# Higgs physics at the LHC: an experimental perspective



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Young experimentalist and theorist institute 13 January 2016



• Motivation

- SM Higgs boson measurements
  - Mass, width, spin, CP
  - Couplings to fermions and bosons
- BSM Higgs boson probes
  - EFT coefficients
  - Non-SM couplings
  - Additional Higgs bosons

### Motivation

• The Higgs boson is the only fundamental scalar in the SM

- Introduces ad-hoc terms in the Lagrangian
  - Not the result of an underlying symmetry in the SM



• Half the SM Lagrangian can now be studied directly!

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# **SM Higgs boson measurements**

Steps in the SM Higgs boson measurement program

- Discover
- Measure mass, spin and CP
- Measure width 🗶
- Confirm mass-coupling correlation



TLAS

Exp.

Obs. yy+4l Exp.  $\dot{\gamma}\dot{\gamma}+4l$ 

Obs. lvlv+tt+bb -- Exp. |v|v+ττ+bb

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2011 + 2012 Data

2σ

√s = 7 TeV: Ldt = 4.6-4.8 fb

√s = 8 TeV: Ldt = 5.8-5.9 fb

### **Higgs boson production**



# Higgs boson decay



# Higgs boson decays to ZZ $\rightarrow$ llll and $\gamma\gamma$



# Higgs boson mass

#### Fully reconstructed final states allow precise mass measurement



# Higgs boson width

#### Direct width measurements limited by experimental resolution

(Width sensitive to non-SM decays)	Channel	ATLAS	CMS	
	H→ZZ	Γ < 2.6 GeV	Γ < 3.4 GeV	
	Н→үү	$\Gamma$ < 5.0 GeV	$\Gamma$ < 2.4 GeV	

Standard Model prediction  $\Gamma \sim 4$  MeV Run 2 combined direct limit could achieve  $\Gamma \leq 1$  GeV

Interference between Higgs production and background shifts the mass in the  $\gamma\gamma$  channel Shift is dependent on the width of the Higgs boson: SM predicts ~60 MeV shift

Difference in masses between  $\gamma\gamma$  and ZZ channels sensitive to Higgs width ATLAS+CMS combination:  $m_{\gamma\gamma} - m_{ZZ} = -0.1 \pm 0.5 \text{ GeV}$ 

Run 2 combined mass difference could have precision of ~200 MeV Could translate into  $\Gamma \le 15$  MeV? Sensitive to higher order corrections

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# Higgs boson width

• Off-shell production indirectly sensitive to width

$$|\mathcal{M}_{i \to H \to f}|^2 = \frac{|\mathcal{M}_i|^2 \ |\mathcal{M}_f|^2}{|p_H^2 - M_H^2 + iM_H\Gamma_H|^2}$$

Most cross section is in the region  $(p_{H}^{2} - M_{H}^{2}) \sim M_{H}\Gamma_{H}$ : cross section inversely proportional to  $\Gamma_{H}$ 

For  $p_{_{\rm H}}^{_{-2}} >> M_{_{\rm H}}^{^{-2}}$  cross section independent of  $\Gamma_{_{\rm H}}$ 

CMS (ZZ) and ATLAS (ZZ & WW) set limits of  $\Gamma \leq 22$  MeV

Model dependent: new particles affecting the width could also affect the high-p<sup>2</sup> measurement

VBF production less model-dependent: Expect first results in this channel in Run 2

CMS also sets a lower bound by looking for displaced vertices  $\Gamma > 3.5 \times 10^{-9} \text{ MeV}$ 



#### Higgs boson spin and CP

Must be a scalar to be called a Higgs boson

- Study angular distributions of ZZ,  $\gamma\gamma$ , and WW decays
  - Use multivariate analysis in each channel
- Spin-0 CP-even state preferred to spin-2 or CP-odd state at ~99% C.L.



