Gradient flow running coupling: SU(2) with 6 fundamental flavors

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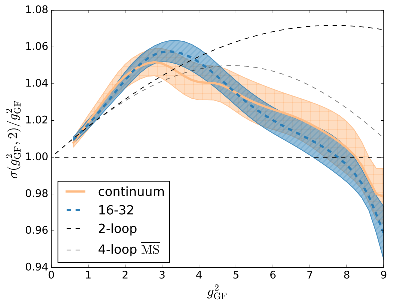
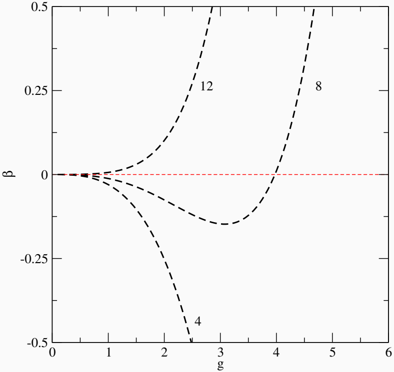
Motivation

- Nearly conformal theories can have walking behavior needed by technicolor
- SU(2) with 8 massless fermions has a fixed point [1]



- Previous studies at N_f=6 inconclusive [2] [3] [4]
- 4-loop $\overline{\mathrm{MS}}$ IRFP $g^2 \sim 30$



[1] V. Leino et al. Lattice 2015 (hep-lat/1511.03563) ,
[2] T. Karavirta et al. JHEP 1205 (2012) 003 (hep-lat/1111.4104) ,
[3] T. Appelquist et al. Phys. Rev. Lett. 112, 111601 (2014) (hep-lat/1311.4889)
[4] M. Hayakawa et al. Phys. Rev. D 88, 094504 (2013) (hep-lat/1307.6997)

Model

- HEX-smeared[1] Wilson-clover action
- Schrödinger functional
  - Use trivial (Dirichlet) boundaries (no background field)
  - Used to reach zero mass (Tune the κ_{cr} at $L = 24$)
  - Allows the measurement of mass anomalous dimension
- Lattice sizes: 8,12,16,18,20,24,30,(36)
  - Use step scaling step $s = 3/2$ (8-12, 12-18, 16-24, 20-30)
  - Can compare to $s = 2$ at 8-16 and 12-24
- β between 8 and 0.5
- We run into bulk phase transition at $\beta < 0.5$
- Smaller lattices $\sim 80\,000$ trajectories, larger $\sim 15\,000$

[1] S. Capitani, S. Durr and C. Hoelbling, JHEP 0611 (2006) 028

Gradient Flow

- Use the gradient flow [1] [2]

$$g_{GF}^2 = \frac{t^2}{\mathcal{N}} \langle E(t + \tau_0 a^2) \rangle$$

- Flow can be evolved using both Wilson plaquette (W) and Lüscher-Weisz (LW) actions
- Energy can be measured with both clover and plaquette definitions
- We use LW and clover unless otherwise specified
- Fix flow time t to L by setting scale: $c = \sqrt{8t}/L = 0.3$
- Use τ_0 correction to tune down the a^2 effects [3]
- Measuring also the topological charge:

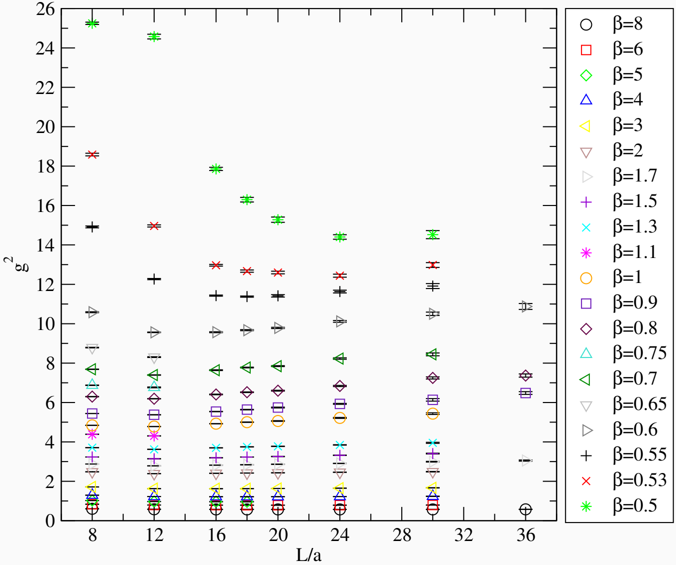
$$Q = \frac{1}{32\pi^2} \sum_x \epsilon_{\mu\nu\alpha\beta} G^a_{\mu\nu}(x;t) G^a_{\alpha\beta}(x;t)$$

