Diagrammatic Monte-Carlo for non-Abelian field theories and resurgence

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Diagrammatic Monte-Carlo for dense QCD and sign problem

So far lattice strong-coupling expansion:

(leading order or few lowest orders)

[de Forcrand,Philipsen,Unger, Gattringer,...]

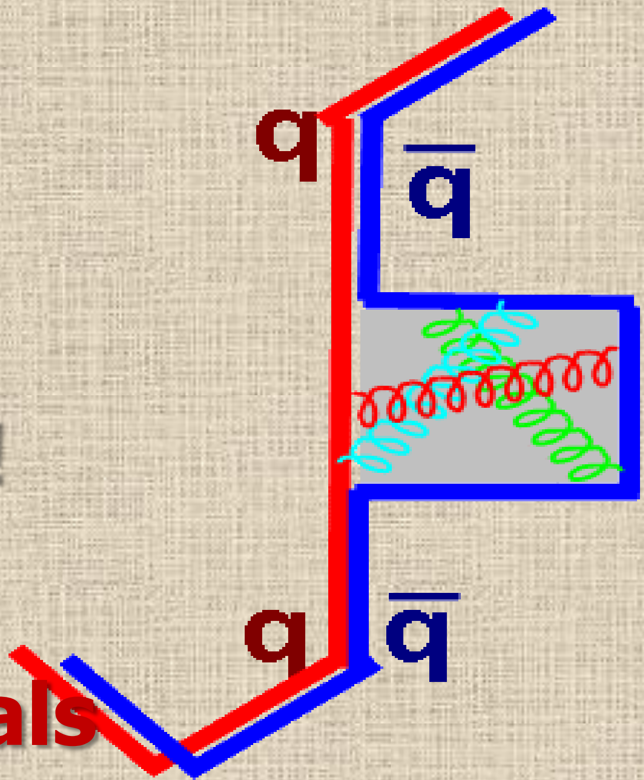
- **Worldlines of quarks/mesons/baryons**
- **Confining strings**

Very good approximation!

Physical degrees of freedom!

✓ **Phase diagram, tri-critical**

✗ **Hadron spectrum, potentials**



Lattice strong-coupling expansion

- **Confinement**
- **Dynamical mass gap generation**

<u>ARE NATURAL, BUT…</u>

Continuum physics is at weak-coupling!

DiagMC @ Weak-coupling?

Non-perturbative physics via RESURGENCE

DiagMC algorithms from Schwinger-Dyson

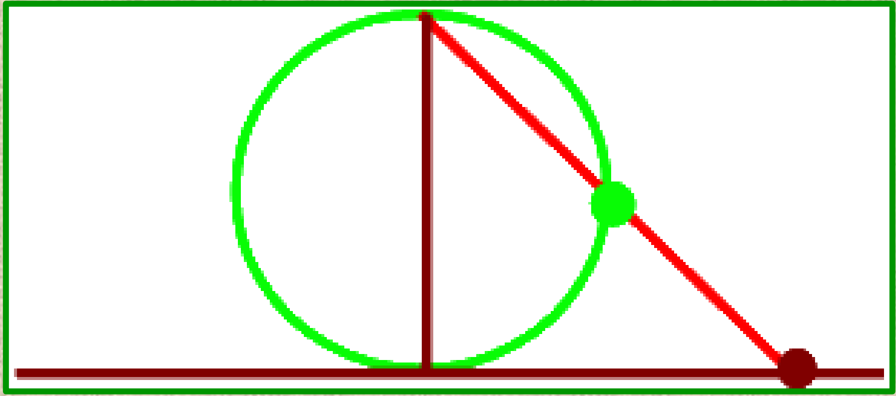
Perturbative DiagMC

it should be first-principle and automatic

- **Take N->∞ to reduce diagram space**
- **Small fluctuations of SU(N) fields**
- **Map SU(N) to Hermitian matrices**

