



Review on
Hadron
Spectroscopy
and
Resonances

Chuan Liu

Introduction

The
conventional
spectrum

Single
channel
scattering

Summary and
outlooks

Review on Hadron Spectroscopy and Resonances

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Disclaimer

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■ I would like to thank...

- LOC for the kind invitation
- my collaborators at CLQCD:
Y. Chen, M. Gong, N. Li, Z. Liu, J.P. Ma, Y.B. Liu,
J.B. Zhang
- people who sent me information/plots:
S. Aoki, G. Bali, S. Dürr, D. Leinweber, K.-F. Liu,
G. Schierholz, C. Urbach, etc.



Things to be covered

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- A good way to start preparing a plenary talk
 - looking at previous plenary talks
 - However, there was no plenary talks on spectrum last year
 - that means that I should cover two years...
-  There is no way to cover so much in 45 min.
- the conventional spectrum computations ✓
 - single-channel meson-meson scattering ✓
 - multi-channel scattering.....see Wilson's talk!
 - baryon-baryon scattering.....see Savage's talk
 -



Outline

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- **Methodologies**
 - The conventional one (the GEVP)
 - The not so conventional ones
 - Lüscher formalism
 - Not of Lüscher: HEFT, HALQCD, etc.
- **The Conventional spectrum**
 - QCD+QED computations
 - Simulations near the physical point
 - Excited nucleon states
- **Single-channel scattering**
 - pion-pion scattering
 - charmed meson scattering
 - others...
- **Summary and outlooks**
 - Where we stand and what to expect next



1. METHODS

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The first step: GEVP

in a typical lattice spectrum calculation

- A set of interpolating operators with the “right” quantum numbers: $\{\mathcal{O}_\alpha : \alpha = 1, 2, \dots, N_{op}\}$
- Compute the correlation matrix:

$$\mathcal{C}_{\alpha\beta}(t, 0) = \langle \mathcal{O}_\alpha(t) \mathcal{O}_\beta^\dagger(0) \rangle, \quad (1)$$

- Solve the so-called Generalized Eigen-Value Problem (GEVP) for the eigenvalues λ_α 's,

$$\mathcal{C}(t, 0) \cdot v_\alpha = \lambda_\alpha(t, t_0) \mathcal{C}(t_0, 0) \cdot v_\alpha, \quad (2)$$

for some appropriately chosen t_0

- From the eigenvalues $\lambda_\alpha(t, t_0)$, extract the corresponding eigenvalues of the Hamiltonian: E_α via

$$\lambda_\alpha(t, t_0) \sim e^{-E_\alpha(t-t_0)}. \quad (3)$$

- Pass the E_α 's to the second step



Complications

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- 1 E_α 's are NOT hadron mass values!
 - E_α is the eigenvalue of the QCD Hamiltonian
 - (in a latticized finite box!)
 - Most hadrons are resonances
- 2 Many types of operators enter (operator mixing)!
 - single hadron operators
 - multi-hadron operators (esp. beyond the threshold)...



The second step

Relate the E_α 's to the spectral quantity

- 0 E_α 's are "approximate" hadron masses
 - only if the hadron is stable or the resonance is "narrow" enough
- 1 Using a version of the Lüscher formalism
 - single channel version has matured over the years
 - multi-channel applications, see Dr. Wilson's talk
 - more channels, rather complicated
- 2 Using other approaches
 - the effective Hamiltonian approach
 - the HALQCD approach
 - the optical potential approach



Lüscher's approach

in theory (e.g. M. Lüscher, NPB354, 531, 1991)

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- original: single-channel spinless two-particle elastic scattering in COM frame,

$$E_\alpha(L) \Leftrightarrow \delta(E_\alpha) . \quad (4)$$

$$\left\{ \begin{array}{l} \tan \delta(\bar{k}) = \frac{\pi^{3/2} q}{Z_{00}(1, q^2)} , \\ 2\sqrt{\bar{k}^2 + m^2} = E(L) , \quad q = kL/(2\pi) . \end{array} \right. \quad (5)$$

- extensions over the years
 - to particles with spin
 - to multi-channels
 - different BC's,
 - different frames,
 - ...



Lüscher's approach in practice

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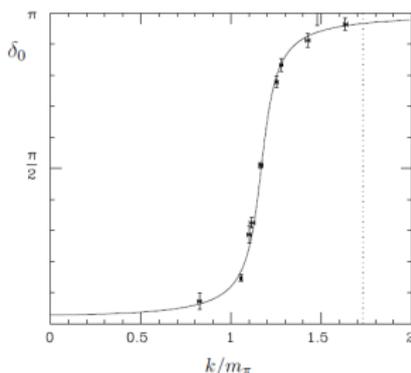
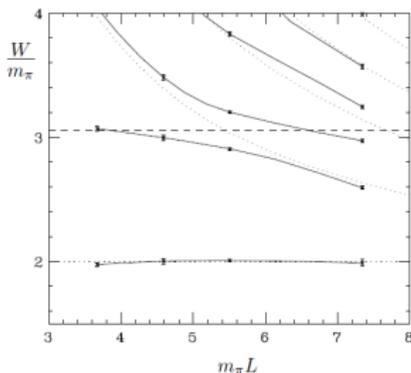
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- scalar $\lambda\phi^4$ theory [F. Zimmermann et al, hep-lat/9211029; NPB425, 413, 1994](#)
- pion-pion scattering
 - quenched 1992; [Gupta et al, PRD48, 388, 1993](#)
 - unquenched since 2005 or so
- has matured in recent years



complicated for multi-channels



Other approaches: the HEFT approach

see e.g. J.M.M. Hall et al, PRD87,094510,2013; arXiv:1303.4157

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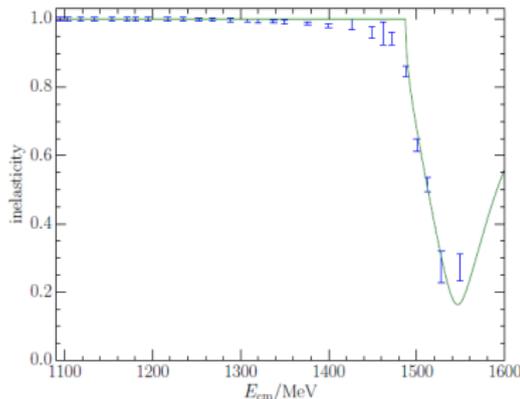
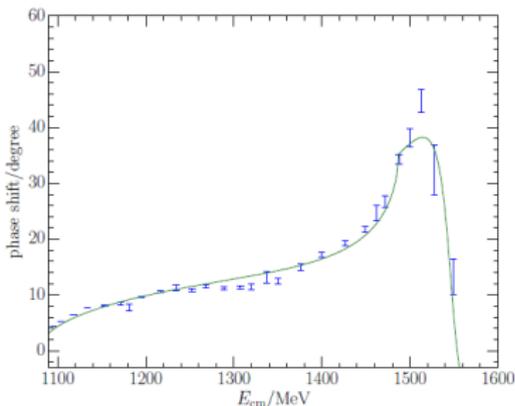
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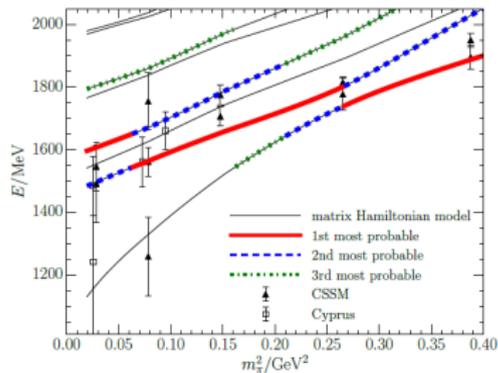
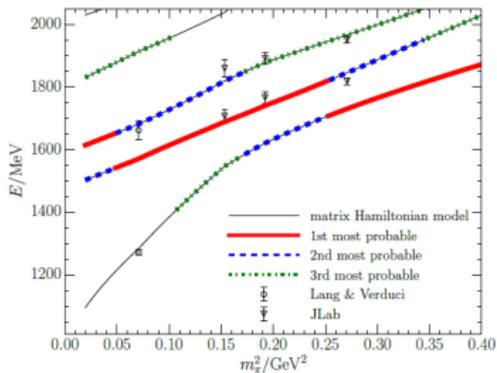


- Needs to construct the appropriate hamiltonian
- model parameters are determined by fitting low-energy data, e.g. for $N^*(1535)$ study, Z.-W. Liu et al, PRL116, 082004, 2016; 1512.00140
- effective Hamiltonian in a finite volume



comparison of the levels

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■ The finite volume levels from [Z.-W. Liu et al, PRL116, 082004, 2016;](#)

1512.00140



Other approaches: the HALQCD method

see e.t. N. Ishii et al, PRL99, 022001,2007; PLB712,437,2012.

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- starts from the so-called NBS wavefunction (a four-point function)

$$F_{N\Omega}(\mathbf{x} - \mathbf{y}, t - t_0) = \langle 0 | N_\alpha(\mathbf{x}, t) \Omega_{\beta,l}(\mathbf{y}, t) \bar{J}_{N\Omega}(t_0) | 0 \rangle \quad (6)$$

- the potential is obtained via the time-dependent HALQCD approach,

$$V_C(r) \simeq \frac{1}{2\mu} \nabla^2 R(r, t) / R(r, t) - \frac{\partial}{\partial t} \ln R(r, t), \quad (7)$$

with $R(r, t) = F_{N\Omega}(t, t) / e^{-(m_N + m_\Omega)t}$.

 no need for GEVP



Other approaches: the optical potential

see e.g. D. Agadjanov et al, arXiv: 1603.07205

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- measure the optical potential directly
 - analytically continue $W(E)$ to $W(E + i\varepsilon)$
 - taking $L \rightarrow \infty$, then $\varepsilon \rightarrow 0$
 - done by smoothing
- can handle multi-channels, or more than 2 particles
- relatively new, needs further study
- In particular, what is the relation with HALQCD approach?