[1] M. Luscher and P. Weisz , JHEP 1102 (2011) 051 (hep-th/1101.0963) ,

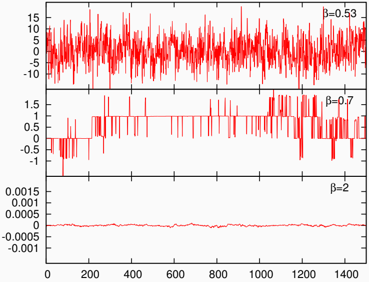
[2] P. Fritzsch and A. Ramos , JHEP 1310 (2013) 008 (hep-lat/1301.4388) ,

[3] A. Cheng, A. Hasenfratz, Y. Liu, G. Petropoulos and D. Schaich. JHEP 1405 (2014) 137 (hep-lat/1404.0984)

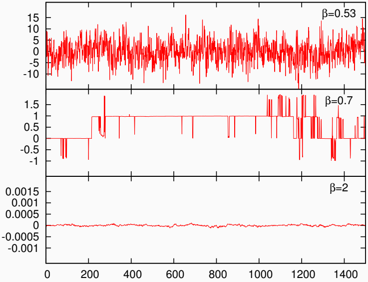
Measured couplings



Topology



L-W evolved flow



Wilson evolved flow

- Topology frozen at small couplings, becomes unfrozen at largest couplings
- LW evolved flow fluctuates more
- Don't use configurations that are frozen to nonzero values
- Projecting $\delta_{Q,0}$ [1] could work, but for $N_f = 8$ effects were small

[1] P. Fritzsch et al. PoS Lattice **2013**, 461 (2014) (hep-lat/1311.7304)

Step scaling function

- Interpolate couplings using a rational function, $m = 7$, $n = 2$

$$g^2_{GF}(g^2_0, L/a, t) = g^2_0 \frac{1 + \sum_{i=1}^m a_i g_0^{2i}}{1 + \sum_{j=1}^n b_j g_0^{2j}}.$$

- Estimate systematic errors by changing the fit parameters
- Step scaling function:

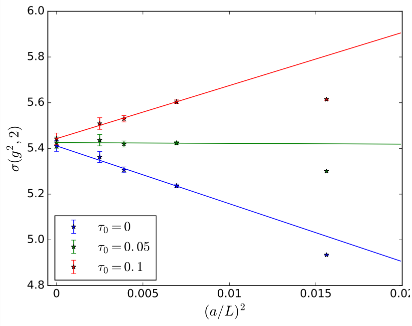
$$\Sigma(u, s, a/L) = g^2_{GF}(g_0, s\frac{L}{a})\Big|_{g^2_{GF}(g_0, \frac{L}{a}) = u},\ \sigma(u, s) = \lim_{a/L \to 0} \Sigma(u, s, a/L)$$

- Extrapolate to continuum limit:

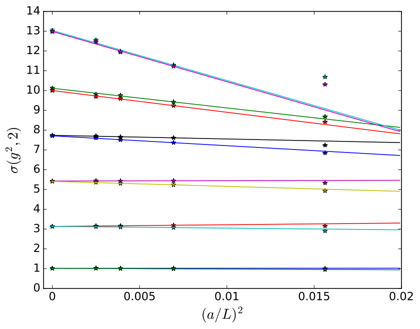
$$\Sigma(u, s, a/L) = \sigma(u, s) + c(u)(\frac{L}{a})^{-2}$$

