Scalar QCD at nonzero density

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- gain better understanding of the nature of the sign problem

Action

$$S = \sum_x \sum_f \left(\sum_\nu \left(e^{\mu \delta_{\nu,\hat{0}}} \phi_x^{(f)\dagger} U_{x,\nu} \phi_{x+\hat{\nu}}^{(f)} + e^{-\mu \delta_{\nu,\hat{0}}} \phi_{x+\hat{\nu}}^{(f)\dagger} U_{x,\nu}^\dagger \phi_x^{(f)} \right) \right.$$
$$\left. -2d \phi_x^{(f)\dagger} \phi_x^{(f)} - m^2 \phi_x^{(f)\dagger} \phi_x^{(f)} \right)$$

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- no γ structure
- periodic boundary conditions
- complex action for $\mu \neq 0 \to$ sign problem

Rewriting the Action

$$S = \sum_x \left(\sum_\nu \mathrm{Tr} \left(J_{x,\nu} U_{x,\nu} + K_{x,\nu} U^\dagger_{x,\nu} \right) - \sum_f \left(2d\phi_x^{(f)\dagger}\phi_x^{(f)} + m^2 \phi_x^{(f)\dagger}\phi_x^{(f)} \right) \right)$$

$$J_{x,\nu} = e^{\mu\delta_{\nu,\hat{0}}} \sum_f \phi_{x+\hat{\nu}}^{(f)} \phi_x^{(f)\dagger} \qquad \text{forward hopping}$$

$$K_{x,\nu} = e^{-\mu\delta_{\nu,\hat{0}}} \sum_f \phi_x^{(f)} \phi_{x+\hat{\nu}}^{(f)\dagger} \qquad \text{backward hopping}$$

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$$\int_{\mathrm{SU}(3)} dU \, \exp(\mathrm{Tr}\,(JU + KU^\dagger)) =$$

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- $Z = \det(KJ) \qquad \Delta = \det J \qquad \bar{\Delta} = \det K$

Partition function with dual variables

$$Z = \sum_{\{j,k,l,n,\bar{n}\}} \int \mathcal{D}\phi\mathcal{D}\phi^\dagger \rho(|\phi|) \prod_{x,\nu} 2 \frac{X_{x,\nu}^{j_{x,\nu}} Y_{x,\nu}^{k_{x,\nu}} Z_{x,\nu}^{l_{x,\nu}} \Delta_{x,\nu}^{n_{x,\nu}} \bar{\Delta}_{x,\nu}^{\bar{n}_{x,\nu}}}{j_{x,\nu}! k_{x,\nu}! l_{x,\nu}! n_{x,\nu}! \bar{n}_{x,\nu}! f_{x,\nu}^{(1)}! f_{x,\nu}^{(2)}!}$$

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- need at least 3 flavors to build baryons $\rightarrow$ focus on $N_f = 3$

Resolving the sign problem

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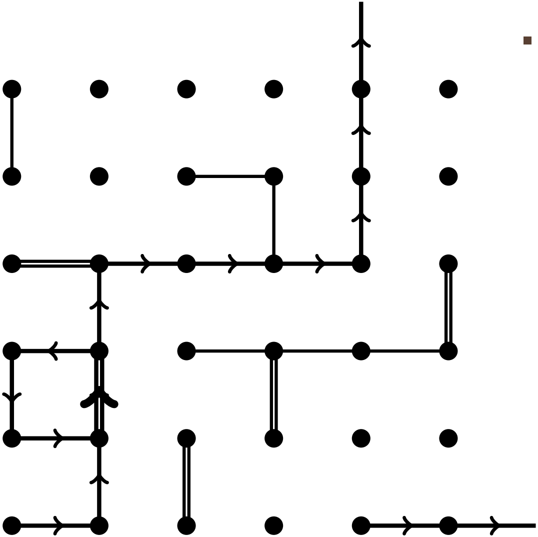
$\Rightarrow$ no sign problem on closed loop configurations

Configurations



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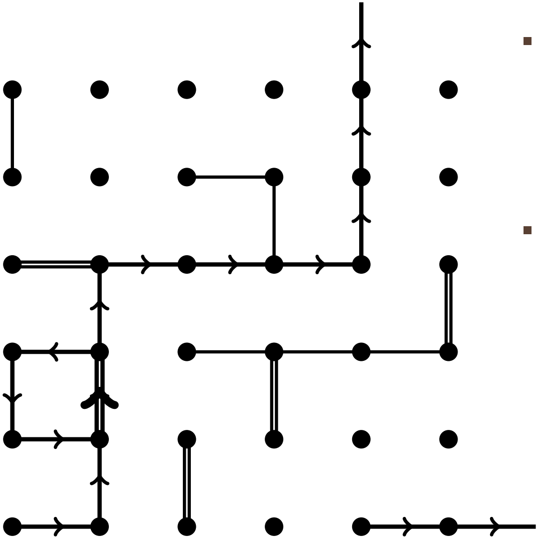

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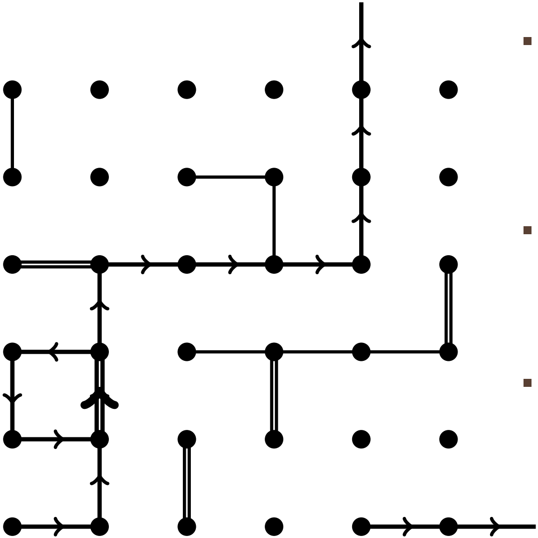

- arbitrary occupation of dual variables belonging to X, Y, Z (unoriented, no arrows) $\rightarrow$ 'mesonic'
- only closed loops of dual variables corresponding to $\Delta, \bar{\Delta} \rightarrow$ 'baryonic'
- different from fermionic case (zero- and multi-occupation possible)

Simulation strategy

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- integrate ϕ stochastically

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- only for $N_f = 3$ nontrivial μ-dependence (presumably also for higher N_f)
- recall sign problem in MDP-formulation (fermionic) due to: boundary conditions, spin structure (staggered phases), closed fermion loops, backward hopping
(all these are absent in the scalar case)

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- go beyond strong coupling