Testing dynamic stabilization in complex Langevin simulations

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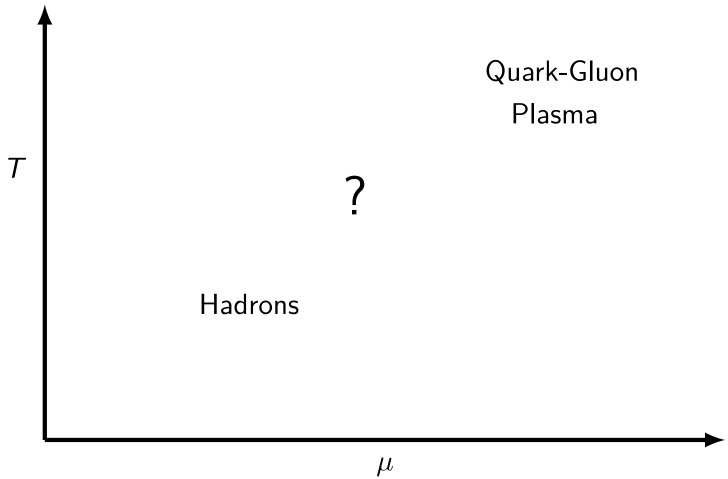
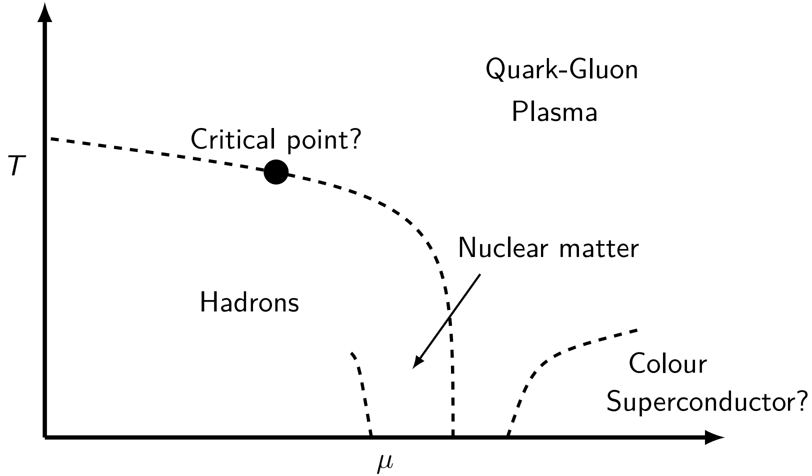
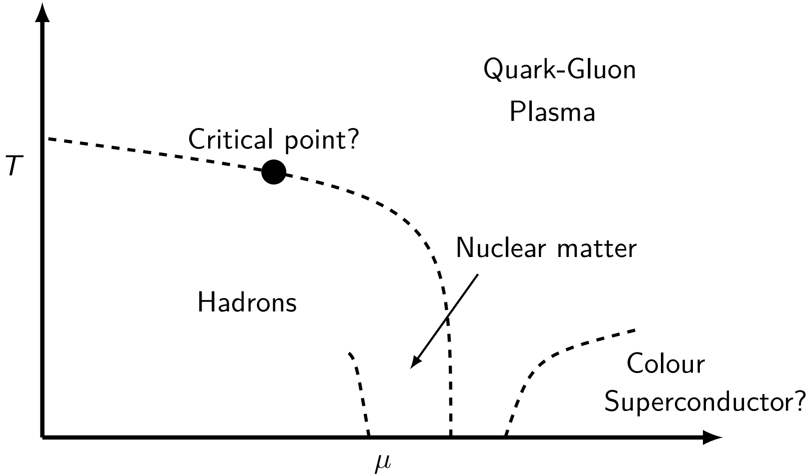
Phase diagram for QCD



