Proton spin decomposition and its perturbative renormalization

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July 25 2016

Proton Spin decomposition

Frame independent decomposition

X.D. Ji., Phys. Rev. Lett. 78, 610-613 (1997).

$$\vec{J} = \underbrace{\int d^3x\, \bar{\psi}\, \{\vec{x} \times \tfrac{i}{2}(\gamma^4 \vec{D} + \vec{\gamma} D^4)\}\, \psi}_{\text{quark AM}} + \int d^3x \{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

glue AM

$$= \underbrace{\int d^3x\, \tfrac{1}{2}\, \overline{\psi}\, \vec{\gamma}\, \gamma^5\, \psi}_{\text{quark spin}} + \underbrace{\int d^3x \psi^\dagger\, \{\vec{x} \times (i\vec{D})\}\, \psi}_{\text{quark OAM}} + \int d^3x \{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\},$$

From the symmetric energy momentum tensor, gauge invariant, frame independent, and well defined on the lattice

Proton Spin decomposition

Calculation through the EMT form factors

X.D. Ji., Phys. Rev. Lett. 78, 610-613 (1997).

Ji's angular momentum (AM) can be written in terms of the symmetrized energy momentum tensor (EMT) as,

$$J^{q,g} = \langle p, s| \int d^3x \, x \times \mathcal{T}^{\{0i\}q,g} |p, s\rangle, \qquad \mathcal{T}^{\{0i\}q} = \frac{1}{4}\bar{\psi}\gamma^{(0}\overleftrightarrow{D}^{i)}, \; \mathcal{T}^{\{0i\}g} = \vec{E} \times \vec{B}.$$

, with the form factors of the off-diagonal part of EMT defined by,

$$\begin{aligned}(p', s'|\mathcal{T}^{\{0i\}q,g}|p, s) = \left(\frac{1}{2}\right) \bar{u}(p', s') \Big[& T_1(q^2)(\gamma^0\bar{p}^i + \gamma^i\bar{p}^0) + \frac{1}{2m}T_2(q^2)\left(\bar{p}^0(i\sigma^{i\alpha}) + \bar{p}^i(i\sigma^{0\alpha})\right) q_\alpha \\ & + \frac{1}{m}T_3(q^2)q^0q^i \Big]^{q,g} u(p, s),\end{aligned}$$

Ji's quark and glue AM correspond to the forward limit of the form factor combination,

