Double Parton Distributions of the Pion

Christian Zimmermann for RQCD,
and Markus Diehl

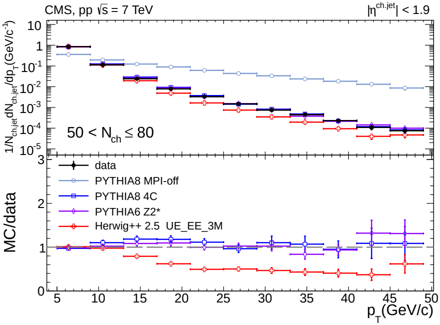
Universität Regensburg

34th International Symposium on Lattice Field Theory

July 29, 2016

Why Double Hard Interactions?

CMS 1310.4554 : Inclusive charged particle jets



One needs double hard interactions for calculating the background of new physics discovery at LHC

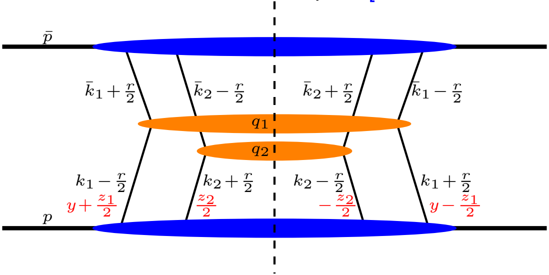
Naive description :

$$d\sigma_{\text{DPS}} = \frac{d\sigma_{\text{SPS}} d\sigma_{\text{SPS}}}{\sigma_{\text{eff}}}$$

⇒ Calculate DPS contributions, test to what extent the naive ansatz above works

DPS: Feynman Graph and Cross Sections

Assume factorization into soft and hard part [arXiv:1111.0910] :



$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1 \mathrm{d}\bar{x}_1 \mathrm{d}x_2 \mathrm{d}\bar{x}_2} = \sum_{\substack{\text{polarization} \\ \text{flavour}}} \frac{{\sigma_1}^{X_1 \bar{X}_1} {\sigma_2}^{X_2 \bar{X}_2}}{C} \int \mathrm{d}^2\boldsymbol{y}\; F_{X_1 X_2}(x_i, \boldsymbol{y}) F_{\bar{X}_1 \bar{X}_2}(\bar{x}_i, \boldsymbol{y})$$

$$+ \{\text{interference}\} + \{\text{higher twist}\}$$

with collinear Double Parton Distributions (DPDs):

$$F_{X_1 X_2}(x_1, x_2, \boldsymbol{y}) = \left[\prod_{j=1}^{2} \int \frac{\mathrm{d}z_i^-}{2\pi} e^{i x_j z_j^- p^+} \right] 2p^+ \int \mathrm{d}y^- \langle h(p) | \mathcal{O}_{X_1}(0, z_2) \mathcal{O}_{X_2}(y, z_1) | h(p) \rangle \,|_{\boldsymbol{z}_i = \boldsymbol{0}}$$

$$\mathcal{O}_X(y, z) = \bar{q}(y - \tfrac{z}{2}) \Gamma_X q(y + \tfrac{z}{2}) \,|_{z^+ = y^+ = 0}$$

DPDs: Mellin Moments and Decompositions

First Mellin Moment:

$$M_{X_1 X_2}(\boldsymbol{y}) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \Big[F_{X_1 X_2}(x_1, x_2, \boldsymbol{y}) - a_{X_1} F_{\bar{X}_1 X_2}(x_1, x_2, \boldsymbol{y}) - a_{X_2} F_{X_1 \bar{X}_2}(x_1, x_2, \boldsymbol{y}) + a_{X_1} a_{X_2} F_{\bar{X}_1 \bar{X}_2}(x_1, x_2, \boldsymbol{y})\Big]$$

$$= 2(p^+)^{-1} \int \mathrm{d}y^- \underbrace{\langle h(p)| \overbrace{\mathcal{O}_{X_1}(0)\, \mathcal{O}_{X_2}(y)}^{\text{local operators}} |h(p)\rangle}_{\mathcal{M}^p_{X_1 X_2}}\Big|_{y^+=0}$$

$\Rightarrow$ $\mathcal{M}^p_{X_1 X_2}$ can be obtained from the lattice.

