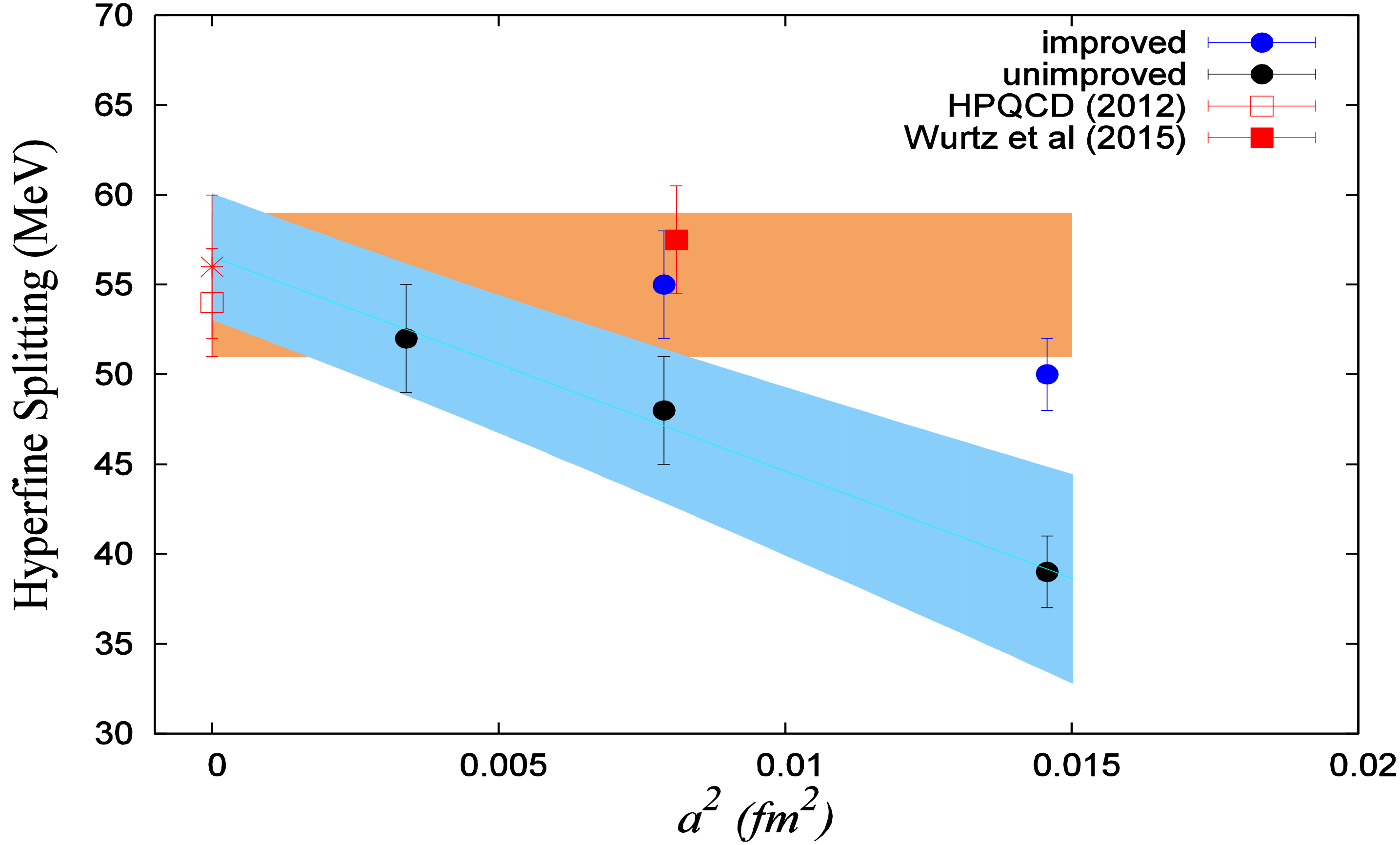
Prediction : B_c*

- Only pseudoscalar B_c is known experimentally (also possibly $B_c(2s)$?)
- Hyperfine splittings (MeV)
  - bu : 45
  - bs : 48
  - bb : 62
  - cc : 113
  - bc : ?