$$\boxed{J^{q,g} = \frac{1}{2}\left[T_1(0) + T_2(0)\right]^{q,g}}$$

Proton Spin decomposition

The lattice ensembles

Smaller lattice spacing

L ~ 4.3 fm
m_π ~ 170 MeV
32^3 x 64, a =0.143 fm

L ~ 2.8 fm
m_π ~ 330 MeV
24^3 x 64, a =0.111 fm

L ~ 5.6 fm
m_π ~ 140 MeV
48^3 x 96, a =0.114 fm

Larger Volume

Lighter sea quark

L ~ 2.8 fm
m_π ~ 300 MeV
32^3 x 64, a =0.086 fm

L ~ 2.0 fm
m_π ~ 370 MeV
32^3 x 64, a =0.063 fm

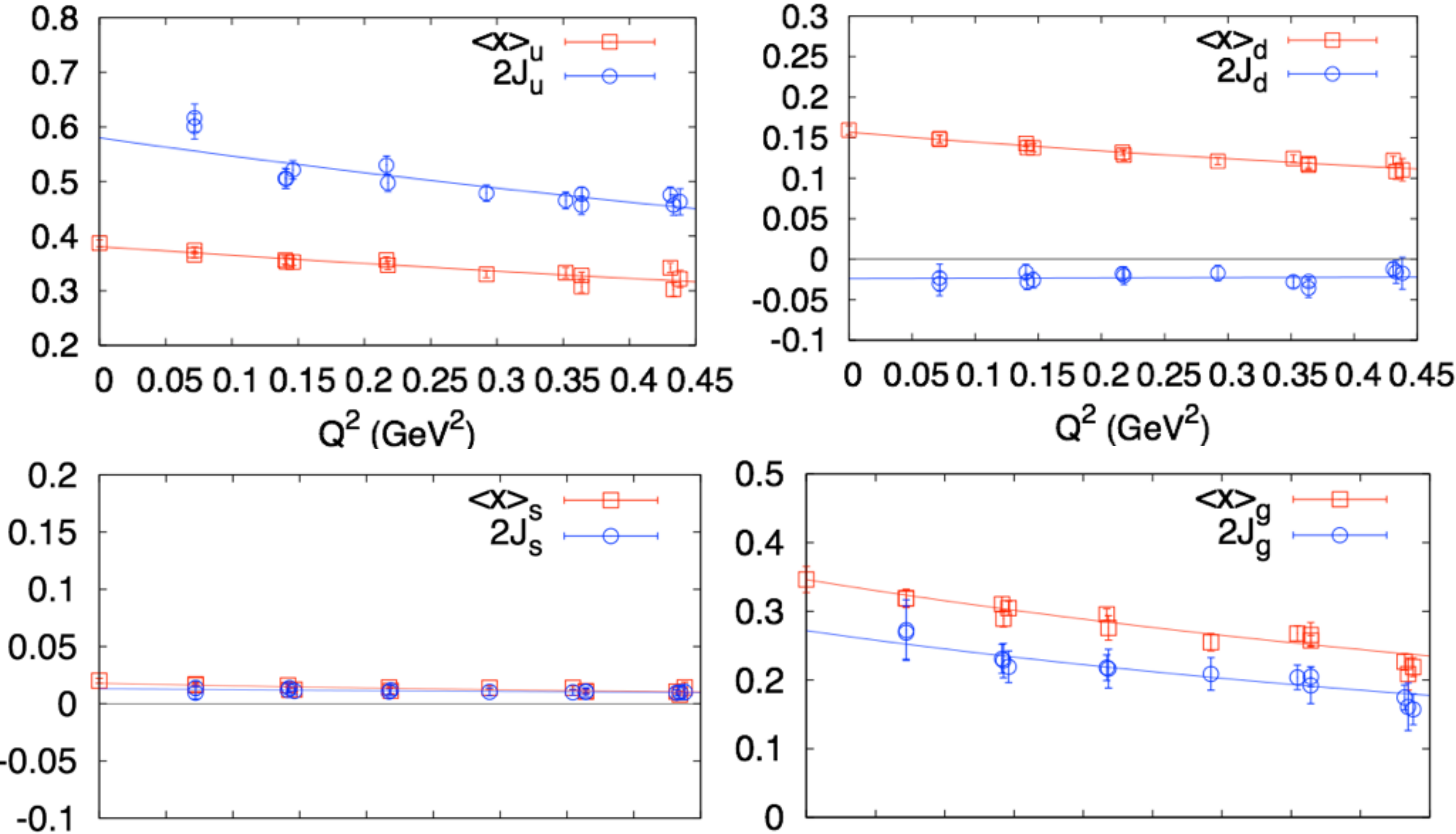
2+1 flavor DWF configurations (RBC-UKQCD)

T. Blum et al. (RBC, UKQCD), Phys. Rev. D93, 074505 (2016)

Quark and glue angular momentums

m_π=400 MeV

The bare results on lattice



Next steps?

- *Repeat the calculation on the other ensembles to access the systematic uncertainty (lattice spacing, volume, sea quark mass, etc.)*

  Costly but the framework has been set up.

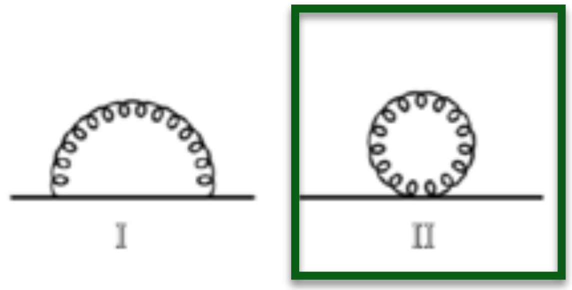
- *Matching the lattice bare results to that under MS-bar scheme at 2GeV.*

  A non-trivial lattice perturbative calculation (will be addressed in the following a few pages).

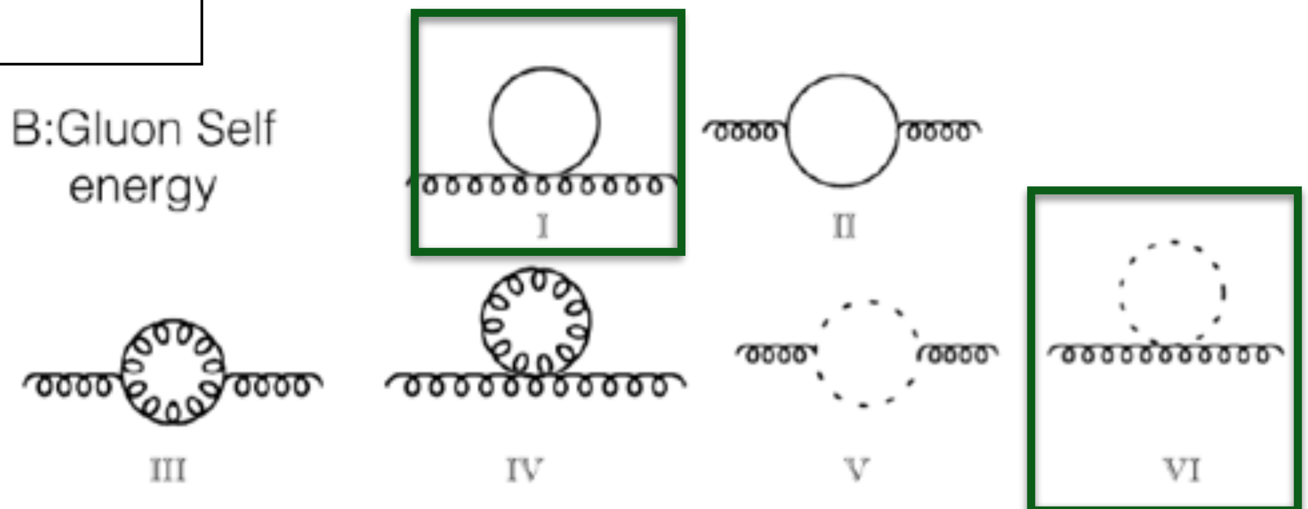
The renormalization of AM for quark and glue

