Determination of charm quark mass from temporal moment of charmonium correlator with Mobius Domain Wall fermion

[arxiv:1606.01002]

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◇ Charm quark mass from the temporal moment

- Determination of charm quark mass with $\delta m_c < 1.0\%$

◇ New calculation with Domain-Wall fermion

(at a: 0.083 ~ 0.044 fm).

Previous works (HPQCD 2008-14)	This work
Staggered fermion	Domain-Wall fermion
Lattice spacings: 0.15 ~ 0.06 fm	0.083 ~ 0.044 fm

- $m_c(m_c) = 1.272(10)$ GeV
- Truncation of perturbative expansion is a significant source of uncertainty in moment method

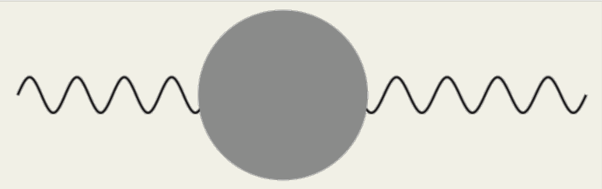
◇ Update since Lattice 2015

- Estimations of perturbative truncation error
- Consistency check with the experimental data (Vector)
- Improved measurement with the Z_2 noize source
  $\Longrightarrow$ Less statistical error
- Leading effect subtracted using an effective theory
  $\Longrightarrow$ Flatter continuum extrapolation

What is the Moment?

◇ Correlator and moment

Current correlator



$$q^2\Pi(q^2) = i\int \mathrm{d}x e^{iqx}\langle j_5(x) j_5(0)\rangle$$

Moment: Derivative in terms of q^2

$$g_{2n} = \frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(q^2\Pi(q^2)\right)_{q^2=0}$$

Perturbative expansion available as a function of $m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}}$.

[K. G. Chetyrkin et al. (2006)]
[R. Boughezal et al. (2006)]
[A. Maier et al. (2009)]

Moment on the lattice

- Coordinate space $i\int \mathrm{d}x \frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \mathrm{e}^{iqt} \longrightarrow a^4 \sum_x t^{2n}$

Correlator $G(t)$ on the lattice

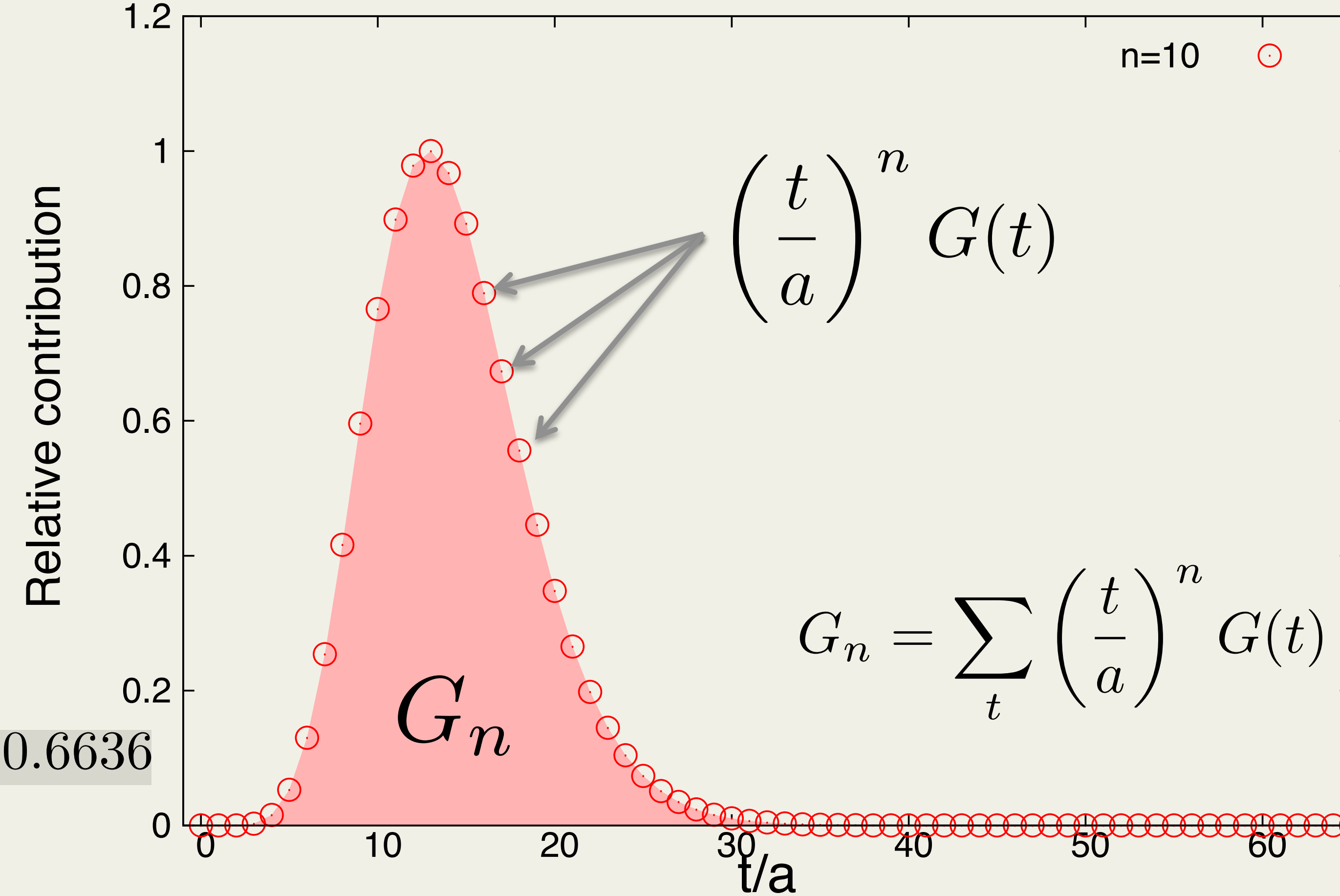
$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0c})^2 \langle j_5(x) j_5(0) \rangle$$

- Moment is easily calculated from $G(t)$

$$G_n = \sum_t \left(\frac{t}{a}\right)^n G(t)$$

What's the Moment?

- Typical energy scale of the moment $G_m \sim m/n$



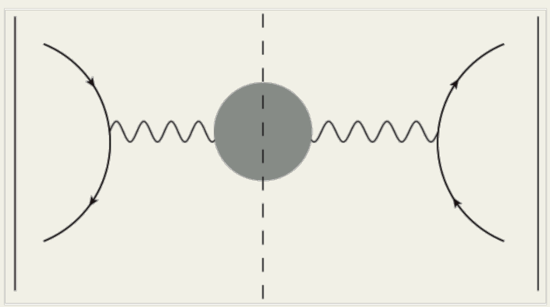
$am_{\eta_c} \sim 0.6636$

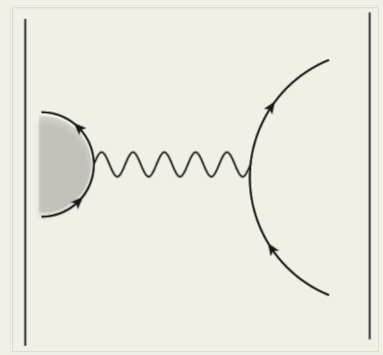
HIGH ⟵ Energy scale ⟶ LOW

What's the Moment? (From comparison with experiment)

- Vector moment can be measured in the experiment.

$$\frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(\Pi(q^2)\right)_{q^2=0} = \frac{1}{12\pi Q_f^2}\int \mathrm{d}s \frac{1}{s^{n+1}} R(s)$$



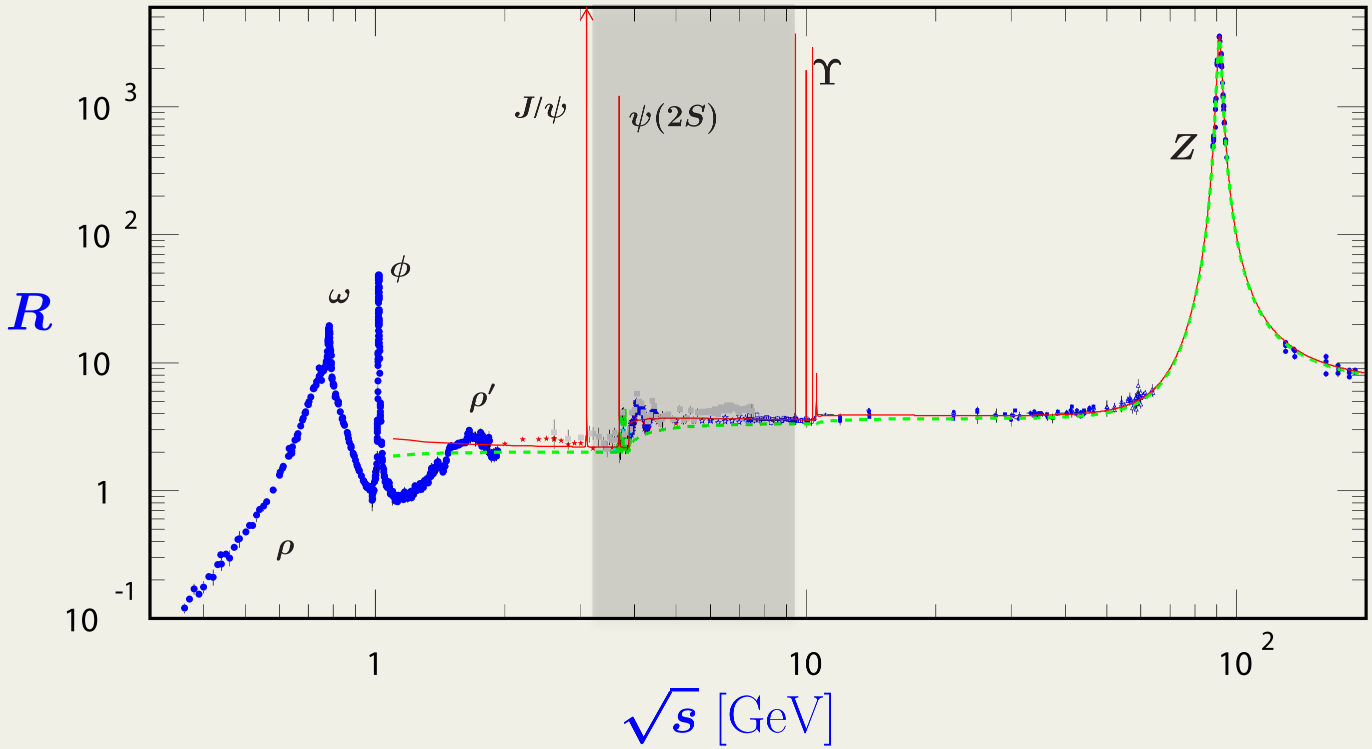


- Moment is a weighted integral of the R-ratio.

