Charmed Bottom Mesons from Lattice QCD

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Why Charmed-bottom mesons?

<table>
<thead>
<tr>
<th>$n \ 2s+1\ell_J \ J^{PC}$</th>
<th>$l = 0$</th>
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<th>$l = \frac{1}{2}$</th>
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<tbody>
<tr>
<td></td>
<td>$c\bar{c}$</td>
<td>$b\bar{b}$</td>
<td>$c\bar{u}, c\bar{d}; \bar{c}u, \bar{c}d$</td>
<td>$c\bar{s}; \bar{c}s$</td>
<td>$b\bar{u}, \bar{b}d; \bar{b}u, \bar{b}d$</td>
<td>$b\bar{s}; \bar{b}s$</td>
<td>$b\bar{c}; \bar{b}c$</td>
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<tr>
<td>$1^1S_0$</td>
<td>0--</td>
<td>$\eta_c(1S)$</td>
<td>$\eta_b(1S)$</td>
<td>$D$</td>
<td>$D_s^\pm$</td>
<td>$B$</td>
<td>$B_s^0$</td>
<td>$B_c^\pm$</td>
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<tr>
<td>$1^3S_1$</td>
<td>1--</td>
<td>$J/\psi(1S)$</td>
<td>$\Upsilon(1S)$</td>
<td>$D^*$</td>
<td>$D_s^{*\pm}$</td>
<td>$B^*$</td>
<td>$B_s^*$</td>
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<tr>
<td>$1^1P_1$</td>
<td>1--</td>
<td>$h_c(1P)$</td>
<td>$h_b(1P)$</td>
<td>$D_1(2420)$</td>
<td>$D_{s1}(2536)^\pm$</td>
<td>$B_1(5721)$</td>
<td>$B_{s1}(5830)^0$</td>
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<tr>
<td>$1^3P_0$</td>
<td>0++</td>
<td>$\chi_{c0}(1P)$</td>
<td>$\chi_{b0}(1P)$</td>
<td>$D_0^*(2400)$</td>
<td>$D_{s0}^*(2317)^\pm\dagger$</td>
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<tr>
<td>$1^3P_1$</td>
<td>1++</td>
<td>$\chi_{c1}(1P)$</td>
<td>$\chi_{b1}(1P)$</td>
<td>$D_1(2430)$</td>
<td>$D_{s1}(2460)^\pm\dagger$</td>
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<td>$1^3P_2$</td>
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<td>$\chi_{c2}(1P)$</td>
<td>$\chi_{b2}(1P)$</td>
<td>$D_2^*(2460)$</td>
<td>$D_{s2}^*(2573)^\pm$</td>
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<td>$1^3D_1$</td>
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<td>$\psi(3770)$</td>
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<td>$D_{s1}^*(2860)\pm\dagger$</td>
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<tr>
<td>$1^3D_3$</td>
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<td>$D_{s3}^*(2860)^\pm$</td>
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<td>$2^1S_0$</td>
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<td>$\eta_c(2S)$</td>
<td>$\eta_b(2S)$</td>
<td>$D(2550)$</td>
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<tr>
<td>$2^3S_1$</td>
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<td>$\psi(2S)$</td>
<td>$\Upsilon(2S)$</td>
<td>$D_{s1}^*(2700)^\pm\dagger$</td>
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<tr>
<td>$2^1P_1$</td>
<td>1--</td>
<td></td>
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<td>$h_b(2P)$</td>
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<tr>
<td>$2^3P_{0,1,2}$</td>
<td>0++, 1++, 2++</td>
<td>$\chi_{c0,2}(2P)$</td>
<td>$\chi_{b0,1,2}(2P)$</td>
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<td>$\text{@PDG}$</td>
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<tr>
<td>$3^3P_{0,1,2}$</td>
<td>0++, 1++, 2++</td>
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<td>$\chi_b(3P)$</td>
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Set up

- **Gauge Configurations**: HISQ 2+1+1

- **Valence quark propagators**:
  
  -- **Light to charm**: Overlap

  -- **Bottom**: NRQCD with improved coefficients
Overlap Fermions

- **Some desirable features:**

  - **No** $O(a)$ **error.**
  
  - The **effective propagator**:

    $$ (1 - \frac{1}{2} D)D(m)^{-1} = (D_c + ma)^{-1} $$

  - $D_c = D/(1 - D/2)$ is chirally symmetric, i.e., $\{\gamma_5, D_c\} = 0$.
  