See
S. Capitani and G. Rossi, Nuclear Physics B 433 (1995) 351–389, as example

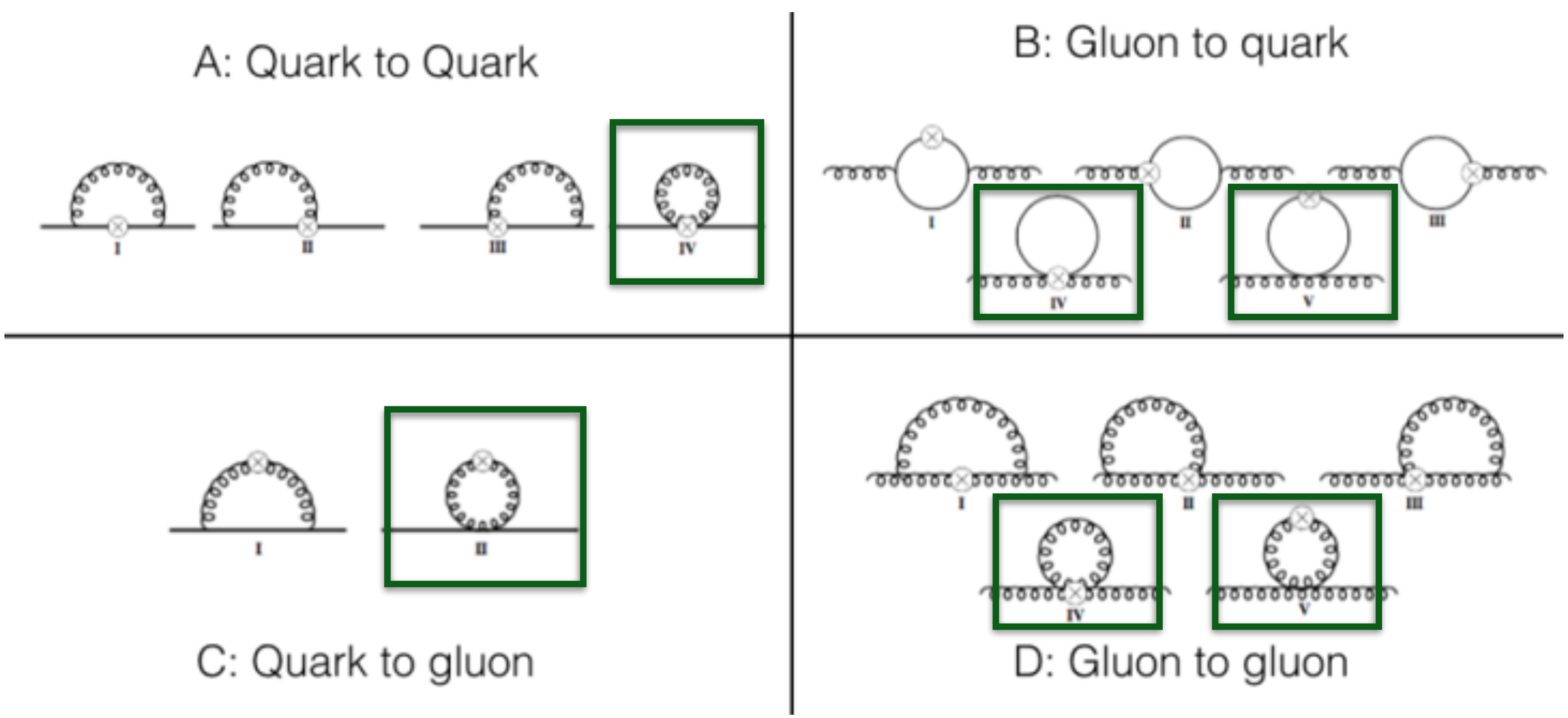
A:Quark Self energy



B:Gluon Self energy



Diagrams don't exist or contribute in the continuum

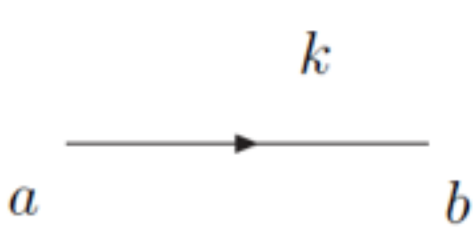


The Feynman rules of LatPT with the extra vertices

See
S. Capitani, Phys.Rept. 382 (2003) 113-302, as example

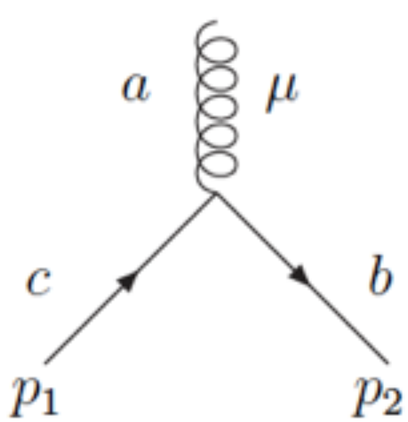
Taking the simplest Wilson fermion as example,