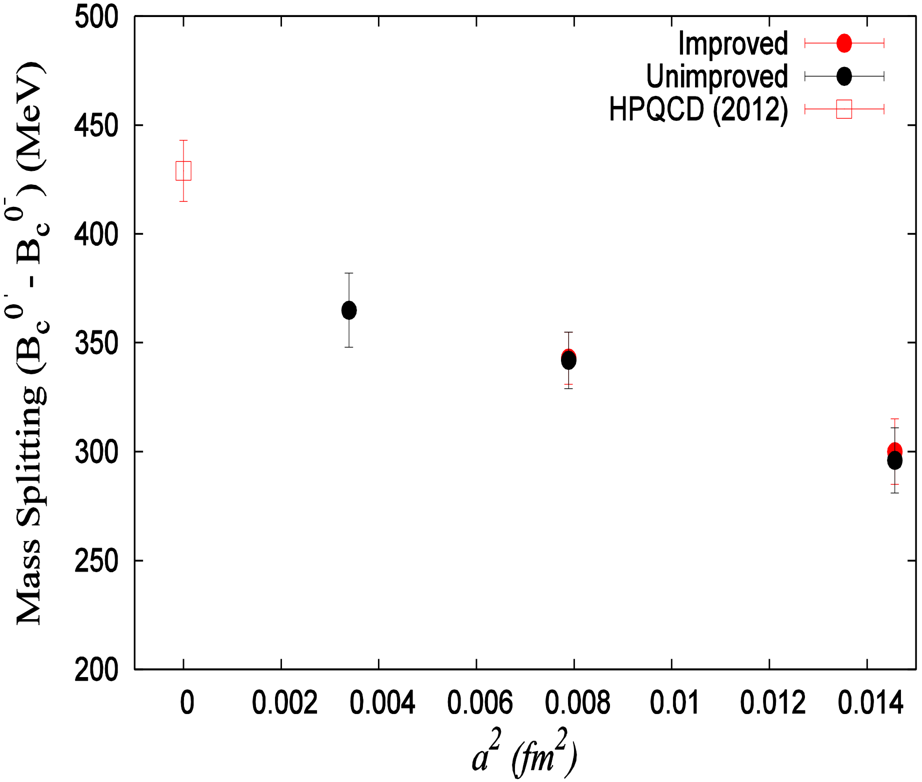
B_c – Onium or B meson type?