My list again

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- QCD+QED computations: BMW, QCDSF+UKQCD
- simulation at the physical point by ETMC
- excited nucleon spectrum: CSSM and χ QCD
- pion-pion scattering: ETMC & other groups
- charmed meson scattering: XYZ particles



2.1 THE NORMAL

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n**o**rmal



proton & neutron

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&





QCD+QED by BMW

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QCD+QED

by

BMW



QCD+QED by BMW: The formalism

$N_f = 1 + 1 + 1 + 1$ simulations, non-compact QED

BMW, Science 347:1452,2015; 1406.4088

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- Use non-compact QED description
- Fix to Coulomb gauge (Feynman \rightarrow Coulomb)
- treating the zero-mode of $A_\mu(x)$ field in QED_L prescription
- The Dirac operator,

$$\begin{aligned}(D)_{x,y} &= (4 + m) \delta_{x,y} - \frac{1}{2} \sum_{\nu > \mu} (F_{\mu\nu,x}^{(U)} + eqF_{\mu\nu,x}^{(A)}) \sigma_{\mu\nu} \delta_{x,y} \\ &- \frac{1}{2} \sum_{\mu} \left[(1 + \gamma_{\mu}) e^{ieqA_{\mu,x}} U_{\mu,x} \delta_{x+\hat{\mu},y} \right. \\ &\left. + (1 - \gamma_{\mu}) e^{-ieqA_{\mu,x-\mu}} U_{\mu,x-\mu}^{\dagger} \delta_{x-\hat{\mu},y} \right] \quad (8)\end{aligned}$$

 Main focus: octet baryons



QCD+QED by BMW

$N_f = 1 + 1 + 1 + 1$ simulations, non-compact QED

BMW, Science 347:1452,2015; 1406.4088

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- lowest pion mass 195MeV, largest volume extent 8fm
- finite volume corrections due to QED in Coulomb gauge
- treating zero-mode in QED_L prescription
- Using ΔM_Σ to separate QED and QCD corrections
- Akaike's information criterion (AIC) used

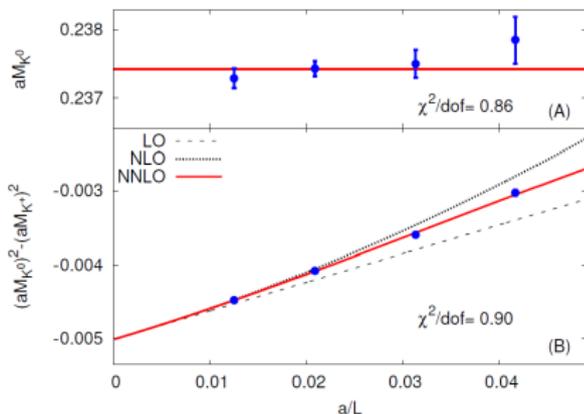


Figure 1: **Finite-volume behavior of kaon masses.** (A) The neutral kaon mass, M_{K^0} , shows no significant finite volume dependence; L denotes the linear size of the system. (B) The mass-squared difference of the charged kaon mass, M_{K^+} , and M_{K^0} indicates that M_{K^+} is strongly dependent on volume. This finite-volume dependence is well described by an asymptotic expansion in $1/L$ whose first two terms are fixed by QED Ward-Takahashi identities (17). The solid curve depicts a fit of the lattice results (points) to the expansion up to and including a fitted $O(1/L^3)$ term. The dashed and dotted curves show the contributions of the leading and leading plus next-to-leading order terms, respectively. The computation was performed by using the following parameters: bare $\alpha \sim 1/10$, $M_\pi = 290$ MeV, and $M_{K^0} = 450$ MeV. The mass difference is negative because a larger-than-physical value of α was used. The lattice spacing a is ~ 0.10 fm.



QCD+QED by BMW

$N_f = 1 + 1 + 1 + 1$ simulations, non-compact QED

BMW, Science 347:1452,2015; 1406.4088

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- lowest pion mass 195MeV, largest volume extent 8fm
- post-dictions for ΔN , $\Delta \Sigma$, ΔM_D
- predictions for $\Delta \Xi$, $\Delta \Xi_{CC}$ and Δ_{CG}

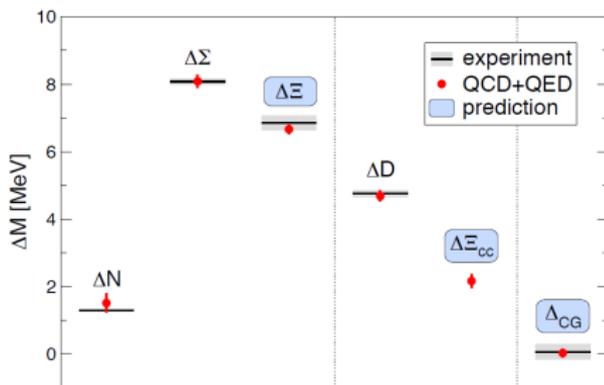


Figure 2: Mass splittings in channels that are stable under the strong and electromagnetic interactions. Both of these interactions are fully unquenched in our 1+1+1+1 flavor calculation. The horizontal lines are the experimental values and the grey shaded regions represent the experimental error (2). Our results are shown by red dots with their uncertainties. The error bars are the squared sums of the statistical and systematic errors. The results for the ΔM_N , ΔM_Σ , and ΔM_D mass splittings are post-dictions, in the sense that their values are known experimentally with higher precision than from our calculation. On the other hand, our calculations yield ΔM_Ξ , $\Delta M_{\Xi_{CC}}$ splittings, and the Coleman-Glashow difference Δ_{CG} , which have either not been measured in experiment or are measured with less precision than obtained here. This feature is represented by a blue shaded region around the label.



QCD+QED by BMW

$N_f = 1 + 1 + 1 + 1$ simulations, non-compact QED

BMW, Science 347:1452,2015; 1406.4088

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- lowest pion mass
- 195MeV, largest volume
- extent 8fm



Fine-tuning

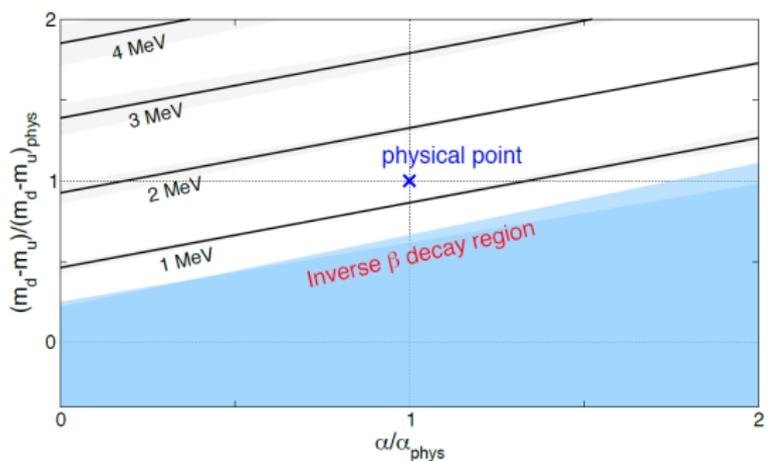


Figure 3: **Contour lines for the neutron-proton mass splitting.** The contours are shown as a function of quark mass difference and the fine structure constant, both normalized with their real world, physical values. Because these two effects compete, by increasing α at fixed quark mass difference one can decrease the mass difference between the neutron and the proton to 0.511 MeV, at which inverse β -decay sets in, as depicted by the blue region. The blue cross shows the physical point. The shaded bands around the contours represent the total statistical and systematic uncertainties on these predictions. A constraint on the neutron-proton mass difference obtained from other considerations leads to a constraint on $m_d - m_u$ and/or α , which can be directly read off from the figure.



Other related BMW computations:

sigma term, f_K/f_π , quark masses

- 1 sigma term via Feynman-Hellmann theorem in 2+1 flavor QCD, [1510.08013](#)),

$$\sigma_{\pi N} = 38(3)(3) \text{ MeV} , \quad (9)$$

- Still much lower than Roy-Steiner equation determination of $\sigma_{\pi N} = 59.1(3.5) \text{ MeV}$
- lattice computations by χ QCD and ETMC are also lower
- Reasons:??

- 2 f_K/f_π in 2+1 flavor QCD ([1601.05998](#))

- important quantity for flavor physics (V_{ud} & V_{us})

$$f_K/f_\pi = 1.182(28) . \Rightarrow V_{us} = 0.2282(54) . \quad (10)$$

- 3 quark masses (qed quenched confs, [1604.07112](#))

$$\begin{aligned} m_u(\overline{MS}, 2 \text{ GeV}) &= 2.27(6)_{stat}(5)_{syst}(4)_{qQED} \text{ MeV} , \\ m_d(\overline{MS}, 2 \text{ GeV}) &= 4.67(6)_{stat}(5)_{syst}(4)_{qQED} \text{ MeV} . \end{aligned} \quad (11)$$



QCD+QED by QCDSF-UKQCD

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by

QCDSF-UKQCD



QCD+QED by QCDSF-UKQCD

1 + 1 + 1 flavor QCD+QED (non-compact)

- One of the most consequential quantity in physics.
- Subtly fine-tuned for the stability of matter

courtesy of Prof. G. Schierholz

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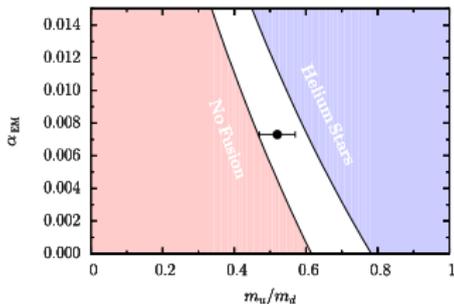
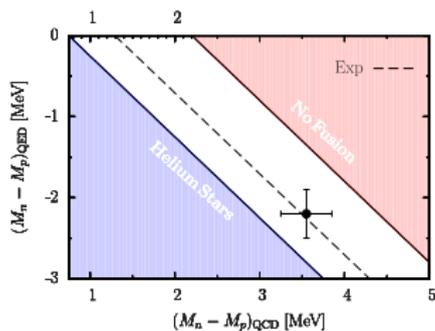
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The Challenge



Having an analytic expression for the nucleon mass as a function of quark masses and





QCD+QED by QCDSF-UKQCD

1 + 1 + 1 flavor QCD+QED (non-compact)

- starting from the $SU(3)$ symmetric point:
$$m_u = m_d = m_s = \bar{m}_q$$
- use mass of π^0 , K^0 and K^+ to set bare parameters
 - Better understanding for flavor violations
 - interpolate between 1 + 1 + 1, 2 + 1 and 3 flavors
- mass of hadron is scheme-independent
- however, the separation of the mass into QCD+QED contributions is scheme (and scale) dependent!
- QED corrections can be absorbed into the quark masses &/or meson masses, depending on its typical scale
- They used the so-called Dashen scheme of renormalization