$$S_W = a^4 \sum_x \Big[-\frac{1}{2a} \sum_\mu \big[\overline{\psi}(x)(r-\gamma_\mu)U_\mu(x)\psi(x+a\hat{\mu}) + \overline{\psi}(x+a\hat{\mu})(r+\gamma_\mu)U_\mu^\dagger(x)\psi(x) \big] + \overline{\psi}(x)\left(m_0 + \frac{4r}{a}\right)\psi(x) \Big]$$



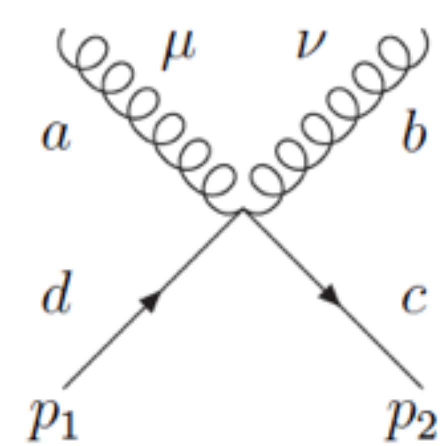
$$\delta^{ab}\cdot a\, \frac{-\mathrm{i}\sum_\mu \gamma_\mu \sin ak_\mu + am_0 + 2r\sum_\mu \sin^2\frac{ak_\mu}{2}}{\sum_\mu \sin^2 ak_\mu + \left(2r\sum_\mu \sin^2\frac{ak_\mu}{2} + am_0\right)^2}$$

Ordinary vertices



$$-g_0(T^a)^{bc}\left(\mathrm{i}\gamma_\mu \cos\frac{a(p_1+p_2)_\mu}{2} + r\sin\frac{a(p_1+p_2)_\mu}{2}\right)$$

Extra vertex



$$-\frac{1}{2}ag_0^2\,\delta_{\mu_1\mu_2}\left(\frac{1}{N_c}\delta^{ab} + d^{abe}T^e\right)^{cd}\left(-\mathrm{i}\gamma_\mu \sin\frac{a(p_1+p_2)_\mu}{2} + r\cos\frac{a(p_1+p_2)_\mu}{2}\right)$$

The renormalization under MS-bar scheme

for the lattice bare quantities

The renormalization of the quark EMT $\mathcal{T}^{\{0i\}q} = \frac{1}{4}\bar{\psi}\gamma^{(0}\overleftrightarrow{D}^{i)}$ with the lattice regularization and under RI-MOM scheme is,

$$Z_L^{MOM} = 1 - \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\log(a^2p^2) + B_{QQ} + \xi] + O(g^4),$$

where B_{QQ} with $B_{QQ}|_{a\to 0} \neq 0$ is the gauge independent finite piece which is sensitive to the lattice quark and gluon actions.

The continuum field renormalization with the dimensional regularization and under RI-MOM and $\overline{MS}$ scheme is,

$$\begin{aligned} Z_{DR}^{\overline{MS}} &= 1 + \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\frac{1}{\epsilon}] + O(g^4), \\ Z_{DR}^{MOM} &= 1 + \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\frac{1}{\epsilon} + \frac{8}{3}\log(\mu^2/p^2) + \frac{40}{9} - \xi] + O(g^4). \end{aligned}$$

So the final renormalization under $\overline{MS}$ scheme for the lattice quantity is,

$$\begin{aligned} Z_L^{\overline{MS}}(a,\mu) &= \frac{Z_{DI}^{\overline{MS}}}{Z_{DI}^{MOM}} Z_L^{MOM}(a,\mu) \\ &= 1 - \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\log(a^2\mu^2) + \frac{40}{9} + B_{QQ}] + O(g^4). \end{aligned}$$

The renormalization of AM

the formulas

From the lattice bare quantities to that under the MOM scheme,

$$\begin{pmatrix} O^{MOM}_{Q,(1)} \\ O^{MOM}_{G,(1)} \end{pmatrix} = \begin{pmatrix} 1-\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(a^2p^2)+B_{QQ}+\xi] & -\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(a^2p^2)+B_{GQ}] \\ +\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(a^2p^2)+B_{QG}] & 1+\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(a^2p^2)+B^f_{GG}]+\frac{g^2N_c}{16\pi^2}[B_{GG}+2\xi-\frac{\xi^2}{4}] \\ +O(g^2)O_{E.O.M.}+O(g^2)O_{G.V.}+O(g^4) & \end{pmatrix} \begin{pmatrix} O^{L}_{Q,(1)} \\ O^{L}_{G,(1)} \end{pmatrix}$$