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadron})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

What's the Moment? (From comparison with experiment)

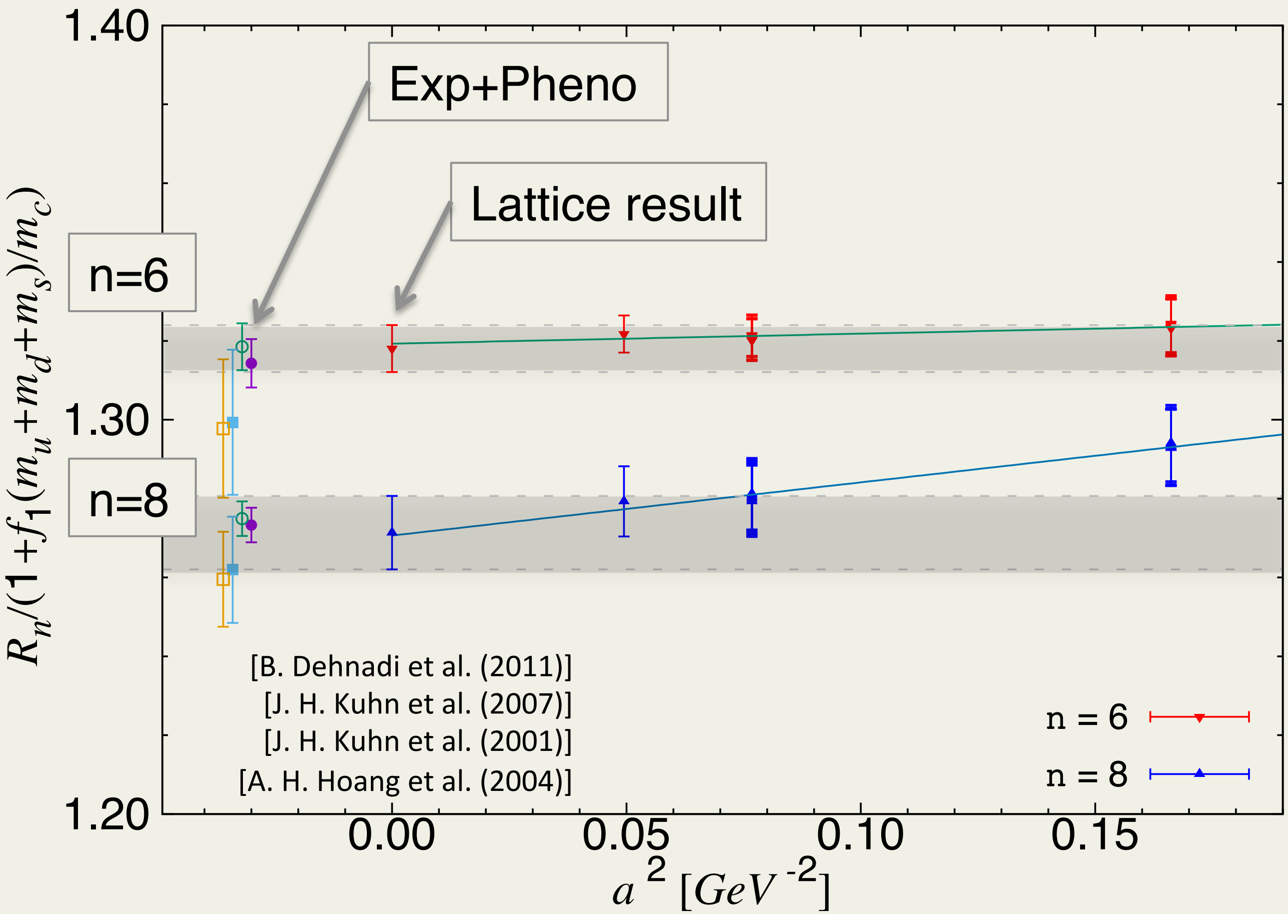
- Vector moment can be measured in the experiment.



- Moment is a weighted integral of the R-ratio.

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadron})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Consistency with experiment (Vector)



- Good agreement with the experimental + phenomenological analysis.

m_c Extraction

Correspondence between lattice and continuum

$$G_n^{(\mathrm{Lat})} = \frac{g_n^{(\mathrm{cont})}(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})}{(am_c^{\overline{\mathrm{MS}}})^{n-4}}$$

- Define the reduced moment R_n
  with G_n and $G_n^{(0)}$, the counterparts at the tree-level,

Lattice

$$R_n = \frac{am_{\eta_c}}{2am_c}\left(\frac{G_n}{G_n^{(0)}}\right)^{\frac{1}{n-4}}$$

Continuum

$$r_n = \left(\frac{g_n}{g_n^{(0)}}\right)^{\frac{1}{n-4}}$$

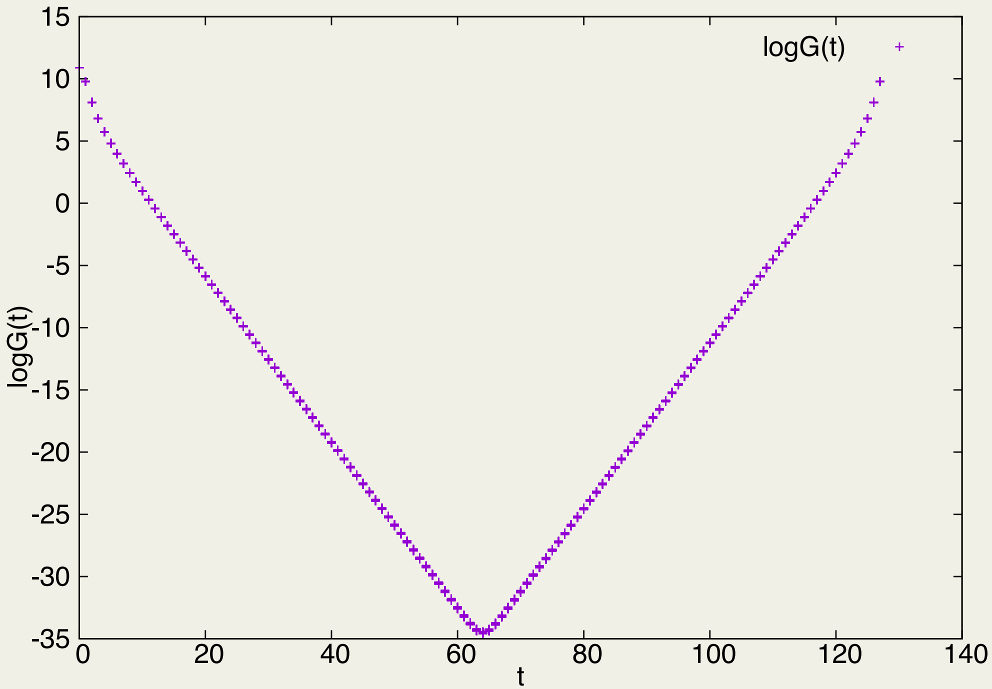
$$R_n = \frac{m_{\eta_c}^{\mathrm{exp}}}{2m_c^{\overline{\mathrm{MS}}}} r_n(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})$$

Lattice setup

- Mobius Domain Wall Fermion formalism
- $n_f = 2 + 1$ in the sea;
  3 different lattice spacings for taking continuum limit.