- Fix τ_0 to minimize a^2 effects

Fixing τ_0



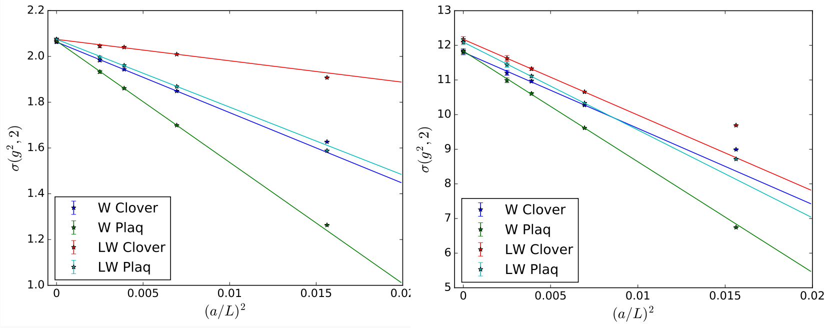
$u = 5$



$\tau_{\rm optimal}$ vs. $\tau_0 = 0$

- Drop the smallest lattice from continuum extrapolation
- Estimate: $\tau_{\rm optimal} = 0.012 \log(1 + 20g^2)$ (Preliminary)
- Logarithm makes sure the τ_0 doesn't grow too large

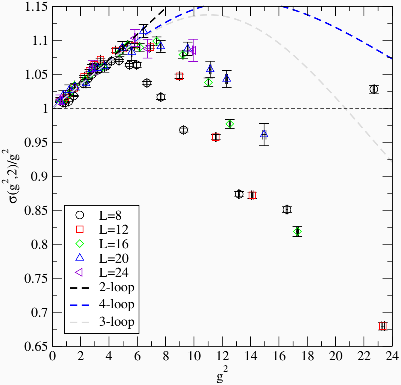
Different discretizations $c = 0.3\,,\ \tau_0 = 0$



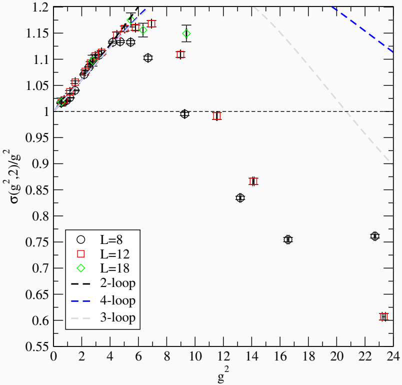
$u = 2$ $\qquad\qquad$ $u = 11$

- Plaquette and Clover agree on continuum limit, plaquette has stronger discretization effects
- LW and W diverge slightly on large couplings, W has stronger discretization effects