# SM Higgs boson measurements

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#### **Higgs boson production**

#### Run 1 coupling results use "kappa" framework

- Multiplicative factors for Higgs terms in the Lagrangian

For a given production process or decay channel:

$$\kappa_j^2 = \sigma_j / \sigma_j^{\text{SM}} \qquad \kappa_j^2 = \Gamma^j / \Gamma_{\text{SM}}^j$$

Connect to measurements via " $\mu$ " factors (notation:  $i \rightarrow H \rightarrow f$ )

$$\mu_i = \frac{\sigma_i}{(\sigma_i)_{\text{SM}}}$$
 and  $\mu^f = \frac{\text{BR}^f}{(\text{BR}^f)_{\text{SM}}}$ .

$$\mu_i^f = \frac{\sigma_i \cdot \mathbf{BR}^f}{(\sigma_i)_{\mathrm{SM}} \cdot (\mathbf{BR}^f)_{\mathrm{SM}}} = \mu_i \times \mu^f$$

ATLAS-CONF-2015-044, 13 January 2015 CMS-PAS-HIG-15-002

Production	Loops	Interference	Multip	blicative factor
$\sigma(ggF)$	$\checkmark$	b-t	$\kappa_g^2 \sim$	$1.06 \cdot \kappa_{\rm t}^2 + 0.01 \cdot \kappa_{\rm b}^2 - 0.07 \cdot \kappa_{\rm t} \kappa_{\rm b}$
$\sigma(VBF)$	_	_	$\sim$	$0.74 \cdot \kappa_{\rm W}^2 + 0.26 \cdot \kappa_{\rm Z}^2$
$\sigma(WH)$	_	_	~	$\kappa_{\rm W}^2$
$\sigma(qq/qg \to ZH)$	_	_	~	$\kappa_{\rm Z}^2$
$\sigma(gg\to ZH)$	$\checkmark$	Z - t	~	$2.27 \cdot \kappa_Z^2 + 0.37 \cdot \kappa_t^2 - 1.64 \cdot \kappa_Z \kappa_t$
$\sigma(ttH)$	_	_	~	$\kappa_{\rm t}^2$
$\sigma(gb \to WtH)$	_	W - t	~	$1.84 \cdot \kappa_{\rm t}^2 + 1.57 \cdot \kappa_{\rm W}^2 - 2.41 \cdot \kappa_{\rm t} \kappa_{\rm W}$
$\sigma(qb \to tHq)$	_	W - t	~	$3.4 \cdot \kappa_t^2 + 3.56 \cdot \kappa_W^2 - 5.96 \cdot \kappa_t \kappa_W$
$\sigma(bbH)$	_	_	~	$\kappa_{\rm b}^2$
Partial decay width				
$\Gamma^{ZZ}$	_	_	~	$\kappa_{\rm Z}^2$
$\Gamma^{WW}$	_	_	~	$\kappa_{\rm W}^2$
$\Gamma^{\gamma\gamma}$	$\checkmark$	W - t	$\kappa_{\gamma}^2 \sim$	$1.59 \cdot \kappa_{\rm W}^2 + 0.07 \cdot \kappa_{\rm t}^2 - 0.66 \cdot \kappa_{\rm W} \kappa_{\rm t}$
$\Gamma^{\tau\tau}$	_	_	•~	$\kappa_{\tau}^2$
$\Gamma^{bb}$	_	_	~	$\kappa_{\rm b}^2$
$\Gamma^{\mu\mu}$	_	_	~	$\kappa_{\mu}^2$
Total width for $BR_{BSM} = 0$				
				$0.57 \cdot \kappa_{\rm b}^2 + 0.22 \cdot \kappa_{\rm W}^2 + 0.09 \cdot \kappa_{\rm g}^2 +$
$\Gamma_{\rm H}$	$\checkmark$	_	$\kappa_{\rm H}^2 \sim$	$+ 0.06 \cdot \kappa_{\tau}^2 + 0.03 \cdot \kappa_Z^2 + 0.03 \cdot \kappa_c^2 +$
				$+ 0.0023 \cdot \kappa_{\gamma}^2 + 0.0016 \cdot \kappa_{Z\gamma}^2 +$
				$+ 0.0001 \cdot \kappa_{s}^{2} + 0.00022 \cdot \kappa_{\mu}^{2}$

# Higgs boson coupling to EW bosons

Z= - 4 Fre Friv + i typy + h.c. + Ψ: Y: 4: 4: + h.c.  $+(D_{\mu}\phi)^{2}-V(\phi)$ 