From the MOM scheme to the MS-bar scheme,

$$\begin{pmatrix} O^{\overline{MS}}_{Q,(1)} \\ O^{\overline{MS}}_{G,(1)} \end{pmatrix} = \begin{pmatrix} 1-\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(\mu^2/p^2)+\frac{40-9\xi}{9}] & -\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(\mu^2/p^2)+\frac{4}{9}] \\ +\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(\mu^2/p^2)+\frac{22}{9}] & 1+\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(\mu^2/p^2)+\frac{10}{9}]+\frac{g^2N_c}{16\pi^2}(\frac{4}{3}-2\xi+\frac{\xi^2}{4}) \\ +O(g^2)O_{E.O.M.}+O(g^2)O_{G.V.}+O(g^4) & \end{pmatrix} \begin{pmatrix} O^{MOM}_{Q,(1)} \\ O^{MOM}_{G,(1)} \end{pmatrix}$$

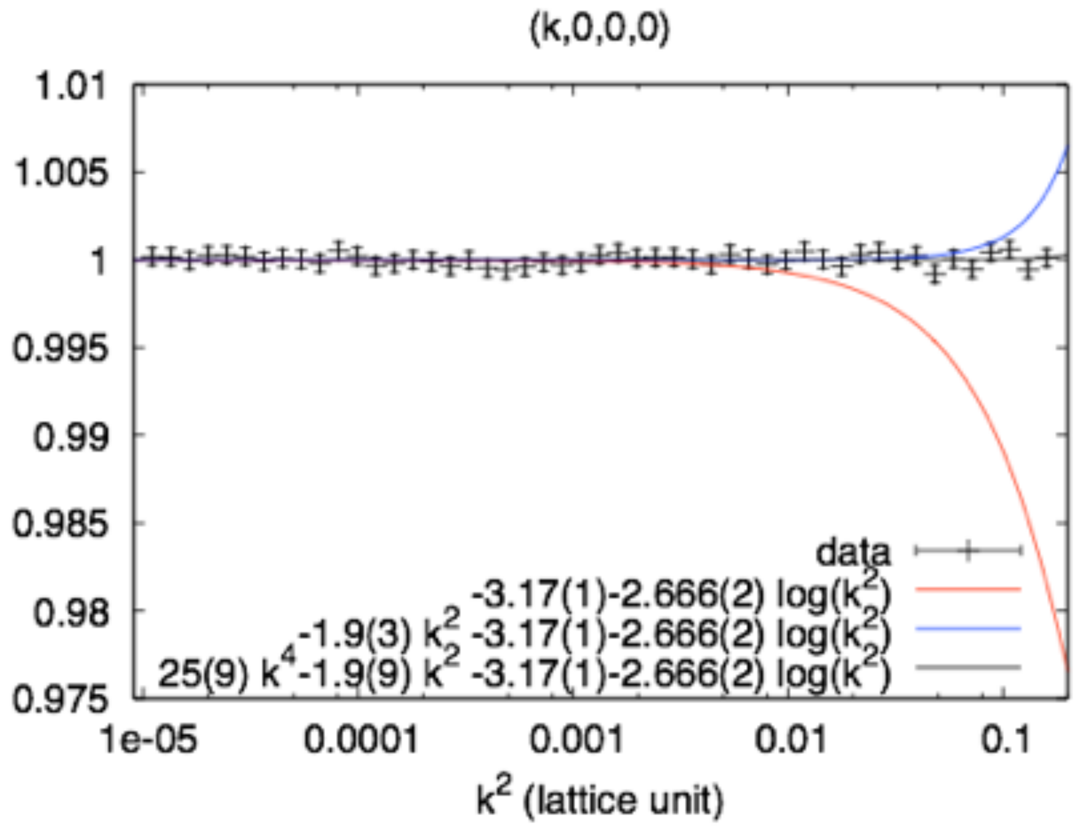
Based on Package-X described in H. H. Patel, Comput.Phys.Commun. 197 (2015) 276-290

B_{XY} are sensitive to the fermion and gauge action, but ξ independent. One can focus on the case under the Feynman gauge to simplify the calculation.

Do the loop integration numerically…

$$Z_L^{MOM} = 1 - \boxed{\frac{g^2 C_F}{16\pi^2}}\left[\frac{8}{3}\log(a^2p^2) + B_{QQ}\right] + O(g^4)$$

~0.02



The ratio of the fit v.s. the numerical integration on different $k^2 = a^2p^2$

- With the higher order of a^2p^2, the larger a^2p^2 region can be well described with the constant part unchanged.
- $B_{QQ}\big|_{a\to 0}$=3.17(1) is precise enough given our statistical error in the simulation.
- We will focus on the constant part of B_{XY} in the following discussions.

