# Top quark physics

## a theoretical perspective

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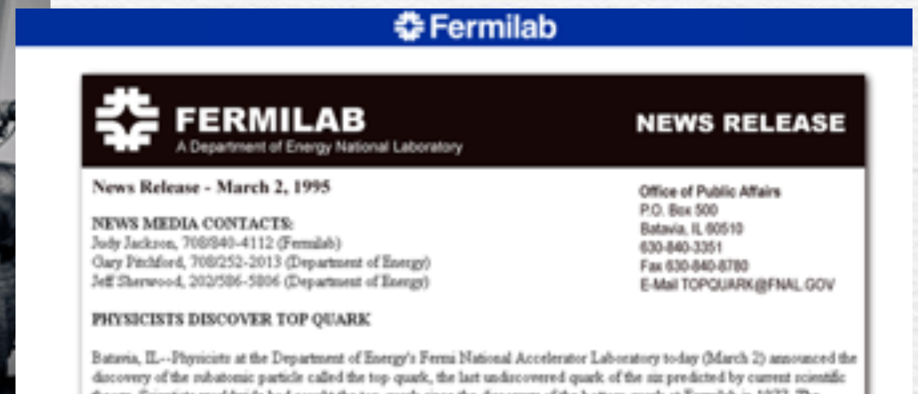
YETI , IPPP Durham, 11-14 January 2016

# Top quark: this is your life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NEWS RELEASE

News Release - March 2, 1995

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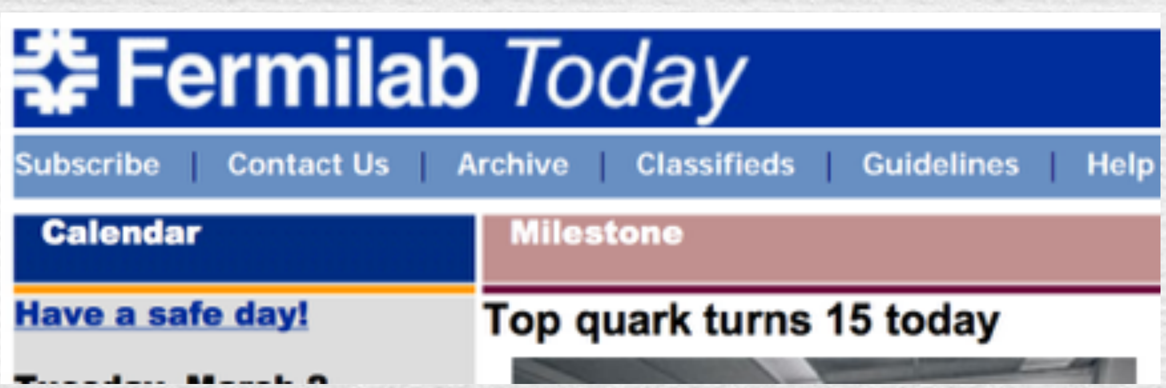
PHYSICISTS DISCOVER TOP QUARK

Batavia, IL--Physicists at the Department of Energy's Fermi National Accelerator Laboratory today (March 2) announced the discovery of the subatomic particle called the top quark, the last undiscovered quark of the six predicted by current scientific theory.

Exciting time..

Birth, 1995, now 21 years old *)

Ten years old, 2005

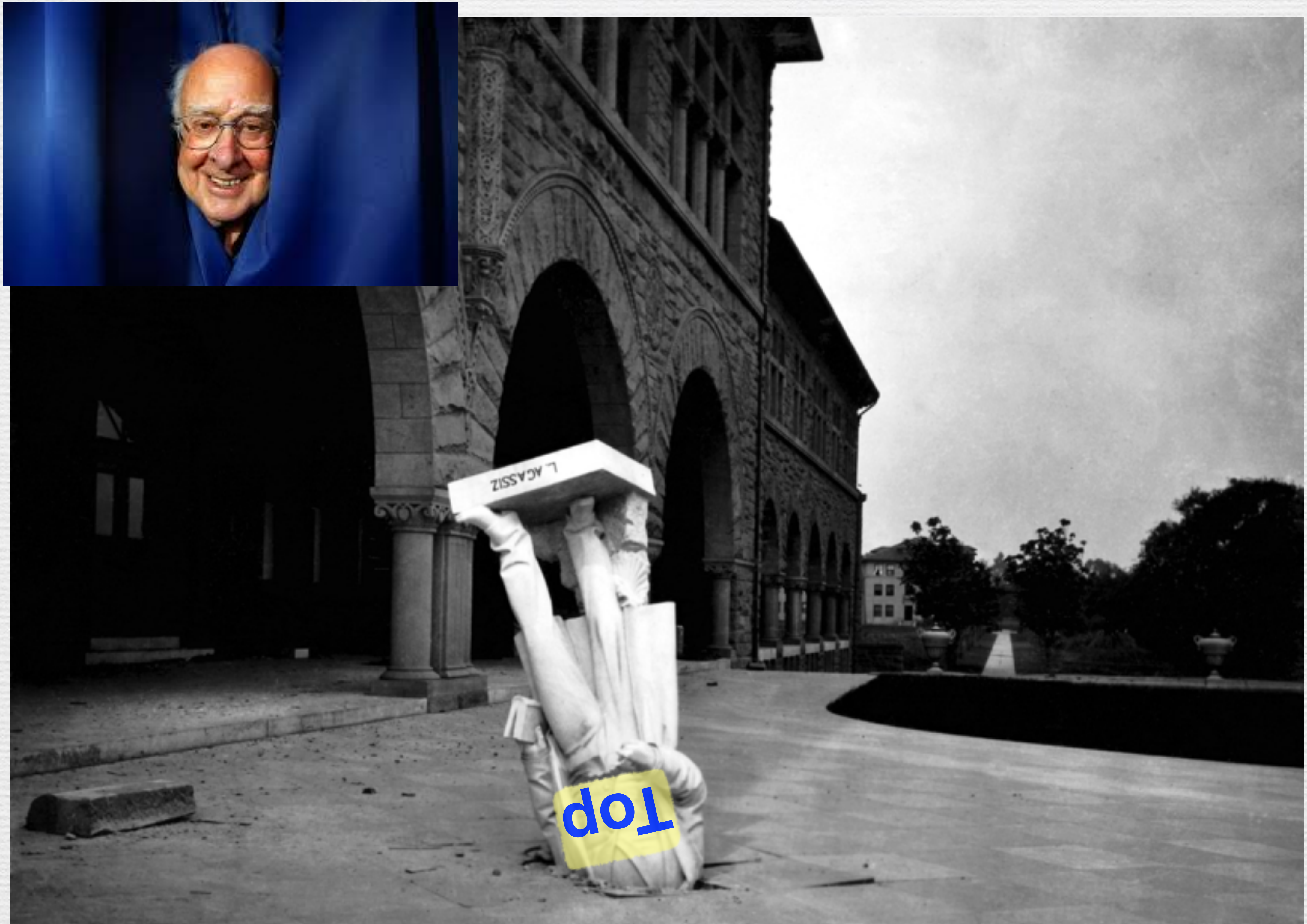




Fifteen, 2010

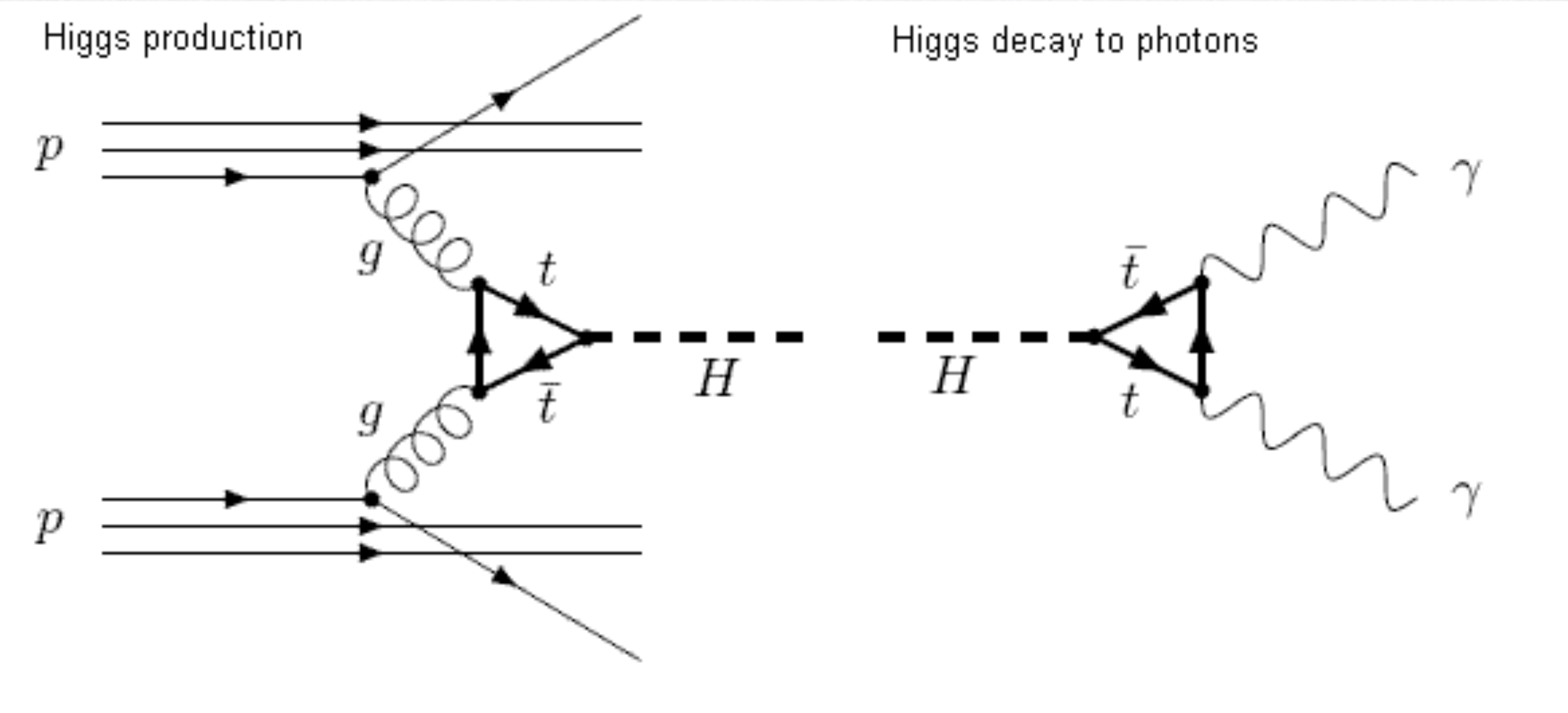
Until recently, the King of the Particles

