B_c decays from highly improved staggered quarks and NRQCD

Andrew Lytle (HPQCD Collaboration)
University of Glasgow

Lattice 2016
Southampton, UK
27.07.16

Intro & Motivation

- Obtain $|V_{cb}|$ from $b \to c$ transitions in semileptonic decays.
- Treatment of c and especially b quarks challenging in lattice simulations due to lattice artifacts which grow as $(am_q)^n$.
- We use two complementary approaches:
  - Highly improved relativistic action at small a, extrapolate $m_h \to m_b$.
  - Improved non-relativistic formalism (NRQCD) at m_b.
- First study:
  - $B_c \to \eta_c$
  - $B_c \to J/\psi$ [accessible at LHCb]
- More precise $b \to c$ currents used in $B \to D$, $B \to D^*$.

Outline

1. Intro & Motivation.
2. Calculation Framework.
   - HISQ action.
   - Improved NRQCD.
3. Semileptonic Decays.
   - Correlation functions.
   - $B_c \to \eta_c$ and results.
   - $B_c \to J/\psi$ and results.
4. Discussion & Future Work.

DiRAC II computing

Computations carried out on the Darwin cluster at Cambridge.

Includes:

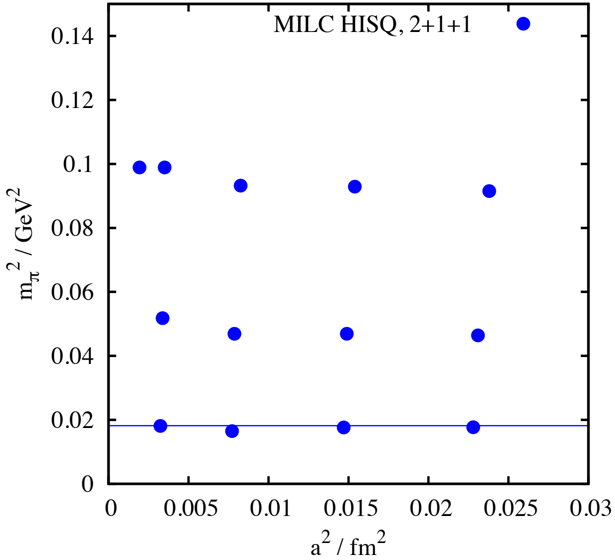
- 9600 Intel Sandy Bridge cores
- 2.6 GHz, 4 GB RAM/core
- 2 PB storage

Part of STFC's HPC facility for theoretical particle physics and astronomy.

MILC ensembles [1004.0342, 1212.4768]

- HISQ fermion action.
- Symanzik-improved gauge action, takes into account $\mathcal{O}(N_f \alpha_s a^2)$ effects of HISQ quarks in sea. [0812.0503]
- Multiple lattice spacings down to ~ 0.045 fm.
- Effects of u/d, s, and c quarks in the sea.
- Multiple light-quark input parameters down to physical pion mass.
  - Chiral fits.
  - Reduce statistical errors.

MILC ensemble parameters



NRQCD

Heavy quark propagators are calculated using a non-relativistic formalism.

Improved Non-relativistic QCD action

- Accurate through $\mathcal{O}(\alpha_s v^4)$.
- Discretisation corrections through $\mathcal{O}(\alpha_s v^2 a^2 p^2)$.
- $v^2 \sim 0.1$ bottomonium, ~ 0.3 charmonium.
- $am > 1 \rightarrow b$ quarks on $a = 0.15 - 0.06$ fm (down to $m_b/2$ on $a = 0.15$ fm).

Propagators constructed via an evolution equation,
$G(\mathbf{x}, t + a) = e^{-aH_{\text{eff}}} G(\mathbf{x}, t)$.

NRQCD

$$
\begin{aligned}
aH_{\mathrm{NRQCD}} &= aH_0 + a\delta H \\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b} \\
a\delta H &= -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2} \left(\nabla \cdot \mathbf{E} - \mathbf{E} \cdot \nabla\right) \\
&\quad - c_3 \frac{1}{8(am_b)^2} \sigma \cdot \left(\nabla \times \mathbf{E} - \mathbf{E} \times \nabla\right) \\
&\quad - c_4 \frac{1}{2am_b} \sigma \cdot \mathbf{B} + c_5 \frac{\Delta^{(4)}}{24am_b} \\
&\quad - c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2}
\end{aligned}
$$