Focus: vacuum structure, octet meson, baryon, quark masses



The lattice action

1 + 1 + 1 flavor QCD+QED (non-compact)

Action

$$S = S_G + S_{QED} + S_F^u + S_F^d + S_F^s$$

$$S_G = \frac{6}{g^2} \sum_{x, \mu < \nu} \frac{1}{3} \text{Tr} \left\{ c_0 [1 - U_{\mu\nu}(x)] + c_1 [1 - R_{\mu\nu}(x)] \right\}$$

$$S_{QED} = \frac{1}{2e^2} \sum_{x, \mu < \nu} (A_\mu(x) + A_\nu(x + \mu) - A_\mu(x + \nu) - A_\nu(x))^2 \quad \text{noncompact}$$

$$S_F^q = \sum_x \left\{ \sum_\mu \left[\bar{q}(x) \frac{\gamma_\mu - 1}{2} e^{-ieqA_\mu(x)} \tilde{U}_\mu(x) q(x + \hat{\mu}) \right. \right. \\ \left. \left. - \bar{q}(x) \frac{\gamma_\mu + 1}{2} e^{ieqA_\mu(x - \hat{\mu})} \tilde{U}_\mu^\dagger(x - \hat{\mu}) q(x - \hat{\mu}) \right] \right. \\ \left. + \frac{1}{2\kappa_q} \bar{q}(x) q(x) - \frac{1}{4} c_{SW} \sum_{\mu\nu} \bar{q}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) q(x) \right\}$$

Lattice spacing a implicit

$$e_u = \frac{2}{3}, \quad e_d = e_s = -\frac{1}{3}$$



Dashen scheme of renormalization

1 + 1 + 1 flavor QCD+QED (non-compact)

The electromagnetic correction to meson mass,

$$M_\gamma^2 = M^2(g^2, e_\star^2, m_u^\star, m_d^\star, m_s^\star) - M^2(g^2, 0, m_u^{QCD}, m_d^{QCD}, m_s^{QCD}). \quad (12)$$

However, what are the values of $m_{u/d/s}^{QCD}$? No experimental inputs are available here.

The so-called Dashen scheme is defined to be that neutral meson masses exactly matches the real-world mass values (after tuning m_q^{QCD} of course):

$$M_{q\bar{q}}^2(g^2, e_\star^2, m_u^\star, m_d^\star, m_s^\star) = M_{q\bar{q}}^2(g^2, 0, m_u^{QCD}, m_d^{QCD}, m_s^{QCD}). \quad (13)$$

which means that the electromagnetic self-energy for the neutral meson mass vanishes exactly (R. Dashen).

Scheme change is accomplished by one-loop lattice perturbation theory.



Pure QCD case: the fan plot

1 + 1 + 1 octet meson and baryon mass

QCD

Gell-Mann–Okubo

\geq quadratic

1

8

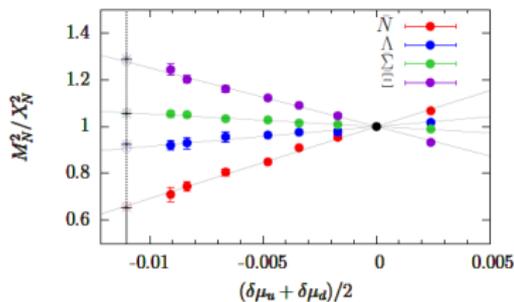
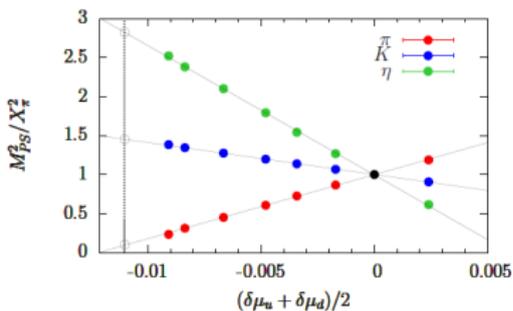
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$$M^2(a\bar{b}) = M_0^2 + \alpha (\delta\mu_a + \delta\mu_b)$$

$$+ \dots \quad \delta\mu_q = \mu_q - \bar{m}$$

$$M^2(aab) = M_0^2 + \alpha_1(2\delta\mu_a + \delta\mu_b) + \alpha_2(\delta\mu_a - \delta\mu_b) + \dots$$

$$\delta\mu_u + \delta\mu_d + \delta\mu_s = 0$$



$$X_\pi^2 = (M_{K^0}^2 + M_{K^+}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2) / 3$$

$$X_N^2 = (M_n^2 + M_p^2 + M_{\Sigma^-}^2 + M_{\Sigma^+}^2 + M_{\Xi^-}^2 + M_{\Xi^0}^2) / 6$$

courtesy of Prof. G. Schierholz



QCD+QED: Dashen scheme

1 + 1 + 1 octet meson and baryon mass

- Define symmetric point by $M_{PS}(u\bar{u}) = M_{PS}(d\bar{d}) = M_{PS}(s\bar{s}) = M_{PS}(d\bar{s}) = M_{PS}(s\bar{d})$

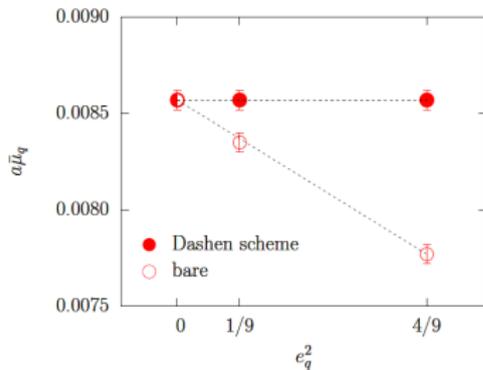
- Renormalize (rescale) quark masses

$$\delta\mu_q \rightarrow \delta\mu_q = \delta\mu_q(1 + K e_q^2)$$

such that

$$\bar{\mu}_u = \bar{\mu}_d = \bar{\mu}_s$$

at symmetric point



of Prof. G. Schierholz



QCD+QED: quark masses

$N_f = 1 + 1 + 1$ octet meson and baryon mass

Quark Masses

$$\begin{aligned}
 M_{\pi^0}^2 &= M_0^2 + \alpha(\delta\mu_u + \delta\mu_d) = \alpha(\mu_u + \mu_d) \\
 M_{K^0}^2 &= M_0^2 + \alpha(\delta\mu_d + \delta\mu_s) = \alpha(\mu_d + \mu_s)
 \end{aligned}
 \left. \vphantom{\begin{aligned} M_{\pi^0}^2 \\ M_{K^0}^2 \end{aligned}} \right\} \mu_u + \mu_d + \mu_s = \text{constant}$$

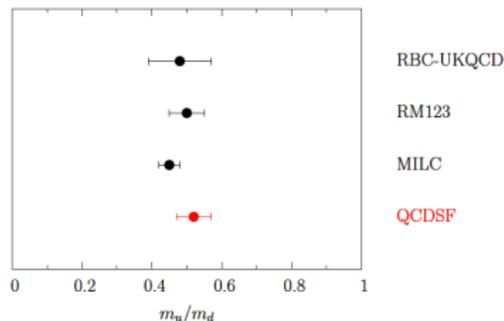
$$m_q = Z_m^{\overline{\text{MS}}}(2 \text{ GeV}) \Delta Z_D^{\overline{\text{MS}}} \mu_q$$

$$m_u = 2.49(14) \text{ MeV}$$

$$m_d = 4.80(27) \text{ MeV}$$

$$m_s = 94.5(52) \text{ MeV}$$

$$\frac{m_u}{m_d} = 0.52(2), \quad \frac{m_s}{m_d} = 19.7(9)$$

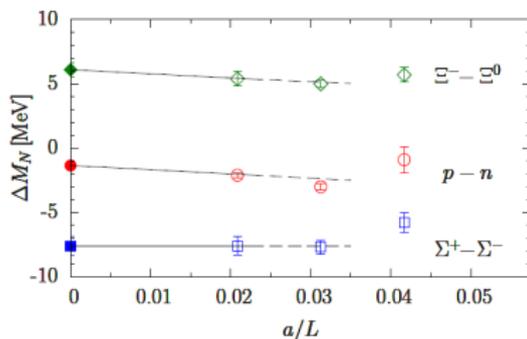
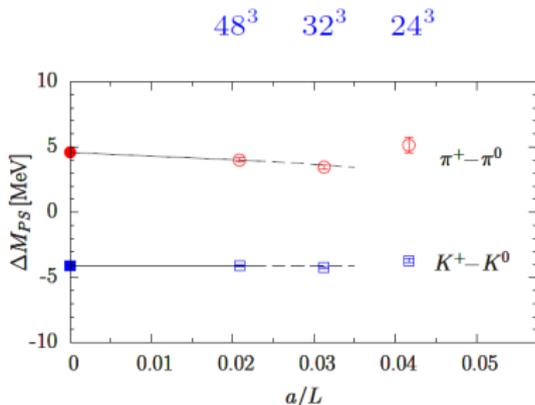


courtesy of Prof. G. Schierholz



Finite volume corrections

1 + 1 + 1 octet meson and baryon mass



■ Using Davoudi&Savage

$$\Delta M_{\pi^+} = \frac{\alpha_{EM}}{2L} c_1 \left[1 + \frac{2}{M_{\pi^+} L} \right] + \frac{2\pi\alpha_{EM}}{3L^3} \langle r^2 \rangle_{\pi} \left[1 + \frac{4\pi c_{-1}}{M_{\pi^+} L} \right]$$