*) actually, it's 13,798,000,000+21

# The king is dead



# Top: matter of life and death for Higgs



# This lecture

- Top is interesting
- Top mass
- Top production (pairs, singly, in association)
- Top decay and spin

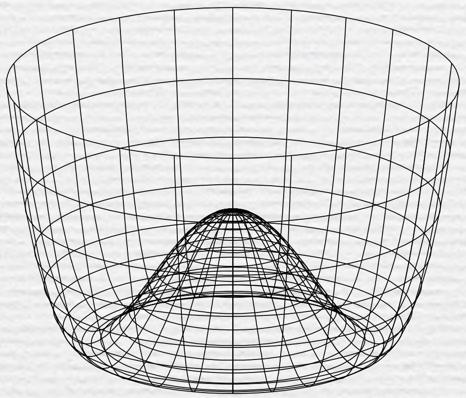
# Top in the Standard Model

# Electroweak symmetry breaking in SM

Symmetries forbid explicit mass terms of type $m^2\phi^2$ $\quad m\bar{\psi}\psi$ $\quad m^2 Z_\mu Z^\mu$

- Include calar field doublet, with potential $V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$

For $\mu^2 < 0$ it looks like



*Assumption*

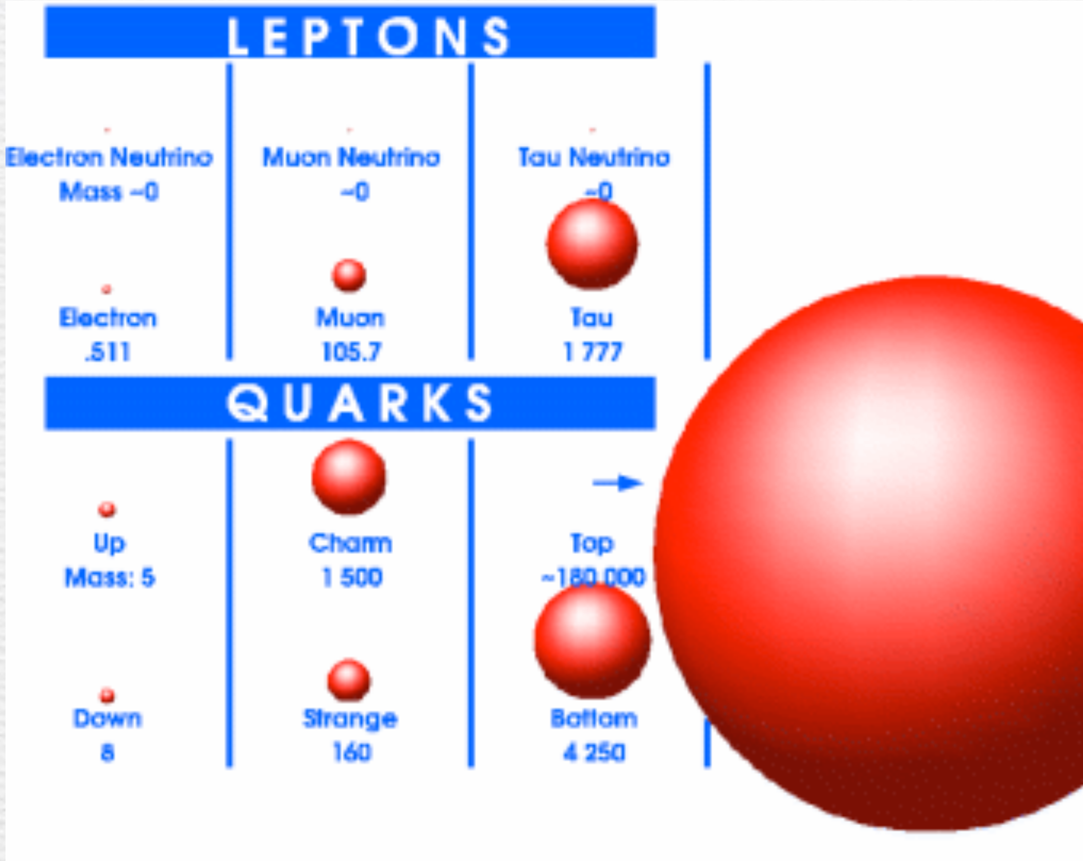
In minimum $\langle\Phi\rangle_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad v = \sqrt{-\mu^2/2\lambda}$

- Add Yukawa interactions

$$\mathcal{L}_{Yukawa} = y_u \bar{Q}_L \epsilon \Phi^* u_R + y_d \bar{Q}_L \Phi d_R + \ldots + h.c.$$

# Heavy quarks: what they taught us

- We learned much from Charm
  - SM consistent SM
- and from Bottom
  - 3rd family, allows for flavour mixing (Cabbibo-Kobayashi-Maskawa, Nobel 2008)
- What do we learn from Top?
  - Its the most expensive quark
  - Interacts strongly with all forces (gauge +Higgs) in SM
  - Nothing fundamental, yet. It owes us.



# Mass generation

- Expanding around the true groundstate $\Phi(x) = e^{i\xi^i(x)\sigma_i} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

*Higgs boson field*

*Fermion mass term* *Higgs-fermion-fermion interaction*

$$y_f[v + h(x)]\bar{\psi}_f\psi_f = m_f\bar{\psi}_f\psi_f + y_f h(x)\bar{\psi}_f\psi_f$$

All SM masses are so generated, and have form: coupling × v

Same couplings that determine masses determine interactions

- Rotation to mass states, with $\theta_W$, $V_{CKM}$, $V_{MNS}$

# Top coupling to other SM particles

Exp. tested?

- to W boson: flavor mixing, lefthanded
  - gw ~ 0.45 $\frac{g}{\sqrt{2}} V_{tq} \, (\bar{t}_L \gamma^\mu q_L) W^+_\mu$ √
- to Z boson: parity violating
  - gz ~ 0.14 $\frac{g}{4\cos\theta_w} \, \bar{t} \left( (1 - \frac{8}{3} \sin^2\theta_w) \gamma^\mu - \gamma^\mu \gamma^5 \right) t \, Z_\mu$ ?
- to photon: vectorlike, has charge 2/3
  - $e_t$ ~ 2/3 $e_t \, \bar{t} \gamma^\mu t A_\mu$ √?
- to gluon: vectorlike, non-trivial in color
  - $g_s$ ~ 1.12 $g_s \left[ T_a^{SU(3)} \right]^{ji} \bar{t}_j \gamma_\mu t_i A^a_\mu$ √
- to Higgs: Yukawa type
  - $y_t$ ~ 1 $y_t \, h \bar{t} t$ √?

Top physics: check structure and strength of all these couplings

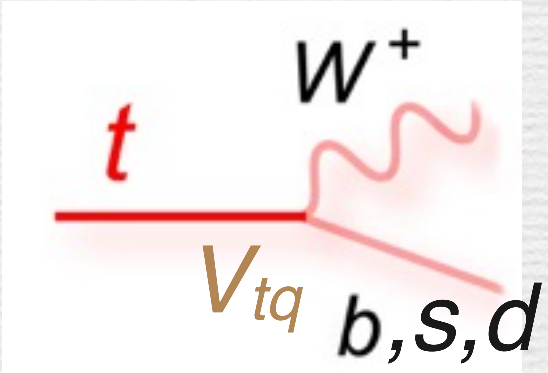
# Top is special:
# 3 reasons

# Why is top special? 1. It's very heavy

Strong coupling to Higgs boson, with coupling constant

$$y_t = \frac{\sqrt{2} m_t}{v} = \frac{\sqrt{2}173}{246} \simeq 0.99$$

Perhaps top has a special role in the EWSB mechanism?