Can decompose DPD matrix elements into invariant functions e.g. :

$$\mathcal{M}^p_{\mathrm{SS/PP}} = 2m_h^2 A_{\mathrm{SS/PP}}(py, y^2)$$

$$\mathcal{T}\mathcal{M}^{p,\{\mu\nu\}}_{\mathrm{VV/AA}} = \Big[2p^\mu p^\nu - \tfrac{1}{2} g^{\mu\nu} p^2\Big] A_{\mathrm{VV/AA}}(py, y^2) + m_h^2 \Big[2p^{\{\mu} y^{\nu\}} - \tfrac{1}{2} g^{\mu\nu} py\Big] B_{\mathrm{VV/AA}}(py, y^2) + m_h^4 \Big[2y^\mu y^\nu - \tfrac{1}{2} g^{\mu\nu} y^2\Big] C_{\mathrm{VV/AA}}(py, y^2)$$

$$\xrightarrow{+\text{components}} \mathcal{T}\mathcal{M}^{p,++}_{\mathrm{VV/AA}} = 2(p^+)^2 A_{\mathrm{VV/AA}}(py, y^2)$$

Relation between Mellin moments and invariant functions, e.g. :

$$M_{qq/\Delta q \Delta q}(\boldsymbol{y}) = \int \mathrm{d}(py) A_{\mathrm{VV/AA}}(py, y^2)$$

Consider DPD Mellin moments , start with first moment

$\Rightarrow$ local operators $\mathcal{O}_X(y) = \bar{q}(y)\Gamma_X q(y)$, no Wilson lines, no derivatives (first moment)

Go to Euclidean space ($y = (\boldsymbol{y}, y^4)$ with $\boldsymbol{y} = (y^1, y^2, y^3)$), $y^4 = iy^0$:

$\Rightarrow$ operators must be at the same time, $y^4 = 0$.

For a pilot study start with pion at zero momentum.

$\Rightarrow$ obtain $\mathcal{M}_{X_1X_2} = \langle \pi^+(\mathbf{0}) | \mathcal{O}_{X_1}(\mathbf{0})\ \mathcal{O}_{X_2}(\boldsymbol{y}) | \pi^+(\mathbf{0}) \rangle$ from the lattice for several channels:

$$\Gamma_X \in \{\underbrace{\mathbb{1}}_{\text{scalar}}, \underbrace{\gamma_5}_{\text{pseudoscalar}}, \underbrace{\gamma^\mu}_{\text{vector}}, \underbrace{\gamma^\mu\gamma_5}_{\text{axialvector}}, \underbrace{\sigma^{\mu\nu}}_{\text{tensor}}\}$$

Ground state matrix element:

$$\langle \pi^+ | \mathcal{O}_{X_1}(\mathbf{0})\mathcal{O}_{X_2}(\boldsymbol{y}) | \pi^+ \rangle = \left. \frac{C^{X_1X_2}{}_{4\text{pt}}(t, \tau, \boldsymbol{y})}{2m_\pi C_{2\text{pt}}(t)} \right|_{t \gg \tau \gg 0}$$

with 4pt function $C^{X_1X_2}{}_{4\text{pt}}(t, \tau, \boldsymbol{y}) = \langle \mathcal{O}^{\boldsymbol{p}}_{\pi^+}(t) \mathcal{O}_{X_1}(\mathbf{0})\mathcal{O}_{X_2}(\boldsymbol{y}) \mathcal{O}^{\dagger\boldsymbol{p}}_{\pi^+}(0) \rangle$

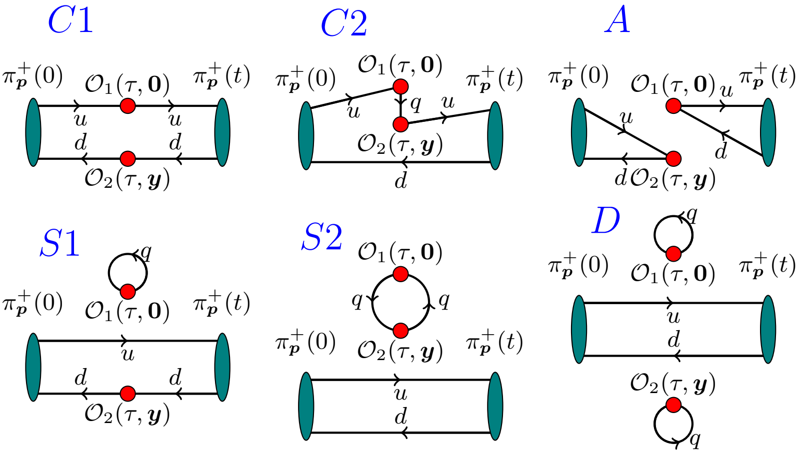
and 2pt function $C_{2\text{pt}}(t) = \langle \mathcal{O}^{\boldsymbol{p}}_{\pi^+}(t) \mathcal{O}^{\dagger\boldsymbol{p}}_{\pi^+}(0) \rangle$