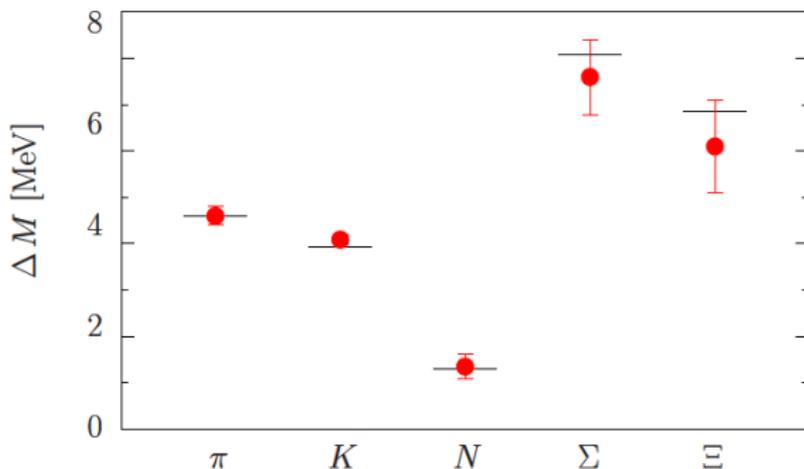
$$\Delta M_p = \frac{\alpha_{EM}}{2L} c_1 \left[1 + \frac{2}{M_p L} \right] + \frac{2\pi\alpha_{EM}}{3L^3} \langle r^2 \rangle_p \left[1 + \frac{4\pi c_{-1}}{M_{\pi^+} L} \right]$$



Mass splittings

1 + 1 + 1 octet meson and baryon mass

ΔM	QCD + QED	QED	Experiment
$M_{\pi^+} - M_{\pi^0}$		4.60(20)	4.59
$M_{K^0} - M_{K^+}$	4.09(10)	-1.66(6)	3.93
$M_n - M_p$	1.35(18)(8)	-2.20(28)(10)	1.30
$M_{\Sigma^-} - M_{\Sigma^+}$	7.60(73)(8)	-0.63(8)(6)	8.08
$M_{\Xi^-} - M_{\Xi^0}$	6.10(55)(45)	1.26(16)(13)	6.85





QCD vs. QED

1 + 1 + 1 octet meson and baryon mass

arXiv:1508.05916

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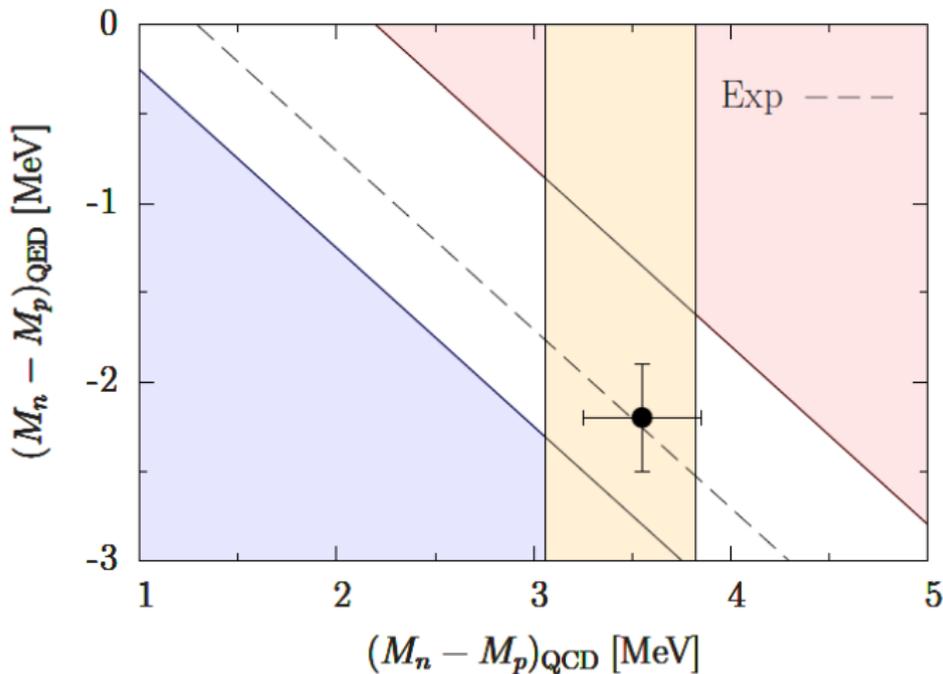
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Physical point simulation by ETMC

$N_f = 2$ simulations, clover term added

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Various quantities studied

- lattice scale from gluonic observables
- pseudo-scalar masses & decay constants
- nucleon and delta masses
- quark masses

$$m_{ud}^{\overline{MS}}(2\text{GeV}) = 3.88(6)(21)(10) \text{ MeV}$$

$$m_s^{\overline{MS}}(2\text{GeV}) = 107(2)(6)(3) \text{ MeV}$$

$$m_c^{\overline{MS}}(2\text{GeV}) = 1.33(3)(7)(3) \text{ GeV}$$

1st: statistical; 2nd: systematic of scale & Z_P ; 3rd:
RI'-MOM to \overline{MS} .

- contrib. to anomalous magnetic moment



Physical point simulation by ETMC

Chiral extrapolations

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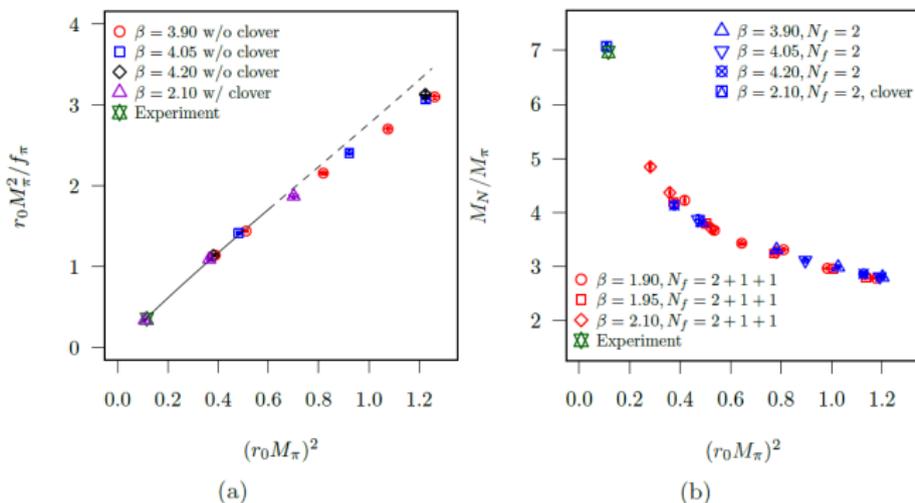


Figure 3: **(a)** $r_0 M_\pi^2 / f_\pi$ as a function of $(r_0 M_\pi)^2$ comparing $N_f = 2$ results w/o clover term [15] with the new results presented in this paper. The line is a NLO χ PT fit to the data as explained in the text. **(b)** Ratio of the nucleon mass to the pion mass as a function of the pion mass squared in units of r_0 . We show data for $N_f = 2$ w/o clover term, $N_f = 2 + 1 + 1$ and the new physical point result.



Physical point simulation by ETMC

comparison with experimental values

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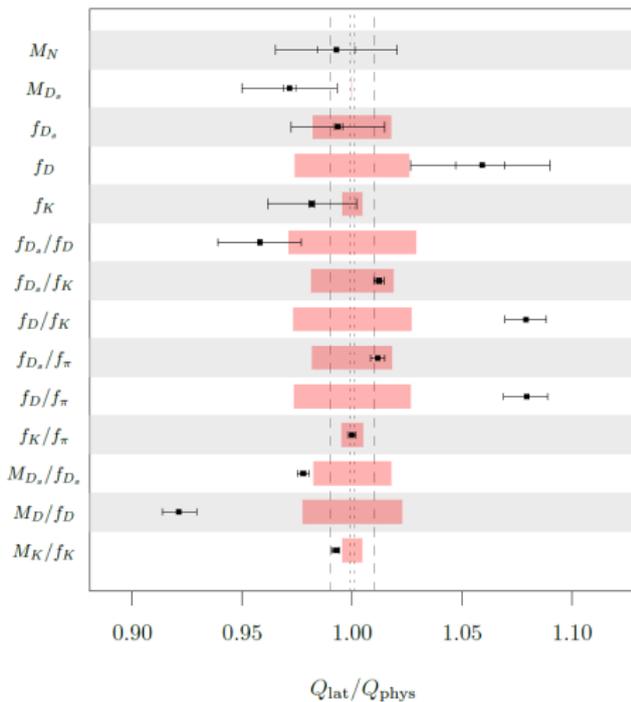


Figure 5: Ratios of lattice results and phenomenological values of the quantities in the legend with lattice decay constants computed via the continuum definition. For dimensional quantities, the inner error bar combines the statistical and systematic errors in quadrature while the outer error bar stems from the estimate of the lattice spacing from gluonic scales. The red bands show the phenomenological uncertainty on Q_{phys} separately (the respective experimental error on M and



Excited Nucleons

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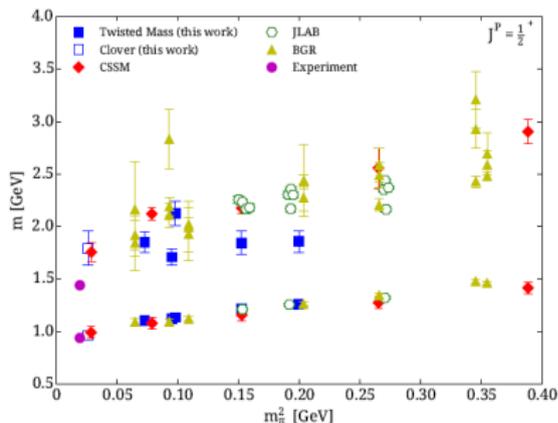
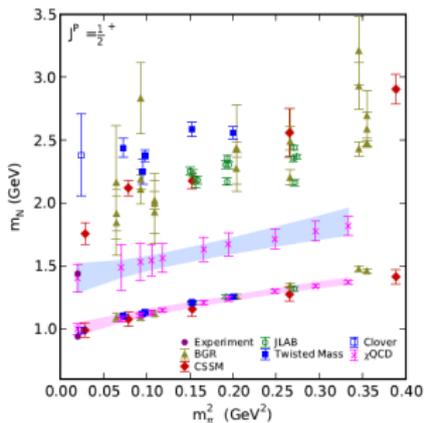
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The roper: a long-lasting issue