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### Higgs boson decay to ZZ

#### • Uncertainty statistics dominated

- Loosely split by production mode





ATLAS and CMS

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# Higgs boson decay to WW

ATLAS and CMS

Observed

LHC Bun 1 Preliminary

SM Higgs boson

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#### • Most complicated of the measured channels

- Constrain cocktail of backgrounds with data



# Higgs boson decay to WW

#### • Fit mass-based discriminant in multiple measurement bins

 $m_{\rm T} = \sqrt{(E_{\rm T}^{\ell\ell} + p_{\rm T}^{\nu\nu})^2 - |\mathbf{p}_{\rm T}^{\ell\ell} + \mathbf{p}_{\rm T}^{\nu\nu}|^2}$ 

ATLAS gg→H→WW measurement has smallest relative production uncertainty in any single channel (27%)

Expect <20% ggF uncertainty in Run 2

 $\begin{array}{ll} \mu_{ggF} = 1.02 \pm 0.19 & \begin{tabular}{c} +0.22 \\ -0.18 \end{tabular} = 1.02 & \begin{tabular}{c} +0.29 \\ -0.26 \end{tabular} \\ \mu_{VBF} = 1.27 & \begin{tabular}{c} +0.44 \\ -0.40 \end{tabular} & \begin{tabular}{c} +0.30 \\ -0.21 \end{tabular} = 1.27 & \begin{tabular}{c} +0.29 \\ -0.45 \end{tabular} \\ (stat) \end{tabular} & (syst) \end{array}$ 

Measurement of vector-boson fusion optimizes selection and discriminant using the radiating jets

VBF production

Uncertainty statistics-dominated:

~25% precision per experiment possible for VBF in Run 2

~50% precision possible for VH in Run 2



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180

Observed

 $1.13_{-0.31}^{+0.37}$ 

 $1.16^{+0.63}_{-0.58}$ 

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# **Differential Higgs boson production**

- Sufficient number of events in ZZ, γγ, and WW for singledifferential cross section measurements
  - ZZ+yy combined; WW separate (not fully reconstructed)



### Higgs boson decay to Zy

#### • Sensitive to similar loop contributions to $\gamma\gamma$



CMS limits 10x SM ATLAS limits 19x SM

Combined ATLAS & CMS could get to 4x SM prediction in Run 2



# Higgs boson coupling to fermions

Z= - 4 Fre Friv + ご サダサ + ん. c. + \u00ed y ; y ; y ; + h. c. +  $D\phi l^2 - V(\phi)$ 

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### Higgs boson decay to $\tau\tau$

#### • Challenge to reconstruct invariant mass



 $\mu_{ggF}^{\tau\tau} = 1.24 \begin{array}{c} +0.49 \\ -0.45 \\ (\text{stat.}) \begin{array}{c} -0.8 \\ -0.29 \\ (\text{syst.}) \end{array} \pm 0.08 \\ (\text{theory syst.}) \end{array}$   $\mu = 1.43 \begin{array}{c} +0.27 \\ -0.26 \\ (\text{stat.}) \begin{array}{c} +0.32 \\ -0.25 \\ (\text{syst.}) \end{array} \pm 0.09 \\ (\text{theory syst.}) \end{array}$ 

If the tau pair does not fall along a line, the mass can be resolved (neutrinos from the tau decay are approximately collinear)

Gluon fusion measured by requiring the  $\mu_{P_T}^{\mu=1.0 \pm 0.5}$  to be large ("boosted")

Dominant  $Z \rightarrow \tau \tau$  background modelled byreplacing muons in  $Z \rightarrow \mu \mu$  data with taus from simulaiton

CMS: combined  $\mu$ =0.78 ± 0.27

Run 2: Expect ~60% ggF and ~30% VBF uncertainty



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ATI AS and CMS

Observed

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LHC Bun 1 Preliminary

Z 1

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Particle mass [GeV]

### Higgs boson decay to bb

• Use VH production to suppress QCD b-jet production



# ttH production with decay to b quarks

- Many possible final states depending on tt decay
  - One/two leptons; categorize by number of jets and b-jets