Cayley map

$$g = \frac{1 + i\alpha\phi}{1 - i\alpha\phi}$$



$$\int_{SU(N)} dg \Rightarrow \int_{\mathbb{H}_{N\times N}} d\phi \, \det\left(1 + \alpha^2\phi^2\right)^{-N} =$$

$$= \int_{\mathbb{H}_{N\times N}} d\phi \, \exp\left(-N\alpha^2 \mathrm{Tr}\, \phi^2 + O\left(\alpha^4\phi^4\right)\right)$$

SU(N) principal chiral model

$$\mathcal{Z} = \int\limits_{U(N)} dg_x \exp\left(-\frac{N}{\lambda}\sum_{<x,y>} \mathrm{Tr}\,\left(g_x^\dagger g_y\right)\right)$$

$$\alpha^2 = \frac{\lambda}{8}$$

Expand action and Jacobian in *φ*

Infinitely many interaction vertices

$$S\left[\phi_x\right] = \frac{1}{2}\sum_{x,y}\left(D_{xy} + \frac{\lambda}{4}\delta_{xy}\right)\mathrm{Tr}\,\left(\phi_x\phi_y\right) +$$

$$+ \sum_{n=2}^{+\infty}\left(-\frac{\lambda}{8}\right)^{n-1}\left(\frac{\lambda}{8n}\sum_x \mathrm{Tr}\,\phi_x^{2n} + \right.$$

$$\left.\frac{1}{2}\sum_{l=1}^{2n-1}(-1)^{l-1}\sum_{x,y} D_{xy}\mathrm{Tr}\,\left(\phi_x^{2n-l}\phi_y^l\right)\right)$$

SU(N) principal chiral model

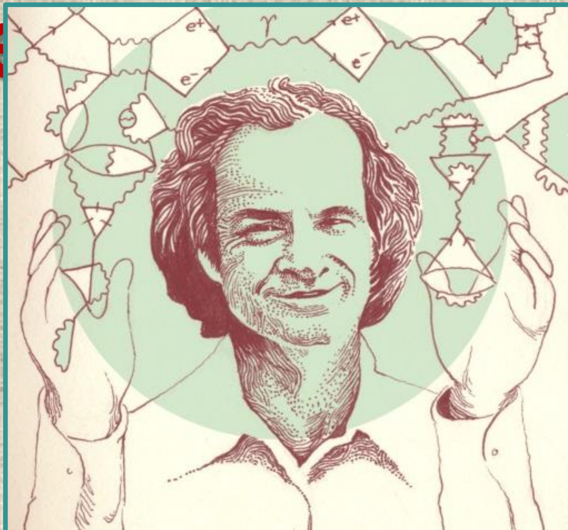
Power series in t'Hooft λ?

Factorial growth even at large N
due to IR renormalons … [Bali, Pineda]
Can be sampled, but resummation difficult

…Bare mass term ~λ from Jacobian???
[a-la Fujikawa for axial anomaly]

- ✓ **Massive planar field the**
- ✓ **Suitable for DiagMC**
- ? **How to expand in λ?**

➡ **Count vertices !??**



Minimal working example:
2D O(N) sigma model @ large N

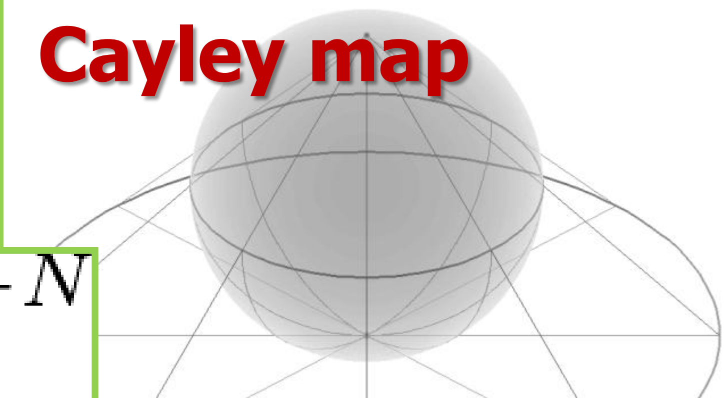
$$\int\limits_{S_N} d\vec{n}_x \exp\left(-\frac{1}{\alpha^2}\sum_{<x,y>}\vec{n}_x\cdot\vec{n}_y\right)$$

$$\langle \vec{n}_x \cdot \vec{n}_y \rangle \sim$$
$$\sim \exp\left(-m^2|x-y|\right)$$