The finite pieces

with kinds of actions

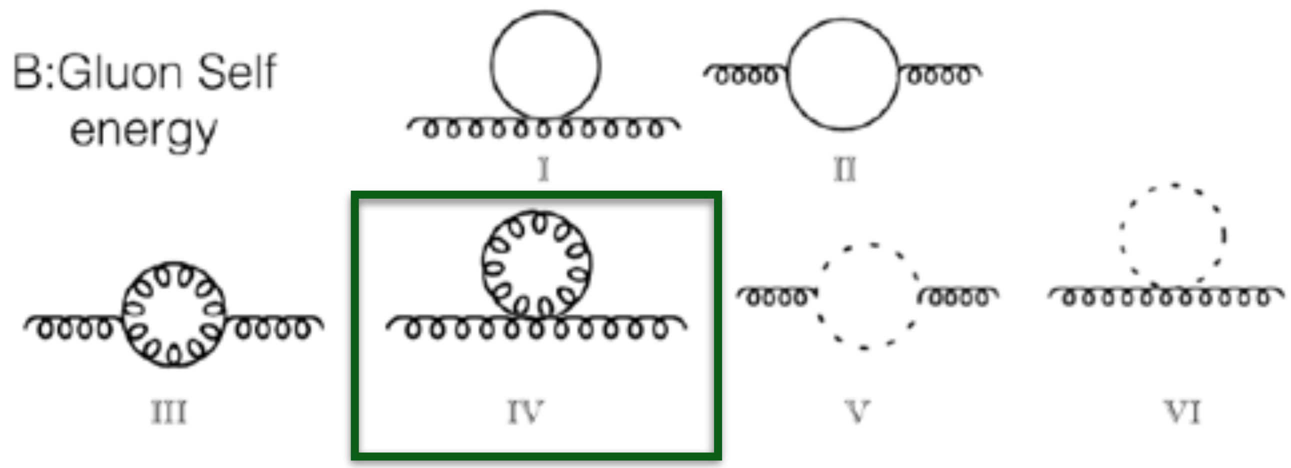
$$Z_L^{\overline{MS}}(a,\mu) = \frac{Z_{DI}^{\overline{MS}}}{Z_{DI}^{MOM}} Z_L^{MOM}(a,\mu) = (1 - \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\log(a^2\mu^2) + \frac{40}{9} + B_{QQ}] + O(g^4)$$

	B_{QQ}	Wilson	Iwasaki	Iwasaki^{HYP}	*The gluon actions*
The quark actions	wilson	-3.17	-2.59	-1.53	
	overlap	-34.90	-18.83	-4.89	
	D_c	-42.10	-24.25	-8.63	

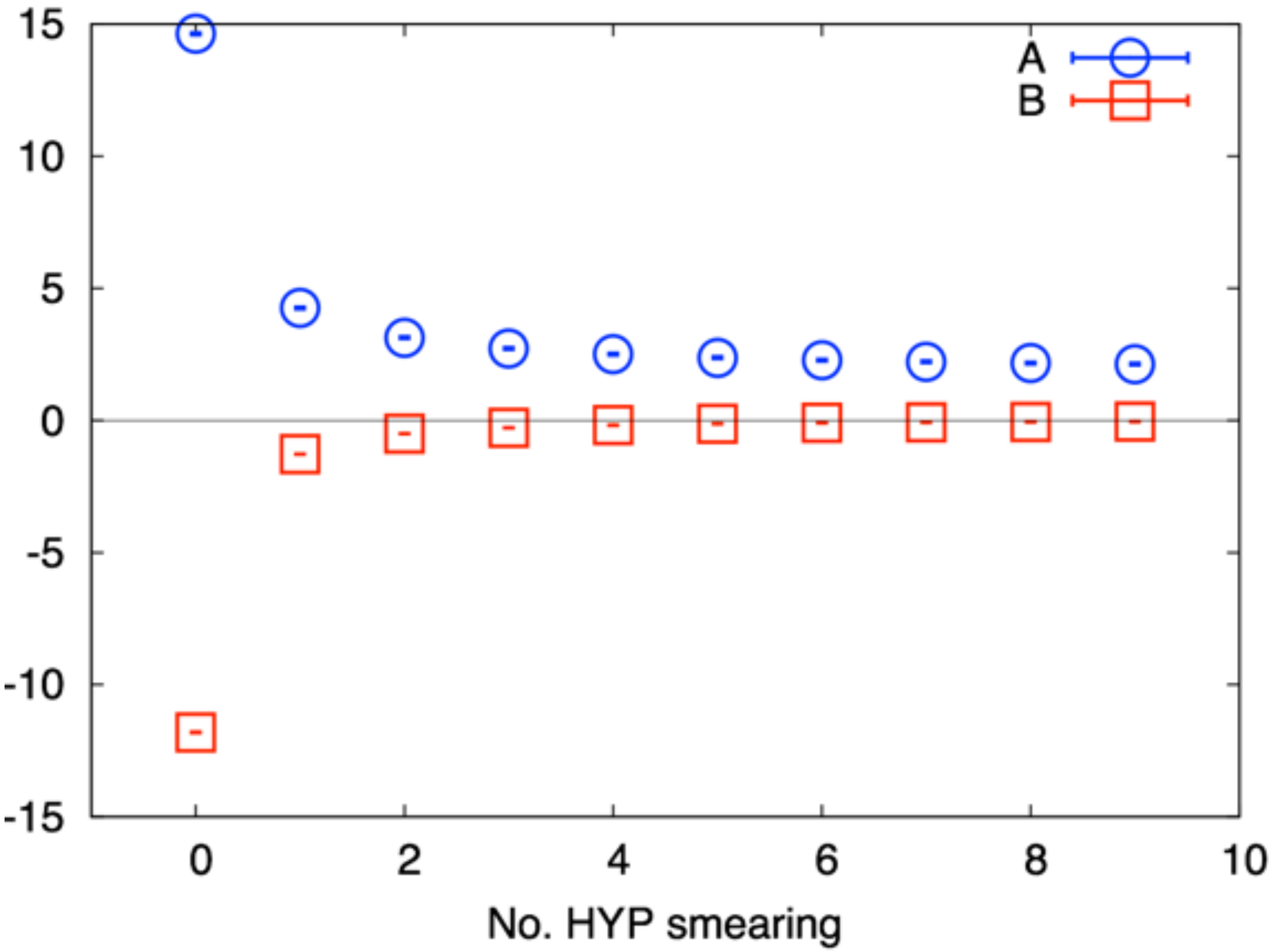
- The values are sensitive to both the quark and gluon actions.
- The values with the unimproved Wilson glue action can be very large.
- The HYP smearing can make the values smaller and become less sensitive to the quark action.