Phase diagram for QCD



Phase diagram for QCD



- Sign problem $\leftrightarrow \det D(\mu \neq 0) \in \mathbb{C}$

Complex Langevin simulations

- Complexify degrees of freedom $\mathrm{SU}(3) \to \mathrm{SL}(3,\mathbb{C})$

$$U_{x,\nu} = \exp\left[\mathrm{i}\, a\, \lambda^c \left(A^c_{x,\nu} + \mathrm{i}\, B^c_{x,\nu}\right)\right]$$

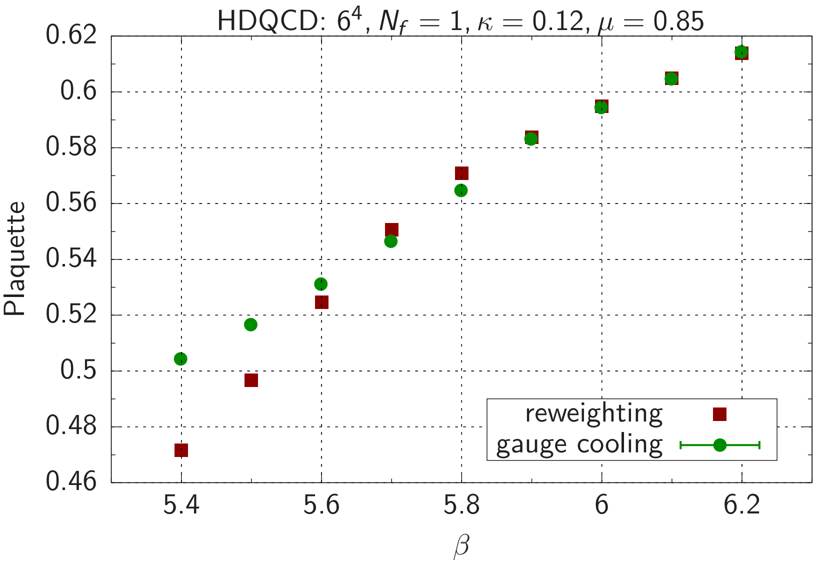
- Evolve links according (1st order) Langevin equation

$$U_{x,\nu}(\theta+\varepsilon) = \exp\left[\mathrm{i}\, \lambda^a \left(\varepsilon\, D^a_{x,\nu}\, S + \sqrt{\varepsilon}\, \eta^a_{x,\nu}\right)\right] U_{x,\nu}(\theta)$$

- Gauge cooling is essential, but sometimes not sufficient..

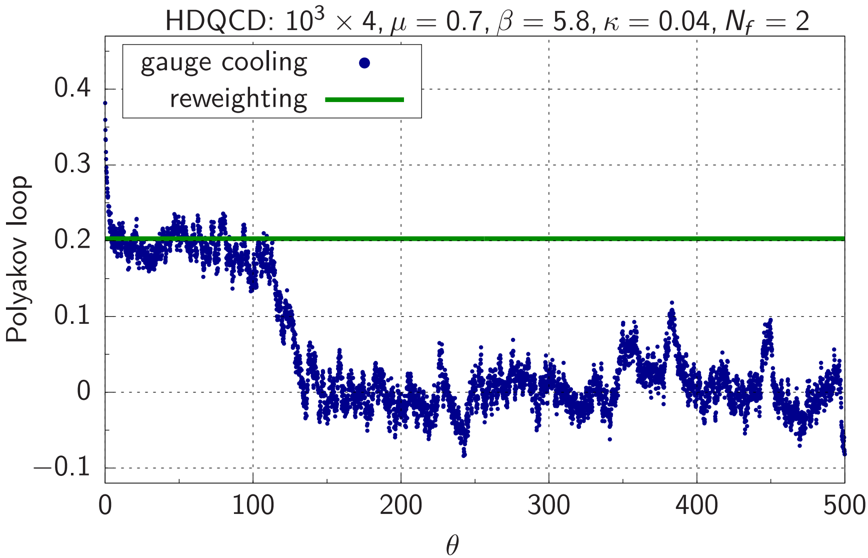
$$U_{x,\nu} = \Omega_x\, U_{x,\nu}\, \Omega^{-1}_{x+\nu}$$