Large mass makes for a *really* short lifetime

$$\tau_{top} = \hbar/\Gamma_t = 5 \times 10^{-25}\, s$$

Compare to other lifetimes

$\tau_{bottom} = 10^{-12}\, s$ $\tau_{\pi} = 10^{-8}\, s$ $\tau_{\mu} = 10^{-6}\, s$ $\tau_{talk} = 10^{3} s$

# Mass implications

- Top will decay before it hadronizes fully
  - the only bare quark: no time to collect haze of gluons and light quarks

$$\tau_{\text{hadronization}} = \hbar/\Lambda_{QCD} = 2 \times 10^{-24}\, s \qquad \tau_{top} = \hbar/\Gamma_t = 5 \times 10^{-25}\, s$$

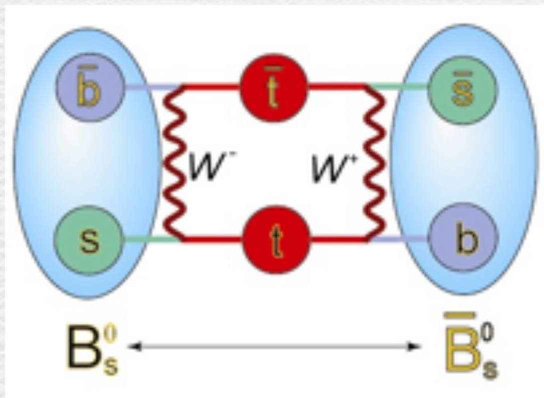
  - this gives us access to its spin (later)
- Typical strength of QCD interactions with Top;

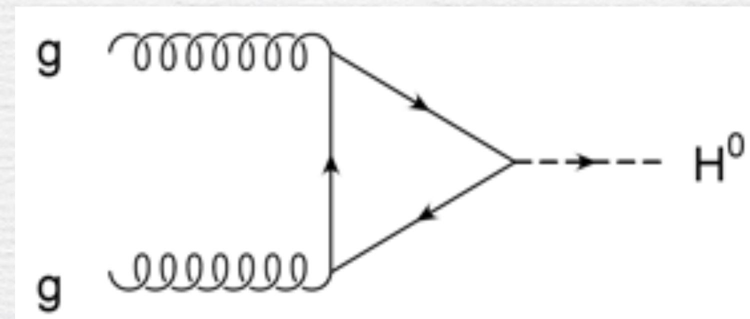
$$\alpha_s(m_t) \simeq 0.1$$

  - very small, so good for perturbative approach

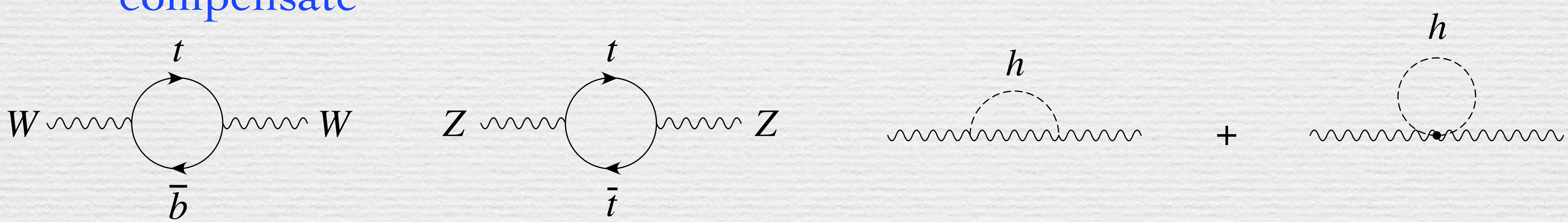
# Why is top special? 2. Very noisy in loops

- Even if top is virtual, it makes itself loudly known




  - in a loop integral a mass scale always occurs in the result
  - very noticeable if there is no particle with (roughly) equal mass to compensate



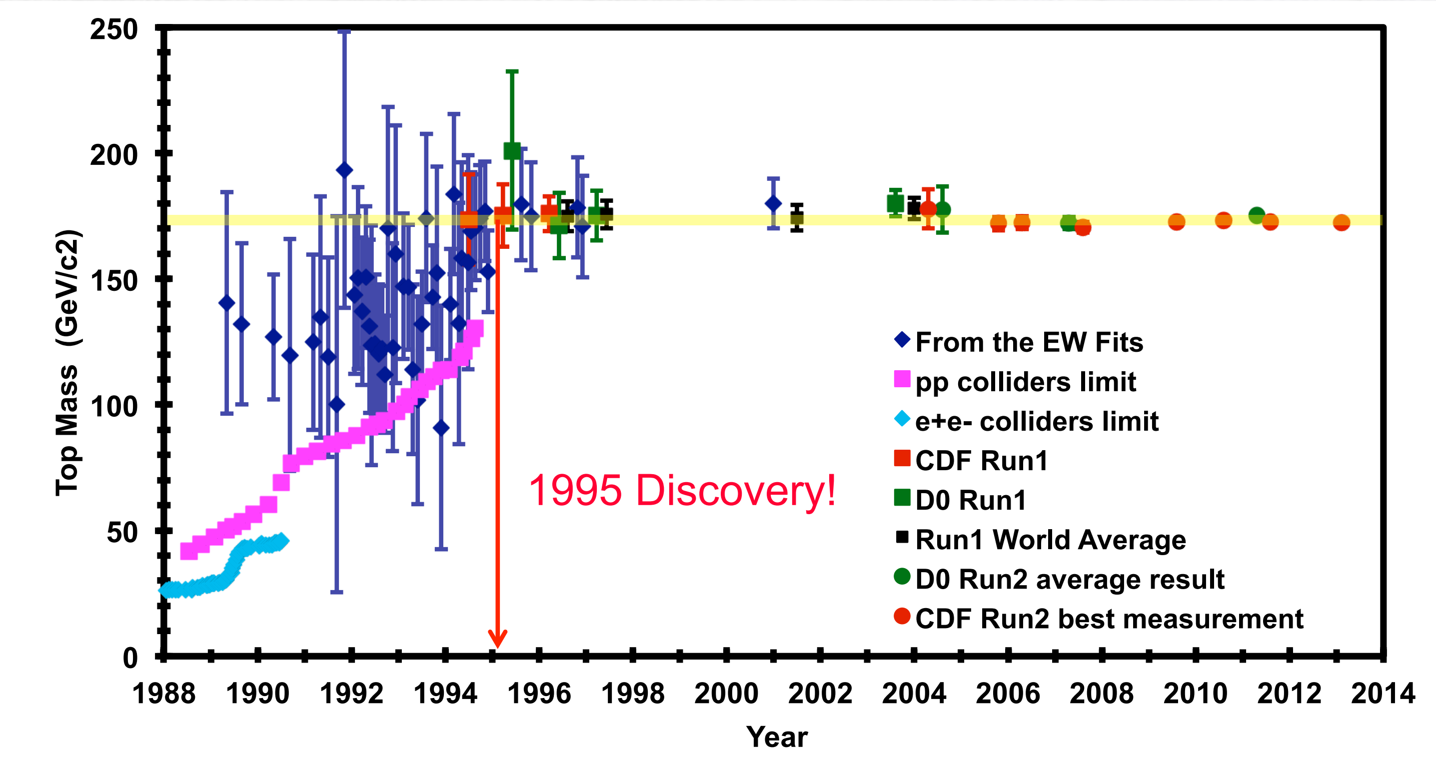

- Express the W mass in terms of 3 fundamental weak parameter, with loop corrections

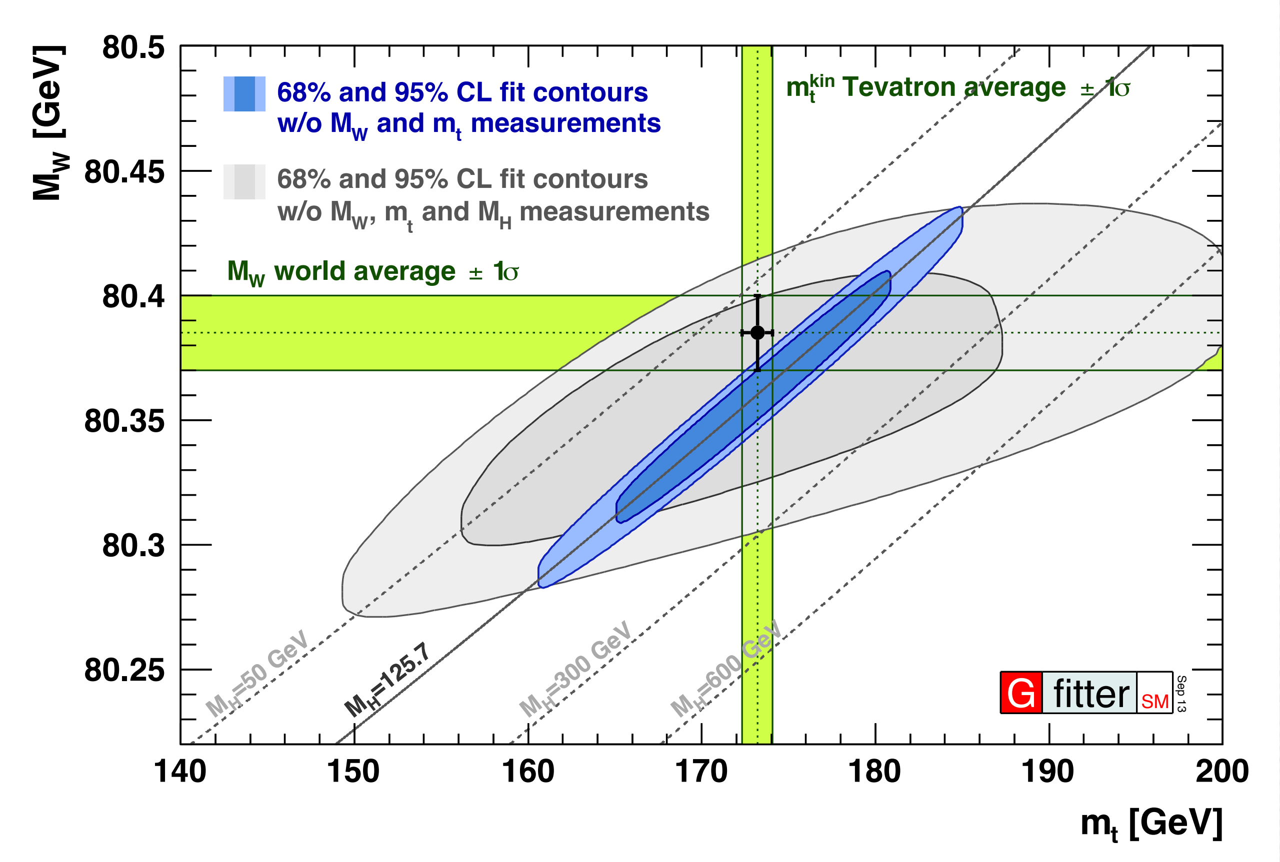
$$\Delta r_{top} = -\frac{3}{8\pi^2}\frac{G_F}{\sqrt{2}\tan^2\theta_w}m_t^2$$