The HYP smearing

for the gluon operators



$$\frac{g^2 N_c}{16\pi^2}[A + \frac{B}{N_c^2}]$$



- Taking the 4-gluon vertex tadpole contribution in the gluon self energy as example.

- The HYP smearing can make the finite piece smaller and converge to fix values.

The renormalization of AM

the results

From the lattice bare quantities with the chiral fermion and HYP smeared Iwasaki gluon to that under the MS-bar scheme, at a scale μ=1/a,

$$\begin{pmatrix} O^{\overline{MS}}_{Q,(1)} \\ O^{\overline{MS}}_{G,(1)} \end{pmatrix} = \begin{pmatrix} 1-\frac{g^2 C_F}{16\pi^2}[\frac{40}{9}-8.63(1)] & -\frac{g^2 N_f}{16\pi^2}[\frac{4}{9}+0.20(1)] \\ +\frac{g^2 C_F}{16\pi^2}[\frac{22}{9}+3.56(1)] & 1+\frac{g^2 N_f}{16\pi^2}[\frac{10}{9}-3.52(1)] \\ & +\frac{g^2 N_c}{16\pi^2}[\frac{4}{3}+1.54(1)+V.T.] \end{pmatrix} \begin{pmatrix} O^{L}_{Q,(1)} \\ O^{L}_{G,(1)} \end{pmatrix} + O(g^4)$$

$$\xrightarrow[g^2\sim 3]{} \begin{pmatrix} 1.1060(2) & -0.0122(2)N_f \\ 0.1521(2) & 1.1637(6)-0.0458(2)N_f+0.0570V.T.] \end{pmatrix} \begin{pmatrix} O^{L}_{Q,(1)} \\ O^{L}_{G,(1)} \end{pmatrix} + O(g^4)$$

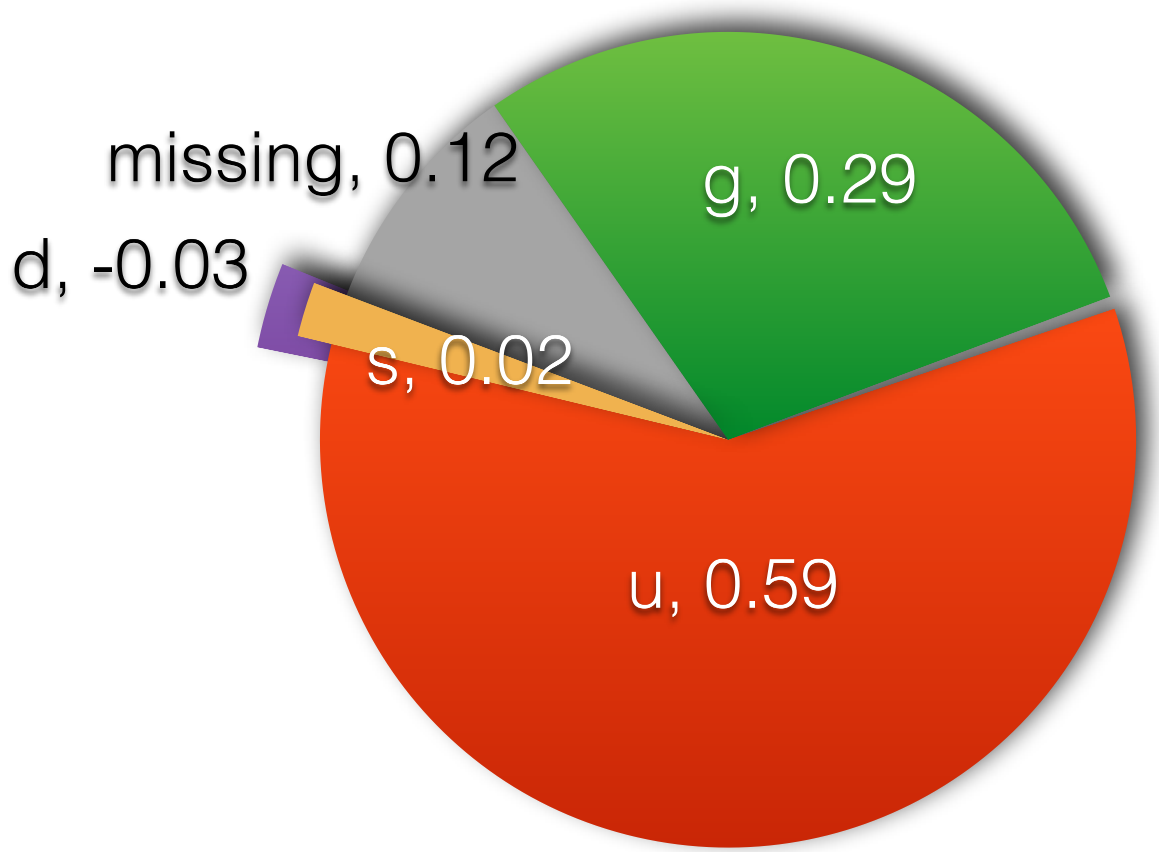
V.T.: The 4-gluon vertex tadpole contribution, in progress.