  - $D_c + m$ is like in the continuum formalism.
  
  - **Multi-mass algorithm** (more than 20 masses -10-15% overhead)
  
  - Renormalization may be relatively simple (e.g. with chiral Ward identity).

- **Undesirable feature:**

  **Cost**
Overlap fermions on 2+1+1 Flavors
HISQ Configurations

Lattices used for this study:
HISQ gauge configurations from MILC

- $24^3 \times 64$, $a = 0.12$ fm, $m_l/m_s = 1/5$, $m_\pi L = 4.54$, $m_\pi = 305$ MeV
- $32^3 \times 96$, $a = 0.089$ fm, $m_l/m_s = 1/5$, $m_\pi L = 4.5$, $m_\pi = 312$ MeV
- $48^3 \times 144$, $a = 0.058$ fm, $m_l/m_s = 1/5$, $m_\pi L = 4.51$, $m_\pi = 319$ MeV

PHYSICAL REVIEW D 87, 054505 (2013) (MILC)

- HYP smearing on gauge fields
- Both point source and coulomb gauge fixed wall source are used
- No of eigenvectors projected: 350 ($a = 0.012$ fm)
  : 350 ($a = 0.09$ fm)
  : 75 ($a = 0.058$ fm)
Rest mass Vs Kinetic mass

Charm mass is tuned by meson kinetic mass and not from rest mass
……a la FermiLab formulation

Expanding the energy momentum relation in powers of $pa$

$$E(p)^2 = M_1^2 + \frac{M_1}{M_2} p^2 + O(p^4)$$

$$= M_1^2 + c^2 p^2$$

$|p| \ll m_0, 1/a$

Rest mass : $M_1 = E(0)$

Kinetic mass : $M_2 = M_1/c^2$
Dispersion relation (at charm mass)

\[ E^2(p) = E^2(p = 0) + p^2 c^2 \]

Finite momentum wall source is used to project to particular momentum state which reduce errorbars substantially.
Lattice spacings and tuning of charm and strange masses

Lattice spacings are calculated by Omega(sss) mass = 1672 GeV

- $48^3 \times 144 : 0.0582(5)$ fm
- $32^3 \times 96 : 0.0877(10)$ fm
- $24^3 \times 64 : 0.1192(14)$ fm

which are quite consistent with lattice spacings determined by MILC.

- Strange mass is tuned by setting pseudoscalar $ss$ mass at 685 MeV
  - $m_s a = 0.0738 \ (a = 0.0118 \text{fm})$
  - $= 0.048 \ (a = 0.0888 \text{fm})$
  - $= 0.028 \ (a = 0.0582 \text{fm})$

- Charm mass is tuned by
  
  $$\frac{1}{4}(m_{\eta_c} + 3 m_{J/\psi})$$

  - $m_c a = 0.527 \ (a = 0.1192 \text{fm})$
  - $= 0.428 \ (a = 0.0888 \text{fm})$
  - $= 0.29 \ (a = 0.0582 \text{fm})$

Taking $m_s = 100$ MeV
- $m_s a = 0.0450 \ (a = 0.0888 \text{fm})$
- $= 0.0295 \ (a = 0.0582 \text{fm})$

Considering kinetic masses of mesons (a la Fermilab formulation)
Omega(\(ssss\)) effective mass
Lattice spacings and tuning of charm and strange masses

Lattice spacings are calculated by Omega(sss) mass = 1672 GeV

\[ 48^3 \times 144 : 0.0582(5) \text{ fm} \]
\[ 32^3 \times 96 : 0.0877(10) \text{ fm} \]
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which are quite consistent with lattice spacings determined by MILC.