Pion interpolators:

$|\pi^+_{\boldsymbol{p}}(t)\rangle + \cdots = \mathcal{O}^{\boldsymbol{p}\dagger}_{\pi^+}(t)|\Omega\rangle = \frac{1}{V}\sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \bar{u}(x)\gamma_5 d(x)|\Omega\rangle = \mathcal{O}^{-\boldsymbol{p}}_{\pi^-}(t)|\Omega\rangle$

4pt Graphs

Obtain six independent Wick contractions:



Choose $t = 15a$, expect plateau at $6a \lesssim \tau \lesssim 9a$

$\Rightarrow$ fit or average

Lattice Setup and Simulation Details

Used gauge ensemble ($N_f = 2$ Wilson-Clover fermions, c.f. [arXiv:1412.7336]):

Ensemble	β	a[fm]	κ	V	m_π[GeV]	$N(N_{4pt})$	N_{sm}
I	5.29	0.071	0.13632	$40^3 \times 64$	0.2888(11)	2025(984)	400

Renormalization (conversion to $\overline{\mathrm{MS}}$ (2GeV)) [arXiv:1003.5756] :

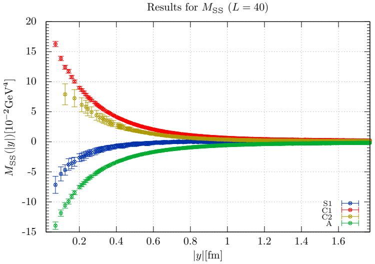
	S	P	V	A	T
Z	0.4577(18)	0.3538(92)	0.7365(48)	0.76487(64)	0.9141(26)
Z_{conv}	1.3543	1.3543	1	1	0.93313

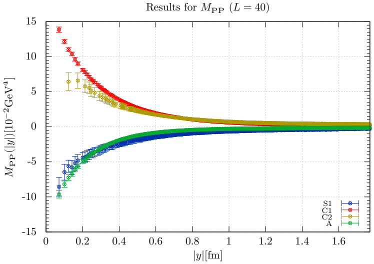
Further Details:

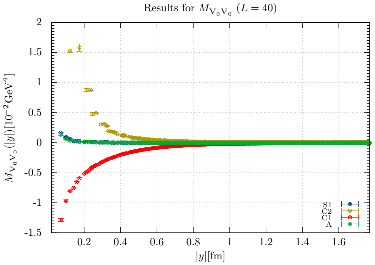
- smooth gauge fields by APE smearing [Nucl. Phys B 251 (1985), pp. 624-632]
- extend interpolators by Wuppertal smearing [Nucl. Phys. Proc. Suppl. 17 (1990), pp. 361-364]
- use stochastic $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ sources (see e.g. [arXiv:0804.1501])
  $\Rightarrow$ One-End-Trick / Two-Hand-Trick

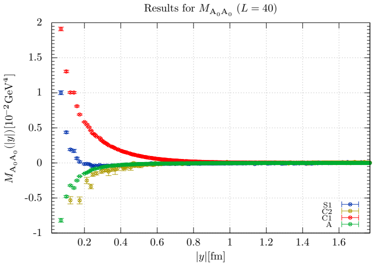
4pt-Results

PRELIMINARY



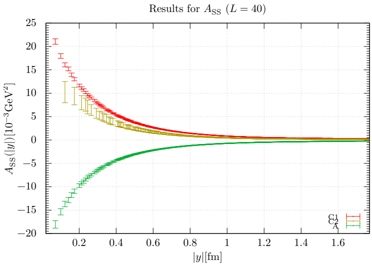


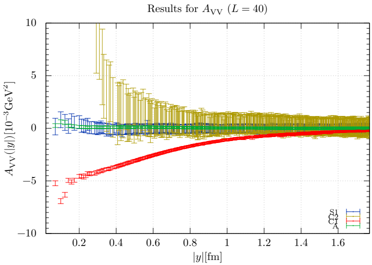




Invariant functions $A_{\mathrm{SS}}(py = 0, \boldsymbol{y}^2)$ and $A_{\mathrm{VV}}(py = 0, \boldsymbol{y}^2)$