*We can force the sum rule of the momentum fractions to avoid the calculation of V.T., and the final normalization factor for the gluon operator is **~1.0.***

The pie charts

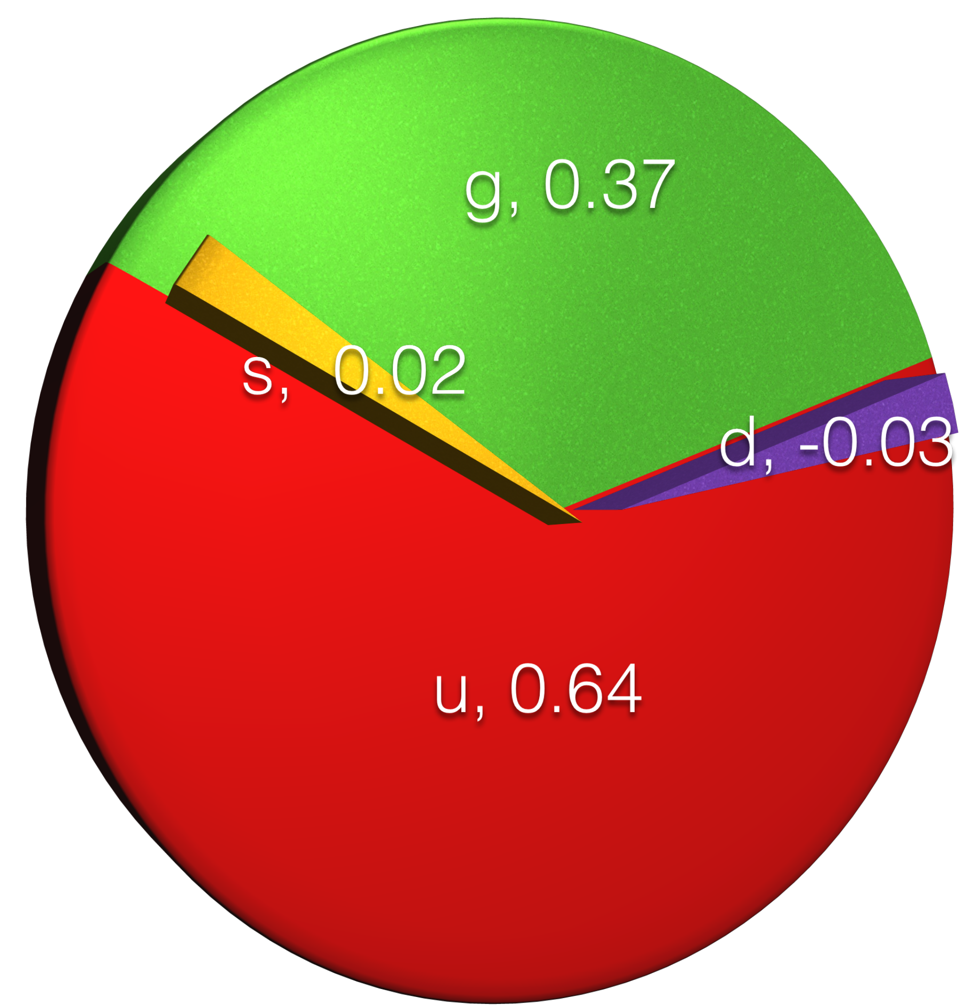
of the quark and gluon AM in proton

m_π=400 MeV, preliminary



Bare values

1-loop renormalized values



The percentage of the angular momentum in proton