Magnetic polarizability of pion

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Introduction

We consider $SU(3)$ lattice gauge theory without dynamical quarks in a constant external abelian magnetic field directed along the third axis z and calculate

1. the energies of the ground states of π^0 and $\pi^\pm$ mesons versus the field value;
2. the dipole magnetic polarizabilities of pions;
3. the magnetic hyperpolarizability of the neutral pion;

Motivation

Magnetic polarizability of a meson

- is the fundamental quantity, characterizing spin interactions of quarks;
- describes the distibution of quark currents inside a meson in an external field;
- the external magnetic field of hadronic scale can be used as a probe of QCD properties.

Technical details

$SU(3)$ lattice gauge configurations:
the tadpole improved Lüscher-Weisz action

$$S = \beta_{imp} \sum_{plq} S_{plq} - \frac{\beta_{imp}}{20u_0^2} \sum_{rt} S_{rt},$$

$S_{plq,rt} = (1/3)Tr(1 - U_{plq,rt})$ is the lattice plaquette or 1×2 rectangular loop,
$u_0 = (W_{1\times1})^{1/4} = \langle (1/3)TrU_{plq} \rangle^{1/4}$ is the tadpole factor, calculated at zero temperature.

Technical details

Add the magnetic field $F_{12} = B_z = B$ into the Dirac operator:

- $A_{\mu\, ij} = A^{gl}_{\mu\, ij} + A^{U(1)}_{\mu} \delta_{ij}, \qquad A^{U(1)}_{\mu}(x) = \frac{B}{2}(x_1 \delta_{\mu,2} - x_2 \delta_{\mu,1})$
- perform the additional twist for fermions

M. H. Al-Hashimi, U. J. Wiese, 2009.

The magnetic field is quantized:

$$eB = \frac{6\pi k}{(aL)^2}, \quad k \in \mathbb{Z}$$

Solve the Dirac equation numerically:

$$D\psi_k = i\lambda_k\psi_k, \quad D = \gamma^\mu(\partial_\mu - iA_\mu).$$

Calculate the propagators:

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}.$$

Correlation functions:

$$\langle\bar{\psi}O_\mu\psi\bar{\psi}O_\nu\psi\rangle_A = -tr[O_\mu D^{-1}(x,y)O_\nu D^{-1}(y,x)]+$$

$$+tr[O_\mu D^{-1}(x,x)]tr[O_\nu D^{-1}(y,y)], \quad x = (\mathbf{n}a, n_t a), \quad y = (\mathbf{n}'a, n_t' a)$$

$$\mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3)|n_i = 0, 1, ..., N-1\}$$

$$\langle\psi^\dagger(\vec{0},n_t)O_\mu\psi(\vec{0},n_t)\psi^\dagger(\vec{0},0)\gamma_5\psi(\vec{0},0)\rangle_A = \sum_k \langle 0|O_\mu|k\rangle\langle k|O_\nu^\dagger|0\rangle e^{-n_t E_k}.$$

The main contributions comes from $\langle 0|O_\mu|k\rangle\langle k|O_\nu^\dagger|0\rangle e^{-n_t E_0}$,

Fit the correlator by the function
$\tilde{C}_{fit}(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T-n_t)aE_0} =$
$2A_0 e^{-N_T a E_0/2}\cosh((N_T-n_t)aE_0)$ at $5 \le n_t \le N_T - 5.$.

Previous results

SERPUKHOV group: $\alpha_{\pi^\pm} = (6.8 \pm 1.4_{stat} \pm 1.2_{syst}) \times 10^{-4}$ fm^3.
Process: $\pi^- Z \to \pi^- Z \gamma$,
Yu. M. Antipov et al., Phys.Lett.B 121, 445 (1983).

MARK II group (SLAC):
$\alpha_{\pi^\pm} = -\beta_{\pi^\pm} = (2.2 \pm 1.6_{stat+syst}) \times 10^{-4}$ fm^3.
Process: $e^+ e^- \to e^+ e^- \pi^+ \pi^-$,
J. Boyer et al., Phys.Rev.D 42, 1350 (1990).
D. Babusci et. al., Phys.Lett.B 277, 158 (1992).

Previous results

Chiral Perturbation Theory (two loops): $\beta = -2.77 \times 10^{-4}$ fm^3, A. Aleksejevs and S. Barkanova, Nucl. Phys. Proc. Suppl. 245, 17 (2013), arXiv:1309.3313 [hep-ph].

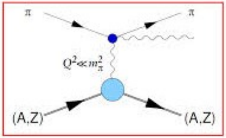
Electric polarizability from COMPASS (CERN): $\alpha_{\pi^\pm} = (2.0 \pm 0.6_{stat} \pm 0.7_{syst}) \cdot 10^4$ fm^3 under the assumption $\alpha_{\pi^\pm} = -\beta_{\pi^\pm}$ of the ChPT.
C. Adolph et al., Phys. Rev. Lett. 114, 062002 (2015).