Gauge cooling



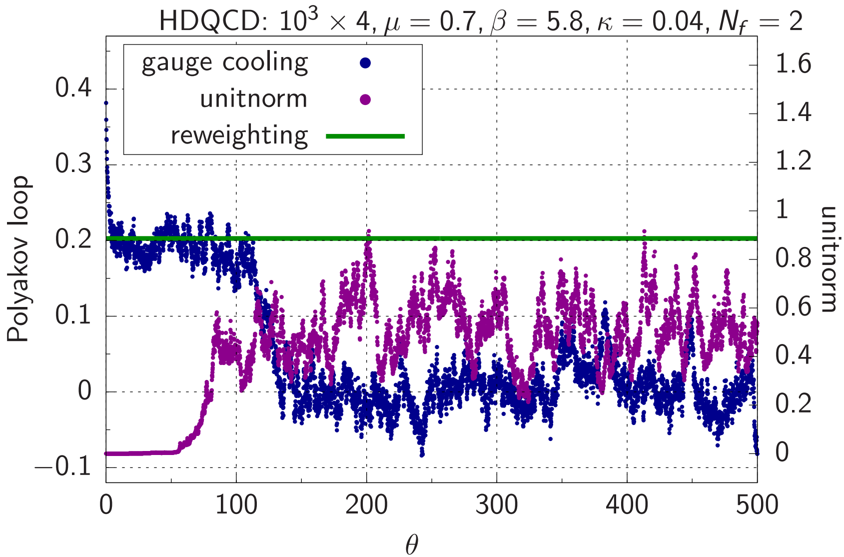
- For small β Complex Langevin results differ from reweighting.

Gauge cooling



- Tunneling to wrong results.

Gauge cooling



- Tunneling to wrong results, when unitnorm grows too large.

Dynamic stablization

- Adding a trivial force to the Langevin dynamics

$$U_{x,\nu}(\theta+\varepsilon) = \exp\left[i\lambda^a \left(\varepsilon\, K^a_{x,\nu} - \varepsilon\, \alpha_{DS}\, M^a_x + \sqrt{\varepsilon}\, \eta^a_{x,\nu}\right)\right] U_{x,\nu}(\theta)$$

  where

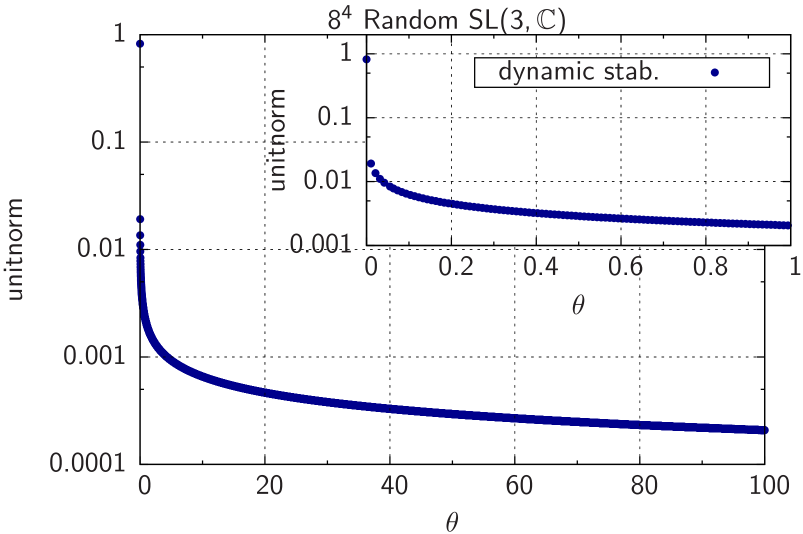
$$M^a_x = b^a_x \left(\sum_c b^c_x\, b^c_x\right)^3 \text{ and } b^a_x = \mathrm{Tr}\Big[\lambda^a \sum_\nu U_{x,\nu} U^\dagger_{x,\nu}\Big].$$