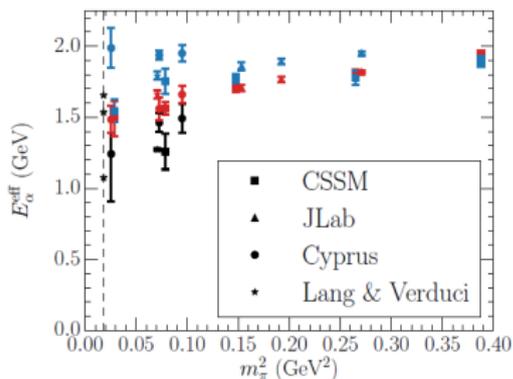
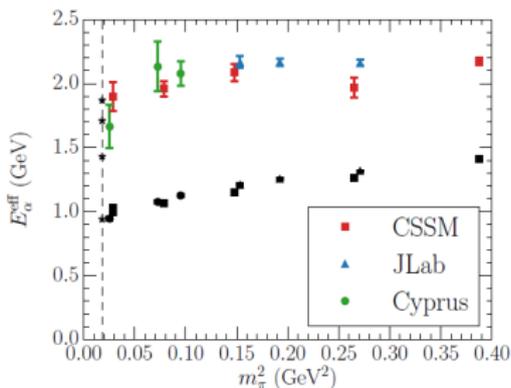
- The Roper state is an example
- from Leinweber et al, arXiv:1511.09146





The roper: a long-lasting issue

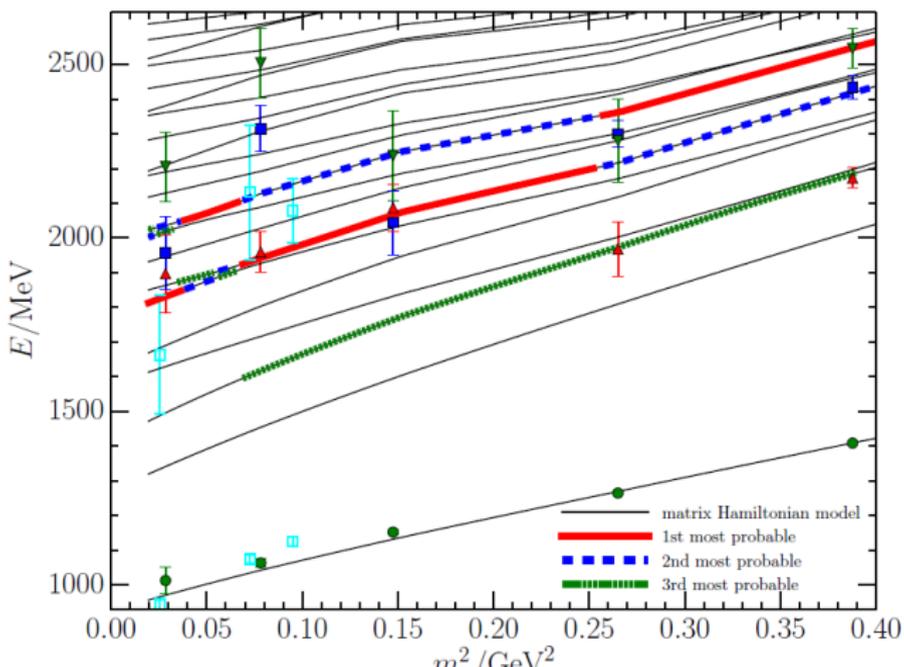
- The question is how to explain
- Leinweber thinks it is due to SEB used by χ QCD





The roper: a long-lasting issue

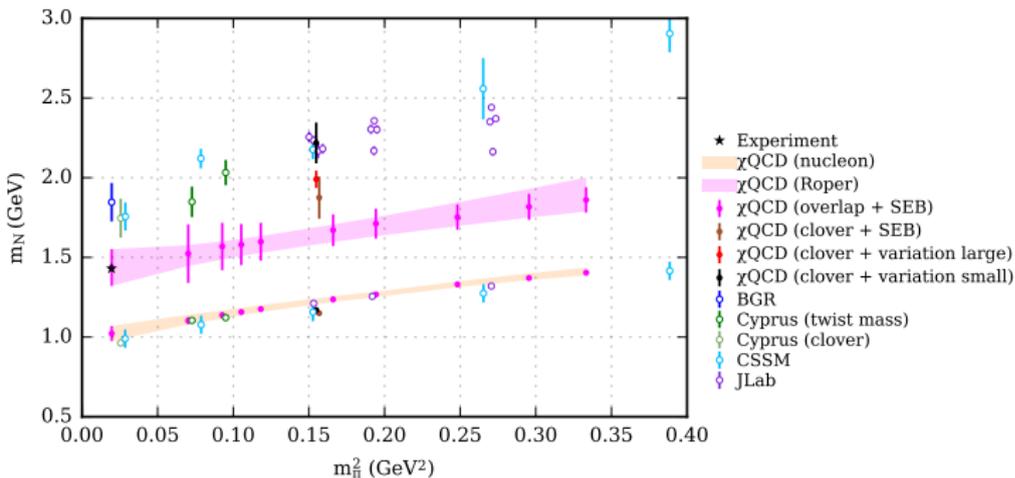
- CSSM claims to be able to generate the right roper energy levels at $L=3\text{fm}$ though the levels themselves do not necessarily appear there





However, χ QCD tells a different story

- χ QCD checked HSC configurations using both the variational method and the SEB
- though SEB does make some difference, it is not that dramatic
- they think chiral property might be the reason





2.2 SCATTERING

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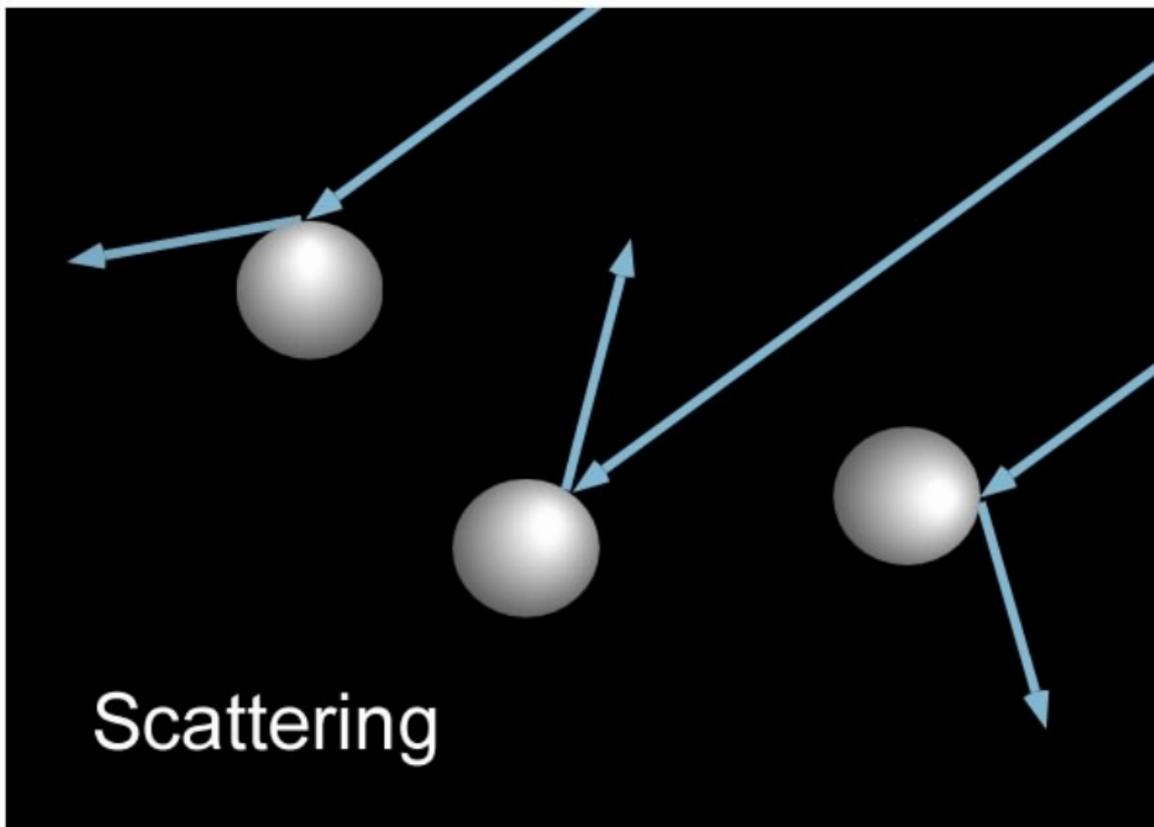
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Pion-pion scattering

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Pion-pion scattering

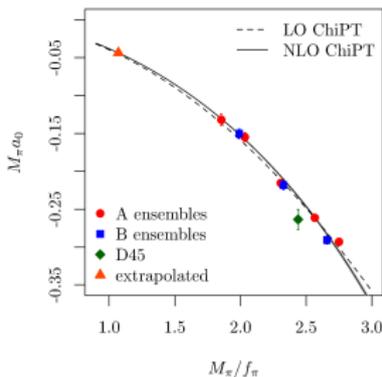


Pion-Pion scattering by ETMC

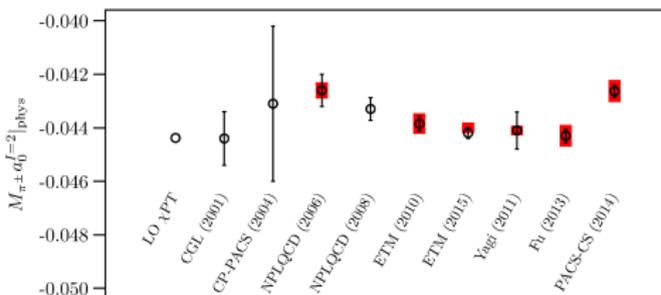
$\pi\pi$ scattering, $N_f = 2 + 1 + 1$ confs., using Lüscher's formalism

- $\pi\pi$ scattering length in $l = 2$ channel C. Helmes et al (ETMC), JHEP 1509 (2015) 109