β	a^{-1} [GeV]	$L \times T$	L_5	am_{ud}	am_s	congfigs	m_π [MeV]	$m_\pi L$
4.17	2.4531(40)	$32^3 \times 64$ ($L = 2.6$ fm)	12	0.0035	0.040	300	230	3.0
				0.007	0.030	300	310	4.0
				0.007	0.040	300	310	4.0
				0.012	0.030	300	400	5.2
				0.012	0.040	300	400	5.2
				0.019	0.030	300	500	6.5
				0.019	0.040	300	500	6.5
		$48^3 \times 96$ ($L = 3.9$ fm)	12	0.0035	0.040	401	230	4.4
4.35	3.6097(89)	$48^3 \times 96$ ($L = 2.6$ fm)	8	0.0042	0.0180	300	300	3.9
				0.0042	0.0250	300	300	3.9
				0.0080	0.0180	301	410	5.4
				0.0080	0.0250	297	410	5.4
				0.0120	0.0180	298	500	6.6
				0.0120	0.0250	300	500	6.6
4.47	4.4961(92)	$64^3 \times 128$ ($L = 2.8$ fm)	8	0.0030	0.015	397	280	4.0

Moment on the lattice



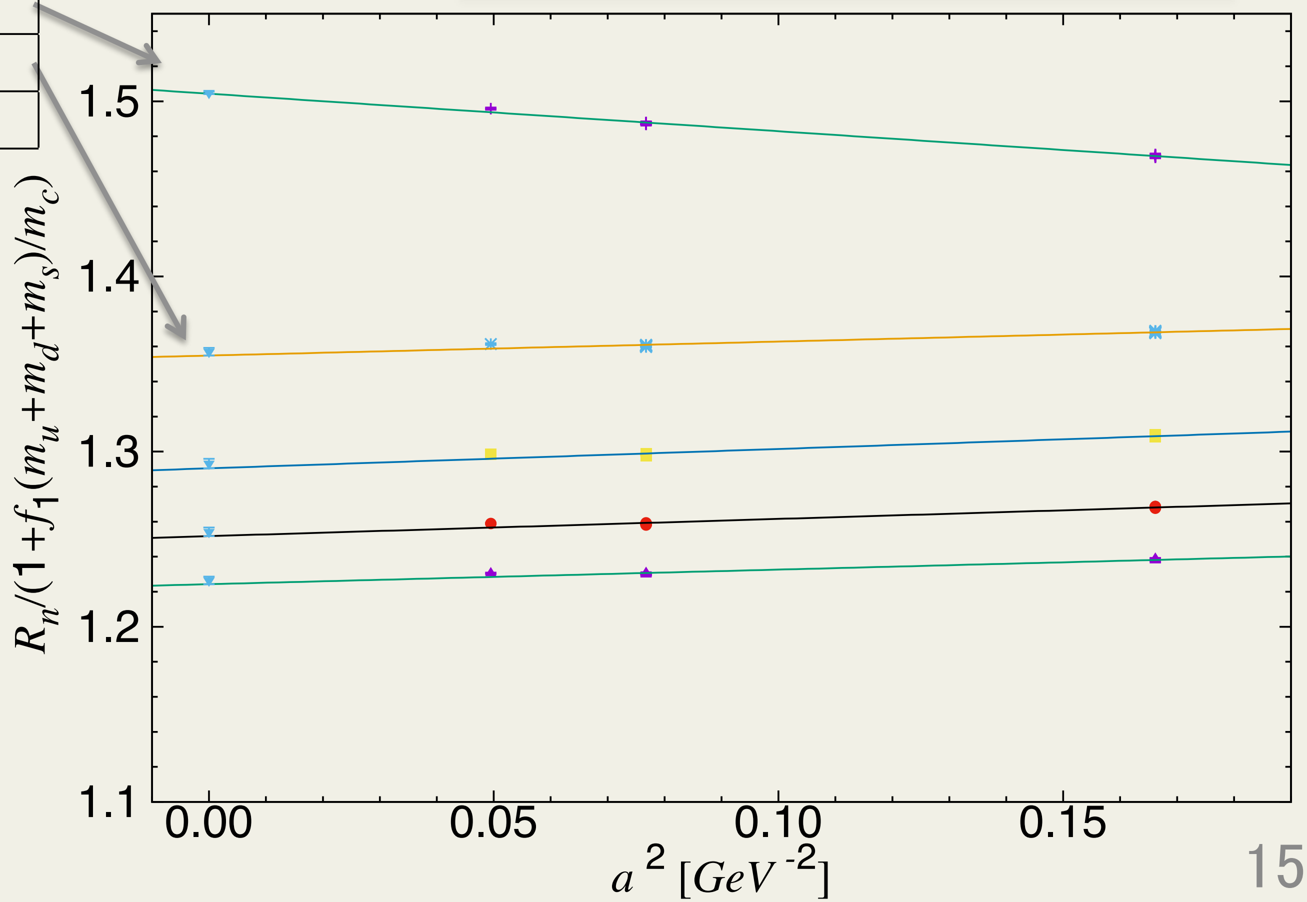
- Construct moments from G(t), $G_n = \sum_t \left(\frac{t}{a}\right)^n G(t)$
- Interpolate to the physical point by tuning $(3m_{J/\psi} + m_{\eta_c})/4$
- Chiral extrapolation

Extrapolation to contimuum ($a = 0$)

$$R_n(a) = R_n(0)\left(1 + c_1 (am_c)^2\right) \times \left(1 + f_1 \frac{m_u + m_d + m_s}{m_c}\right)$$

Essentially flat (~1) term

	(Stat.)($O(a^4)$)(Vol.)
R_6	1.5048(5)(5)(4)
R_8	1.3570(4)(22)(3)
R_{10}	1.2931(4)(27)(5)



Possible systematic errors

$$R_n = \frac{m_{\eta_c}^{\exp}}{2m_c^{\overline{\rm MS}}} r_n(m_c^{\overline{\rm MS}}, \alpha_{\overline{\rm MS}})$$

(1): Truncation error from perturbative expansion of r_n

(2): Input meson mass $m_{\eta_c}^{\exp}$ error

After correcting for… (a) Electromagnetic effect,
(b) Disconnected diagram contributions.

(3): Gluon condensate contribution

(see Backup slides)

Error ❶ Perturbative truncation

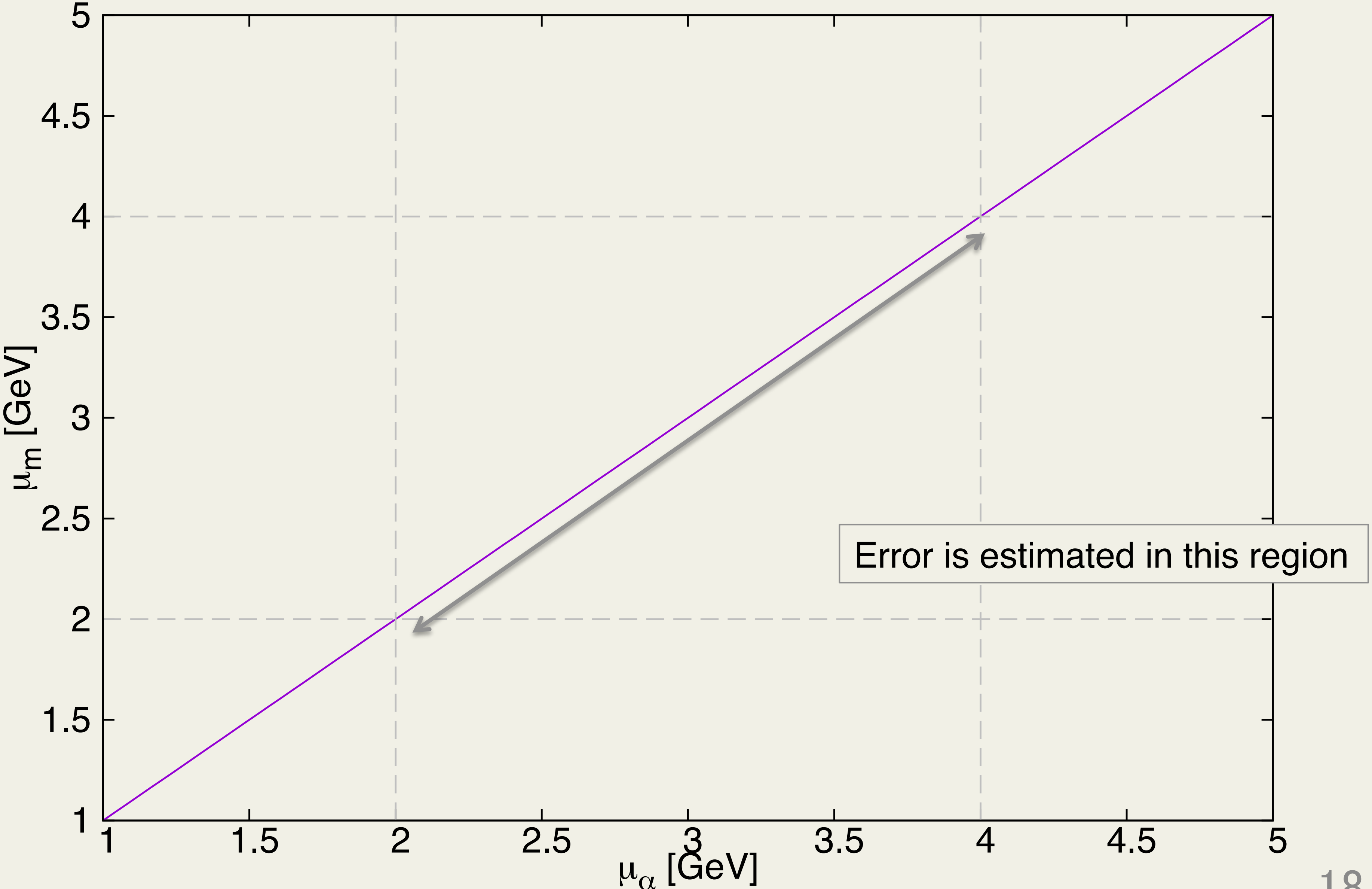
$$R_n = \frac{m_{\eta_c}^{\mathrm{exp}}}{2m_c^{\overline{\mathrm{MS}}}} r_n(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})$$

- $\dfrac{r_n(\mu)}{m_c^{\overline{\mathrm{MS}}}(\mu)}$ should depend on the renormalization scale μ at all order of perturbative series.