- Expanding the force in terms of gauge fields A and B

$$M^a_x \sim a^7 \left(\overline{B}^c_y\, \overline{B}^c_y\right)^3 \overline{B}^a_x + \mathcal{O}(a^8)\,.$$

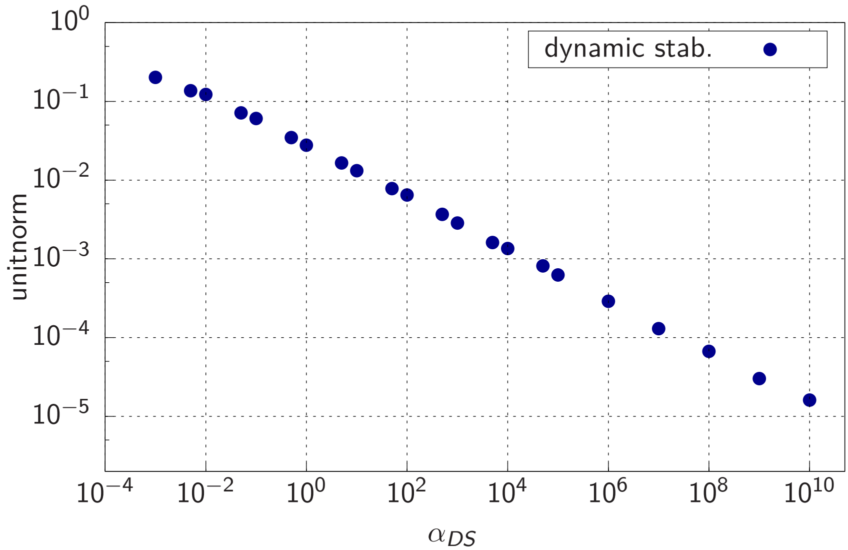
- Dynamic stabilization is numerically cheap and can be combined with gauge cooling (Here: 1 step)

Dynamic stablization



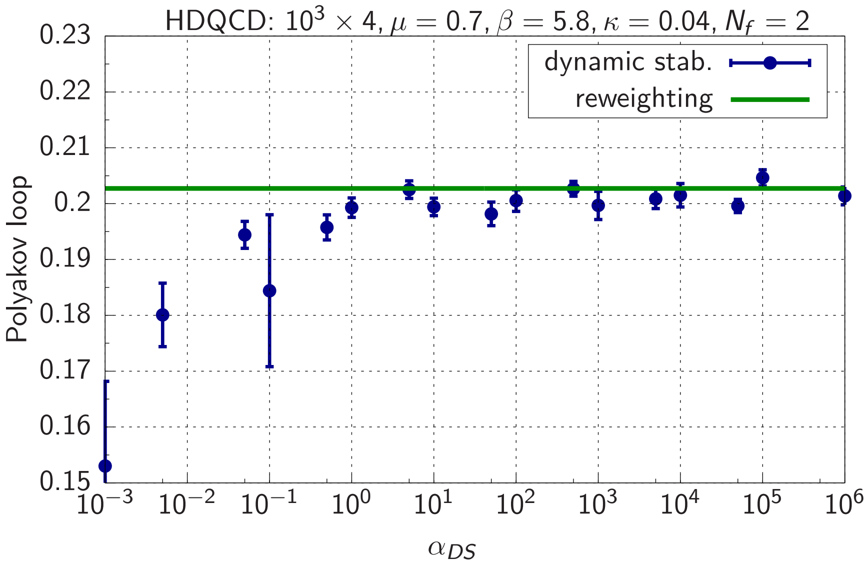
- Start with random $SL(3,\mathbb{C})$ configuration.
- Only Dynamic stablization, no action and noise