- stochastic LapH
- whole series of lattice ensembles



- Comparison with literature:



$$M_{\pi} a_0^2 = -0.0442(2) \begin{pmatrix} +4 \\ -0 \end{pmatrix}$$

$$l_{\pi\pi}^{l=2} = 3.79(0.61) \begin{pmatrix} +1.34 \\ -0.11 \end{pmatrix}$$



πK and KK also computed

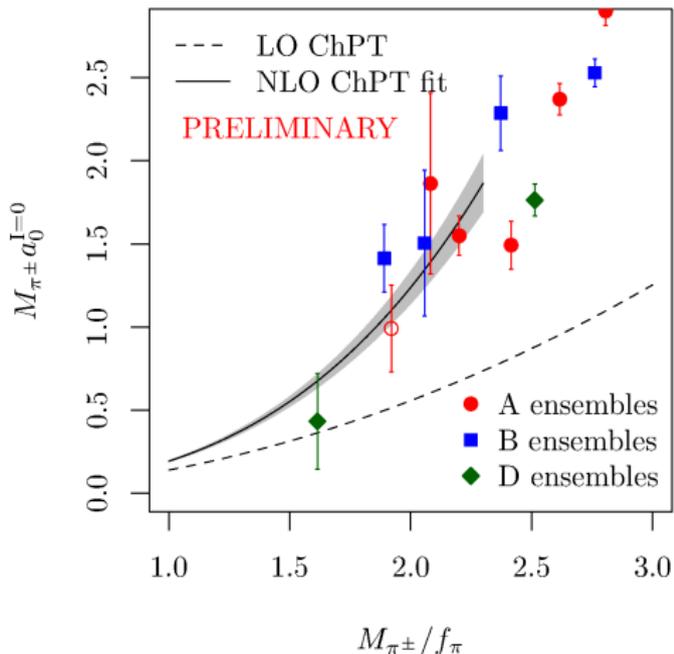
see C. Helmes, Tue. 4:30pm



Pion-Pion scattering by ETMC

$\pi\pi$ scattering $l = 0$

■ Preliminary results in $l = 0$ channel

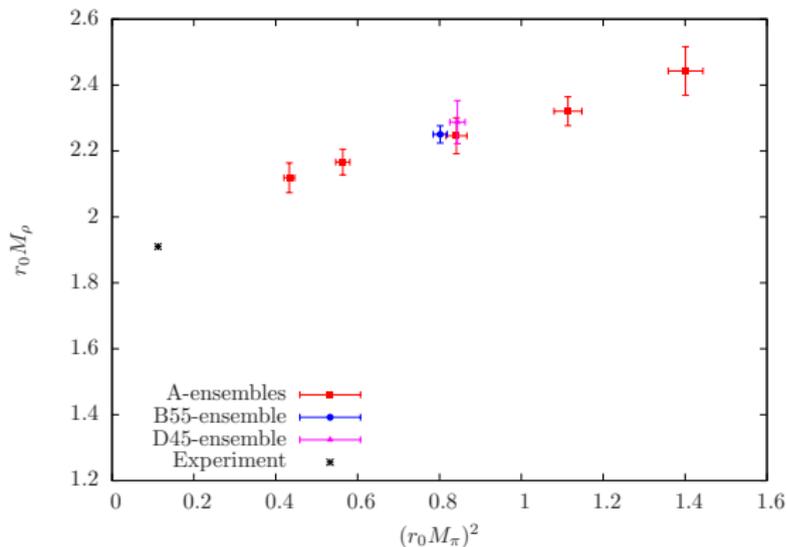




the ρ resonance by various groups

$\pi\pi$ scattering $l = 1$

■ Preliminary results in $l = 1$ channel



M. Werner, The Rho Resonance from Twisted Mass Lattice QCD, Tuesday 14:00, spectrum



Pion-pion scattering: rho

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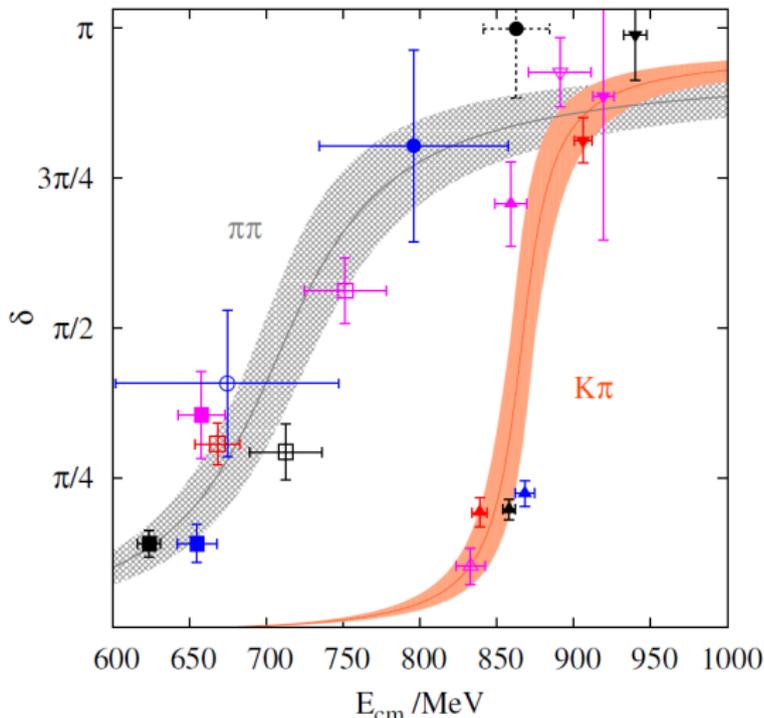
Pion-pion scattering, $\rho(770)$



the ρ resonance by various groups

RQCD: 2 flavors non-perturbatively improved Wilson [Bali et al, 1512.08678](#)

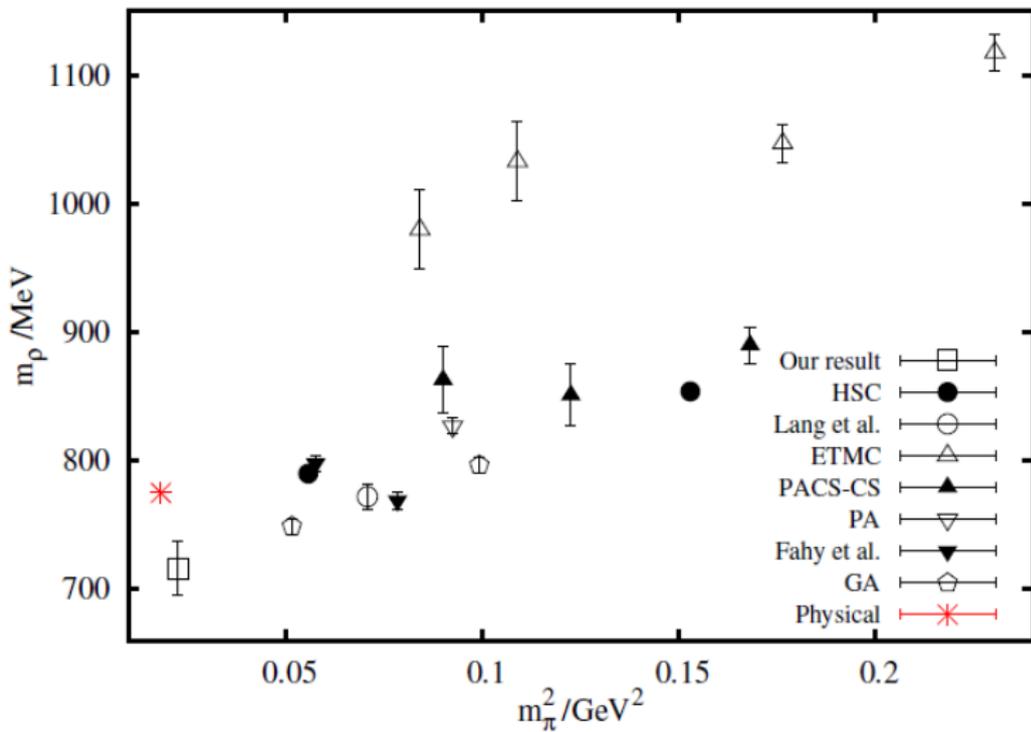
- close to physical pion mass: $m_\pi \simeq 150\text{MeV}$
- moving frames study in ρ and the K^* channels





the ρ mass comparison

RQCD: 2 flavors non-perturbatively improved Wilson



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the ρ resonance by various groups

Morningstar et al: 1604.05593

■ 2 + 1 flavor anisotropic lattice, $m_\pi \simeq 230\text{MeV}$

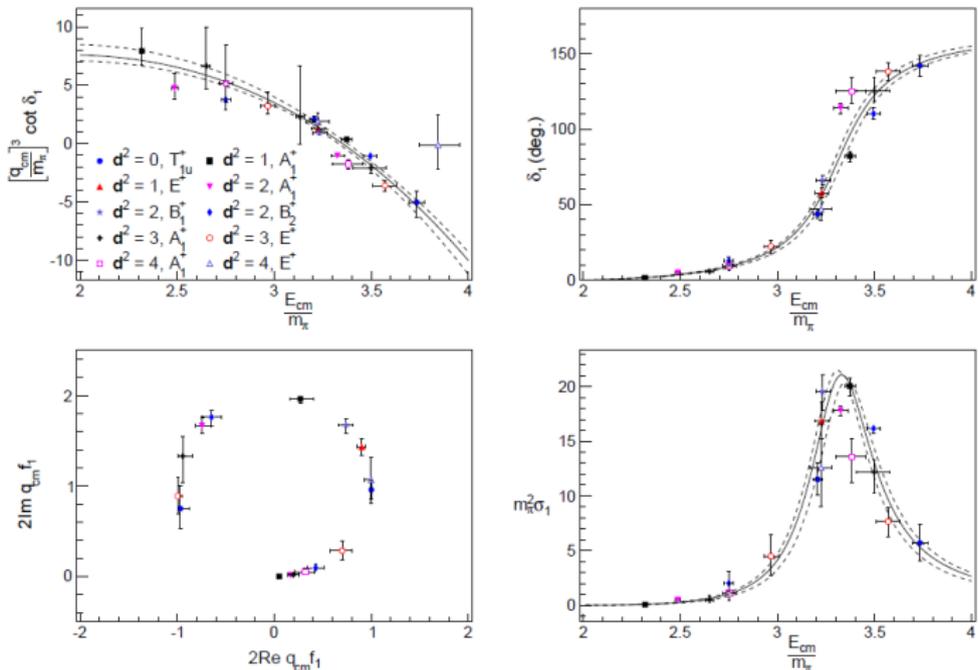


Figure 5: The real part of the inverse scattering amplitude (top left), phase shift (top right), Argand plot (lower left), and partial wave amplitude (lower right) for the $I = 1$,