PRELIMINARY





Naive Factorization Ansatz

Insert complete set of states; assume that the pion states dominate:

$$\langle \pi^+(p)| \mathcal{O}_1 \mathcal{O}_2 |\pi^+(p)\rangle = \sum_Y \langle \pi^+(p)| \mathcal{O}_1 |Y\rangle \langle Y| \mathcal{O}_2 |\pi^+(p)\rangle$$

$$\approx \int \frac{d^4p'}{(2\pi)^4} \langle \pi^+(p)| \mathcal{O}_1 |\pi^+(p')\rangle \langle \pi^+(p')| \mathcal{O}_2 |\pi^+(p)\rangle \, \delta(p'^2 - m_\pi^2)$$

Test 1 : Factorize $\langle \pi^+(p)| \mathcal{O}_1^{uu}(z_1,0)\mathcal{O}_2^{dd}(z_2,y) |\pi^+(p)\rangle$
$\Rightarrow$ can relate invariant function $A_{X_1 X_2}$ to pion form factors $F_X(t)$
$\left(t(\zeta,\boldsymbol{r}^2) = -\frac{\zeta^2 m_\pi^2 + \boldsymbol{r}^2}{1-\zeta}\right)$:

$$A_{\mathrm{VV/SS}}(py{=}0,y^2) \approx \frac{\eta_C^{\mathrm{V/S}}}{\pi} \int_0^1 d\zeta \frac{(1-\frac{\zeta}{2})}{1-\zeta} \int \frac{d^2\boldsymbol{r}}{(2\pi)^2} e^{-i\boldsymbol{y}\cdot\boldsymbol{r}} F_{\mathrm{V/S}}^2(t(\zeta,\boldsymbol{r}^2))$$

Test 2 : Factorize local Matrix element $(t(\boldsymbol{r}) = 2m^2 - 2mE_{\boldsymbol{r}})$:

$$\langle \pi^+(p)| \mathcal{O}_{\mathrm{V}_0}^{uu}(0)\mathcal{O}_{\mathrm{V}_0}^{dd}(y) |\pi^+(p)\rangle \approx -\frac{1}{4\pi^2|\boldsymbol{y}|} \int_0^\infty d(\boldsymbol{r}^2) \frac{\sin(|\boldsymbol{y}||\boldsymbol{r}|)(m+E_{\boldsymbol{r}})^2}{2E_{\boldsymbol{r}}} F_{\mathrm{V}}^2(t(\boldsymbol{r}^2))$$

$$\langle \pi^+(p)| \mathcal{O}_{\mathrm{S}}^{uu}(0)\mathcal{O}_{\mathrm{S}}^{dd}(y) |\pi^+(p)\rangle \approx \frac{1}{4\pi^2|\boldsymbol{y}|} \int_0^\infty d(\boldsymbol{r}^2) \frac{\sin(|\boldsymbol{y}||\boldsymbol{r}|)}{2E_{\boldsymbol{r}}} F_{\mathrm{S}}^2(t(\boldsymbol{r}^2))$$

Both tests trivially fail for the pseudoscalar and axialvector case, since $F_{\mathrm{P}} = F_{\mathrm{A}} = 0$ for the pion in contrast to our lattice results for 4pt-functions.

The Pion Form Factor

The Pion form factor:

$$F_{\mathrm{S}}(Q^2) = \langle \pi(p+Q) | \mathcal{O}_{\mathrm{S}} | \pi(p) \rangle \qquad F_{\mathrm{V}}(Q^2)(2p+Q)^\mu = \langle \pi(p+Q) | \mathcal{O}_{\mathrm{V}}^\mu | \pi(p) \rangle$$

- $\Rightarrow$ obtain FF from 3pt-functions, at first neglect disconnected contributions
- $\Rightarrow$ use momenta $|\boldsymbol{p}| \leq \frac{2\pi}{N_S}\sqrt{3}$