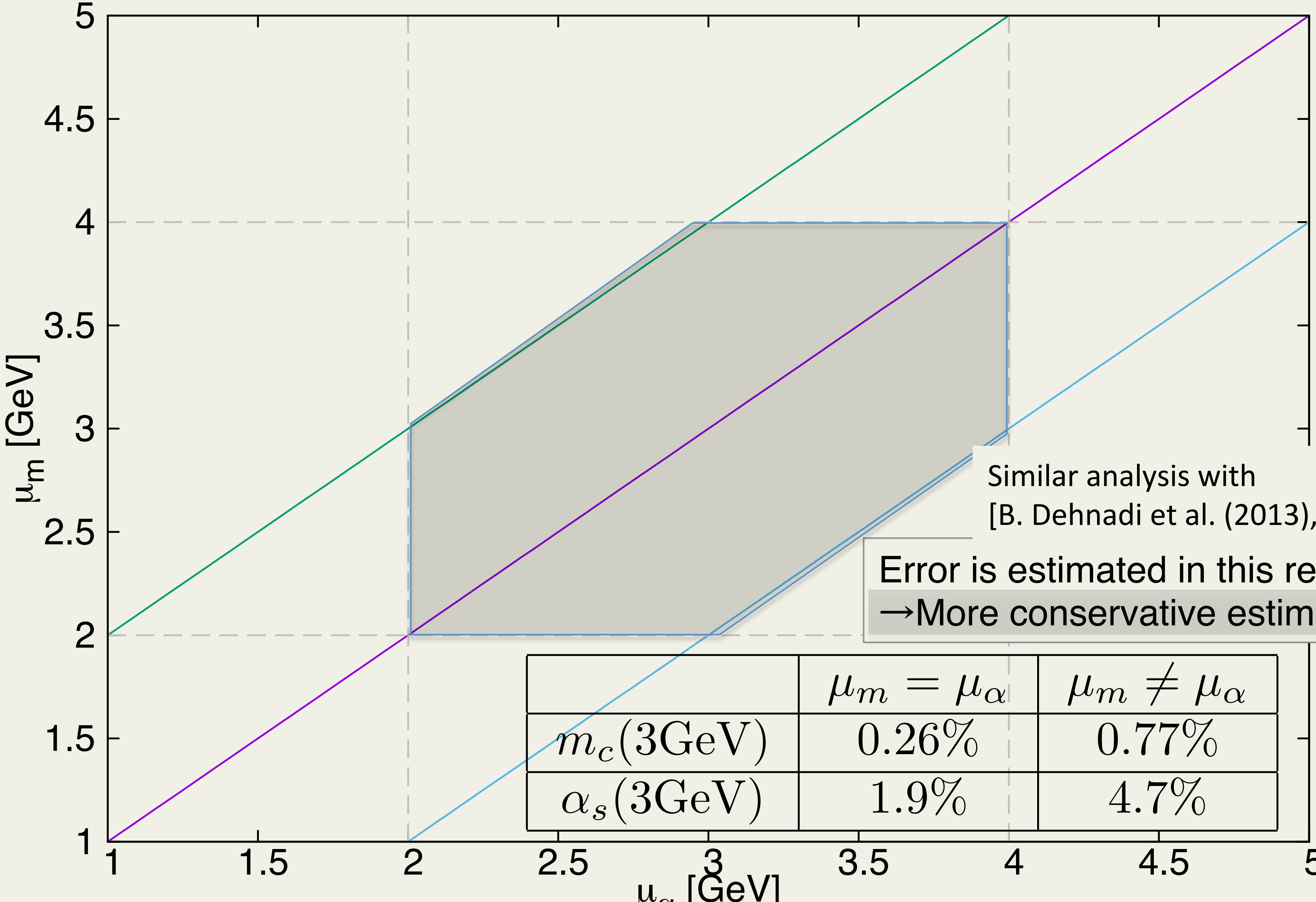
$$\begin{aligned}(r_6)^2 &= 1 \\ &+ \left(3.9 + 2.0 \log\frac{m_c(\mu)^2}{\mu^2}\right)\frac{\alpha_s}{\pi} \\ &+ \left(13.6 + 3.0 \log\frac{m_c(\mu)^2}{\mu^2} - 0.08\left(\log\frac{m_c(\mu)^2}{\mu^2}\right)^2\right)\left(\frac{\alpha_s}{\pi}\right)^2 \\ &+ \left(13.2 + 14.2 \log\frac{m_c(\mu)^2}{\mu^2} + 1.03\left(\log\frac{m_c(\mu)^2}{\mu^2}\right)^2 + 0.06\left(\log\frac{m_c(\mu)^2}{\mu^2}\right)^3\right)\left(\frac{\alpha_s}{\pi}\right)^3\end{aligned}$$

- Estimate the truncation error from μ dependence of $\dfrac{r_n(\mu)}{m_c^{\overline{\mathrm{MS}}}(\mu)}$

Estimation with $\mu_m = \mu_\alpha$.



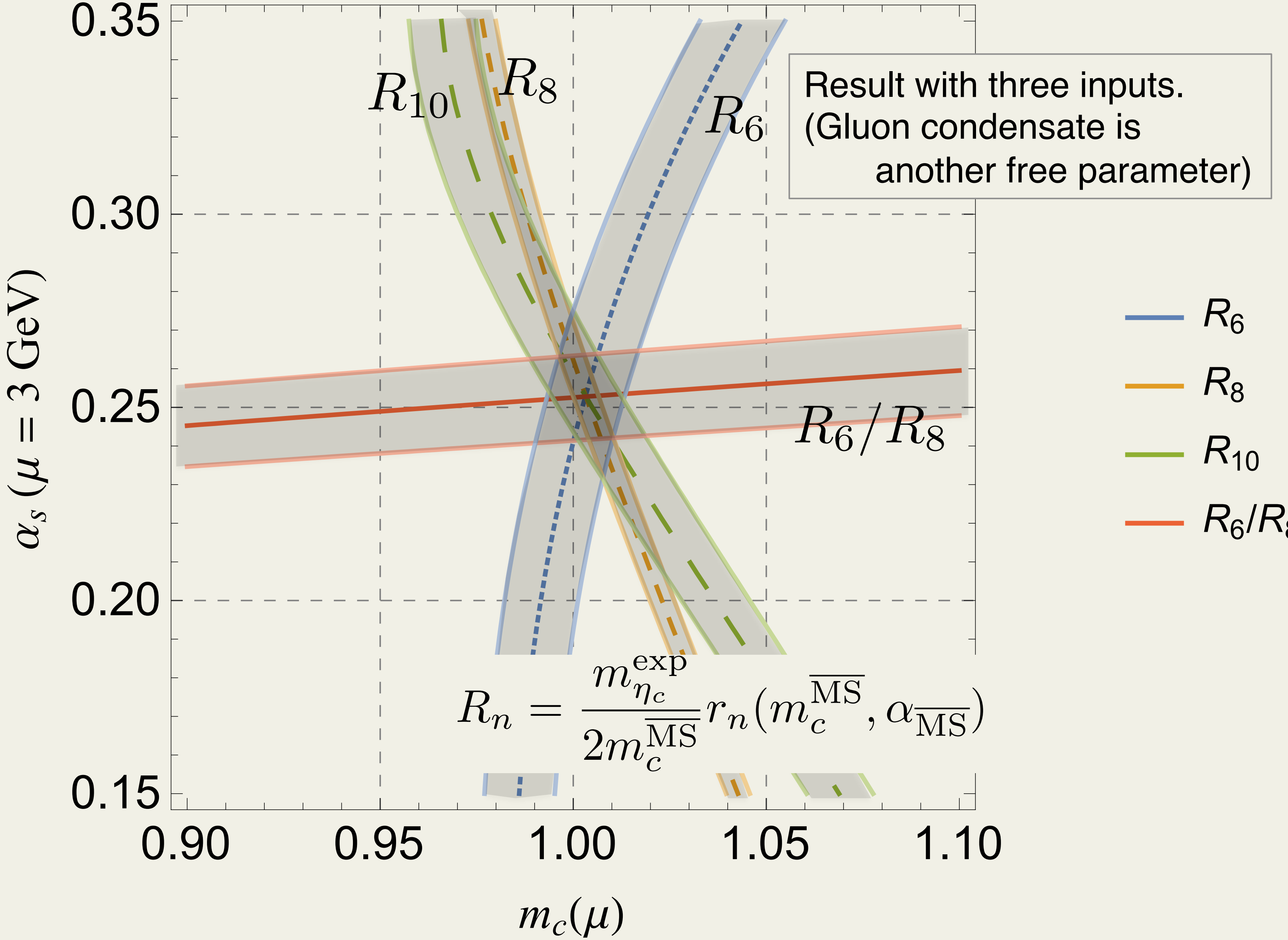
Estimation with $\mu_m \neq \mu_\alpha$ as conservative one



Similar analysis with
[B. Dehnadi et al. (2013),(2015)]

Error is estimated in this region
→More conservative estimation

	$\mu_m = \mu_\alpha$	$\mu_m \neq \mu_\alpha$
$m_c(3\mathrm{GeV})$	0.26%	0.77%
$\alpha_s(3\mathrm{GeV})$	1.9%	4.7%


Result with three inputs.
(Gluon condensate is another free parameter)
R_{10}
R_8
R_6
R_6/R_8
$R_n = \frac{m_{\eta_c}^{\mathrm{exp}}}{2m_c^{\overline{\mathrm{MS}}}} r_n(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})$
$\alpha_s\,(\mu = 3\ \mathrm{GeV})$
$m_c(\mu)$
0.35
0.30
0.25
0.20
0.15
0.90
0.95
1.00
1.05
1.10

Result ($\mu_m \neq \mu_\alpha$)

- More conservative estimation of the perturbative error.