Non-perturbative mass gap

$$m^2 = 32\exp\left(-\frac{4\pi}{\alpha^2}\right)$$

Cayley map



Jacobian reads

$$\mathcal{D}n_x = \mathcal{D}\phi_x\left(1+\frac{\lambda}{4}\phi_x^2\right)^{-N}$$

$$S_N \rightarrow \mathbb{R}^{N-1}$$

Again, bare mass term from the Jacobian… [PB, 1510.06568]

O(N) sigma model @ large N

Full action in new coordinates

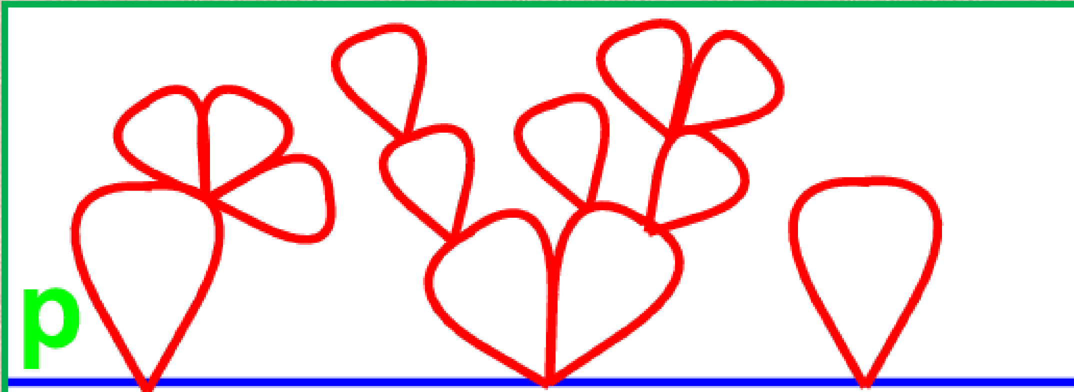
$$S[\phi_x] = \frac{1}{2}\sum_{x,y}\left(D_{xy} + \frac{\lambda}{2}\delta_{xy}\right)\phi_x\cdot\phi_y +$$

$$+ \sum_{k=2}^{+\infty}\frac{(-1)^{k-1}\lambda^k}{4^k\,k}\sum_x\left(\phi_x^2\right)^k +$$

$$+ \sum_{\substack{k,l=0\\k+l\neq 0}}^{+\infty}\frac{(-1)^{k+l}\lambda^{k+l}}{2\cdot 4^{k+l}}\sum_{x,y}D_{xy}\left(\phi_x^2\right)^k\left(\phi_y^2\right)^l\left(\phi_x\cdot\phi_y\right)$$

We blindly do perturbation theory …

Only cactus diagrams @ large N



Trans-series and Resurgence

From our perturbative expansion we get

$$m^2 = \sum_{p,q=0}^{+\infty} c_{p,q}\, \lambda^p\, (\log\lambda)^q$$

Same for PCM!!!

Resurgent trans-series [Écalle,81]