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F\sin^2\theta_w}\frac{1}{1-\Delta r(m_t, m_H)}$$

$$\Delta r_{Higgs} = \frac{3}{8\pi^2}\frac{G_F}{\sqrt{2}\tan^2\theta_w}m_W^2\left(2\ln(m_H/m_Z) - 5/6\right)$$

# Top predicted through its loop presence



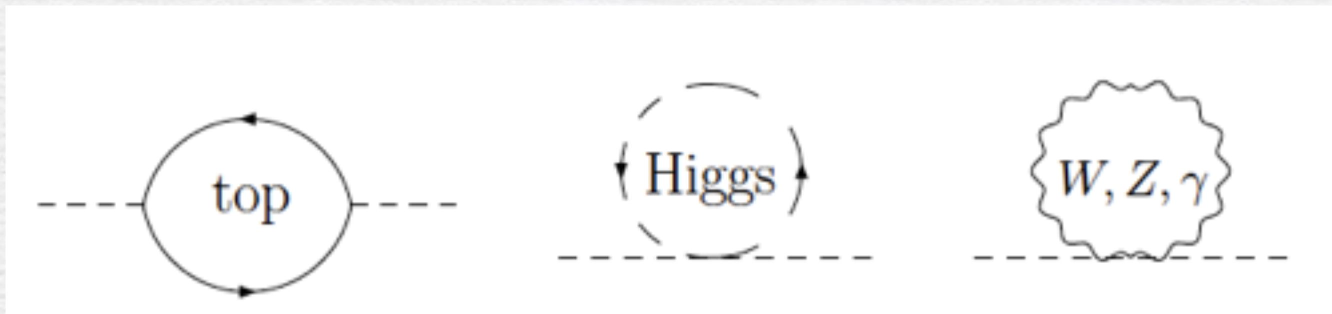




Impressive consistency between Top, Higgs, W mass

# Top loops cause trouble: "naturalness"

- Top is a trouble maker for the Standard Model, if one values **natural** values of parameters.
  - 't Hooft: parameter is naturally small if, when it is zero, a new symmetry emerges
    - electron mass = 0: chiral symmetry
    - but set Higgs mass = 0, no extra symmetry
- Such symmetries "protect" the parameters:
  - corrections to the electron mass are multiplicative, and small
- But the Higgs mass is unprotected, so corrections can be very large
  - **Top is the worst culprit**

# Top and naturalness




$$\delta m_H^2 = -\frac{3}{8\pi^2} y_t^2 \Lambda^2 \text{ [top]} + \frac{1}{16\pi^2} g^2 \Lambda^2 \text{ [gauge]} + \frac{1}{16\pi^2} \lambda^2 \Lambda^2 \text{ [Higgs]}$$

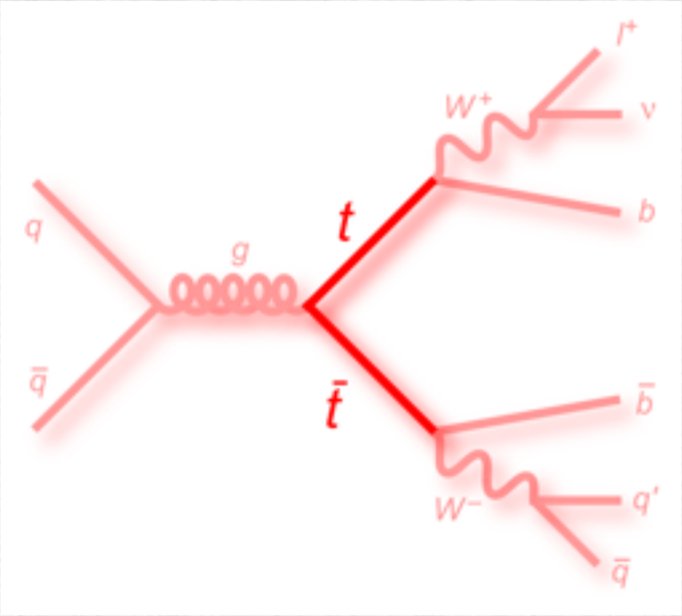
- E.g. for 10 TeV cutoff $\Lambda$ (where New Physics could kick in)

$$m_H^2 = m_{\text{tree}}^2 - [100 - 10 - 5](200 \text{ GeV})^2$$

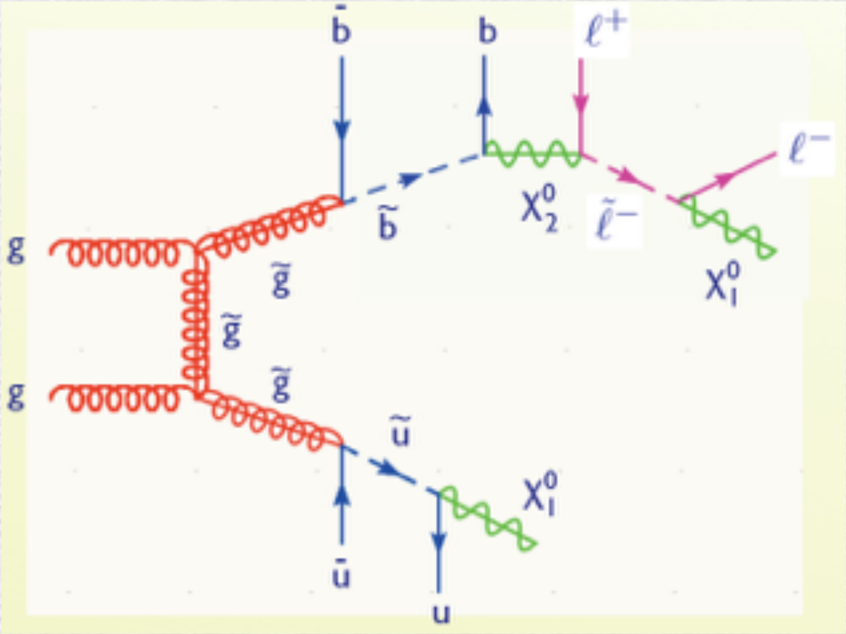
- $m_{\text{tree}}$ should then precisely compensate. This is "fine-tuning", and awkward
- New Physics could "fix" this.
  - That would pay the debt…

# Why is top special? 3. Practice for new methods

- Methods:
  - Top was the first particle whose discovery and study has been due to Monte Carlo simulation programs
    - VECBOS in 1994 - ... - ALPGEN now, many others
  - Learn how to deal with complex final states, with significant missing energy, and taggable particles




Top




Susy

# Top as background

- Top is also a background, e.g. to
  - New Physics
    - $gg \rightarrow H$, $qq \rightarrow Hqq$ ($H \rightarrow WW$), supersymmetry
    - ttj and ttjj for ttH
  - Itself
    - tt is background to single top

# Top in a nut-shell

- Top ubiquitous in high-scale particle physics, central in the duels about the status of the Standard Model
- Top should be extra-sensitive to effects of New Physics, real or virtual
- Top has large mass, short life, gives easy access
- We should, and will
  - scrutinize Top's behaviour very very carefully
  - understand its production and decay
  - and its properties


WEIGHT GAIN FORMULA
mass
GOLD
GIÚP TĂNG CÂN

# The last of the mass problems?

I. Newton (1687)

- We thought we had solved it in the 17th century
  - (i) resistance force and (ii) gravitational coupling