Fit data on the parametrization:

$$F(t) = \frac{F_0}{\left(1 + \frac{t}{M^2}\right)^p}, \quad t = Q^2$$

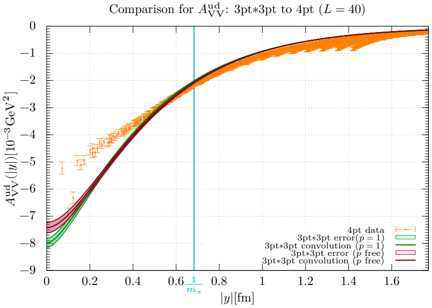
For the vector FF can fix $F_0 = 1$ (charge conservation)
Fit Result (correlated fit):

#	quantity	F_0	$M[\mathrm{GeV}]$	p	χ^2/DOF
1	F_{em}	1(fixed)	0.777(12)	1(fixed)	6.010
2		1(fixed)	0.872(16)	1.173(69)	4.400
3	F_{scal}	2.222(19)GeV	1.314(39)	1(fixed)	7.886
4		2.212(19)GeV	2.023(50)	2(fixed)	9.877

Factorization Test Results

Test 1 for A_{VV}:

PRELIMINARY



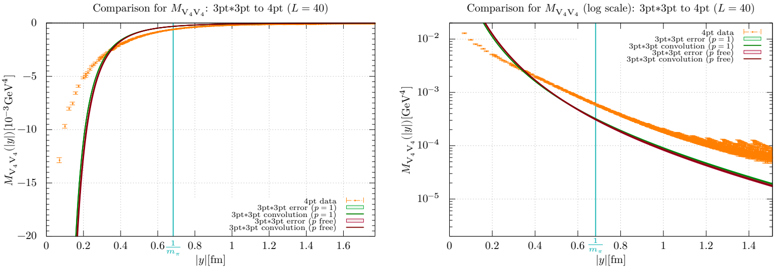
- ❍ good agreement for large distances of the two scattering quarks
- ❍ clear discrepancy for scattering distances smaller than m_π^{-1}

$\Rightarrow$ naive factorization might be a good assumption if the scattering quarks are far apart from each other

Factorization Test Results

Test 2 for $M_{\mathrm{V}_4\mathrm{V}_4}$:

PRELIMINARY

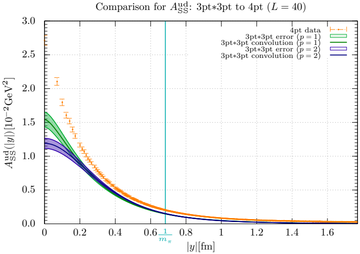


No region where the two curves match

$\Rightarrow$ Bad assumption, only order of magnitude is roughly the same for large distances

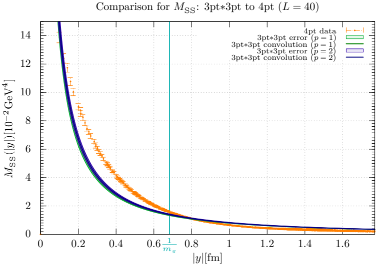
Factorization Test Results

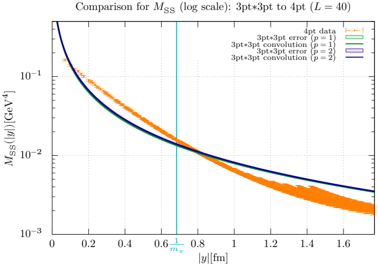
Test 1 for A_{SS} (PRELIMINARY) :



- Test 1: Agreement worse than for A_{VV} (disconnected contributions ?)
- Test 2: Bad agreement as for $M_{\mathrm{V_4 V_4}}$

Test 2 for M_{SS} (PRELIMINARY) :





Summary

Achievements:

- Calculated all 4pt-graphs for the π^+ at $\boldsymbol{p} = \mathbf{0}$
- Good quality of the data (except for S_2 and D)
- Tested naive factorization into convolution of two form factors:
  - Test 1: ok if distances are large
    Clear deviation for distances smaller than the pion wave length
  - Test 2: worse (order of magnitude correct)

  $\Rightarrow$ Calculation of two-current matrix elements essential!