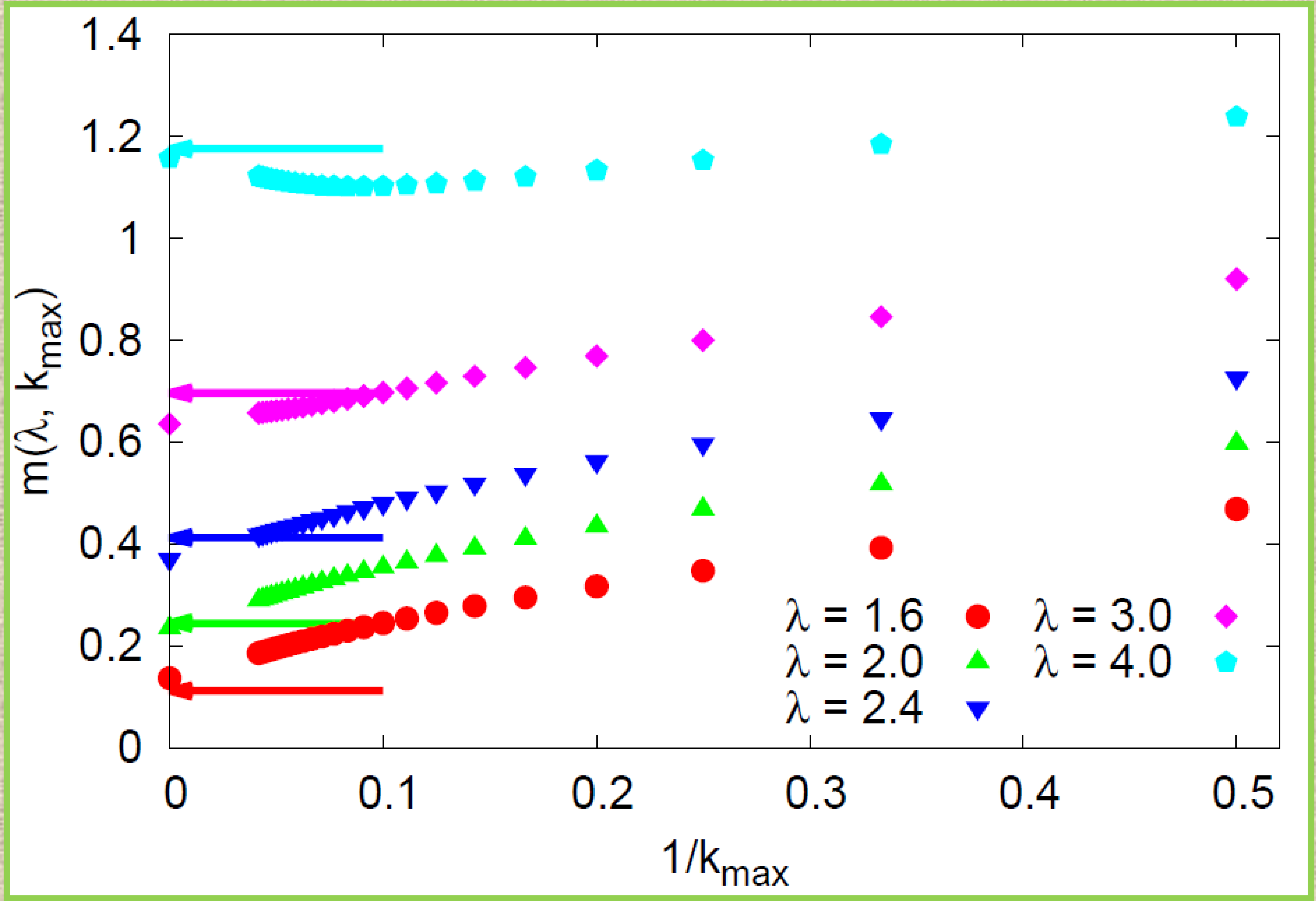
$$f(z) = \sum_{p,q,r} c_{p,q,r}\, z^p\, (\log z)^q \left(e^{-\frac{S}{z}}\right)^r$$

PT **Zero modes** **Classical solutions**

[Argyres,Dunne,Unsal,...,2011-present]

$$\exp\left(-\frac{1}{\lambda}\right) = \exp\left(-e^{-\log(\lambda)}\right) = \sum_k c_k\, (\log\lambda)^k$$