# ttH production with decay to b quarks

#### • Fit multivariate distribution in each final state



# ttH production with decay to photons

#### • ATLAS and CMS limits are ~7x SM cross section

– Expect to be within a factor of  $\sim 2$  in Run 2



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### ttH production with leptons in decay

- Combine Higgs boson decays to WW, ττ and ZZ
  - Categorize by leptonic combinations in the event

Category	Higgs boson decay mode					
	WW*	ττ	ZZ*	Other		
2ℓ0T <sub>had</sub>	80%	15%	3%	2%		
3ℓ	74%	15%	7%	4%		
$2\ell 1\tau_{had}$	35%	62%	2%	1%		
4 <i>ℓ</i>	69%	14%	14%	4%		
$1\ell 2\tau_{had}$	4%	93%	0%	3%		

	ee	$\mathrm{e}\mu$	$\mu\mu$	$3\ell$	$4\ell$
$t\overline{t}H, H \rightarrow WW$	$1.0\pm0.1$	$3.2\pm0.4$	$2.4\pm0.3$	$3.4\pm0.5$	$0.29\pm0.04$
$t\overline{t}H, H \rightarrow ZZ$		$0.1\pm0.0$	$0.1\pm0.0$	$0.2\pm0.0$	$0.09\pm0.02$
ttH, H $\rightarrow \tau \tau$	$0.3\pm0.0$	$1.0\pm0.1$	$0.7\pm0.1$	$1.1 \pm 0.2$	$0.15\pm0.02$
$t\overline{t}W$	$4.3\pm0.6$	$16.5\pm2.3$	$10.4\pm1.5$	$10.3\pm1.9$	
$t\overline{t}Z/\gamma^*$	$1.8\pm0.4$	$4.9\pm0.9$	$2.9\pm0.5$	$8.4\pm1.7$	$1.12\pm0.62$
$t\overline{t}WW$	$0.1\pm0.0$	$0.4\pm0.1$	$0.3 \pm 0.0$	$0.4 \pm 0.1$	$0.04\pm0.02$
$t\overline{t}\gamma$	$1.3\pm0.3$	$1.9\pm0.5$		$2.6\pm0.6$	
WZ	$0.6\pm0.6$	$1.5\pm1.7$	$1.0\pm1.1$	$3.9\pm0.7$	
ZZ		$0.1\pm0.1$	$0.1 \pm 0.0$	$0.3 \pm 0.1$	$0.47\pm0.10$
Rare SM bkg.	$0.4\pm0.1$	$1.6\pm0.4$	$1.1\pm0.3$	$0.8\pm0.3$	$0.01\pm0.00$
Non-prompt	$7.6\pm2.5$	$20.0\pm4.4$	$11.9\pm4.2$	$33.3\pm7.5$	$0.43\pm0.22$
Charge misidentified	$1.8\pm0.5$	$2.3\pm0.7$			
All signals	$1.4\pm0.2$	$4.3\pm0.6$	$3.1\pm0.4$	$4.7\pm0.7$	$0.54\pm0.08$
All backgrounds	$18.0\pm2.7$	$49.3\pm5.4$	$27.7 \pm 4.7$	$59.8 \pm 8.0$	$2.07\pm0.67$
Data	19	51	41	68	1



### Higgs boson decay to µµ

Directly test Higgs couplings to second generation fermions

- ATLAS & CMS limits ~7x SM prediction
- CMS also constrains decay to ee

• 37000x SM prediction



ATLAS splits into seven categories based on dimuon momentum and rapidity, and on production mechanism

Combined ATLAS & CMS could get to 2x SM prediction in Run 2



# Higgs boson decay to $J/\psi \gamma$

#### Sensitive to Hcc coupling

- Predicted branching ratio  $2.8 \times 10^{-6}$ 



Expect Run 2 limits to reach  $5 \times 10^{-4}$  (~180x SM)

Could also probe Hcc coupling using charm tagging in VH production

### Higgs boson self-coupling

Z= - 4 Fre Friv + ご サダサ + ん. c. + Ψ: Y: 4: 4: + h.c. +  $D_{\mu}\phi l^{2} - V(\phi)$ 

### Higgs boson pair production

- Destructively interfering diagrams leads to very low rates
  - Many channels to study