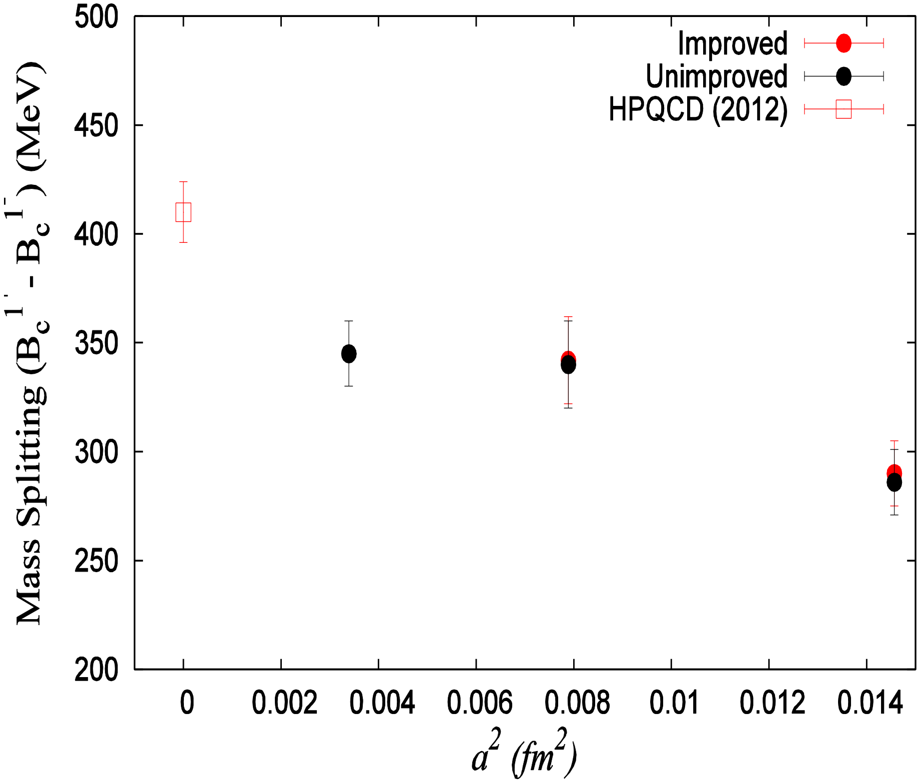


Prediction : B_c*

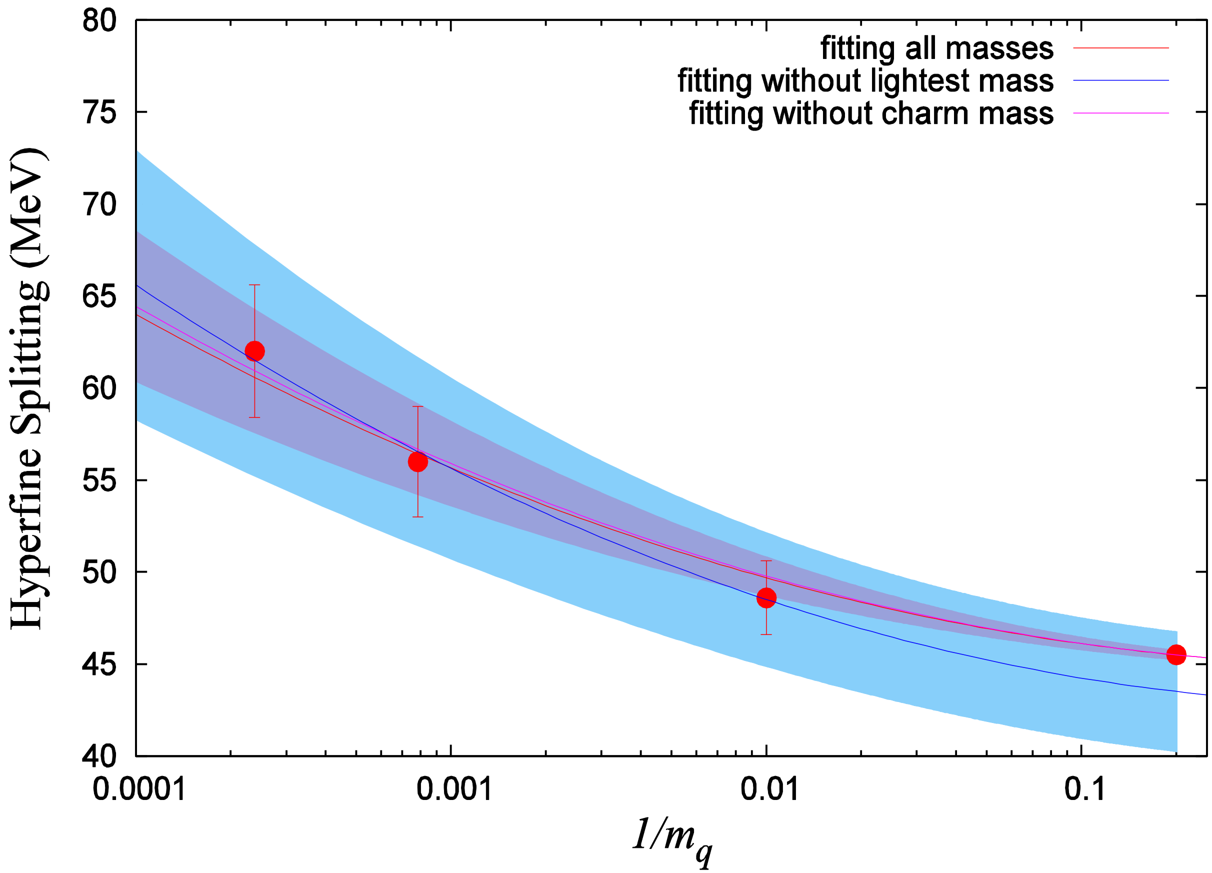


Mass-splittings



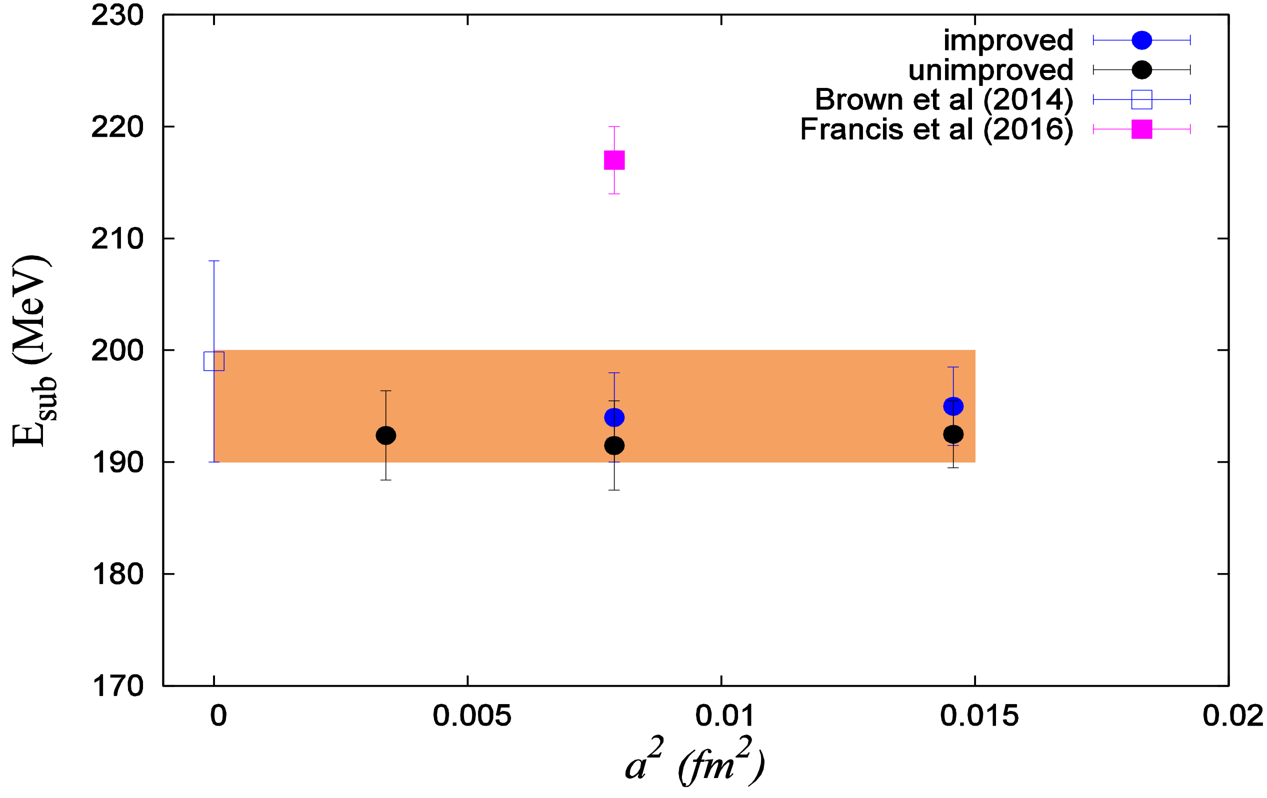


Hyperfine splittings in bottom mesons



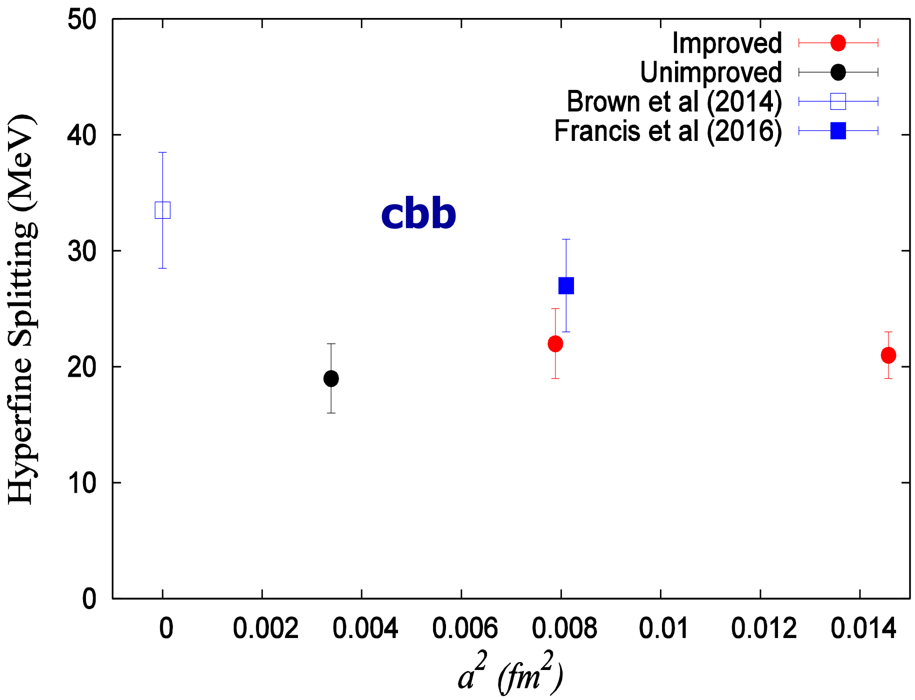
$$\Delta(\boldsymbol{E}_{\boldsymbol{m_q}}) = a + b(\log(\boldsymbol{1}/\boldsymbol{m_q}))^{\mathbf{2}}$$