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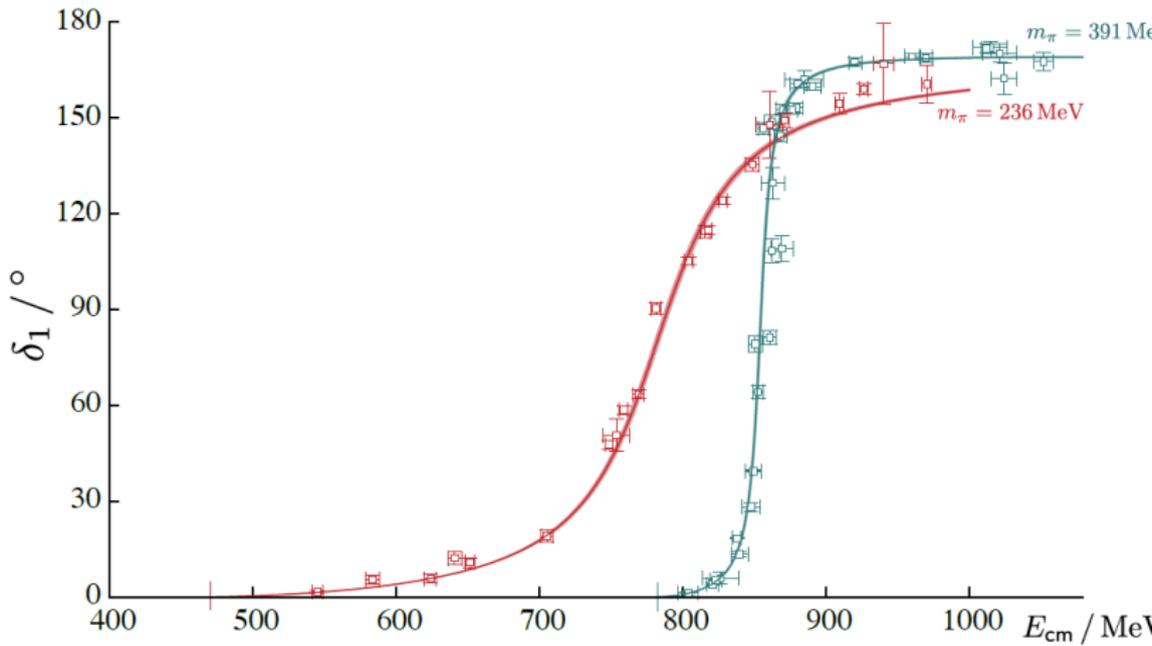
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the ρ resonance by various groups

HSC: 1507.02599

- 2 + 1 flavor anisotropic lattice, $m_\pi \simeq 236\text{MeV}$
- coupled-channel effects taken into account, see Wilson's talk



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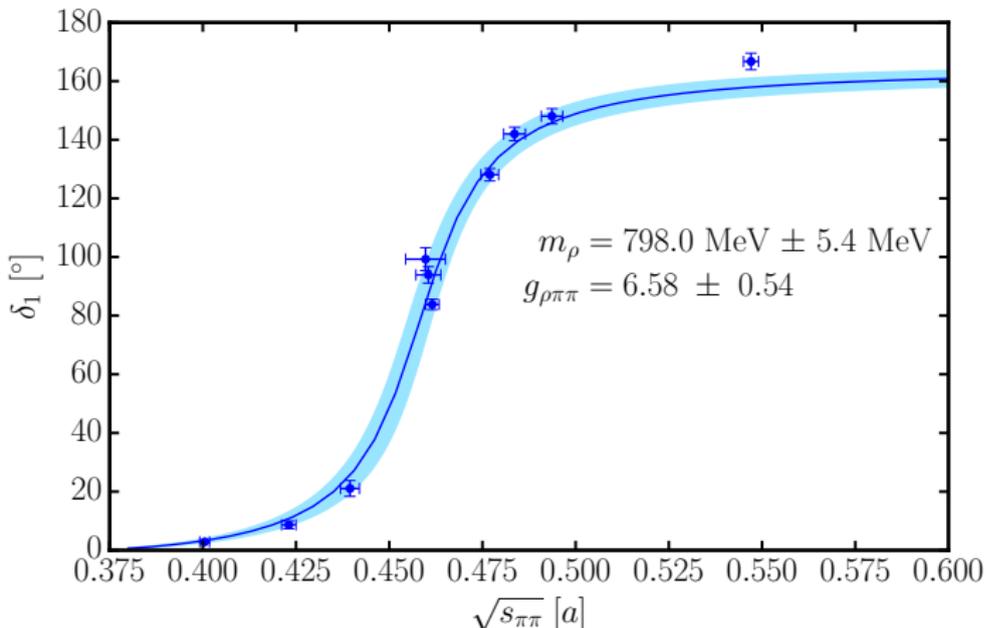
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the ρ resonance by various groups

A study of the radiative transition $\pi\pi \rightarrow \pi\gamma^*$ with lattice QCD

- 2+1 flavors of clover, $m_\pi \simeq 320\text{MeV}$
- $m_\rho = 798(5.4)\text{MeV}$ and $g_{\rho\pi\pi} = 6.58(0.54)$
- L. LESKOVEC, Friday, 17:50, Hadron Structure section





the ρ resonance by various groups

Guo et al: 1605.03993

- 2 flavor of nHYP fermions, $m_\pi \simeq 226, 315\text{MeV}$
- extrapolated value of m_ρ substantially smaller than physical value
- they argued it is due to quenching of the strange quark

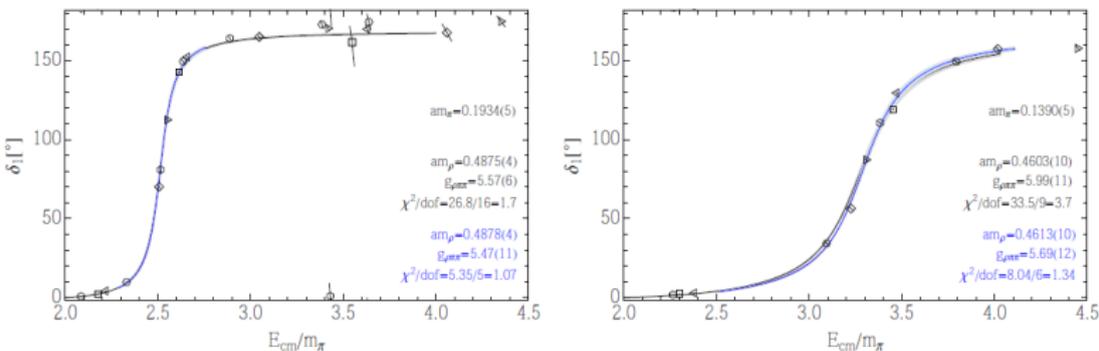


FIG. 5. Phase shifts as a function of the center of mass energy. The errorbars are slanted along the direction of the Lüscher curves. On the left we have the $m_\pi = 315\text{ MeV}$ data and on the right the $m_\pi = 226\text{ MeV}$ data. The triangles, squares, a hexagons correspond to data extracted from $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3$ (left) and $\mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6$ (right) respectively. The black curve, error-bar and fit parameters correspond to Breit-Wigner fit to all data points in the elastic region, $E_{\text{cm}} < 4m_\pi$. Blue color indicates fit to the data in $m_\rho \pm 2\Gamma_\rho$ region. The $\text{U}\chi\text{PT}$ fits are very close to the blue Breit-Wigner curves.



Charmed meson scattering and the XYZ's

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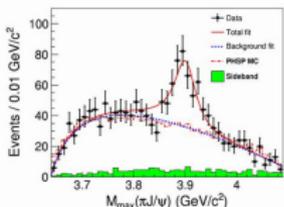
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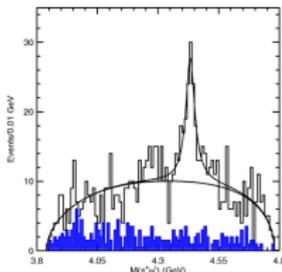
The XYZ particles



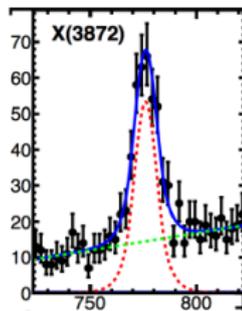
The XYZ particles and other threshold exotics



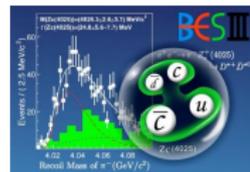
$Z_c(3900)$



$Z(4430)$



$X(3872)$



$Z_c(4025)$

- quarkonium-like states: valence quark structure $Q\bar{Q}q'\bar{q}$
- Neutral ones, $q = q'$, e.g. $X(3872)$, $Y(4260)$, etc.
- Charged ones, $q \neq q'$, $Z_c(3900)$, $Z_c(4025)$, $Z(4430)$, etc.
- Close to thresholds of mesons: $Q\bar{q}$ and $\bar{Q}q'$

Plus the newly discovered pentaquark states: P_c^+ , etc.



Charmed meson near-threshold scattering

$N_f = 2$ twisted mass confs., using Lüscher

■ $(D^* \bar{D}^*)^\pm (Z_c(4025))$

CLQCD, PRD92 054507 (2015)

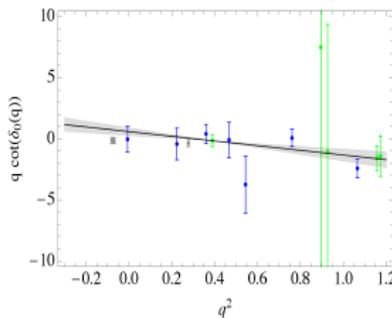
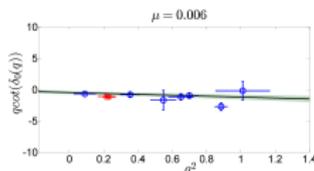
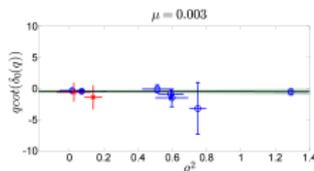
$(D \bar{D}^*)^\pm (Z_c(3900))$

CLQCD, PRD89 094506 (2014)

- TBC utilized
- 3 m_π values:
300, 425, 485 MeV

■ weakly repulsive interaction found

■ no indication of a bound state



$$q \cot \delta(q^2) = \frac{1}{a_0} + \frac{1}{2} r_0 q^2 + \dots,$$



need more ensembles (with $N_f = 2 + 1 + 1$) to inspect chiral & finite volume behavior