Dynamic stablization



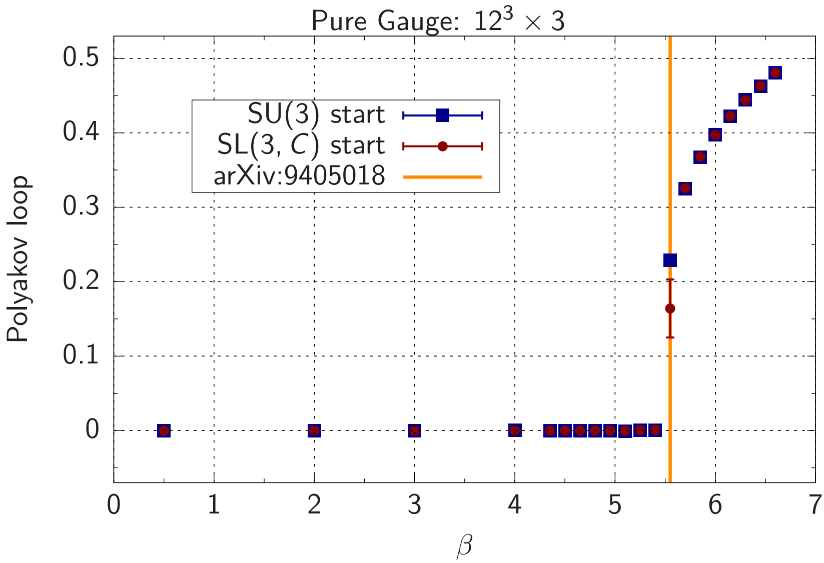
- Strength of dynamic stablization $\rightarrow$ Control of unitnorm

Dynamic stablization



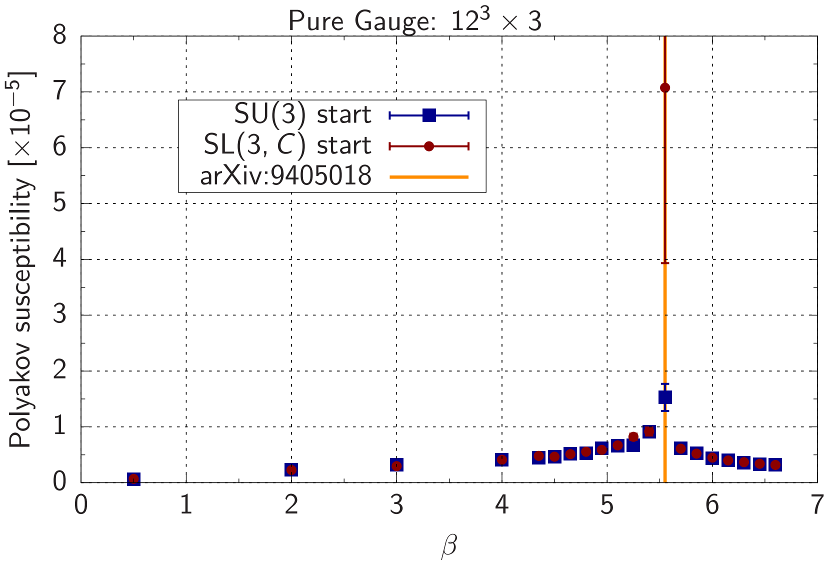
- For suffient large α_{DS} we obtain the correct result.

Pure Gauge



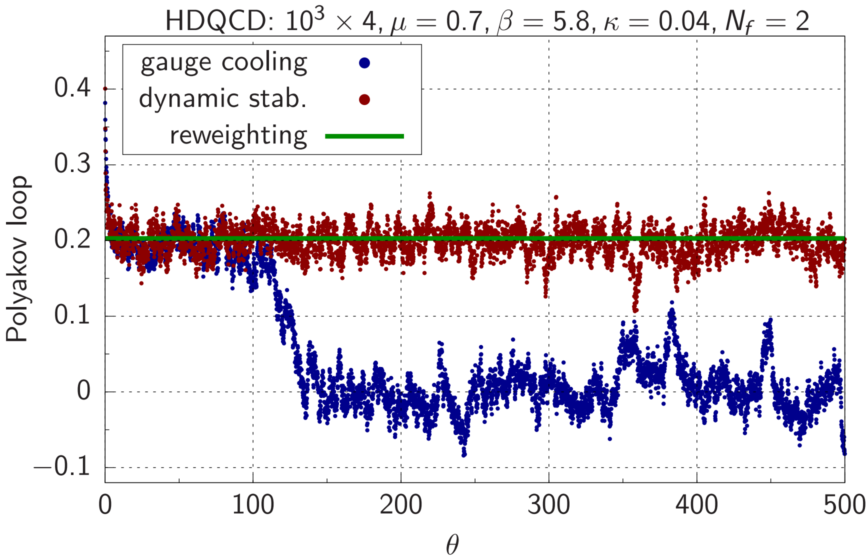
- The correct transition is obtained, even for $SL(3, \mathbb{C})$ start.