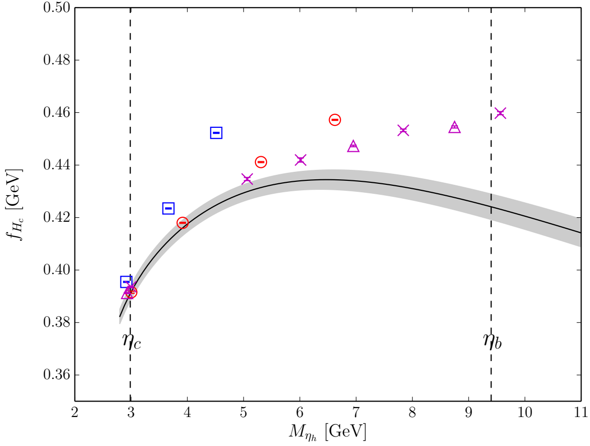
General strategy

Strategy pursued by HPQCD collaboration:

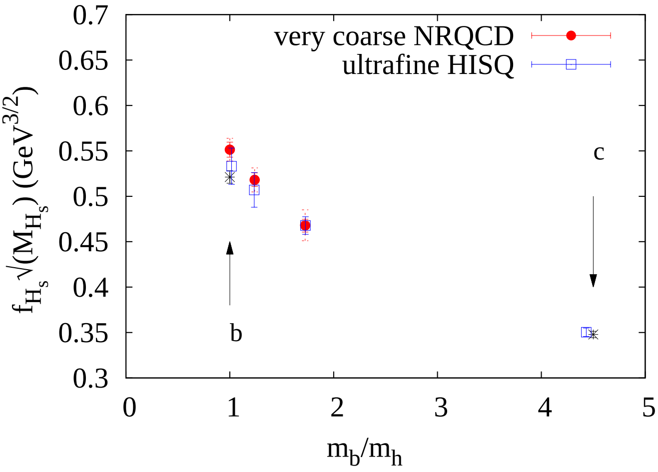
- Staggered quarks → small a, physical pions, multiple lattice spacings..
- Highly improved action
  → discretisation effects under control at m_c
  → reduced taste-splittings
  → physical point ensembles with dynamical u/d, s, and c quarks.
- Compute heavy quarks using (improved) NRQCD.

These two approaches are complementary. Ideally there is a range of overlap in applicability to check the approaches are mutually consistent.

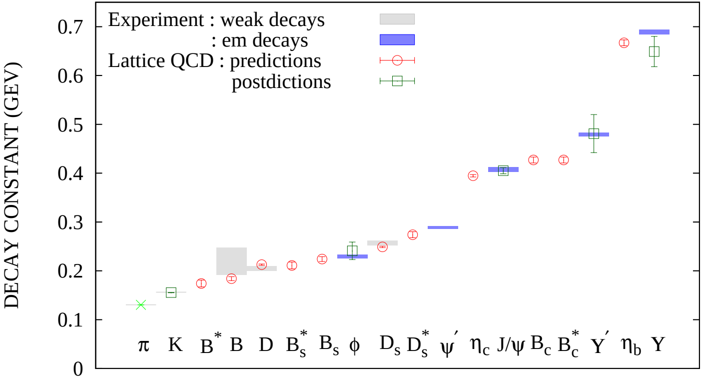
f_{H_c} from HISQ.



From b to c. [1302.2644], [1008.4018]



Decay constants – summary plot.



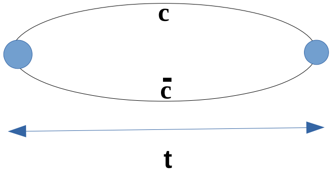
B_c semileptonic decays

Semileptonic decays

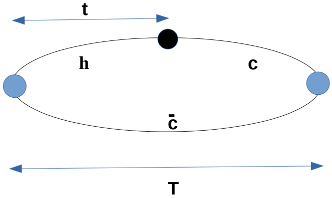
- Study of $B_c \to \eta_c$, $B_c \to J/\psi$ decay matrix elements.
- We work in the frame where the B_c is at rest.
- The form factors which parametrise the matrix elements are functions of q^2, where q is the four-momentum transferred to the leptons.
  - $q^2_{\mathrm{max}} = (M - m)^2$, zero recoil of decay hadron.
  - $q^2 = 0$, maximum recoil of decay hadron.
- Matrix elements are determined by simultaneous fitting of three-point and two-point functions.