#### **Combined Run 1 results**

#### • ATLAS+CMS $\kappa$ and $\mu$ constraints



# Higgs boson beyond the SM

#### Concurrent effort along various tracks

- Parameterize deviations in SM measurements
- Search for BSM decays
- Search for additional Higgs bosons



• Run 2 to probe more terms in the Lagrangian

- Current tools include  $\leq 6$  dimensional operators to LO
  - For example, single-Higgs couplings to the SM:

$$\begin{split} \Delta \mathcal{L}_{h} &= \frac{h}{v} \left[ 2\delta c_{w} m_{W}^{2} W_{\mu}^{+} W_{\mu}^{-} + \delta c_{z} m_{Z}^{2} Z_{\mu} Z_{\mu} \right. \\ &- \sum_{f \in u, d, e} \sum_{ij} \sqrt{m_{f_{i}} m_{f_{j}}} [\delta y_{f}]_{ij} \left[ \cos \phi_{ij}^{f} \bar{f}_{i} f_{j} - i \sin \phi_{ij}^{f} \bar{f}_{i} \gamma_{5} f_{j} \right] \\ &+ c_{ww} \frac{g^{2}}{2} W_{\mu\nu}^{+} W_{\mu\nu}^{-} + \tilde{c}_{ww} \frac{g^{2}}{2} W_{\mu\nu}^{+} \tilde{W}_{\mu\nu}^{-} + c_{w\Box} g^{2} \left( W_{\mu}^{-} \partial_{\nu} W_{\mu\nu}^{+} + h.c. \right) \\ &+ c_{gg} \frac{g_{s}^{2}}{4} G_{\mu\nu}^{a} G_{\mu\nu}^{a} + c_{\gamma\gamma} \frac{e^{2}}{4} A_{\mu\nu} A_{\mu\nu} + c_{z\gamma} \frac{e\sqrt{g^{2} + g^{\prime 2}}}{2} Z_{\mu\nu} A_{\mu\nu} + c_{zz} \frac{g^{2} + g^{\prime 2}}{4} Z_{\mu\nu} Z_{\mu\nu} \\ &+ \tilde{c}_{gg} \frac{g_{s}^{2}}{4} G_{\mu\nu}^{a} \tilde{G}_{\mu\nu}^{a} + \tilde{c}_{\gamma\gamma} \frac{e^{2}}{4} A_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{z\gamma} \frac{e\sqrt{g^{2} + g^{\prime 2}}}{2} Z_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{zz} \frac{g^{2} + g^{\prime 2}}{4} Z_{\mu\nu} \tilde{Z}_{\mu\nu} \end{split}$$

- Derive from measurements of exclusive cross sections
  - e.g.  $\sigma(0\text{-jet}), \sigma(\text{high-p}_{T}^{H})$

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• Run 2 experimental focus will be on binned cross sections

- Effectively production-level fiducial cross sections
  - Extrapolate back to decay vertex



Post-decay fiducial cross sections measured in Run 1

Туре	$n_j = 0$		$n_j = 1$
Preselection	oł	$\begin{array}{l} p_{\mathrm{T}}^{\ell 1} > 22\\ p_{\mathrm{T}}^{\ell 2} > 10\\ \text{pposite-charge}\\ m_{\ell \ell} > 10\\ p_{\mathrm{T}}^{\nu \nu} > 20 \end{array}$	ť
$n_j$ dependent	$\Delta \phi_{\ell\ell,\nu\nu} > \pi$	/2	-
ATLAS	$p_{\mathrm{T}}^{\varepsilon\varepsilon} > 30$		$m_{\mathrm{T}}^{\ell} > 50$
H→WW	$m_{\ell\ell} < 55$ $\Delta \phi_{\ell\ell} < 1.2$	8	$m_{\tau\tau} < 66$ $m_{\ell\ell} < 55$ $\Delta \phi_{\ell\ell} < 1.8$
$\mu^{ggF}_{0j,e\mu} =$	1.39 ±0.27	+0.21 +0.27 -0.19 -0.17	
$\mu^{ m ggF}_{1j,e\mu}$ =	$1.14 \begin{array}{c} +0.42 \\ -0.41 \end{array}$	$\substack{+0.27\\-0.26} \begin{array}{c}+0.42\\-0.17\end{array}$	
	(stat)	(syst) (sig)	H
$\sigma^{\rm ggF}_{{\rm fid},0j} = 27.$	5 + 5.4 + 4.1 = -5.3 - 3.9 =	27.6 <sup>+6.8</sup> <sub>-6.6</sub> f	b
$\sigma_{\mathrm{fid},1j}^{\mathrm{ggF}} = 8.3$	$^{+3.1}_{-3.0}  {}^{+2.0}_{-1.9} =$	8.3 <sup>+3.7</sup> <sub>-3.5</sub> f	b. 35
ALLY.	(stat) (syst)		hatta