Pure Gauge



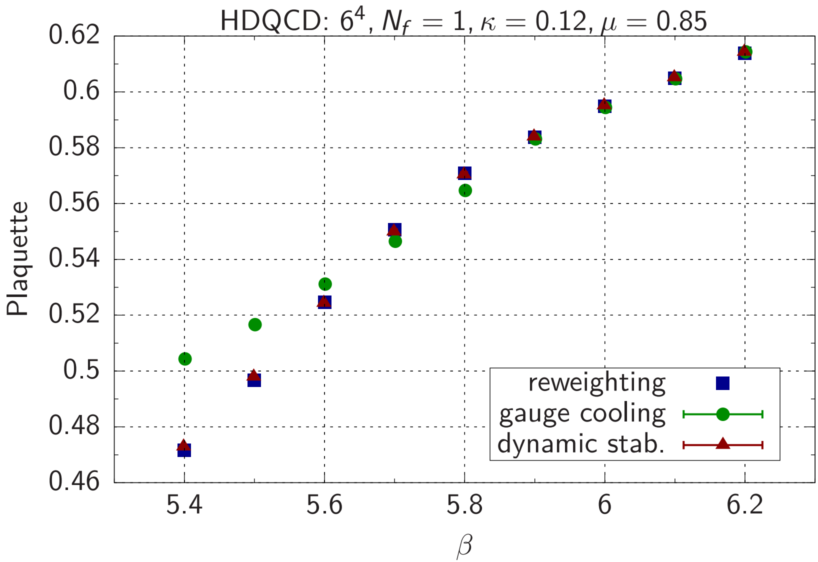
- The correct transition is obtained, even for $SL(3, \mathbb{C})$ start.

Heavy Dense QCD



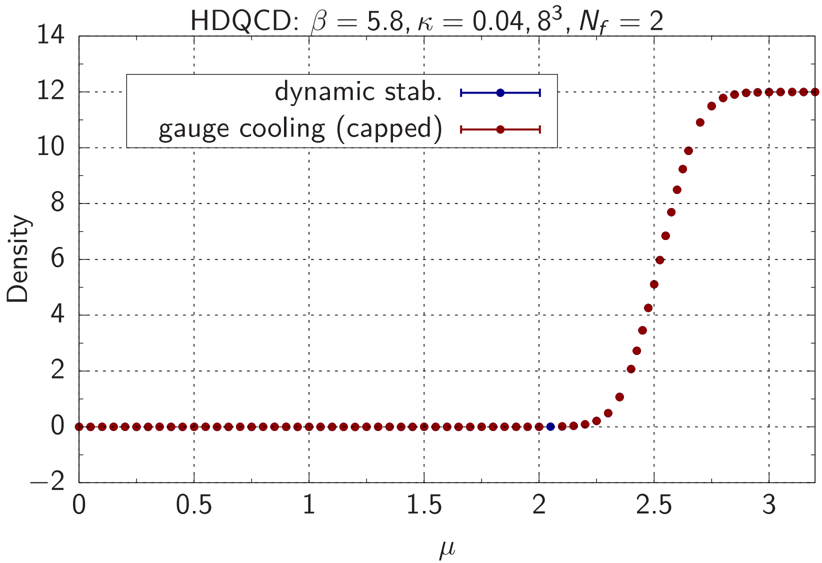
- Improved stability using dynamic stablization