Semileptonic decays – meson correlators

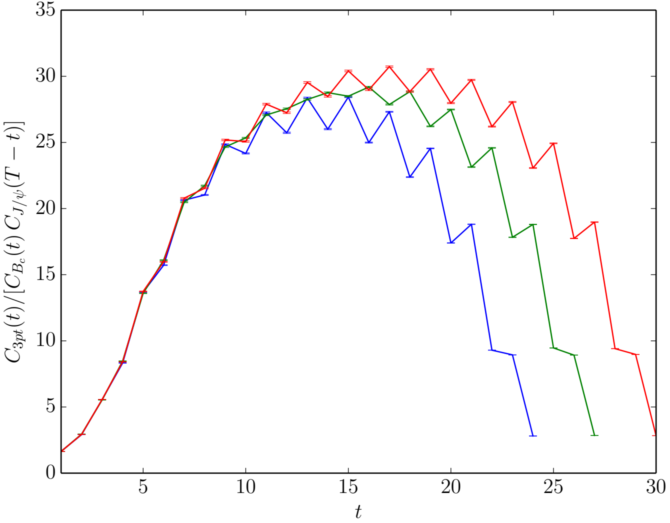
Two-point functions:



Three-point functions:



Semileptonic decays



$B_c \to \eta_c$

$$Z\langle \eta_c(p)|V^\mu|B_c(P)\rangle = f_+(q^2)\left[P^\mu + p^\mu - \frac{M^2-m^2}{q^2}q^\mu\right] + f_0(q^2)\,\frac{M^2-m^2}{q^2}\,q^\mu\,,$$

From PCVC,

$$\langle \eta_c(p)|S|B_c(P)\rangle = \frac{M^2-m^2}{m_{b0}-m_{c0}} f_0(q^2)$$

Find Z by calculating both matrix elements at q^2_{max}.

$B_c \to \eta_c$

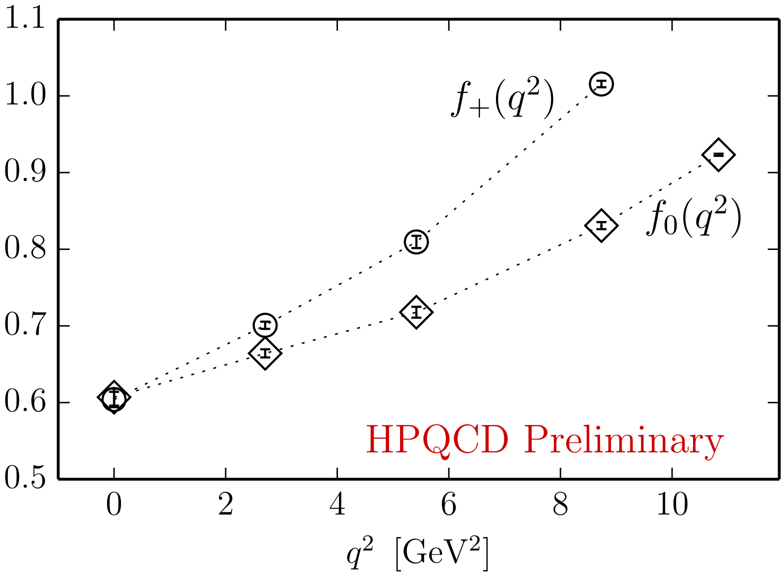
f_0 and f_+ are determined in the NRQCD formalism from matrix elements of the vector current $\langle V_\mu^{\rm nrqcd} \rangle$, where

$$V_0^{\rm nrqcd} = (1 + \alpha_s z_0^{(0)}) \left[V_0^{(0)} + (1 + \alpha_s z_0^{(1)}) V_0^{(1)} + \alpha_s z_0^{(2)} V_0^{(2)} \right]$$

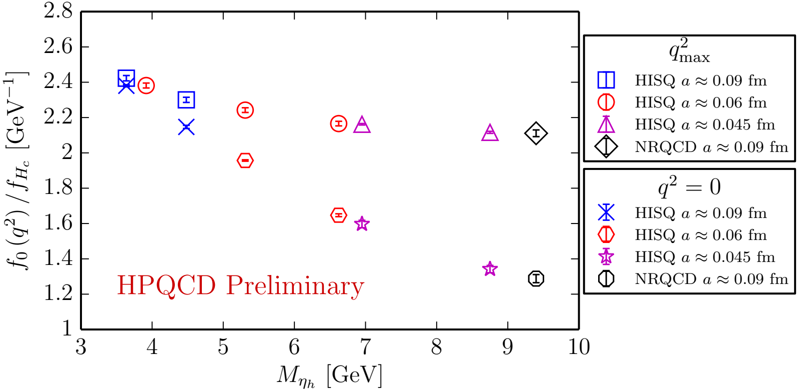
$$V_k^{\rm nrqcd} = (1 + \alpha_s z_k^{(0)}) \Big[V_k^{(0)} + (1 + \alpha_s z_k^{(1)}) V_k^{(1)} + \alpha_s z_k^{(2)} V_k^{(2)} + \alpha_s z_k^{(3)} V_k^{(3)} + \alpha_s z_k^{(4)} V_k^{(4)} \Big] .$$

One goal of the present work is to constrain the coefficients entering $V_\mu^{\rm nrqcd}$ using fully relativistic HISQ data.

NRQCD form factors.



f_0 from HISQ.