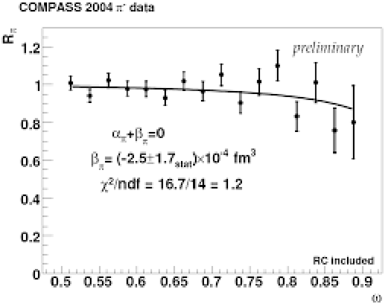
COMPASS

The Primakoff reaction:

$$\pi^- + (A, Z) \to \pi^- + (A, Z) + \gamma$$

quasi-real photon Compton scattering on π^-





$$R_\pi = \sigma_{exp}(\omega)/\sigma_{theory}(\omega), \omega = E_\gamma$$

Energy levels of charged pion

The energy levels of free charged pointlike particles in a background magnetic field parallel to 'z' axis

$$E^2 = p_z^2 + (2n+1)|qH| - gs_z qH + E^2(H=0), \quad H = eB$$

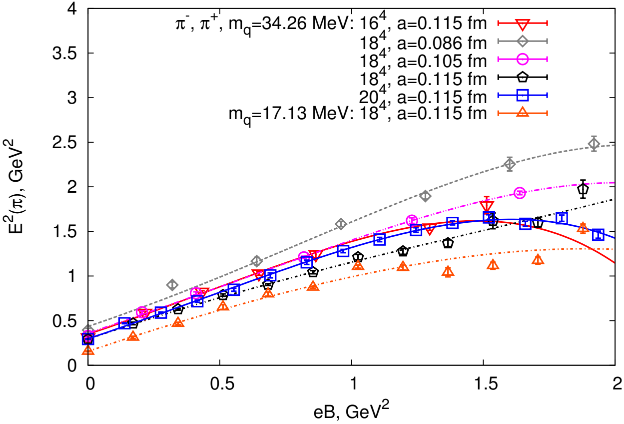
It doesn't take into account polarizability of particle.
In the relativistic case

$$E^2 = |qH| + m^2 - 4\pi m \beta_m H^2 - 4\pi m \beta_m^{1h} H^4$$

.

Energy of $\pi^{\pm}$ meson vs. the magnetic field

$$C^{PSPS} = \langle \bar{\psi}_d(\vec{0}, n_t)\gamma_5\psi_u(\vec{0}, n_t)\bar{\psi}_u(\vec{0}, 0)\gamma_5\psi_d(\vec{0}, 0)\rangle$$



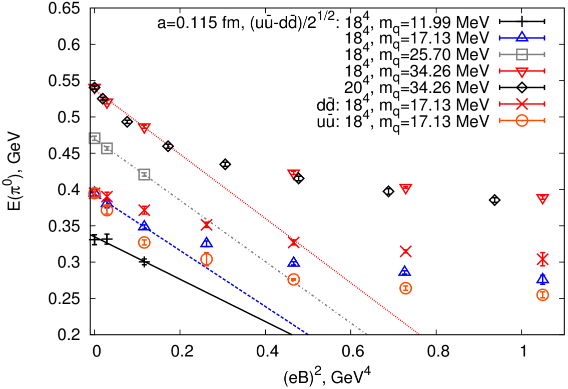
Magnetic dipole polarizability of $\pi^{\pm}$ meson

Fit at $eB \in [0, 2]$ GeV^2, the quark mass is $m_q = 34.26$ MeV.

V_{latt}	a (fm)	β_m (GeV^{-3})	β_m^{1h} (GeV^{-7})	χ^2/d.o.f.
18^4	0.086	-0.027 ± 0.010	0.007 ± 0.004	4.828
20^4	0.115	-0.015 ± 0.004	0.011 ± 0.002	5.121
18^4	0.105	-0.011 ± 0.007	0.005 ± 0.003	3.414
16^4	0.115	-0.021 ± 0.015	0.015 ± 0.009	3.935

$\beta_m = (-2.06 \pm 0.76) \times 10^{-4}$ fm^3 for the lattice volume 18^4, lattice spacing 0.086 fm,
COMPASS (CERN): $\beta_m^{\pi^{\pm}} = -(2.0 \pm 0.6_{stat} \pm 0.7_{syst}) \times 10^4$ fm^3,
ChPT (two loops): $\beta = -2.77 \times 10^{-4}$ fm^3.

Energy of π^0 meson vs. the magnetic field



Fit at $(eB)^2 \in [0 : 0.15\ GeV^4]$: $E = E(B = 0) - 2\pi\beta(eB)^2$

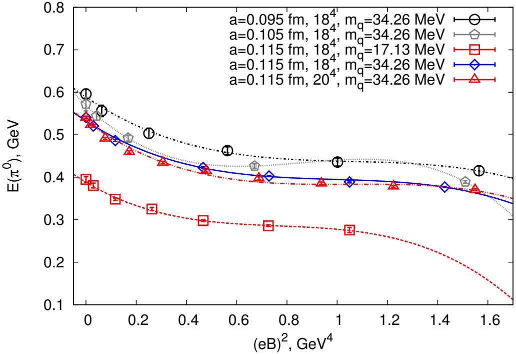
Magnetic dipole polarizability of π^0 meson

$\beta_m = (3.3 \pm 0.4) \cdot 10^{-4}$ fm^3 for the lattice volume 18^4 and lattice spacing 0.115 fm in the chiral limit ($m_q = 0$).

ChPT 2 loops: $\beta_{\pi^0} = (1.5 \pm 0.3) \cdot 10^{-4}\ fm^3$ (0.5 at one loop); $\alpha_\pi + \beta_\pi = 0$ in the leading order of ChPT.

COMPASS from JLab data: $\alpha_{\pi^0} = (-1.0 : -0.3) \cdot 10^{-4}\ fm^3$.

Energy of π^0 meson



$$E = E(B=0) - 2\pi\beta_m (eB)^2 - 2\pi\beta_m^h (eB)^4 - k(eB)^6$$

$\beta_m^h = -0.068 \pm 0.015\ \mathrm{GeV}^{-7}$ for $V_{lat} = 18^4$, $a = 0.115$ fm, $m_q = 17.13$ MeV, χ^2/d.o.f.=1.433.

Conclusions

1. We confirm the result of COMPASS experiment for the magnetic dipole polarizability of charged pion.
2. The magnetic polarizability of neutral pion was calculated. It agrees in sign with the prediction of ChPT and result obtained from JLab data.
3. The contribution of magnetic hyperpolarizability to the energy of neutral pion was found.

Thank you for your attention!