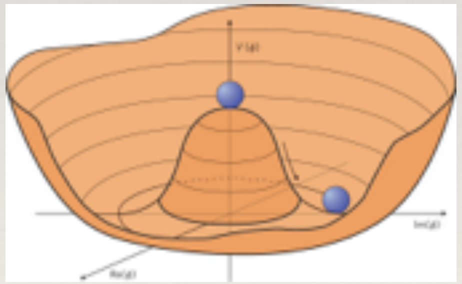
**Gravity holds universe together**

- New insight in 1905: condensed energy
  - Non-trivial for proton

A. Einstein (1905)

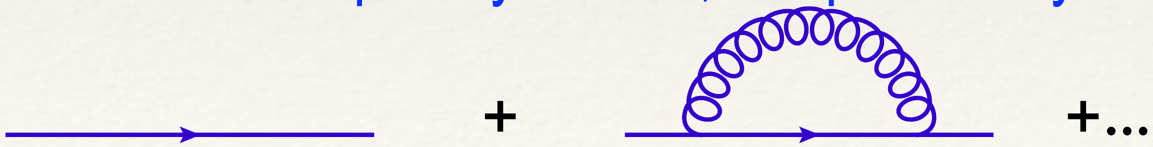
K. Wilson; Durr et al (2008)

- Yet newer insight: coupling to condensate



R, Brout, F. Englert, P. Higgs, Kibble, Hagen, Guralnik (1964 -2012)

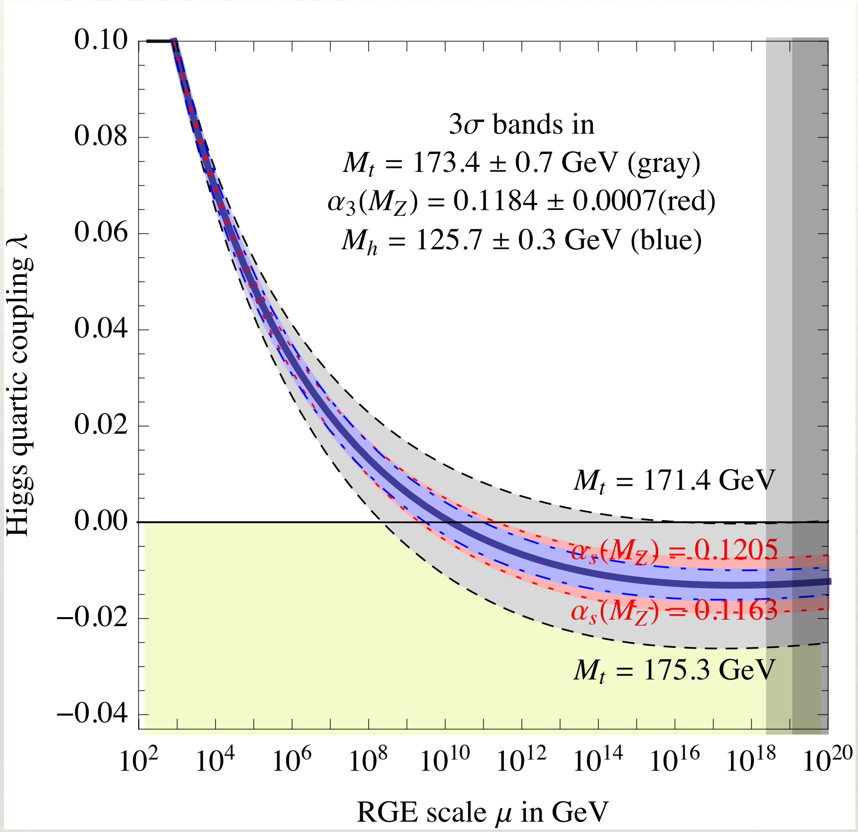
- Finally
  - Mass of confined particle? Conceptually solved, but practically subtle



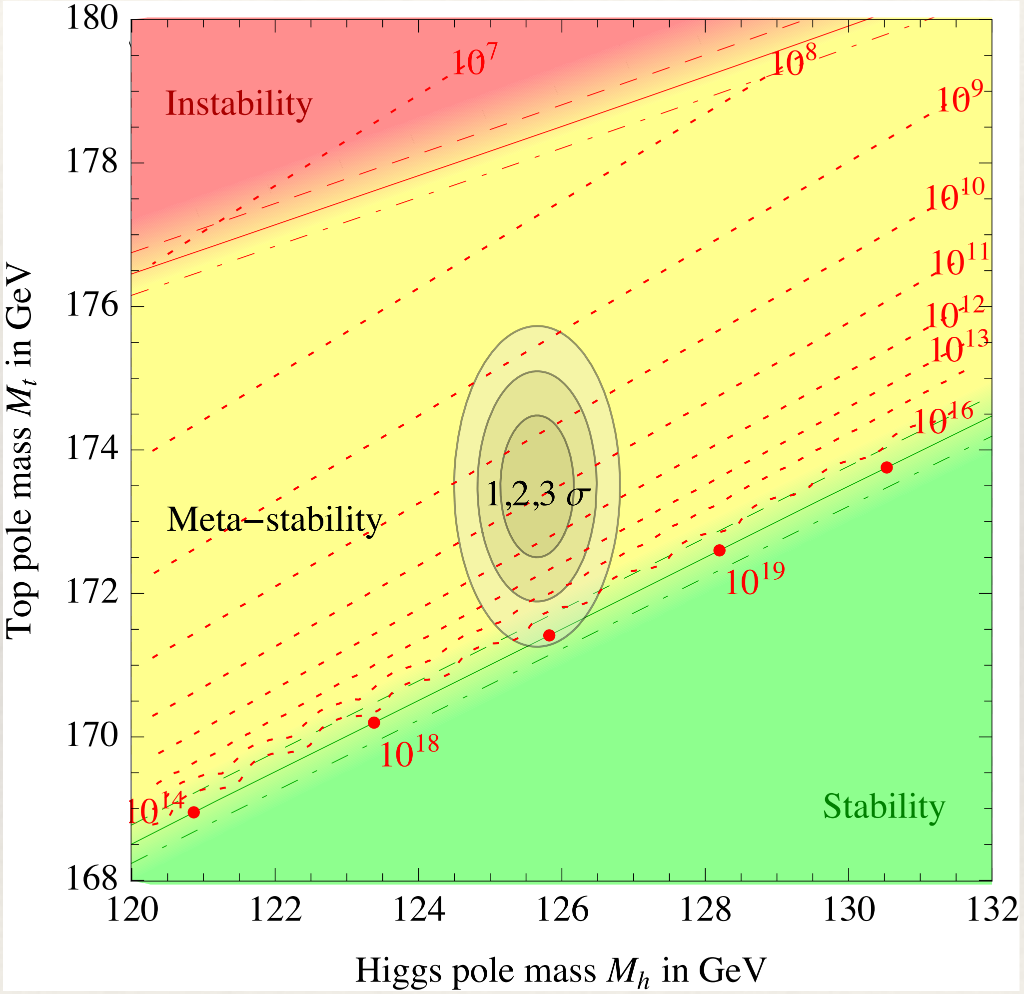

**Does top make the universe fall apart?**

# State of the Vacuum

- Top quark dominant in loop corrections that make the Higgs 4-pt coupling evolve. Full two-loop analysis:



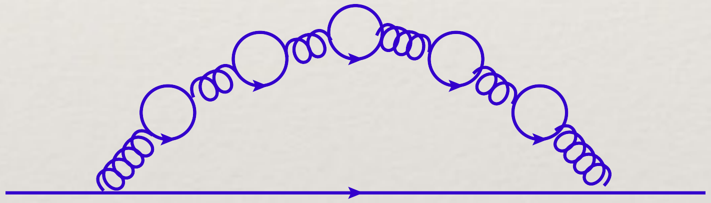

Buttazzo et al (July 2013)



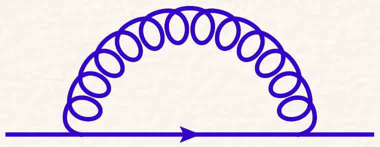

- Depends on precise top quark mass
  - within 300 MeV or so
    - But no practical worries about universe expiring

# Fermion mass

- Electron mass definition is“easy”: defined by pole in full propagator
  - ✓ If particle momentum satisfies pole condition ($p^2=m^2$), can propagate to $\infty$
    - $\Rightarrow$ there is no real ambiguity what electron “pole” mass is
- But: quarks are confined, so physical on-shell quarks cannot exist
  - ✓ Leads to non-perturbative ambiguity of few hundred MeV
    - (revealed by all-order pQCD!)