Heavy Dense QCD



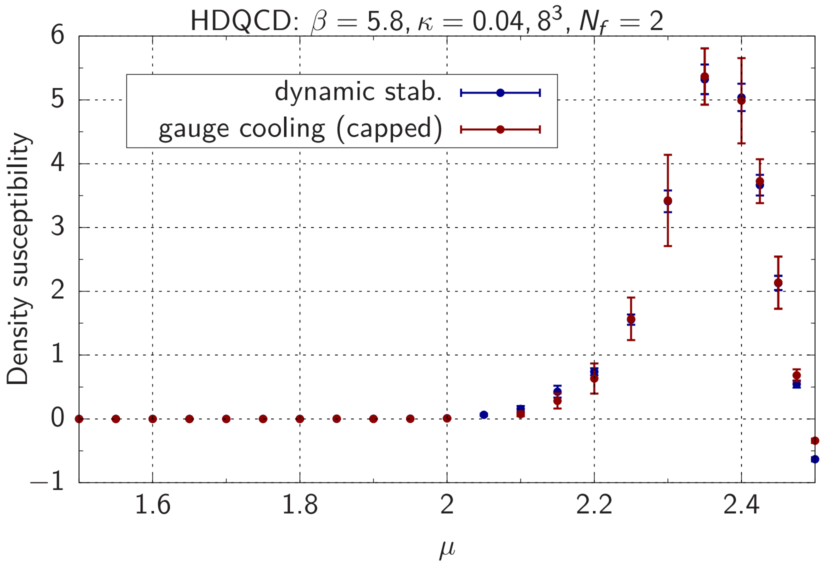
- Correct values recovered for all gauge couplings.

Heavy Dense QCD



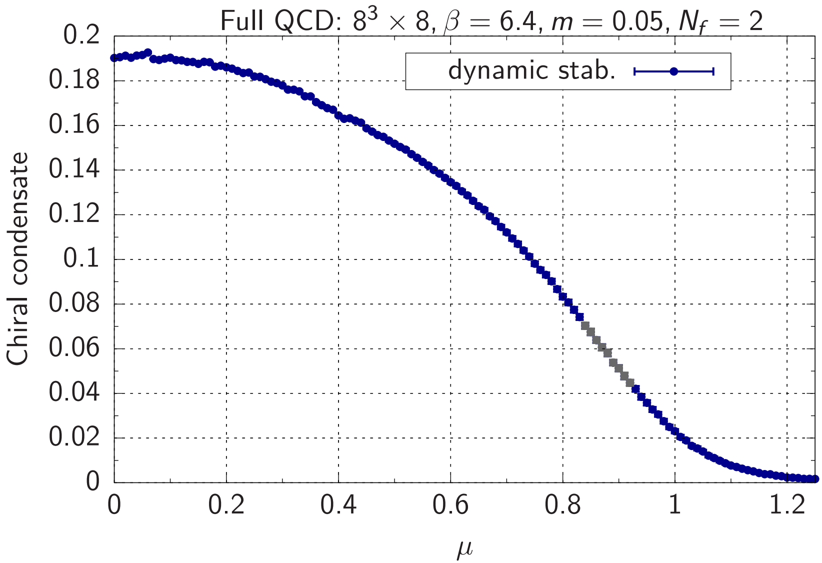
- Comparison of with capped (unitnorm < 0.03) gauge cooling