O(N) sigma model @ large N



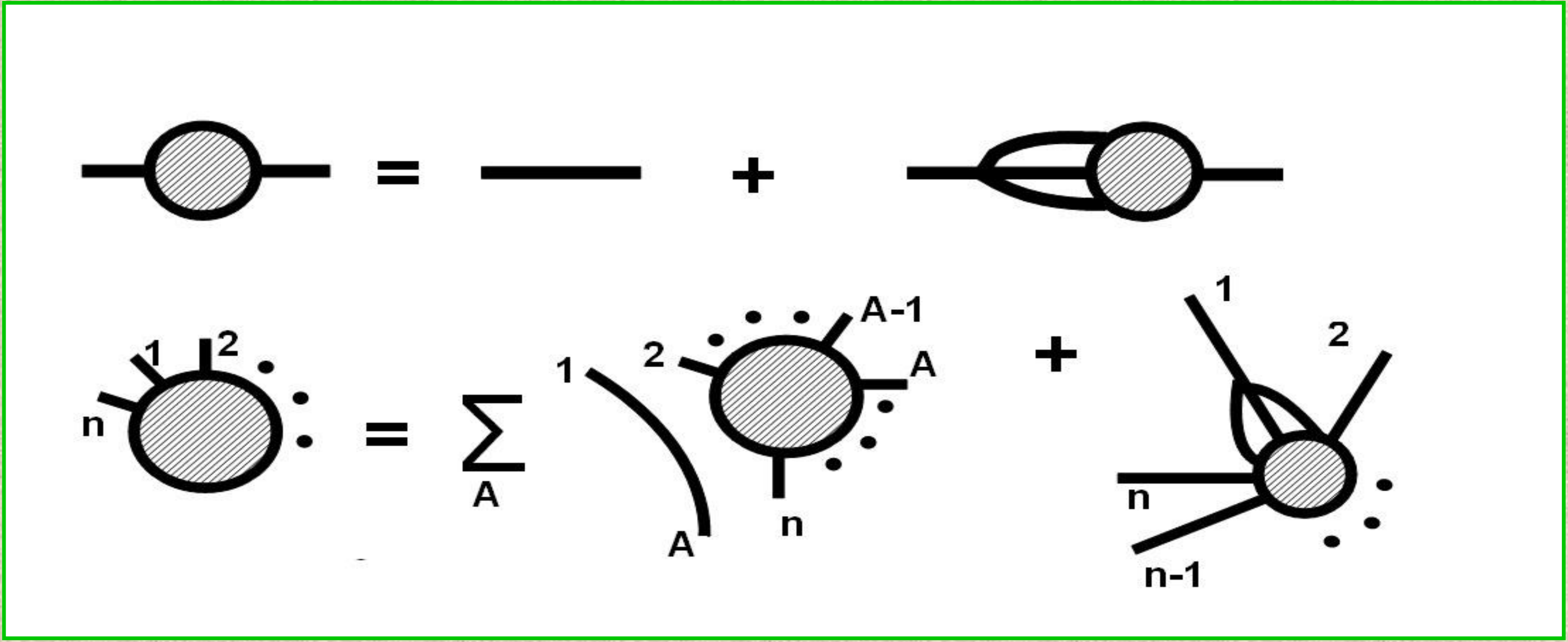
Good convergence in practice

(But no proof of convergence!!!)