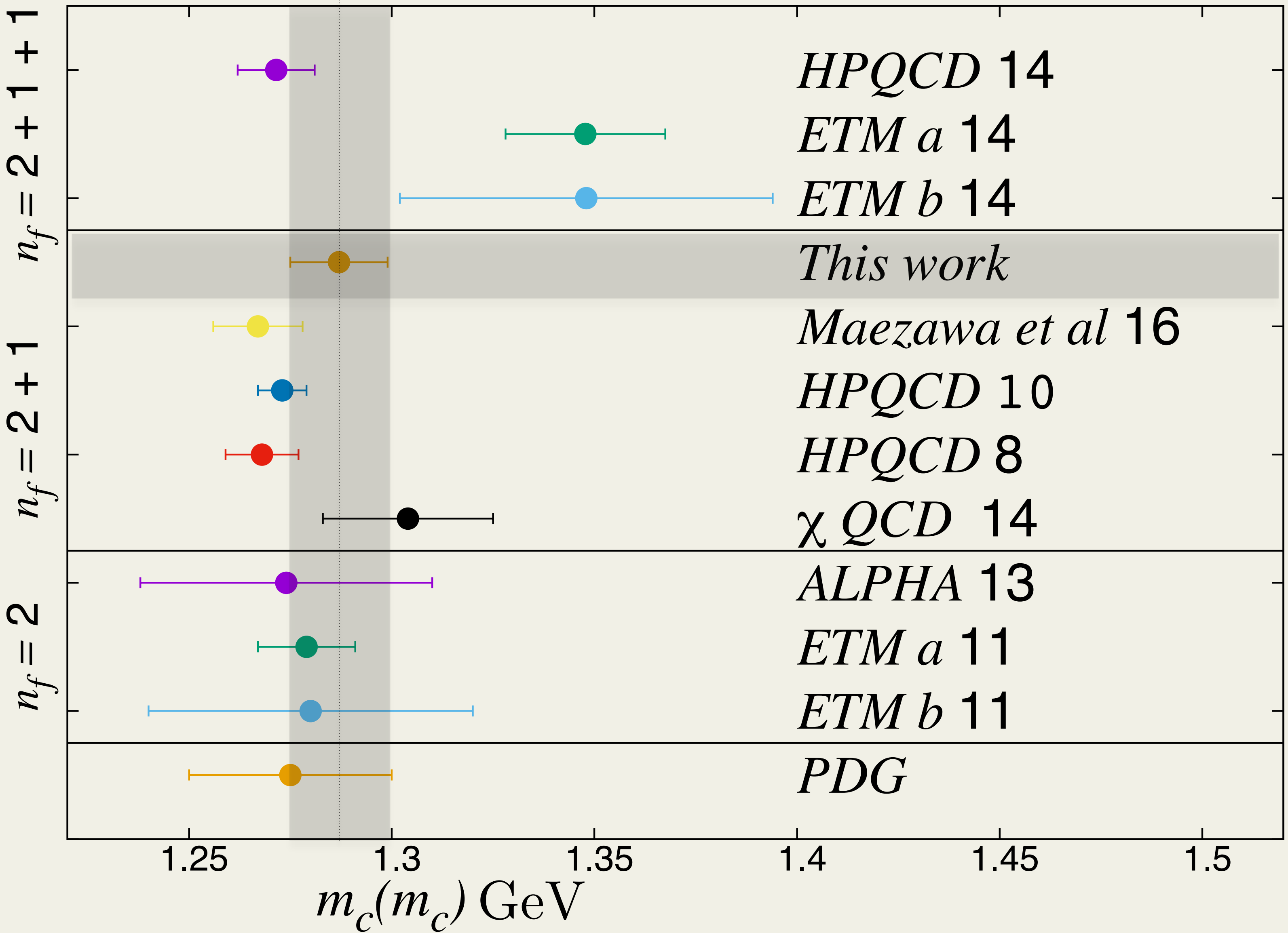
$R_6/R_8, R_8, \& R_{10}$

			Lattice				m_{η_c}(negligible)		
		pert	$t_0^{1/2}$	stat	$O(a^4)$	vol	$m_{\eta_c}^{\rm exp}$	disc	EM
m_c(3GeV) [GeV]	1.0033(96)	(77)	(49)	(4)	(30)	(4)	(3)	(4)	(6)
α_s(3GeV)	0.2528(127)	(120)	(32)	(2)	(26)	(1)	(0)	(0)	(1)

- Perturbative error is $\times 2$ larger than that of $\mu_m = \mu_\alpha$ and dominant source of the systematic error

- ~1% precision is achieved for $m_c^{\overline{\rm MS}}$.

Result



$n_f = 2+1+1$
$n_f = 2+1$
$n_f = 2$
HPQCD 14
ETM a 14
ETM b 14
This work
Maezawa et al 16
HPQCD 10
HPQCD 8
χ QCD 14
ALPHA 13
ETM a 11
ETM b 11
PDG
1.25
1.3
1.35
1.4
1.45
1.5
$m_c(m_c)$ GeV

Result ($\mu_m \neq \mu_\alpha$)

	This work	PDG(2014)
$m_c^{\overline{\mathrm{MS}}}(\mu = 3\ \mathrm{GeV})$	$1.003(10)\ \mathrm{GeV}$	
$m_c^{\overline{\mathrm{MS}}}(\mu = m_c^{\overline{\mathrm{MS}}})$	$1.287(12)\ \mathrm{GeV}$	$1.275(25)\ \mathrm{GeV}$
$\alpha_{\overline{\mathrm{MS}}}(\mu = 3\ \mathrm{GeV})$	$0.253(13)$	$0.2567(34)$
$\alpha_{\overline{\mathrm{MS}}}(\mu = M_Z)$	$0.1177(26)$	$0.1185(6)$
$\Lambda_{\overline{\mathrm{MS}}}^{n_f=4}$	$286(37)\ \mathrm{MeV}$	$297(8)\ \mathrm{MeV}$
$\Lambda_{\overline{\mathrm{MS}}}^{n_f=5}$	$205(32)\ \mathrm{MeV}$	$214(7)\ \mathrm{MeV}$

Summary

- We extract $m_c^{\overline{\rm MS}}$ and $\alpha_{\overline{\rm MS}}$ from the temporal moments of charmonium current correlators.

- Take continuum limit by the data at $a^{-1} = 2.4, 3.6, 4.5\ \mathrm{GeV}$. Discretization effect is significant but controllable, and perturbative truncation is more important.

- In the vector channel, The moments are consistent with the experimental R-ratio.

$$m_c^{\overline{\rm MS}}(3\ \mathrm{GeV}) = 1.003(10)\ \mathrm{GeV}$$

$$\alpha_{\overline{\rm MS}}(3\ \mathrm{GeV}) = 0.253(13)$$

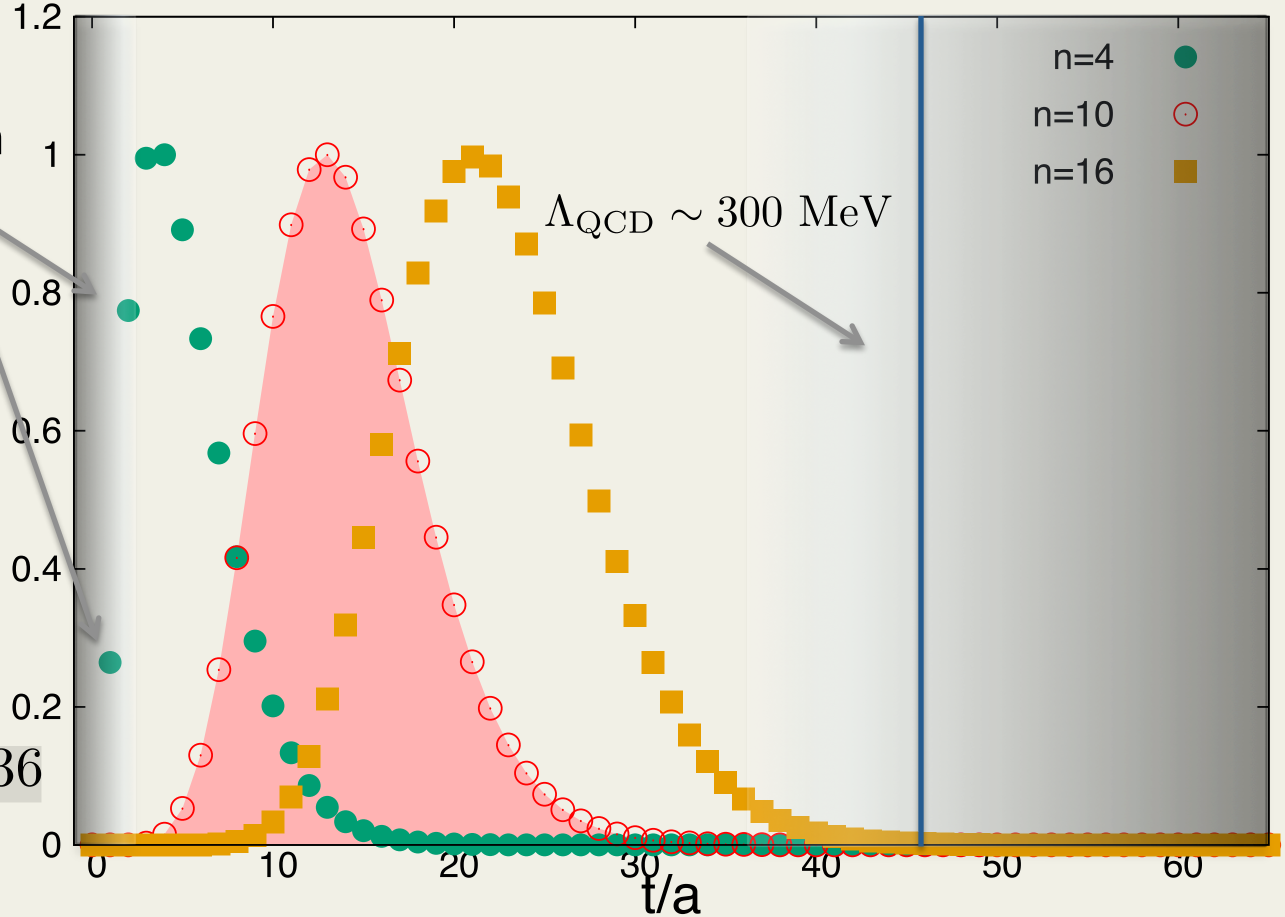
Backup slides

- Typical energy scale depends on the weight factor n

"Window"

$$a^{-1} \gg (\text{Energy scale}) \gg \Lambda_{\text{QCD}} \longrightarrow 6 \leq n \ll 20$$

Contain Discretization effect



$am_{\eta_c} \sim 0.6636$

HIGH ⟵ Energy scale ⟶ LOW

Error ❶ Input meson mass $m_{\eta_c}^{\exp}$ error

- We use PDG value, $m_{\eta_c}^{\exp} = 2.9836(7)\ \mathrm{GeV}$, after correcting for…
  - (a) Electromagnetic effect,
  - (b) Disconnected diagram contributions.