Ongoing discussion in LHC Higgs WG about binning

Likely several stages for measurements to choose from Use the finest binning feasible for the measurement

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- Run 1 results with effective Lagrangian from ATLAS  $H \rightarrow \gamma \gamma$ 
  - Consider only terms relevant for ggF and VBF production

 $\mathcal{L} = \bar{c}_{\gamma} O_{\gamma} + \bar{c}_{g} O_{g} + \bar{c}_{HW} O_{HW} + \bar{c}_{HB} O_{HB}$  $+ \tilde{c}_{\gamma} \tilde{O}_{\gamma} + \tilde{c}_{g} \tilde{O}_{g} + \tilde{c}_{HW} \tilde{O}_{HW} + \tilde{c}_{HB} \tilde{O}_{HB},$ 

arxiv:1508.02507



- Run 1 results with effective Lagrangian from ATLAS  $H \rightarrow \gamma \gamma$ 
  - Obtain constraints in 1- & 2-dimensional coupling planes



# Higgs boson decays to BSM particles

- Decays to a low-mass scalar ("a") or invisible/dark matter (" $\chi$ ")
  - Dark matter needs to be produced with objects (VBF, VH)









# Higgs boson decays to BSM particles

- Decays to a low-mass scalar ("a") or invisible/dark matter (" $\chi$ ")
  - Low-mass scalar: many possible decays, broad mass range



#### • Probe additional EW singlets, doublets, triplets

- Type II two-Higgs doublet model appears in supersymmetry
- Contribution of second Higgs to particle masses small



- New bosons are charged and neutral (CP-even and CP-odd)
  - High tan $\beta$ : preferentially decay to bb,  $\tau\tau$ ,  $\mu\mu$



- Best sensitivity from  $\tau\tau$  decay at high tan $\beta$ 
  - First results from Run 2



No excesses observed

#### Additional CP-even Higgs boson can decay to SM Higgs pairs





Run 2



• Possible to have enhanced production and decay in loops

- Interesting Run 2 excesses in  $\gamma\gamma$  mass distribution



### Summary

- Run 1 saw the discovery of the Higgs boson and a comprehensive program to measure its properties
- Run 2 will probe new processes, search for small deviations in old processes, and expand the reach for additional scalars
- There is a parallel joint experimental and theoretical effort in the context of the LHC Higgs working group: a new yellow report to be released in the summer

### **Higgs boson production**

- Leading-order effective Lagrangian has limitations
  - Parameters have no sensitivity to e.g. pTH
- Ongoing work to extend to NLO and to add dimension-8 operators
  - already 59 operators at dimension-6
- Yellow report will also have prescription for connecting experimental constraints on EFT parameters to specific models

### **Higgs boson production**

 At the LHC Higgs bosons are produced via the same kind of radiative corrections that affect the Higgs mass



Two additional gluon vertices, one fewer Higgs vertex: sensitive to new strongly charged particles contributing to the Higgs mass

### Higgs boson mass

Supersymmetry possible solution to hierarchy problem



 $\Delta m_{\rm H}^2 = -2|y_t|^2/(16\pi) [\Lambda^2 + ...]$ 

The quadratic divergence cancels, leaving:

$$\Delta(m_{h^0}^2) = \frac{3}{4\pi^2} \cos^2 \alpha \ y_t^2 m_t^2 \ln\left(m_{\tilde{t}_1} m_{\tilde{t}_2} / m_t^2\right)$$

This adds to the tree-level expression  $m_{H}^{2} = m_{Z}^{2} \cos^{2}2\beta$