Fermions with long-range interactions

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1 Introduction

The Hamiltonian of the generalised t-V model on a 1D chain is [1]:

$$H = -t\sum_{i=1}^{L}\left(\phi_i^\dagger\phi_{i+1} + \text{h.c.}\right) + \sum_{i=1}^{L}\sum_{m=1}^{p} U_m\phi_i^\dagger\phi_i\phi_{i+m}^\dagger\phi_{i+m}$$

L – system size (periodic), p – max interaction range, $\{U_m\}$ – potentials

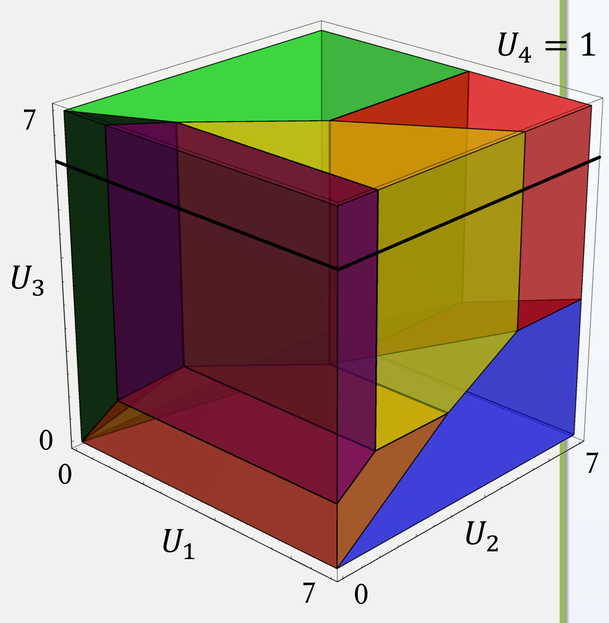
Away from critical density	Critical density $Q_C = \frac{1}{m}; m = 1, \dots, p$
• Luttinger liquid • Highly degenerate GS • Interacting "hard rods", size p	• Mott insulator • Simple ground state • "Rods" filling the lattice or equally overlapping

2 Phase Diagram

At the critical density, if the potential is not decreasing uniformly, we can have the following phases in the system [2]:

- ❖ Charge-density wave (CDW) phases – insulating
- ❖ Luttinger liquid, despite the insulating density
- ❖ Bond-order phase – phase with long-range ordering

At the atomic limit ($t = 0$), one can determine what the CDW phases are for specific systems (e.g. $p = 4, Q = 1/3$):



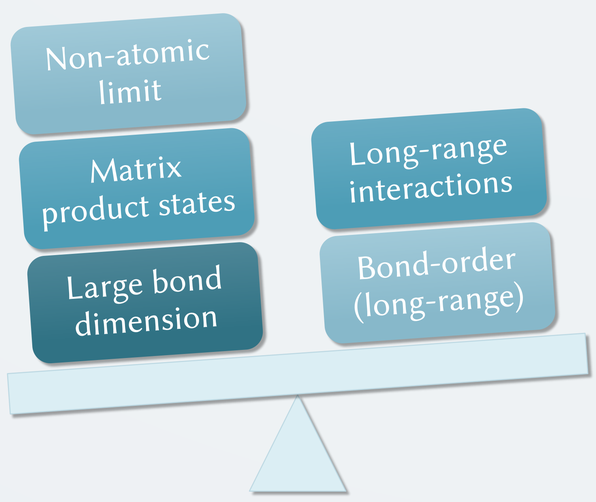
	Ground state unit cell	Energy
	●○○	$\frac{1}{3}U_3$
	●●○○○○	$\frac{1}{6}U_1$
	●○●○○○	$\frac{1}{6}(U_2 + U_4)$
	●●○○○●○○○	$\frac{1}{9}(U_1 + 2U_4)$
	●○●○●○○○○	$\frac{1}{9}(2U_2 + U_4)$
	●○●○○●○●○○○○	$\frac{1}{12}(2U_2 + U_3)$
	●●●○○○○●○●○○○○●○●○○○○	$\frac{1}{21}(2U_1 + 3U_2)$

3 Motivation

Although analysis of the atomic limit provides us with a complete picture of expected CDW phases, these phases may not be present in real life situations ($t \neq 0$).

We have decided to use matrix product states approach[3] in order to see

1. Emergence of the non-CDW phases.
2. How long-range correlations will affect the bond dimension needed.
3. If any CDW phase can never emerge.