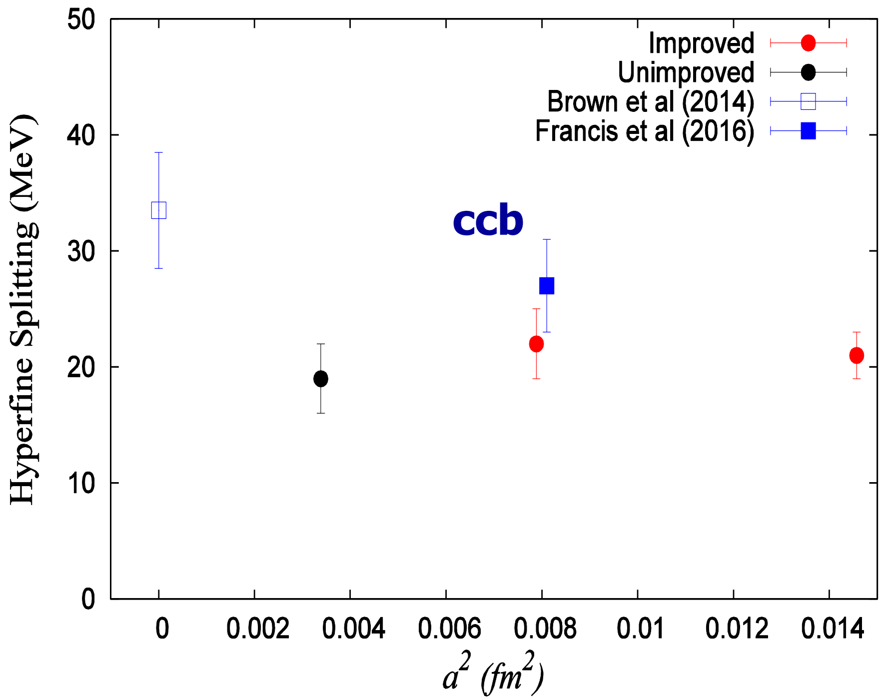
Triply bottom baryon



$$E_X^{(\mathrm{sub})} = E_X - \frac{n_c}{2}\overline{E}_{c\bar{c}} - \frac{n_b}{2}\overline{E}_{b\bar{b}}.$$

Hyperfine splittings in charmed-bottomed baryons





Conclusions and Outlooks

- **Ground state hadron spectra including all flavors are calculated using NRQCD (*b*) and overlap (*l*, *s*, and *c*) valence quarks on HISQ gauge configurations.**
- **Basic mass spectra for charmed-bottom mesons and baryons are presented here.**

- Proper chiral and continuum limits need to be taken

- Decay constants are under study