# Heavy quark mass, definition(s)

 +  +...

$$= \frac{1}{\not{p} - m_0 - \Sigma(p, m_0)}$$

$$m_0 \frac{\alpha_s}{\pi}\left[\frac{1}{\epsilon} + \text{finite stuff}\right]$$

To make finite, substitute $m_0 = m_R\left(1 + \frac{\alpha_s}{\pi}\left[\frac{1}{\epsilon} + z_{\text{finite}}\right]\right)$

Mass definitions differ in the choice of

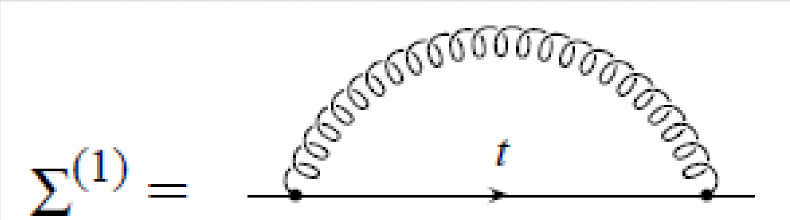
Pole mass: pretend quarks are free and long- $\frac{1}{\not{p} - m_0 - \Sigma(p, m_0)} = \frac{c}{\not{p} - M}$

MSbar mass: treat mass as a coupling $z_{\text{finite}} = 0$

# Pole mass issues

- Most natural definition for a free (stable) particle (electron, Z-boson) Kronfeld
    - gauge invariant and IR safe to all orders
- But quarks are confined, so pole mass has intrinsic uncertainty of order $\Lambda_{QCD}$
    - Full QCD has no pole at the top quark mass
        - ✓ Finite width of top does not "screen" this — Smith, Willenbrock
    - Reproduced in perturbation theory

Bigi, Shifman, Uraltsev, Vainshtein,
Beneke, Braun, Smith, Willenbrock

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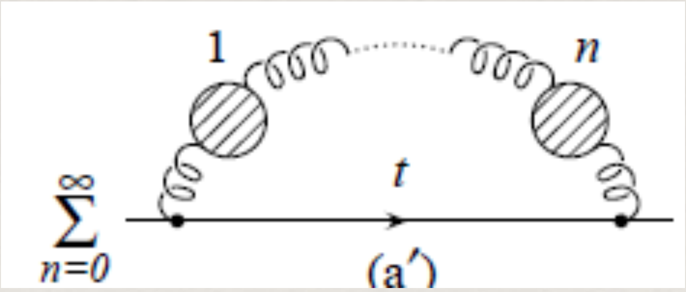



$$\Sigma(m,m) \approx \sum_n \alpha_s^{n+1} \beta_0^n n!$$

Renormalon behaviour →
order $\Lambda_{QCD}$ uncertainty

# Heavy quark mass schemes

- Various definitions other than the pole and MSbar schemes have been made
- PS (potential subtracted) mass
  - Substract from the pole mass the IR part of the ttbar Coulomb potential
    - ✓ The two parts have the same IR sensitivity

$$m^{\mathrm{PS}} = M - \tfrac{1}{2}\int_{|q|<\mu_f} \frac{d^3q}{(2\pi)^3} V(q)$$

Beneke

    - ✓ V known to 3-loop

Beneke, Kiyo, Schuller; Smirnov[2], Steinhauser; Anzai, Kiyo, Zumino

- 1S mass
  - Half the perturbative mass of (fictitious) $1^3S_1$ state

$$m^{1\mathrm{S}} = M + \tfrac{1}{2}E_1^{pt}$$

Hoang, Teubner

# Some m$_{pole}$ observations

- Perturbative ("asymptotic") expansion of pole mass

$$m_{\rm pole} = m_{\overline{\rm MS}} \times (1 + 0.047 + 0.01 + 0.003 + \ldots)$$

Melnikov, van Ritbergen

  - -> uncertainty about 500 MeV (or less)
  - Uncertainty in pole mass about 300 MeV
  - resultant uncertainty in MSbar mass smaller than

$$m_{\overline{\rm MS}}(3 - {\rm loop}) - m_{\overline{\rm MS}}(2 - {\rm loop})$$

    - ✓ $\longrightarrow$ NNNNLO?

# MSbar vs pole mass at 4 loop

- Important progress: 4-loop relations between top quark masses

$$M = c_m(\mu) m(\mu)$$

Marquard, Smirnov, Smirnov, Steinhauser

$$c_m^{(4)}\Big|_{n_l=5} = 827.37 \pm 21.5 + 408.88\, l_{\overline{\rm MS}} + 86.574\, l_{\overline{\rm MS}}^2 + 22.023\, l_{\overline{\rm MS}}^3 + 3.2227\, l_{\overline{\rm MS}}^4\,, \qquad (12)$$

$$l_{\overline{\rm MS}} = \ln(\mu^2/m^2)$$

  - Use of various specialized codes (FORM, FIRE, FIESTA,..), many of the (master) loop integrals done numerically.
  - This is also sufficient, together with N3LO Coulomb potential, for 4-loop relations to PS and 1S masses

$$M = m\Big(1 + 0.4244\,\alpha_s + 0.8345\,\alpha_s^2 + 2.375\,\alpha_s^3 + (8.49 \pm 0.25)\,\alpha_s^4) + \ldots\Big)$$

- Result

$$M = 163.643 + 7.557 + 1.617 + 0.501 + 0.195 \pm 0.005\ \mathrm{GeV}$$

- Numerically: nice progression!! No sign of an impending renormalon

# Impact on MSbar mass

Marquard, Smirnov, Smirnov, Steinhauser

- Study how a different threshold mass measurement leads to MSbar mass

| input #loops | $m^{\mathrm{PS}} =$ 171.792 | $m^{\mathrm{1S}} =$ 172.227 | $m^{\mathrm{RS}} =$ 171.215 |
|---|---|---|---|
| 1 | 165.097 | 165.045 | 164.847 |
| 2 | 163.943 | 163.861 | 163.853 |
| 3 | 163.687 | 163.651 | 163.663 |
| 4 | 163.643 | 163.643 | 163.643 |
| 4 ($\times$1.03) | 163.637 | 163.637 | 163.637 |

- 3-loop still gives 200-250 MeV shifts
- 4-loop only gives further {44,8,20} MeV shifts
  - final remaining uncertainty estimate {23,7,11} MeV

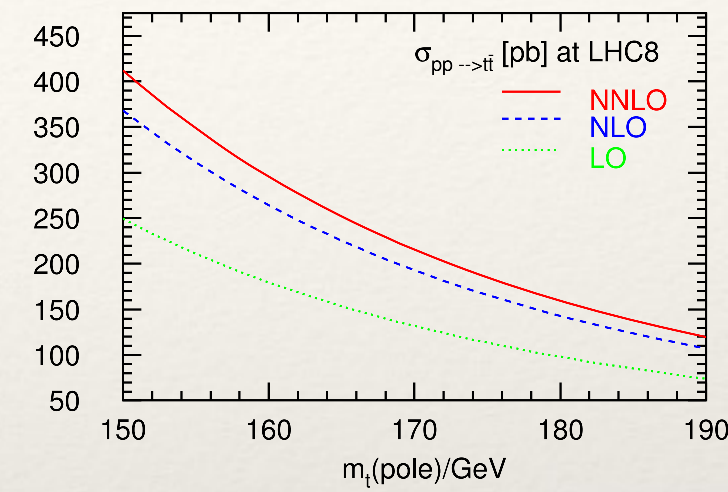
# What top mass is measured?

- Most involve MC's that are LO, so they could never tell the difference between different mass definitions.
- So what mass do hadron colliders determine?
  - Pole mass? "Pythia" mass?
    - ✓ Typically the path from data to a value for m involves a Monte Carlo, itself driven by a mass parameter.
    - ✓ Path goes via (shower) cuts, efficiencies, hadronization models etc

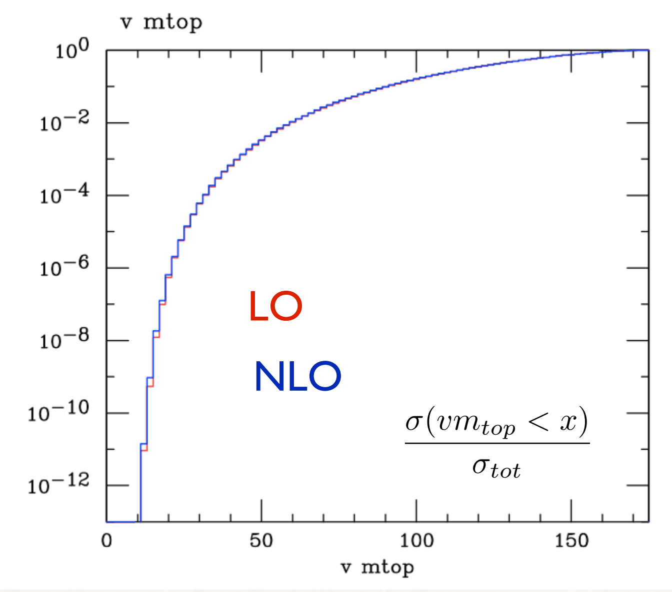
# Mass by proxy

- Of course, one does not need to reconstruct the top quark from its decays. Needs to solve implicit equation

$$\sigma^{\rm exp}(\{Q\}) = \sigma^{\rm th}(m_t, \{Q\})$$




- using an observable σ that is optimally sensitive to $m_t$.
  - Adjust $m_t$ to fit data best.
- When extracting ttbar cross section, IR sensitive region is minute fraction of total result.
  - Pole mass should be fine here; can interpret "$m_{top}$" in MC as pole mass, with small error (unlike $e^+e^-$)




[Mangano at TOP2013]

# Proxy mass: determing the MSbar mass

- How to determine the MSbar mass?
  - Problem: on-shell condition of final state top always gives pole mass

$$\mathrm{Im}\left[\frac{1}{p^2 - M^2 + i\varepsilon}\right] = \pi\,\delta(p^2 - M^2)$$