- Estimates from previous works (lattice, pheno):

Electromagnetic

$$m_{\eta_c} - m_{\eta_c}^{\text{no EM}} = -2.4(8)\ \mathrm{MeV}$$

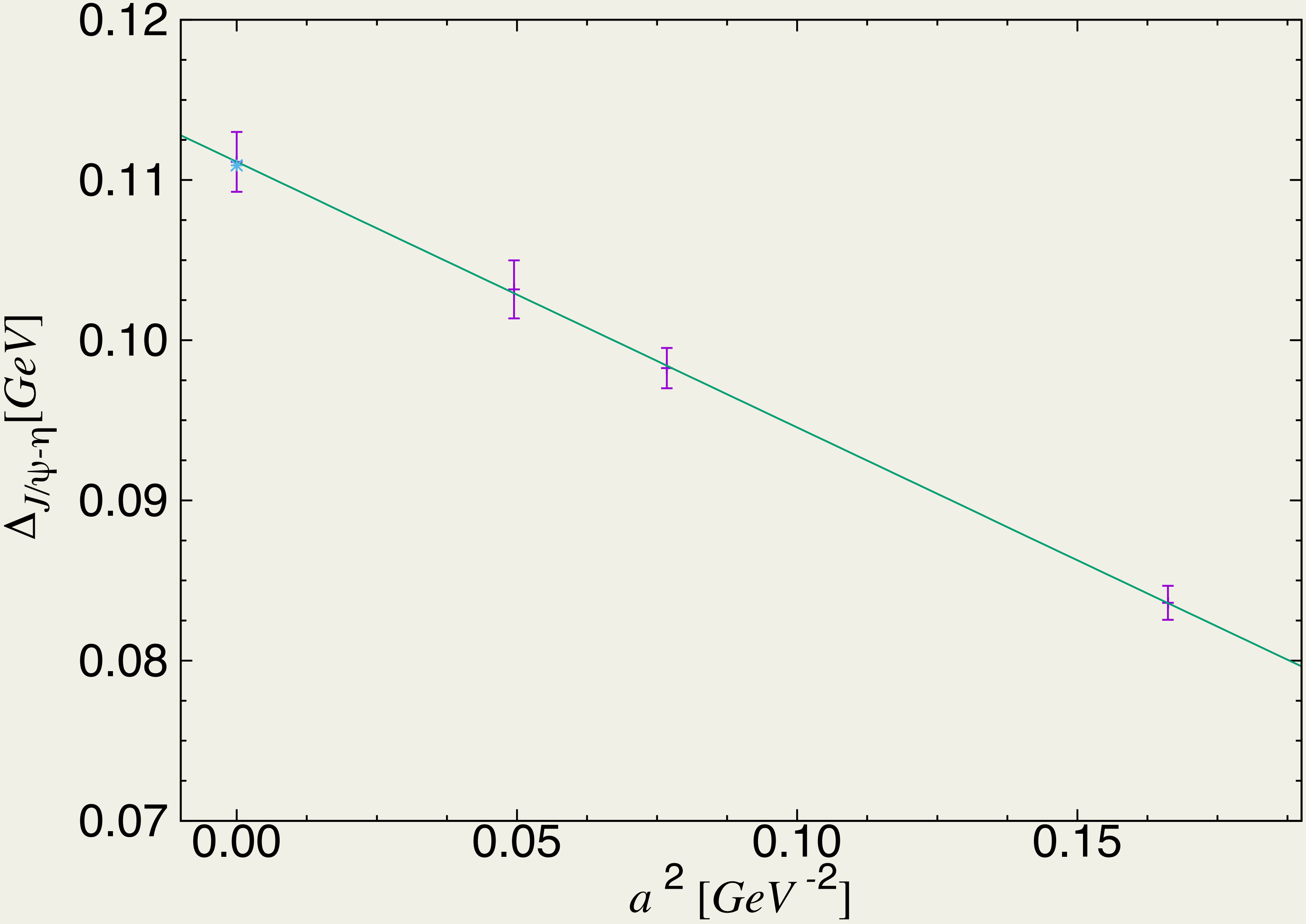
[E. Follana, et al. (2007)]

Disconnected

$$m_{\eta_c} - m_{\eta_c}^{\text{no Disconnect}} = -2.6(13)\ \mathrm{MeV}$$

[C. T. H. Davis, et al. (2007)]

Error ❶ Input meson mass $m_{\eta_c}^{\mathrm{exp}}$ error



Error ❷ Finite volume effect

- Prepare two ensembles (same setup except for the volume)

$$R_n(L=32) \qquad m_\pi L \sim 3.0$$

$$R_n(L=48) \qquad m_\pi L \sim 4.4$$

Finite volume error

$$\delta_L R_n = |R_n(L=48) - R_n(L=32)|$$

Error ❹ Gluon condensate

◉ Perturbative calculation does not contain gluon condensate.

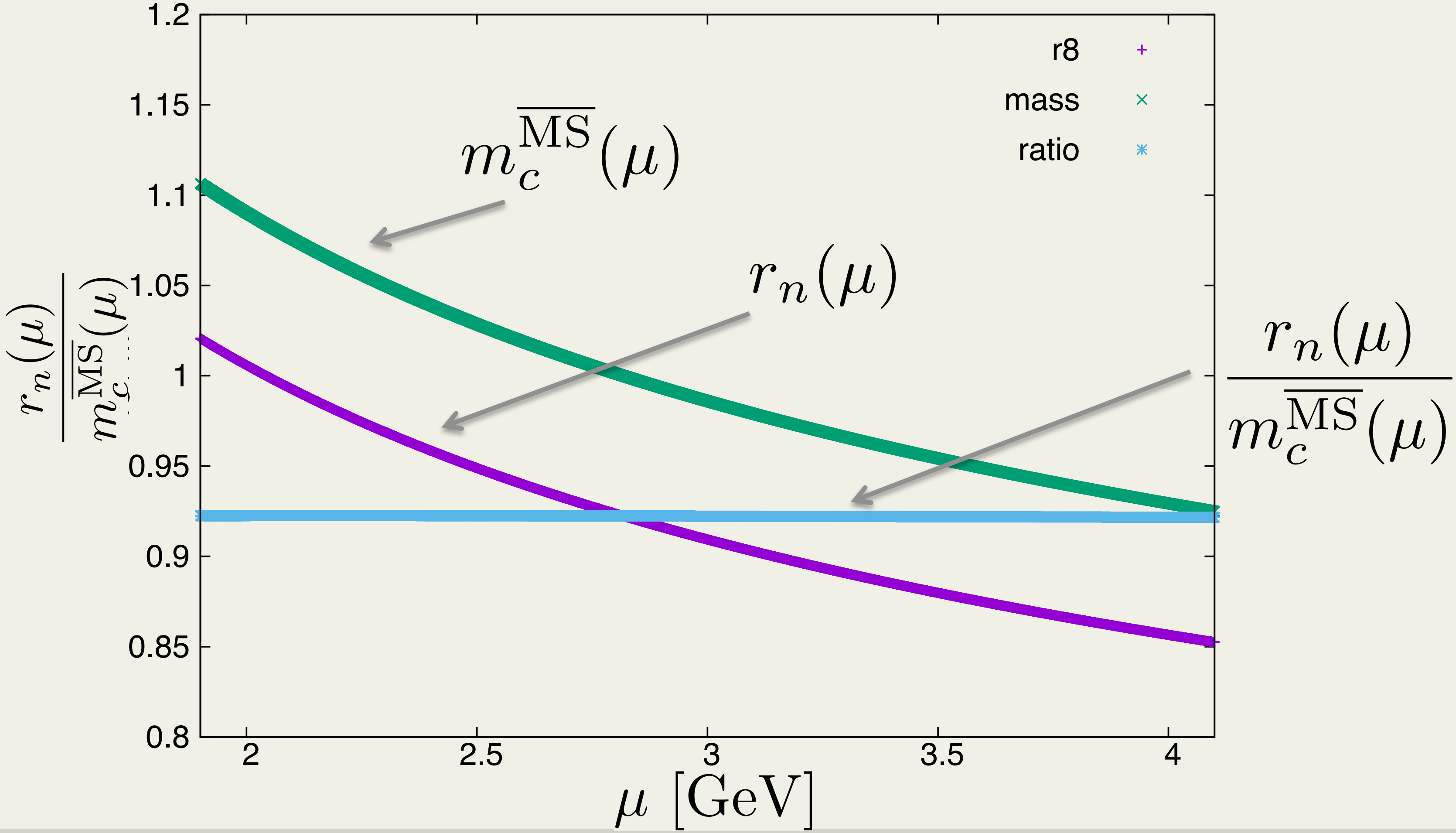
It is known to 2-loop by OPE. [M. A. Shifman, A. I. Vaientshtein, & V. I. Zakharov (1979)] [D. J. Broadhurst, et al (1994)]

$$r_n^{\rm glue} = \left(\frac{\langle (\alpha_s/\pi) G^{\mu\nu} G_{\mu\nu} \rangle}{(2m)^4} A_n(m_c^{\overline{\rm MS}}, \alpha_{\overline{\rm MS}}) \right)^{\frac{1}{n-4}}$$

◇ We may extract it as a solution of the equations, n = 6, 8, & 10.

