Fermions with long-range interactions

**INTRODUCTION**

The Hamiltonian of the generalised $t$-$V$ model on a 1D chain is [1]:

$$H = -t \sum_{i=1}^{L} (\phi_i^\dagger \phi_{i+1} + h.c.) + \sum_{i=1}^{L} \sum_{m=1}^{p} U_{m} \phi_i^\dagger \phi_i^\dagger \phi_{i+m}$$

$L$ - system size (periodic), $p$ - max interaction range, $(U_m)$ - potentials

**PHASE DIAGRAM**

At the critical density, if the potential is not decreasing uniformly, we can have the following phases in the system [2]:
- Charge-density wave (CDW) phases – insulating
- Luttinger liquid, despite the insulating density
- Bond-order phase – phase with long-range ordering

At the atomic limit ($t = 0$), one can determine what the CDW phases are for specific systems (e.g. $p = 4, Q = 1/3$):

**MOTIVATION**

Although analysis of the atomic limit provides us with a complete picture of expected CDW phases, these phases may not be present in real life situations ($t \neq 0$).

We have decided to use matrix product states approach[3] in order to see
1. Emergence of the non-CDW phases.
2. How long-range correlations will affect the bond dimension needed.
3. If any CDW phase can never emerge.

**HAMILTONIAN AS MATRIX PRODUCT OPERATOR**

Firstly, we transform the Hamiltonian to its spin-half equivalent. The potential for $p = 2$:

$$U_1 \sum_{i=1}^{L} \phi_i^\dagger \phi_{i+1} + U_2 \sum_{i=2}^{L} \phi_i^\dagger \phi_{i+2}$$

where $\phi_i^\dagger = |i\rangle \langle i|$. We can construct the Hamiltonian using the states of the automaton [4] on the right.

The full Hamiltonian for any interaction range can be constructed using the following automaton:

**PRELIMINARY RESULTS**

The one-site Hamiltonian in MPO representation has dimensions $(p+4) \times (p+4)$.

**OUTLOOK**

Complete the results: more points near the liquid-insulator transition
Check the existence of bond-order phases
Bond dimension vs. interaction range
Determine which CDW-phases will not appear for $t \neq 0$

**REFERENCES**