For $Z(4430)$

$N_f = 2$ twisted mass confs., using Lüscher

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- $(\bar{D}_1 D^*)^\pm (Z(4430))$ CLQCD, Phys.Rev. D93 (2016)
 - attractive interaction shows up
 - appears to be more attractive than the quenched results
G. Meng et al, PRD80 034503 (2009)
 - some indications of a bound state seen
 - however, needs more volumes



S. Prelovsek et al study on Z_c 's etc.

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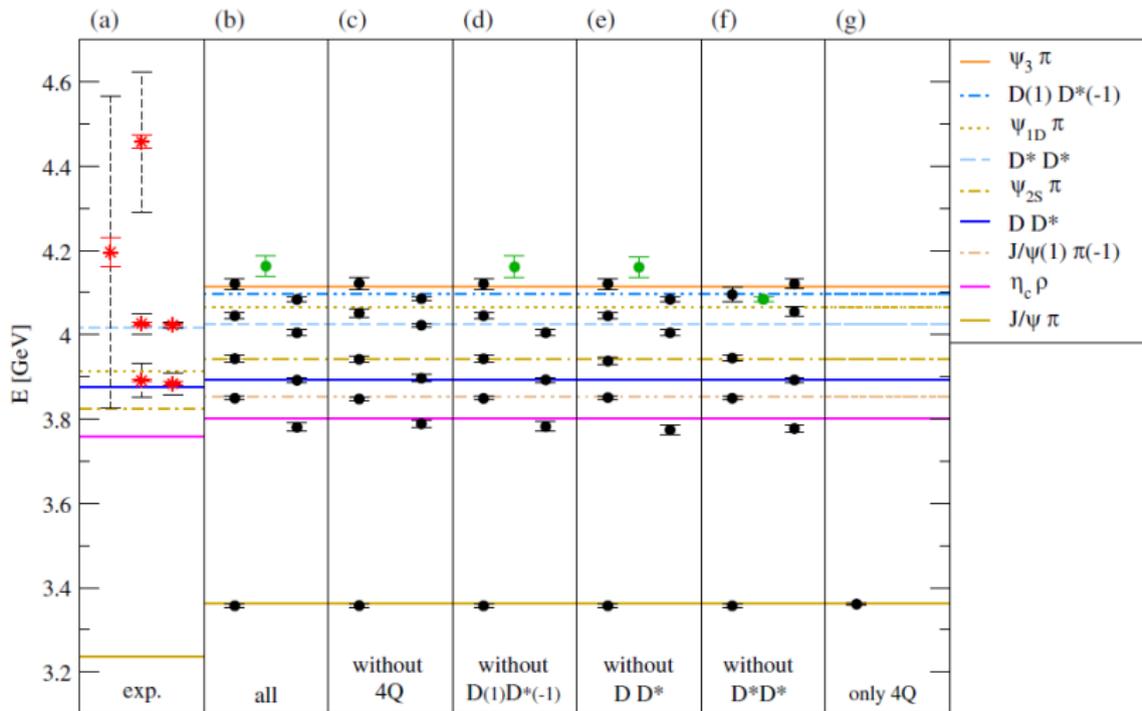
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■ Study the spectrum and compare with the free



Vector and scalar charmonium resonances with lattice QCD

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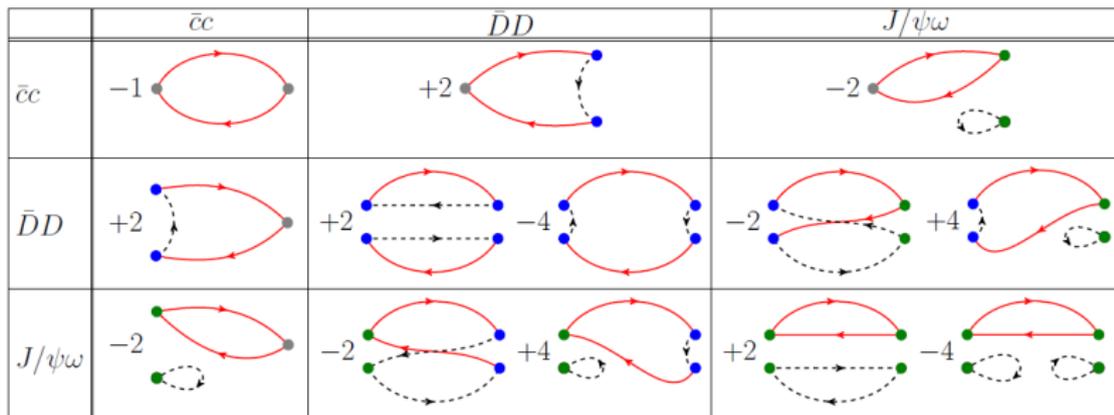
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C.B. Lang et al JHEP 1509 (2015) 089;1503.05363

- $c\bar{c}$, $D\bar{D}$ and $J/\psi\omega$ operators in scalar channel
- $c\bar{c}$, $D\bar{D}$ operators in vector channel

☞ $\psi(3770)$ well described by $D\bar{D}$ p -wave scattering

☞ χ_{c0} is still full of puzzles



X(3872) and Y(4140) using diquark-antidiquark operators

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M. Padmanath et al, PRD92, 034501 (2015); 1503.03257

- use $\bar{c}c$ and $\bar{c}c(\bar{u}u + \bar{d}d)$ $\bar{c}c\bar{s}s$ type diquark-antidiquark operators in $J^{PC} = 1^{++}$ channel

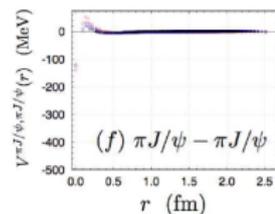
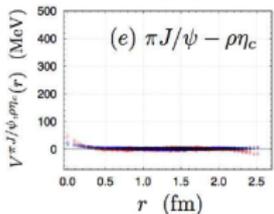
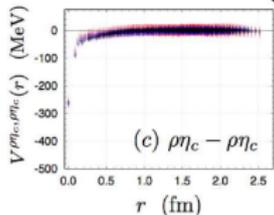
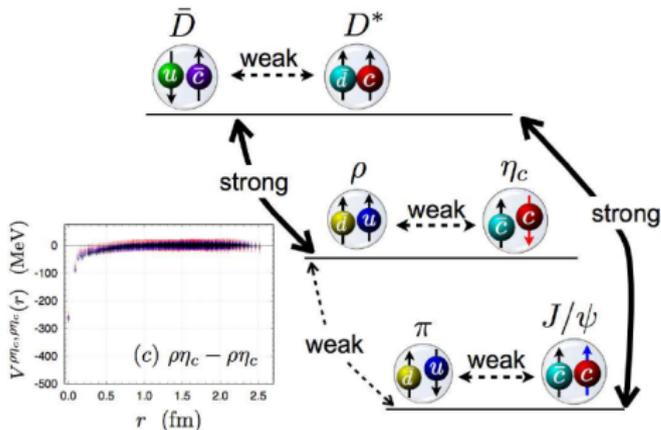
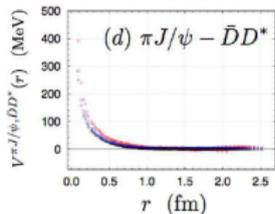
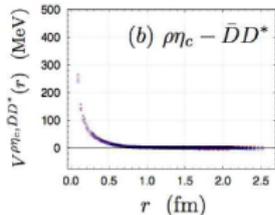
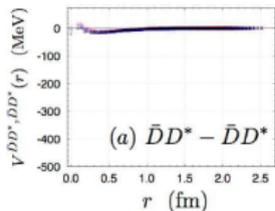
👉 X(3872) with $I = 0$ is observed

👉 Y(4140) is not found



The HALQCD approach

$Z_c(3900)$, 1602.03465



- $Z_c(3900)$ is NOT a conventional resonance
- appears to be a coupled channel effect arising from the $\pi J/\psi - \bar{D}D^*$ coupling



other lattice studies

other near-threshold states, or b quarks, etc.

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Involving charm and strange quarks:

- Near threshold states $D_{s0}^*(2317)$ and $D_{s1}(2460)$ [Antonio Cox for RQCD, Thursday, 15:40](#)

Involving b quarks:

- Predicting positive parity Bs mesons from lattice QCD [C. B. Lang et al PLB750 \(2015\) 17;1501.01646](#)
- $B_s \pi^+$ scattering and search for X(5568) with lattice QCD [C.B. Lang et al; 1607.03185](#)

Tetra-quarks:

- Heavy and light spectroscopy near the physical point, Part II: Tetraquarks [Anthony Francis, Monday, 15:35](#)
- Including heavy spin effects in a lattice QCD study of static-static-light-light tetraquarks [Marc Wagner, Monday, 15:15](#)
- Lattice QCD study of heavy-heavy-light-light tetraquark candidates [Antje Peters, Monday, 15:35](#)



Summary for the XYZ's

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- Overall picture is still far from clear
- Need more systematic studies
- Need more methods to deal with so many open channels
-



scattering involving baryons

meson-baryon & baryon-baryon scattering

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meson and baryons:

- Low energy scattering phase shifts for meson-baryon systems [W. Detmold and A.N. Nicholson, 1511.02275](#)

two baryons:

- $\Omega\Omega$ interaction from 2+1 flavor lattice QCD [HALQCD, 1503.03189](#)
- Spin-2 $N\Omega$ Dibaryon from Lattice QCD [HALQCD, 1403.7284](#)
- Two Nucleon Systems at $m_\pi \simeq 450\text{MeV}$ from Lattice QCD [NPLQCD: 1508.07583](#)

operator construction for hadrons with spin:

- Lattice operators for scattering of particles with spin [S. PRELOVSEK et al, Tuesday afternoon 17:30, spectrum session](#)

Glueball spectrum:

- Glueball spectrum from $N_f=2$ lattice QCD study on anisotropic lattices [Y.Chen, Tuesday afternoon sessions](#)



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- 1** conventional computations have come to the precision era: able to disentangle isospin and QED corrections; close enough to the chiral limit, etc.
 - we begin to understand quantitatively why we can exist!
- 2** single-channel scattering has been studied using Lüscher formalism, both light and heavy
 - for the light sector, quantities are also becoming precise
 - for the heavy sector, especially the exotic ones, studies are still not systematic enough (puzzles remain)
- 3** other strategies should be studied in order to understand the complicated nature of hadronic world!

Thank you for your patience!