- Indirectly
  - compute cross section using pole mass
  - replace pole mass by MSbar mass
  - Now fit to data, extract MSbar mass

$$\sigma_{tt}(M, \alpha_s)$$

$$m = \overline{m}(\mu)(1 + \alpha_s(\mu)d^1 + \alpha_s(\mu)^2 d^2 + \ldots)$$

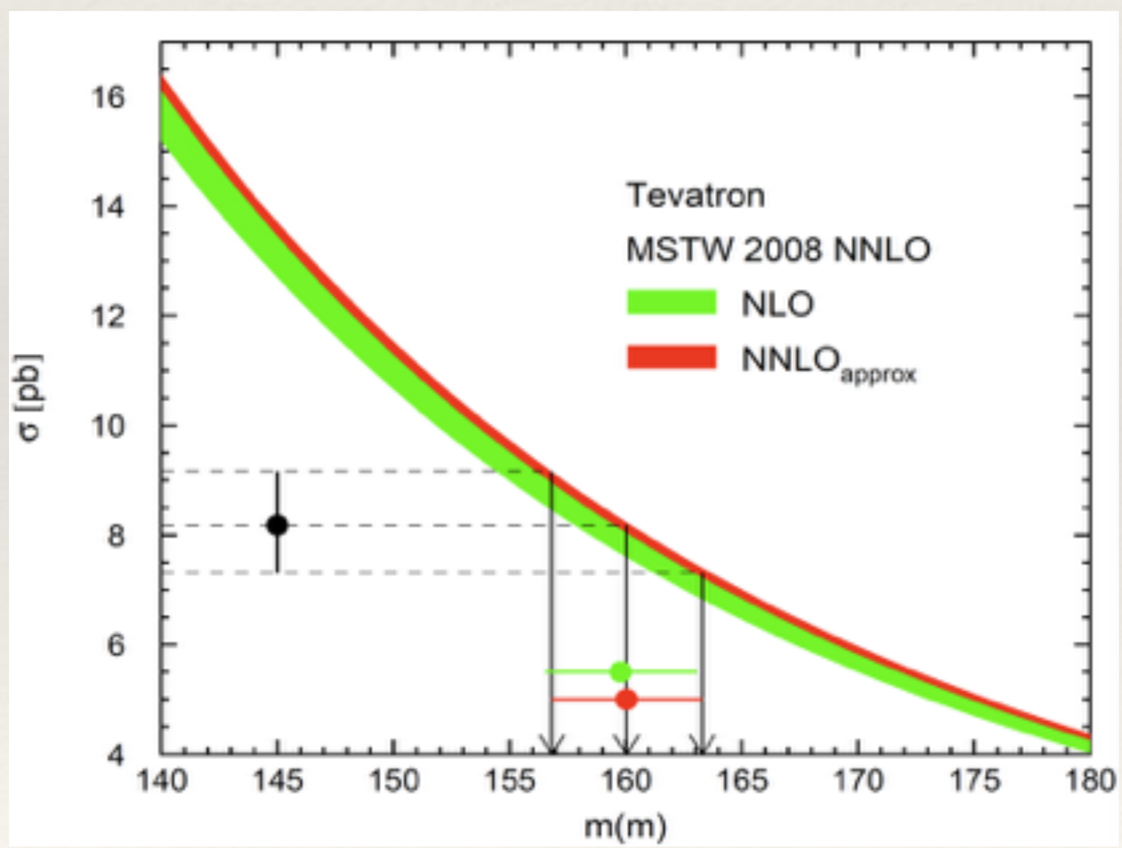
Langenfeld, Moch, Uwer

# MSbar mass extraction

Langenfeld, Moch, Uwer

- Accuracy limited by mt sensitivity and PDF uncertainties
- Other proposals for mass-sensitive observables:
  - (moments of) the invariant mass distribution   Frederix, Maltoni
  - tt+1 jet rate   Alioli, Fernandez, Fuster, Irles, Moch, Uwer

$8.18^{+0.98}_{-0.87}$ pb




| | $\overline{m}$ [GeV] | $m_t$ [GeV] |
|---|---|---|
| LO | $159.2^{+3.5}_{-3.4}$ | $159.2^{+3.5}_{-3.4}$ |
| NLO | $159.8^{+3.3}_{-3.3}$ | $165.8^{+3.5}_{-3.5}$ |
| NNLO | $160.0^{+3.3}_{-3.2}$ | $168.2^{+3.6}_{-3.5}$ |

# Some other LHC mass proxies

Frixione, Mitov

- In dilepton channel, can use shapes of various observables sensitive to top mass
  - study with NLO+PS+MadSpin
  - single-inclusive or mildly correlated (1,4,5) stable under above effects
    - ✓ 2,3 not -> be careful with using NNLO with stable tops
  - about 0.8 GeV theory error in studied scenario, with aMC@NLO

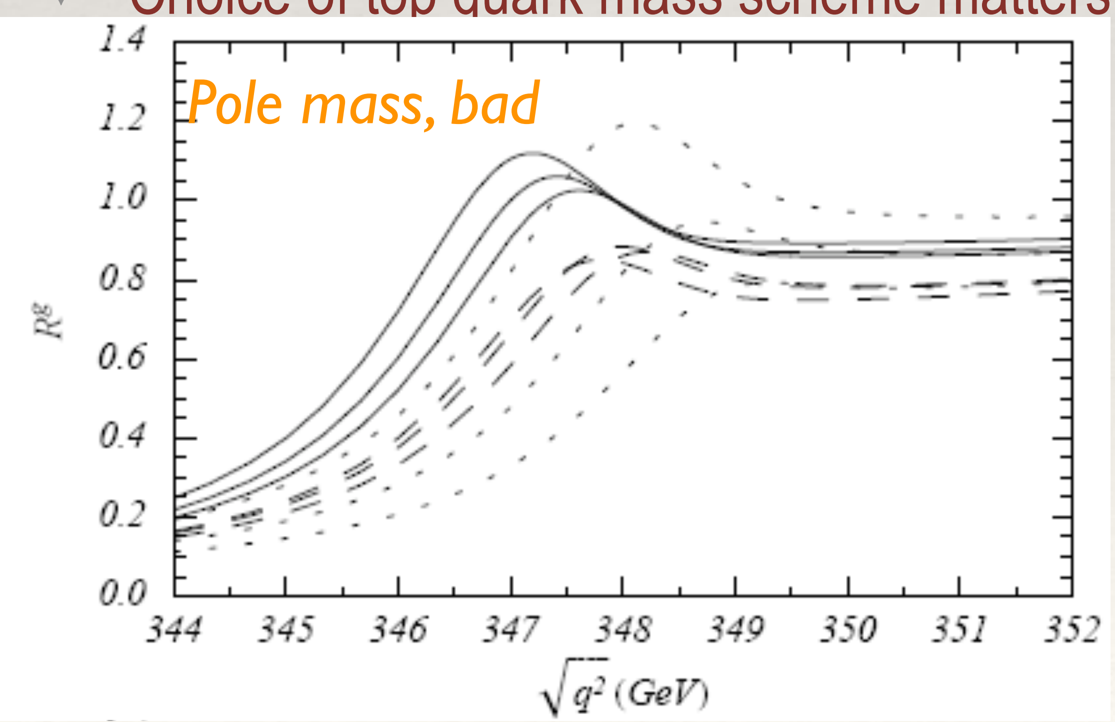
| Label | Observable |
|---|---|
| 1 | $p_T(\ell^+)$ |
| 2 | $p_T(\ell^+\ell^-)$ |
| 3 | $M(\ell^+\ell^-)$ |
| 4 | $E(\ell^+) + E(\ell^-)$ |
| 5 | $p_T(\ell^+) + p_T(\ell^-)$ |

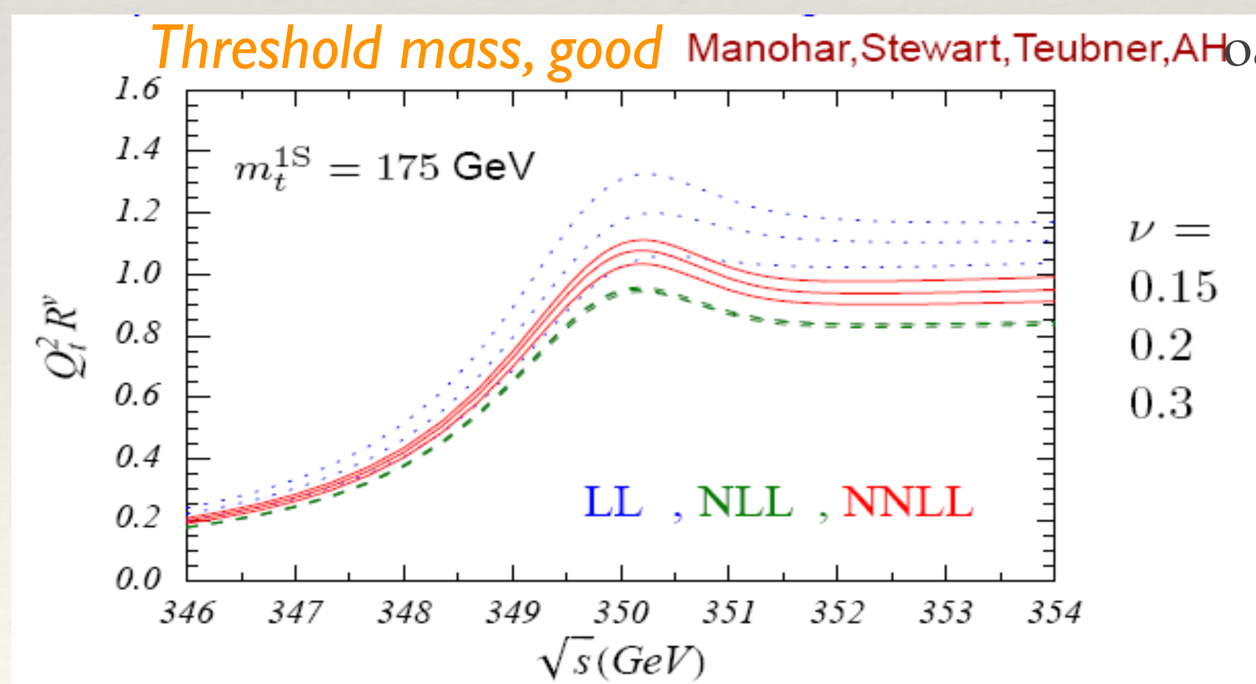
# Top threshold mass

- Scan the ttbar threshold at linear collider by varying beam energy. The opening of the top channel leads to “smooth” theta-function
- Distribution can be measured very precisely. with calculation using Schrodinger equation and appropiate short-distance mass
- Also sensitive to top quark width, allows good measurement
- Calculation non-relativistic effective field theory. Two small parameters: $\alpha_s$ and v.