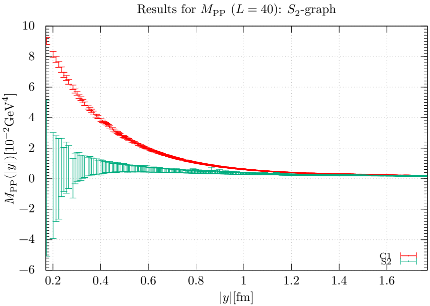
To do:

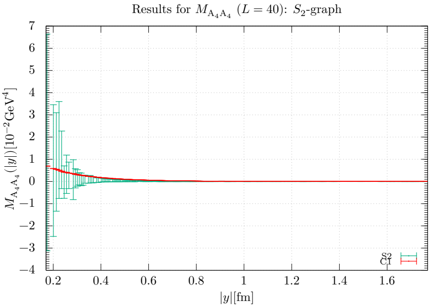
- Get better signals for S_2 and D -graph
- Take derivatives into account
- Go to non-zero momenta
- Investigate other particles: ρ-meson, nucleon (LHC experiments)

Thank you for your attention!

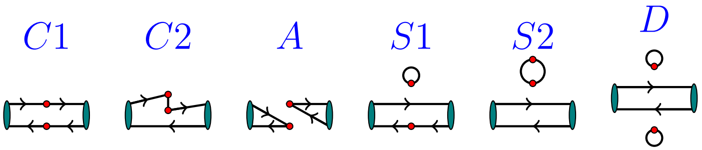
S_2 results:

PRELIMINARY





Isospin Matrix Elements



Depending on the quark flavor of the insertion operators, a certain set of 4pt-graphs must be summed up.

Isovector operators:

$$\mathcal{O}_X^1 = \bar{u}\Gamma_X d + \bar{d}\Gamma_X u$$
$$\mathcal{O}_X^2 = i\bar{u}\Gamma_X d - i\bar{d}\Gamma_X u$$
$$\mathcal{O}_X^3 = \bar{u}\Gamma_X u - \bar{d}\Gamma_X d$$

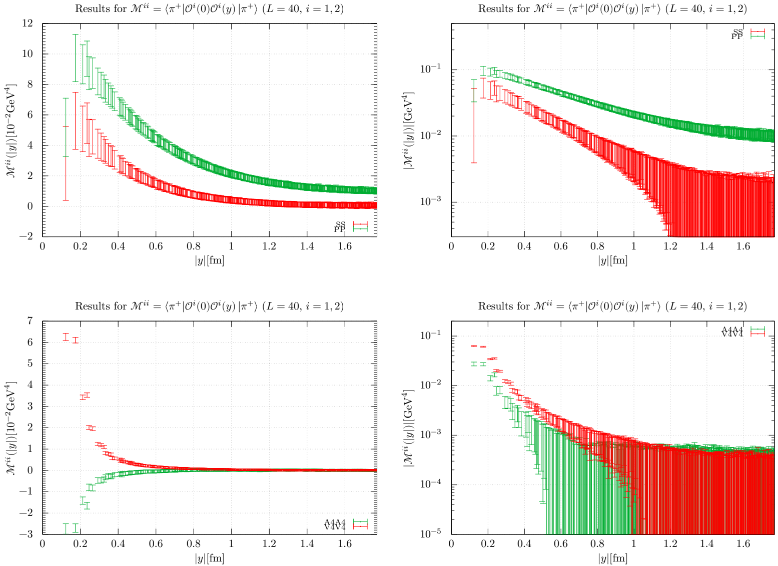
Matrix Elements in terms of 4pt-graphs:

$$\langle\pi^+|\,\mathcal{O}_{X_1}^1\mathcal{O}_{X_2}^1\,|\pi^+\rangle = \langle\pi^+|\,\mathcal{O}_{X_1}^2\mathcal{O}_{X_2}^2\,|\pi^+\rangle \sim 4C_2^{X_1X_2} - 2A^{X_1X_2} - 2S_2^{X_1X_2}$$
$$\langle\pi^+|\,\mathcal{O}_{X_1}^3\mathcal{O}_{X_2}^3\,|\pi^+\rangle \sim 4C_2^{X_1X_2} - 2C_1^{X_1X_2} - 2S_2^{X_1X_2}$$

Neglect contributions of S_2.

Isospin Matrix Elements: Results

PRELIMINARY



Isospin Matrix Elements: Results

PRELIMINARY