Step scaling on the lattice $c = 0.3$

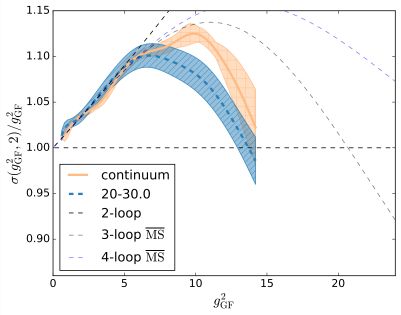


$s = 3/2$

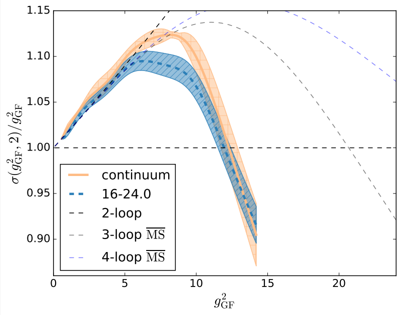


$s = 2$

Continuum limit



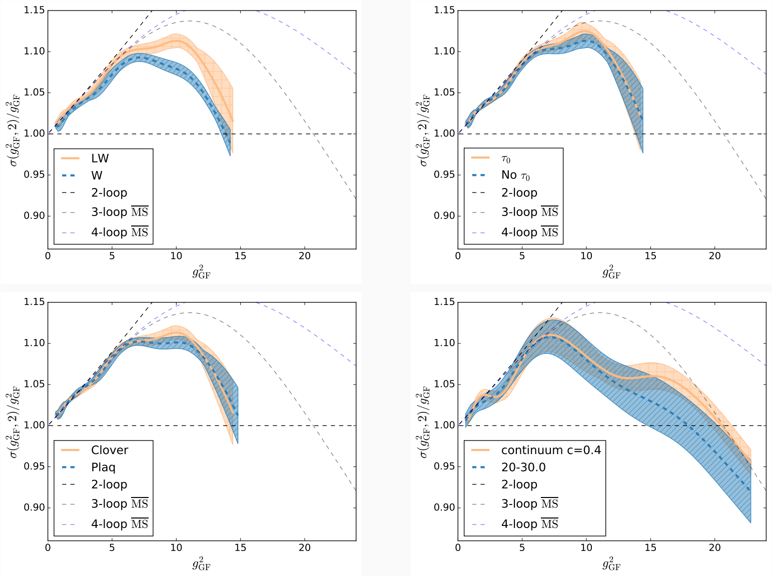
12 – 30



8 – 24

- L20-30 has less statistics than L8-12
- L8-12 behaves oddly on strong coupling and L8 was not used when defining τ_0

Effects of parameters



Mass anomalous dimension

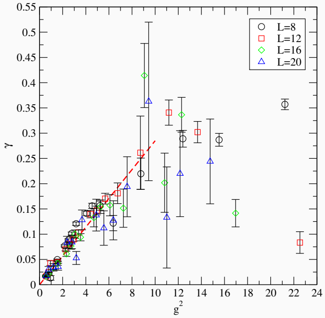
- Schrödinger functional pseudoscalar density renormalization constant allows calculation of γ [1]
- Interpolate Z_P with $Z_P = 1 + \sum_{i=1}^{5} a_i g_0^{2i}$
- Near fixed point approximate as γ^*

$$Z_P(g_0, \frac{L}{a}) = \frac{\sqrt{N f_1}}{f_p(\frac{1}{2}\frac{L}{a})}, \qquad \Sigma_P(u, \frac{a}{L}) = \left.\frac{Z_P(g_0, \frac{sL}{a})}{Z_P(g_0, \frac{L}{a})}\right|_{g_{GF}^2 = u}$$

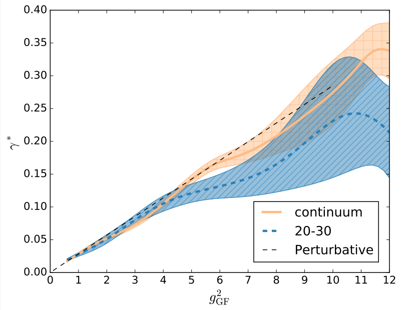
$$\sigma_P(g^2) = \lim_{a \to 0} \Sigma_P(g^2, \frac{a}{L}), \qquad \gamma^* = -\frac{\log \sigma_P(g^2)}{\log s}$$

[1] S. Capitani, M. Luscher, R. Sommer and H. Witting *Nucl. Phys.* B 544 (1999) (hep-lat/9810063)

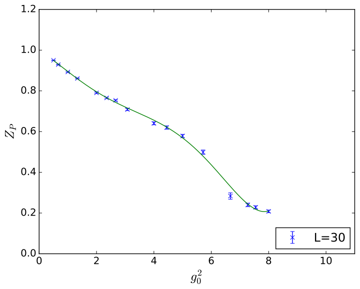
γ^*



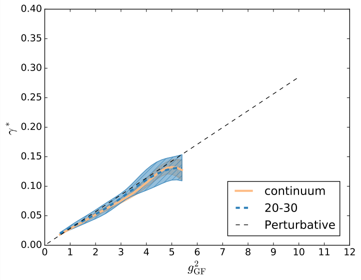
γ
g²
L=8
L=12
L=16
L=20



γ*
g²_GF
continuum
20-30
Perturbative



Z_P
g²_0
L=30



γ*
g²_GF
continuum
20-30
Perturbative

Conclusions

- Finite volume GF step scaling works at strong coupling
- This choice of action, boundaries and smearing allows us to reach relatively small β before running into a bulk phase transition
- Topological freezing mostly problem only on certain range of β's
- $N_f = 6$ seems to approach a IRFP around $g^2 \sim 15$
- Check also the posters:
  - γ with spectral density method – Joni Suorsa
  - Spectrum of $N_f = 2, 4, 6, 8$ – Sara Tähtinen