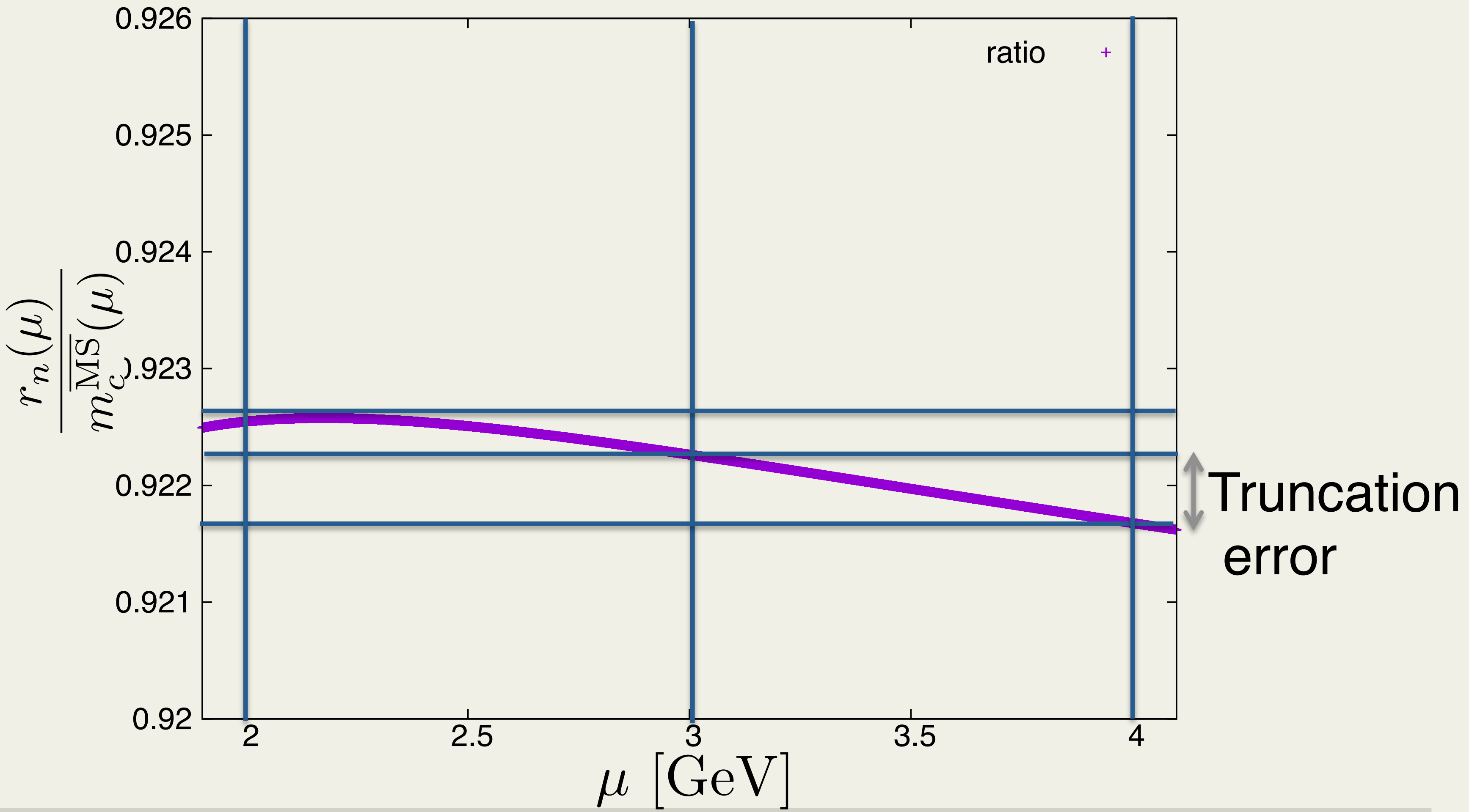
$$\frac{\langle (\alpha_s/\pi) G^{\mu\nu} G_{\mu\nu} \rangle}{m^4} = -0.0006(78)$$

Error ❹ Perturbative truncation



- Dependence is almost canceled out.

Error ❹ Perturbative truncation



- Actually, we consider $\mu_\alpha \neq \mu_m$ $(\alpha(\mu_\alpha), m_c(\mu_m))$,
  not only $\mu_\alpha = \mu_m$ [B. Dehnadi, A. H. Hoang, and V. Mateu (2015)]

Consistency with experiment (Vector)

Pseudo $\longrightarrow$ Vector

$$G_n^{(\mathrm{Lat})} = \frac{g_n^{(\mathrm{conti})}(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})}{(am_c^{\overline{\mathrm{MS}}})^{n-4}} \qquad Z_V^2 G_{Vn}^{(\mathrm{Lat})} = \frac{g_{Vn}^{(\mathrm{conti})}(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})}{(am_c^{\overline{\mathrm{MS}}})^{n-2}}$$

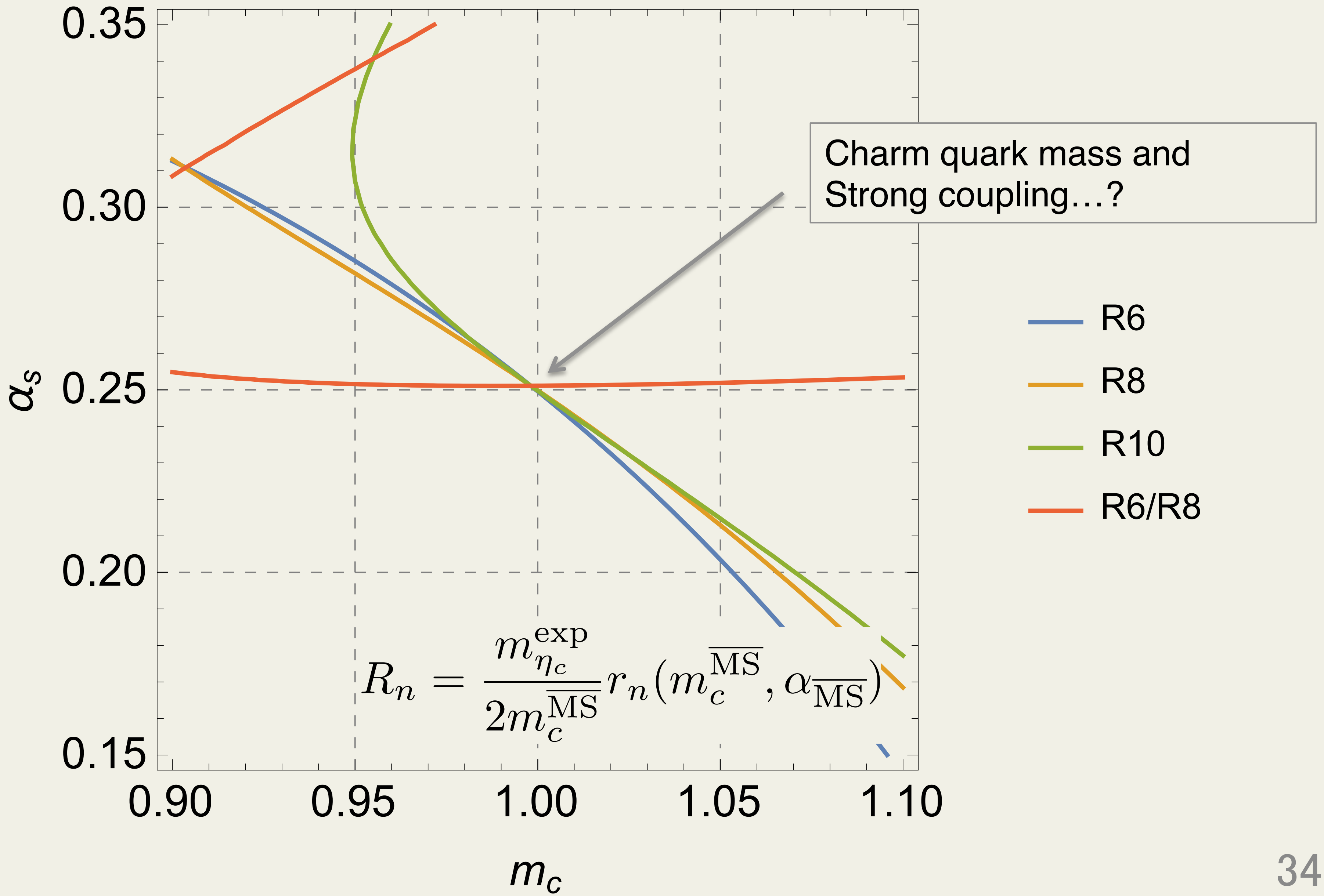
- Input Z_V from $\Pi_V^{\overline{\mathrm{MS}}}(x)$ analysis with OPE.

(JLQCD, M. Tomii et al.)