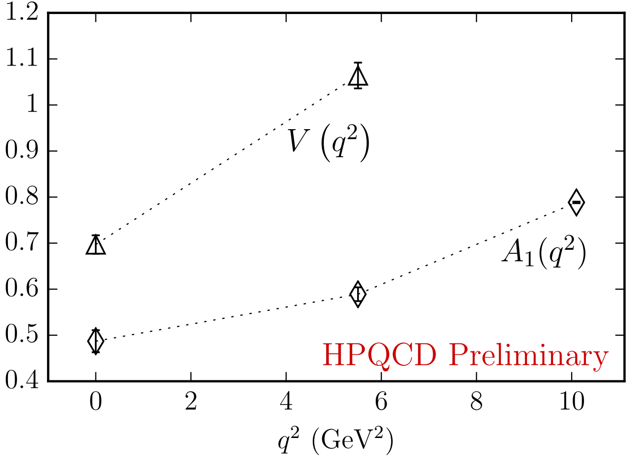


$f_0\,(q^2)\,/\,f_{H_c}$ [GeV^{-1}]
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
3
4
5
6
7
8
9
10
M_{η_h} [GeV]
HPQCD Preliminary
$q^2_{\rm max}$
HISQ $a \approx 0.09$ fm
HISQ $a \approx 0.06$ fm
HISQ $a \approx 0.045$ fm
NRQCD $a \approx 0.09$ fm
$q^2 = 0$
HISQ $a \approx 0.09$ fm
HISQ $a \approx 0.06$ fm
HISQ $a \approx 0.045$ fm
NRQCD $a \approx 0.09$ fm

$B_c \to J/\psi$

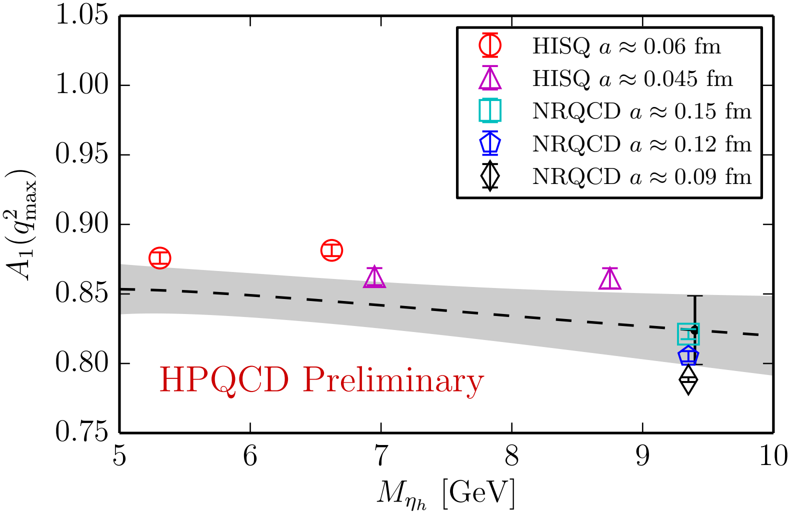
$$\langle J/\psi(p,\varepsilon)|V^\mu - A^\mu|B_c(P)\rangle =$$
$$\frac{2i\epsilon^{\mu\nu\rho\sigma}}{M+m}\varepsilon^*_\nu p_\rho P_\sigma V(q^2) - (M+m)\varepsilon^{*\mu}A_1(q^2) +$$
$$\frac{\varepsilon^*\cdot q}{M+m}\left(p+P\right)^\mu A_2(q^2) + 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_3(q^2) - 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_0(q^2)$$

$B_c \to J/\psi$.



1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0
2
4
6
8
10
q^2 (GeV^2)
$V\left(q^2\right)$
$A_1(q^2)$
HPQCD Preliminary

A_1 from HISQ.



HISQ $a \approx 0.06$ fm
HISQ $a \approx 0.045$ fm
NRQCD $a \approx 0.15$ fm
NRQCD $a \approx 0.12$ fm
NRQCD $a \approx 0.09$ fm
HPQCD Preliminary
$A_1(q^2_{\mathrm{max}})$
M_{η_h} [GeV]
1.05
1.00
0.95
0.90
0.85
0.80
0.75
5
6
7
8
9
10

Summary

- A promising approach to study of $b \to c$ transitions:
  - Lattice NRQCD with HISQ quarks, plus
  - Fully relativistic formulation, extrapolate m_h to m_b.

- Proof-of-principle demonstrated for f_0.
  - Controlled calculation over full q^2 range.
  - Good agreement seen with NRQCD results.

- Outputs:
  - B_c to $J/\Psi \to$ new possible determination of $|V_{cb}|$.
  - Improved understanding of NRQCD currents feeds into additional calculations (B to D, B to D^*, ...).

Thank you!