- Strange mass is tuned by setting pseudoscalar $ss$ mass at 685 MeV

\[
m_s a = 0.0738 \ (a = 0.0118 \text{ fm})
= 0.0485 \ (a = 0.0888 \text{ fm})
= 0.028 \ (a = 0.0582 \text{ fm})
\]

Taking \( m_s = 100 \text{ MeV} \)
\[ m_s a = 0.0450 \ (a = 0.0888 \text{ fm}), \]
\[ = 0.0295 \ (a = 0.0582 \text{ fm}) \]

- Charm mass is tuned by

\[ \frac{1}{4} \left( m_{\eta_c} + 3 m_{J/\psi} \right) \]

\[
m_c a = 0.528 \ (a = 0.1192 \text{ fm})
= 0.428 \ (a = 0.0888 \text{ fm}),
= 0.29 \ (a = 0.0582 \text{ fm})
\]

Considering kinetic masses of mesons (a la Fermilab formulation).
NRQCD Action

\[ aH = aH_0 + a\delta H; \quad aH_0 = -\frac{\Delta^{(2)}}{2am_b}, \]

\[ a\delta H = -c_1 \frac{\left(\Delta^{(2)}\right)^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2}(\nabla \cdot \mathbf{E} - \mathbf{E} \cdot \nabla)
- c_3 \frac{1}{8(am_b)^2} \sigma \cdot (\mathbf{\nabla} \times \mathbf{E} - \mathbf{E} \times \mathbf{\nabla})
- c_4 \frac{1}{2am_b} \sigma \cdot \mathbf{B} + c_5 \frac{\Delta^{(4)}}{24am_b} - c_6 \frac{\left(\Delta^{(2)}\right)^2}{16n(am_b)^2}. \]

\( \nabla \) is the symmetric lattice derivative.

\( \Delta^{(2)} \) and \( \Delta^{(4)} \) \( \rightarrow \) \( \sum_i D_i^2 \) and \( \sum_i D_i^4 \)

Tadpole improvement.

Coefficients are improved \( \mathcal{O}(\alpha_s^2 a^2 \nu^4) \)

Dowdall et all, PRD 85,054509,2012
A. Hart et all, PRD 79, 074008 (2009)
\[ aE(P) = aE(0) + \sqrt{a^2 P^2 + a^2 M_{\text{Kin}}^2} \]

\[ aM_{\text{Kin}} = \frac{a^2 P^2 - (a\Delta E)^2}{2a\Delta E} \]

\[ \bar{M}_{\text{Kin}}(1S) = \frac{(3M_{\text{Kin}}(\Upsilon) + M_{\text{Kin}}(\eta_b))}{4} \]
b-quark mass tuning
We have calculated ground state energy spectra of mesons and baryons for all possible quantum numbers with $l$, $s$, $c$ and $b$ quarks.

Decay constants from two points functions are also being evaluated.

Results for charmed-bottom mesons and baryons will be presented.
Pseudoscalar meson mass
Pseudoscalar meson mass from physical to about 6 GeV
Hyperfine splitting in charmonia

![Graph showing hyperfine splitting vs. \(a^2 (fm^2)\)](image-url)
Hyperfine splitting in bottomonia
Hyperfine splitting : $B_s$
$B_C (0^-)$

![Graph showing $E_{sub}$ vs $a^2 (fm^2)$ with data points and error bars.

$E_X^{(sub)} = E_X - \frac{n_c}{2} \overline{E_{c\bar{c}}} - \frac{n_b}{2} \overline{E_{b\bar{b}}}.\)
Prediction: $B_c^*$

- Only pseudoscalar $B_c$ is known experimentally (also possibly $B_c(2s)$?)

- Hyperfine splittings (MeV)
  - $bu : 45$
  - $bs : 48$
  - $bb : 62$
  - $cc : 113$
  - $bc : ?$
$B_c$ – Onium or B meson type?
Prediction: $B_c^*$
Hyperfine splittings in bottom mesons

\[ \Delta(E_{m_q}) = a + b(\log(1/m_q))^2 \]
Triply bottom baryon

\[ E_{X}^{\text{(sub)}} = E_X - \frac{n_c}{2} \overline{E}_{c\bar{c}} - \frac{n_b}{2} \overline{E}_{b\bar{b}}. \]
Hyperfine splittings in charmed-bottomed baryons
Conclusions and Outlooks

- Ground state hadron spectra including all flavors are calculated using NRQCD ($b$) and overlap ($l$, $s$, and $c$) valence quarks on HISQ gauge configurations.
- Basic mass spectra for charmed-bottom mesons and baryons are presented here.
- Proper chiral and continuum limits need to be taken.
- Decay constants are under study.