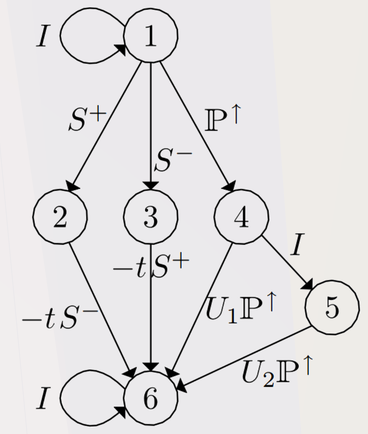


4 Hamiltonian as Matrix Product Operator

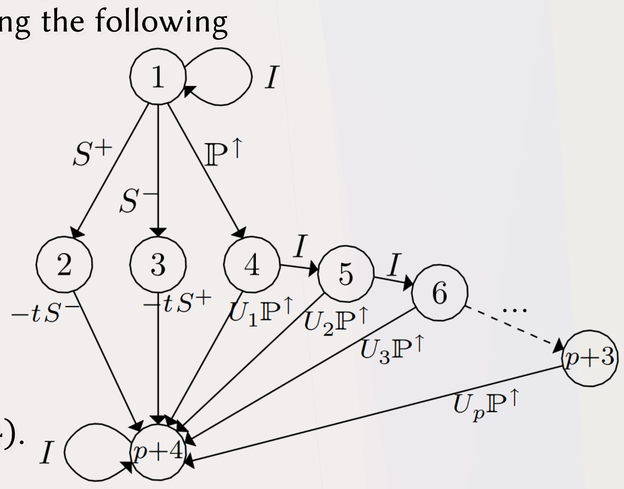
Firstly, we transform the Hamiltonian to its spin-half equivalent. The potential for $p = 2$:

$$U_1\sum_{i=1}^{L}\mathbb{P}_i^\uparrow\mathbb{P}_{i+1}^\uparrow + U_2\sum_{i=1}^{L}\mathbb{P}_i^\uparrow\mathbb{P}_{i+2}^\uparrow,$$

where $\mathbb{P}^\uparrow = |\uparrow\rangle\langle\uparrow|$. We can construct the Hamiltonian using the states of the automaton [4] on the right.

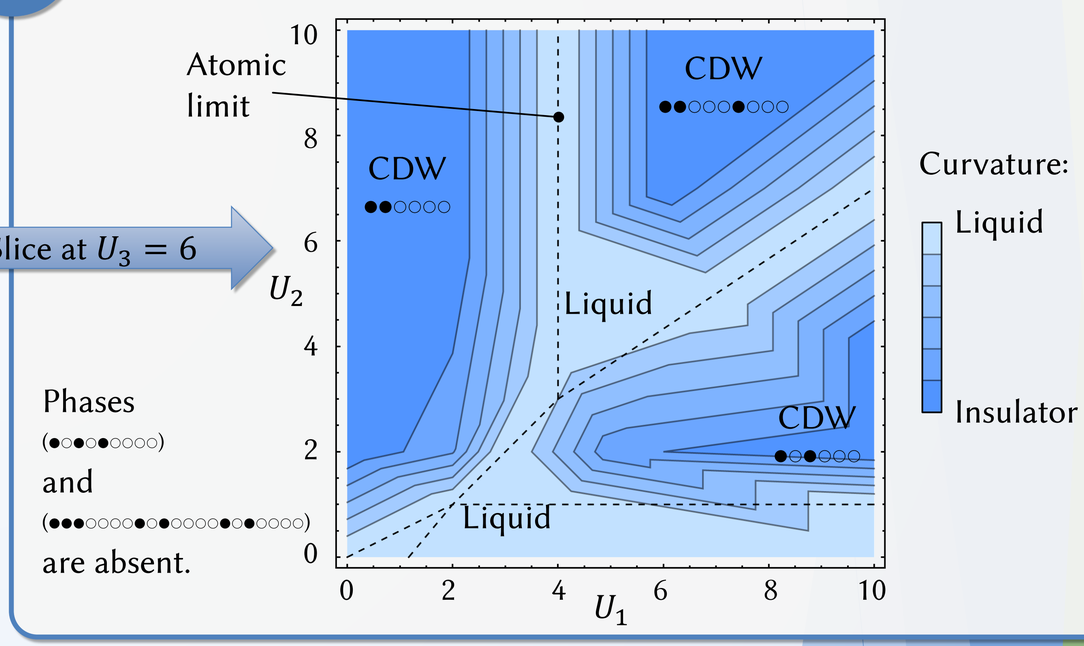


The full Hamiltonian for any interaction range can be constructed using the following automaton:



The one-site Hamiltonian in MPO representation has dimensions $(p + 4) \times (p + 4)$.

5 Preliminary Results



6 Outlook

- Complete the results: more points near the liquid-insulator transition
- Check the existence of bond-order phases
- Bond dimension vs. interaction range
- Determine which CDW-phases will not appear for $t \neq 0$

References:

[1] G. Gómez-Santos, Phys. Rev. Lett. 70, 3780 (1993).

[2] P. Schmitteckert and R. Werner, Phys. Rev. B 69, 195115 (2004); T. Mishra et al., Phys. Rev. B 84, 115135 (2011).

[3] D. Perez-Garcia et al., Quantum Inf. Comput. 7, 401 (2007); F. Verstraete et al., Adv. Phys. 57, 143 (2008).

[4] U. Schollwöck, "DMRG: Ground States, Time Evolution, and Spectral Functions" from "Emergent Phenomena in Correlated Matter".

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