Heavy Dense QCD



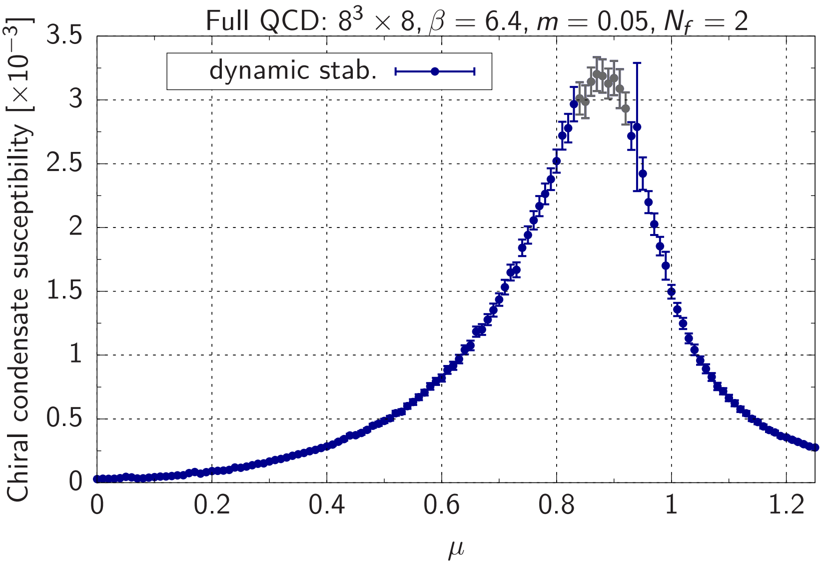
- Comparison of with capped (unitnorm < 0.03) gauge cooling

Last but not least - Full QCD



- Unimproved Staggered quarks

Last but not least - Full QCD



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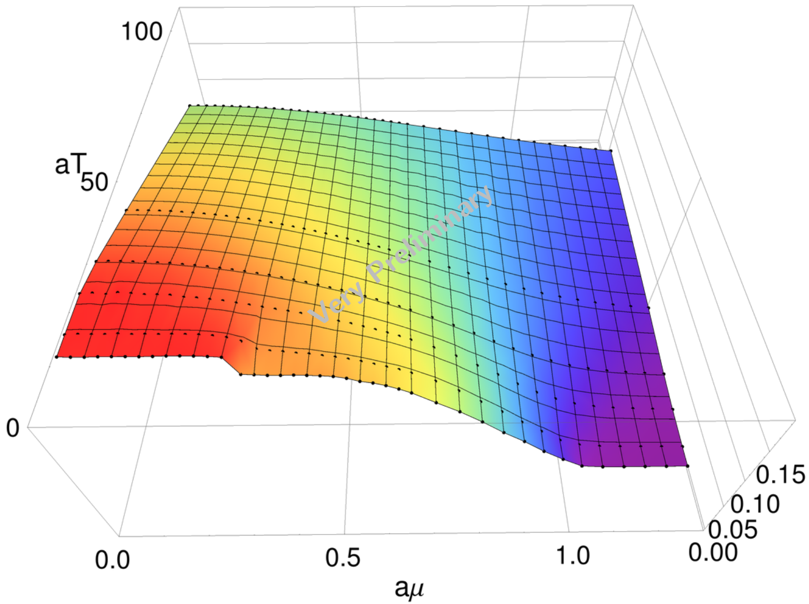


- Unimproved Staggered quarks

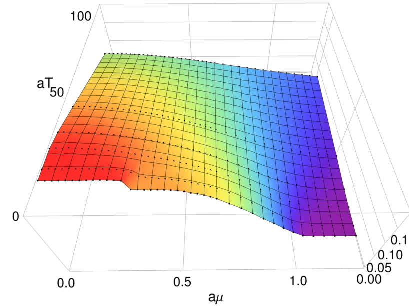
Future work

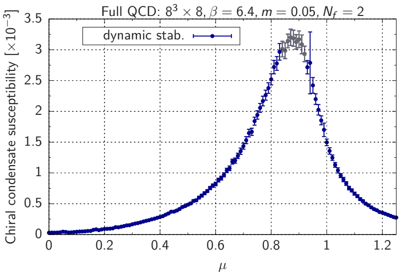
Conclusion

- Dynamical stabilisation allows to control the excursion into imaginary directions of $SL(3, \mathbb{C})$
- Dynamical stabilisation improves convergence

Future work

- More tests to check correct behaviour
- Start Full QCD simulations to identify phase structure of QCD.
- Extend simulations to real time QCD ($e^{iS_{QCD}}$).





Thank you for your attention!