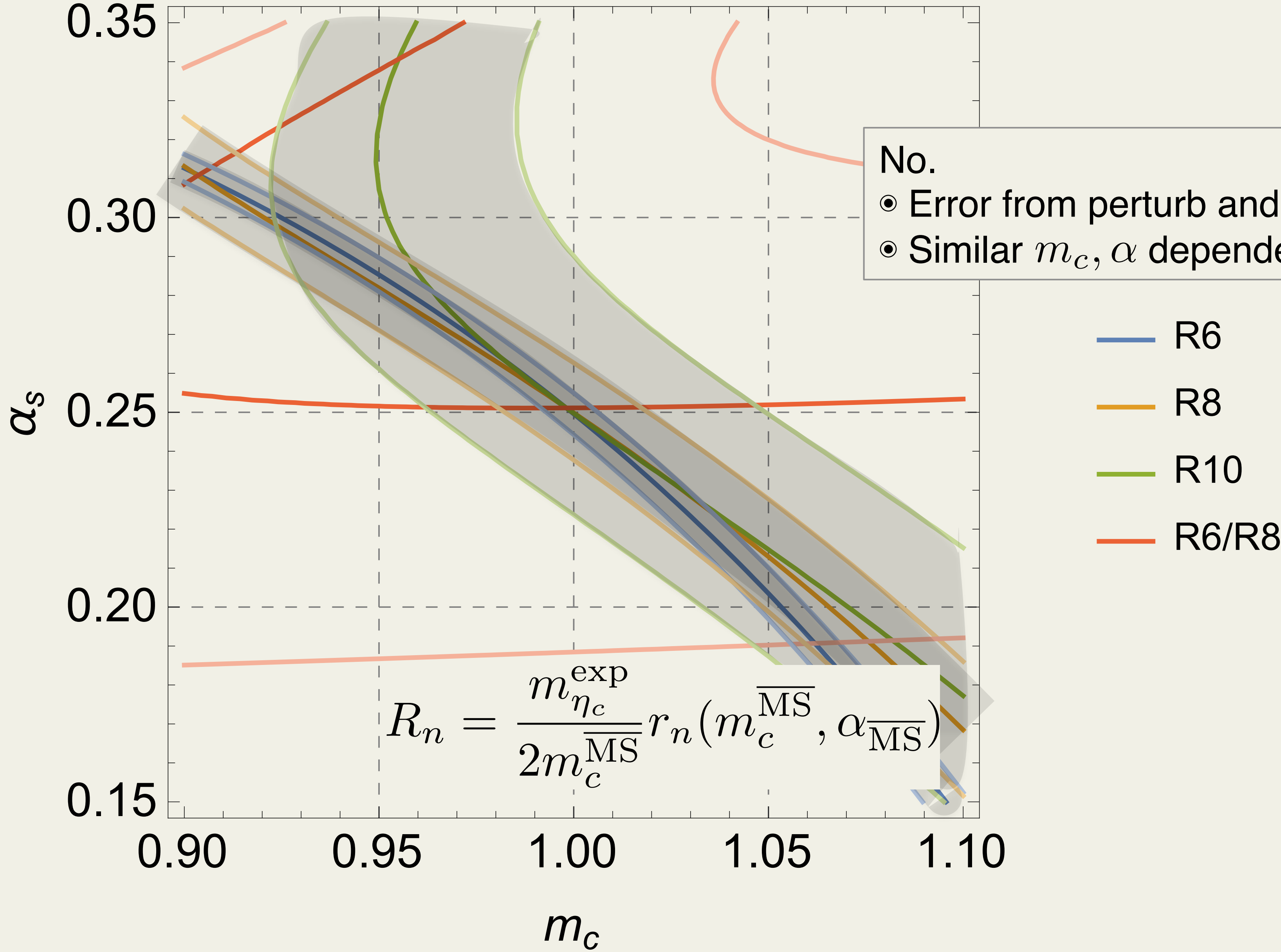
$$\tilde{Z}_{\mathrm{V}}(x) = Z_V + c_{-2}x^{-2} + c_4 x^4 + c_6 x^6 + O(x^8)$$

(e.g.) $Z_V(a^{-1} = 4.47\ \mathrm{GeV}) = 0.9651(46)$

Vector current Moment



Vector current Moment



No.

- Error from perturb and Z_V
- Similar m_c, α dependence

Error ❶ Input meson mass $m_{\eta_c}^{\rm exp}$ error

- Finally we use…

$$m_{\eta_c}^{\rm modified} = 2983.6 + 2.4_{\rm Disc.} + 2.6_{\rm EM} \pm (0.7)_{\rm PDG} \pm (0.8)_{\rm Disc.} \pm (1.3)_{\rm EM} \pm (2.3)_{\rm split}$$

◆Note: All of these error sources are negligible.

Moment and R-ratio

Residue theorem (or Disparsion relation)

$$\frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(\Pi(q^2)\right)_{q^2=Q_0^2} = \oint \frac{\mathrm{d}q^2}{2\pi i}\frac{1}{(q^2-Q_0^2)^{n+1}}\Pi(q^2)$$

Contour integral $$= \int \frac{\mathrm{d}q^2}{2\pi i}\frac{1}{(q^2-Q_0^2)^{n+1}} 2i\mathrm{Im}\left[\Pi(q^2)\right]$$

Optical theorem $$= \int \frac{\mathrm{d}q^2}{\pi}\frac{q^2}{(4\pi\alpha)^2 Q_f^2}\frac{1}{(q^2-Q_0^2)^{n+1}}\sigma_{e^+e^-\to\mathrm{hadron}}(q^2)$$

$$= \frac{1}{12\pi Q_f^2}\int \mathrm{d}q^2 \frac{1}{(q^2-Q_0^2)^{n+1}}\frac{\sigma_{e^+e^-\to\mathrm{hadron}}(q^2)}{\sigma_{e^+e^-\to\mu^+\mu^-}(q^2)}$$

Take $Q_0 = 0$

$$\frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(\Pi(q^2)\right)_{q^2=0} = \frac{1}{12\pi Q_f^2}\int \mathrm{d}s \frac{1}{s^{n+1}} R(s)_{e^+e^-\to\ \mathrm{hadron}}$$

Perturbative moment

k	$C_k^{(0)}$	$C_k^{(10)}$	$C_k^{(11)}$	$C_k^{(20)}$	$C_k^{(21)}$	$C_k^{(22)}$	$C_k^{(30)}$	$C_k^{(31)}$	$C_k^{(32)}$	$C_k^{(33)}$
1	1.3333	3.1111	0.0000	0.1154	-6.4815	0.0000	-1.2224	2.5008	13.5031	0.0000
2	0.5333	2.0642	1.0667	7.2362	1.5909	-0.0444	7.0659	-7.5852	0.5505	0.0321
3	0.3048	1.2117	1.2190	5.9992	4.3373	1.1683	14.5789	7.3626	4.2523	-0.0649
4	0.2032	0.7128	1.2190	4.2670	4.8064	2.3873	13.3285	14.7645	11.0345	1.4589
5	0.1478	0.4013	1.1821	2.9149	4.3282	3.4971		16.0798	16.6772	4.4685
6	0.1137	0.1944	1.1366	1.9656	3.4173	4.4992		14.1098	19.9049	8.7485
7	0.0909	0.0500	1.0912	1.3353	2.2995	5.4104		10.7755	20.3500	14.1272
8	0.0749	-0.0545	1.0484	0.9453	1.0837	6.2466		7.2863	17.9597	20.4750

n=4,6,8,10

Z_V factor extraction

Input Z_V $\longrightarrow$ Predict R_n

then invert it…

Input R_n $\longrightarrow$ Predict Z_V

- Moment is known perturbatively (and experimentally).

(Input Experiment $\longrightarrow$ $\delta Z_V \sim 1\%$)

Input Perturbation $\longrightarrow$ $\delta Z_V \sim 3\%$

(or 2% with PDG $\alpha_{\overline{\mathrm{MS}}}$)

inputs	$m_c(\mu)$ [GeV]	pert	$t_0^{1/2}$	stat	$O(a^4)$	vol	$m_{\eta_c}^{\rm exp}$	disc	EM
R_6, R_8, R_{10}	1.0032(98)	(82)	(51)	(5)	(16)	(4)	(3)	(4)	(6)
R_6, R_6/R_8, R_{10}	1.0031(194)	(176)	(78)	(6)	(18)	(5)	(4)	(4)	(7)
R_6/R_8, R_8, R_{10}	1.0033(96)	(77)	(49)	(4)	(30)	(4)	(3)	(4)	(6)

inputs	$\alpha_s(\mu)$	pert	$t_0^{1/2}$	stat	$O(a^4)$	vol	$m_{\eta_c}^{\rm exp}$	disc	EM
R_6, R_8, R_{10}	0.2530(256)	(213)	(134)	(12)	(38)	(10)	(9)	(10)	(16)
R_6, R_6/R_8, R_{10}	0.2528(127)	(120)	(33)	(2)	(25)	(1)	(0)	(0)	(1)
R_6/R_8, R_8, R_{10}	0.2528(127)	(120)	(32)	(2)	(26)	(1)	(0)	(0)	(1)

inputs	$\frac{<(\alpha/\pi)G^2>}{m^4}$	pert	$t_0^{1/2}$	stat	$O(a^4)$	vol	$m_{\eta_c}^{\rm exp}$	disc	EM
R_6, R_8, R_{10}	−0.0005(99)	(85)	(45)	(4)	(23)	(4)	(3)	(4)	(6)
R_6, R_6/R_8, R_{10}	−0.0006(144)	(133)	(49)	(4)	(23)	(4)	(3)	(3)	(5)
R_6/R_8, R_8, R_{10}	−0.0006(78)	(68)	(29)	(3)	(22)	(3)	(2)	(3)	(5)