(Back to) Principal Chiral Model

Now we need DiagMC, all planar diagrams

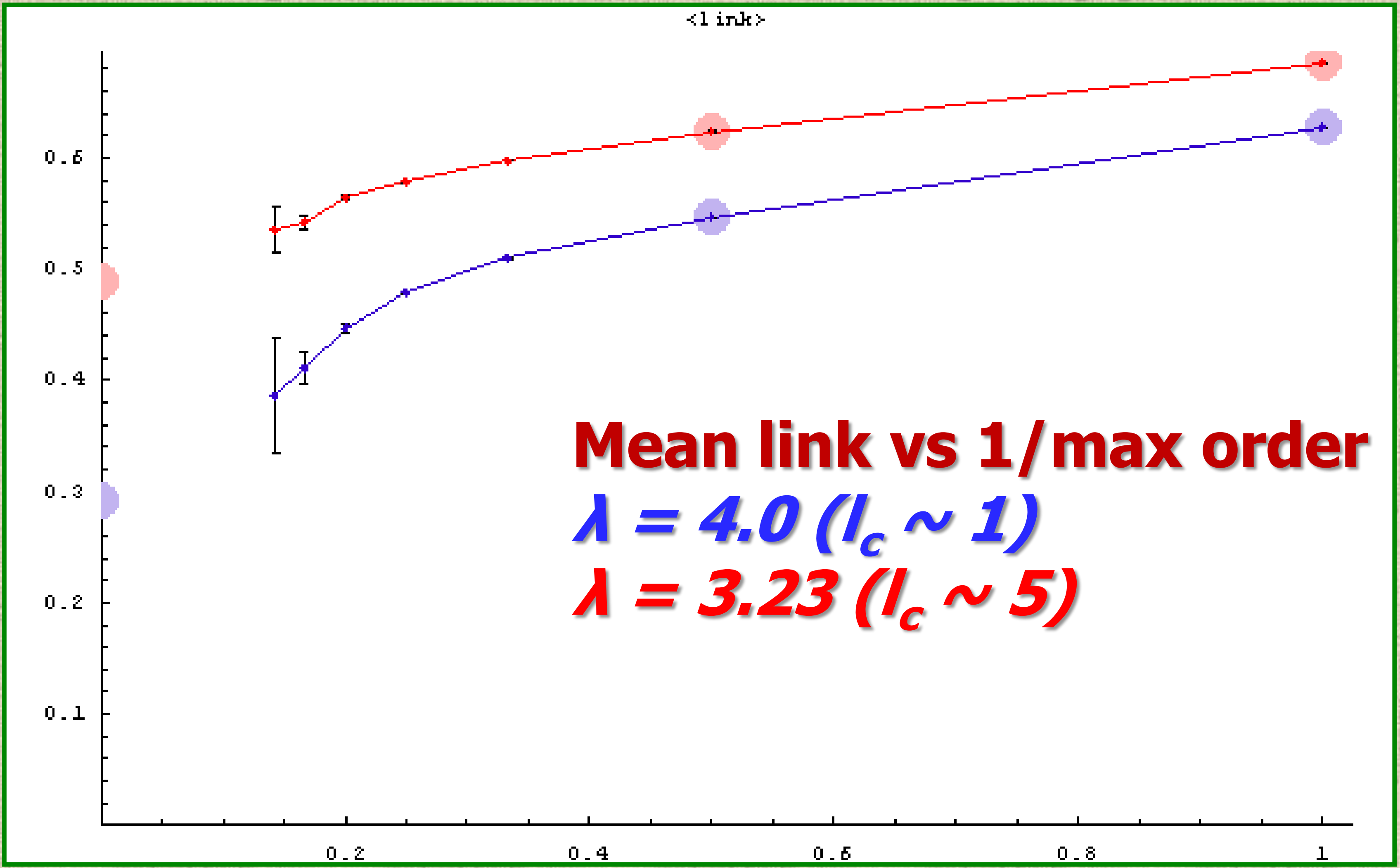
Stochastically solve Schwinger-Dyson equations



Recursive structure for diagrams:

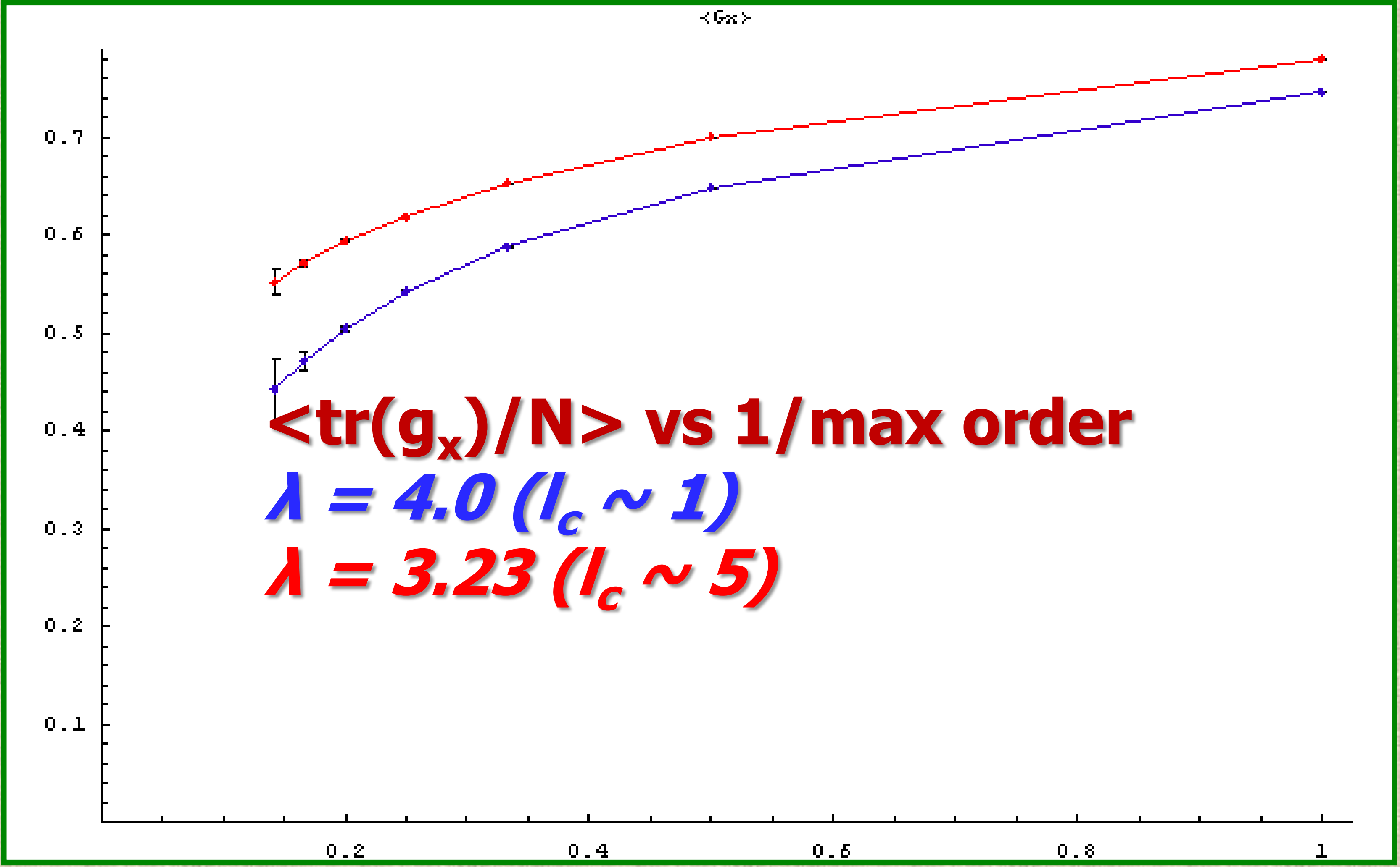
V vertices, L legs -> V-v vertices, L+l legs

Results from DiagMC
(several hours on laptop)



Mean Link vs [Vicari,Rossi, 9307014]

Results from DiagMC



Restoration of SU(N) x SU(N)

Errors due to sign problem at high orders

Discussion

DiagMC in the large-N limit:

- **# of diagrams grows exponentially**
- **All contributions are finite**
- **Good for DiagMC!!!**
- **No IR renormalons at the expense of generating transseries**
- **Numerical evidence of convergence**

BUT: sign problem at high orders

Inevitable for any (naive) approach close to continuum!

No signs-no continuum limit

Discussion

- **With lattice perturbation theory, we have lost the beauty of strong coupling (Strings for gauge theories!)**
- **Another series expansion is possible, basing on Schwinger-Dyson equations with original field variables, so far convergence not so good???**

Thank you for your attention!