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times \left\{1\,(\mathrm{LL});\, \alpha_s, v\,(\mathrm{NLL});\, \alpha_s^2, \alpha_s v, v^2\,(\mathrm{NNLL})\right\}$$

- Choice of top quark mass scheme matters..



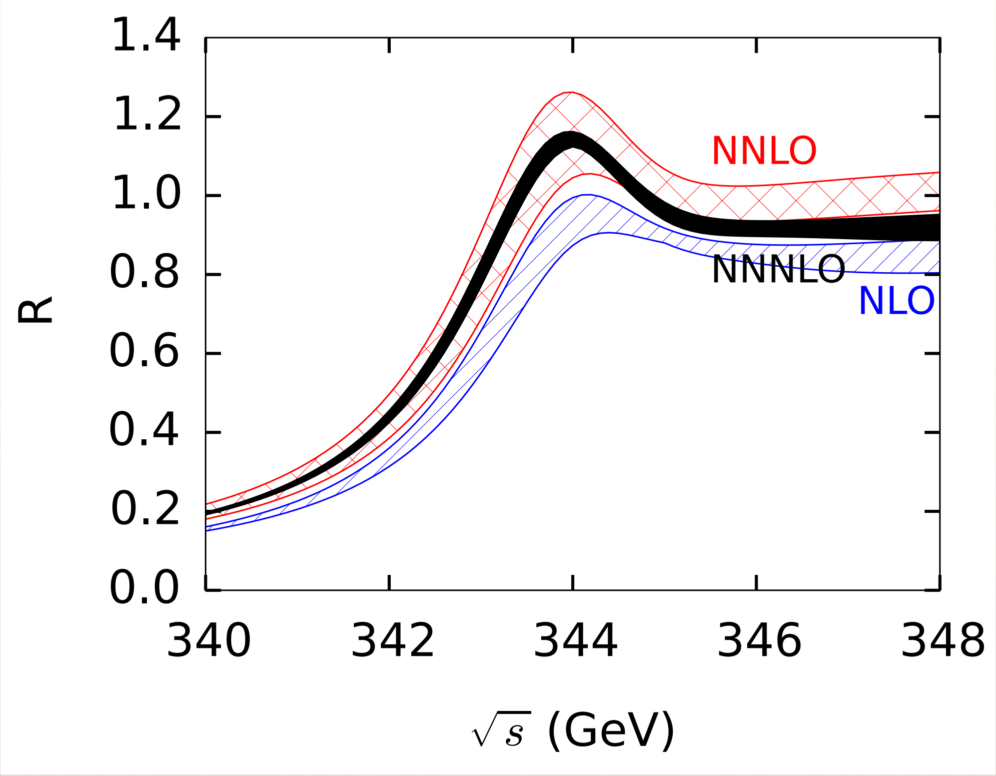




Manohar,Stewart,Teubner,AHoang

# N3LO for ttbar S-wave threshold production at e+e- collider

- Now finally the full N3LO cross section, including the last non-logarithmic terms, is known
  - Heroic effort, and it was worth it! QCD calculation under control

Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser

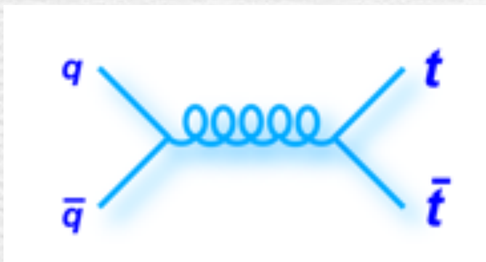


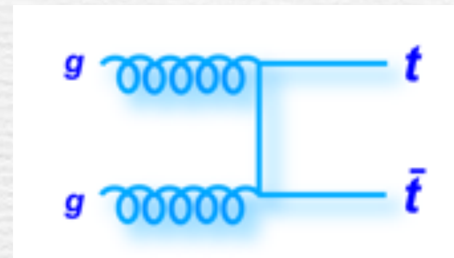

  - Dramatic scale reduction N2LO → N3LO.
  - QCD uncertainty on top quark mass can go below 50 MeV.
    - ✓ But are also non-QCD effects to study: EW, Higgs, Beamstrahlung, non-resonant terms..

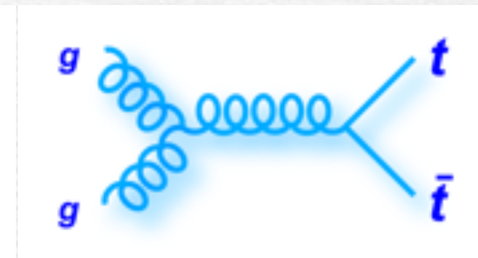
# Producing tops

# Producing top in hadron colliders

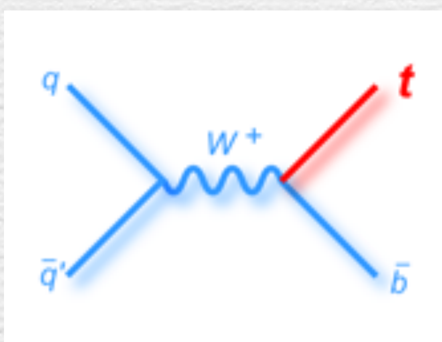
- Tops can be produced via strong interaction, in pairs. These are the LO diagrams

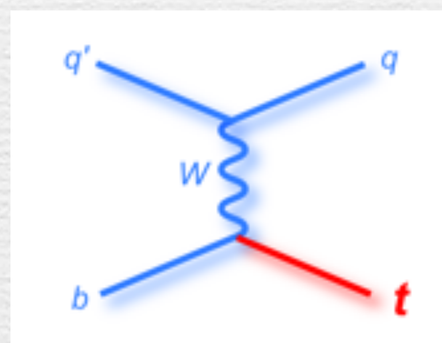


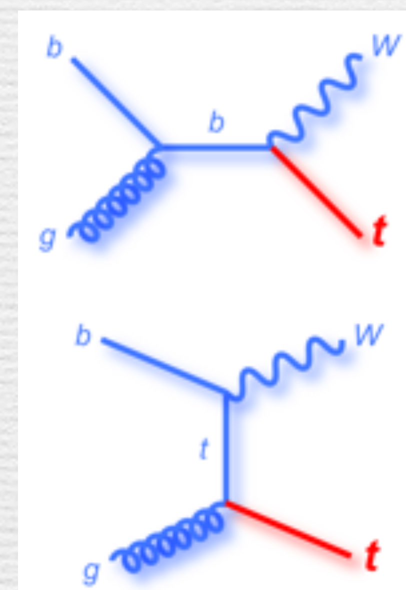




- Top can be also produced singly, via the weak interaction.



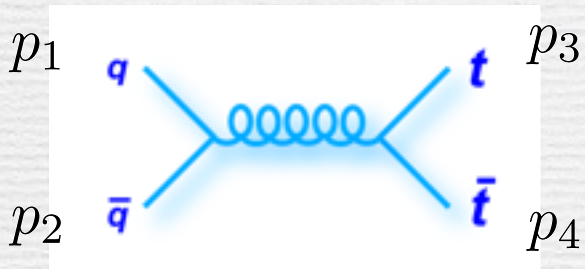



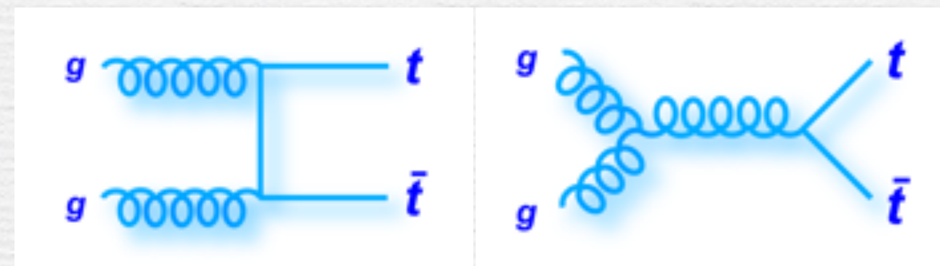

    - All these modes have now been seen

# Top production: Tevatron and LHC

- Tevatron: top foundry
  - about 70K top pairs produced, discovery (1995), first tests of properties
- LHC: top factory
  - So far about 6M top pairs produced
  - Next phase (> 2015) about 90M/year
- Theory
  - Calculated to NNLO accuracy, plus logarithms of all orders (resummation)
  - NLO for many differential distributions, and some already at NNLO

# Pair production cross section at LO




- LO squared, spin and color -summed and averaged matrix elements

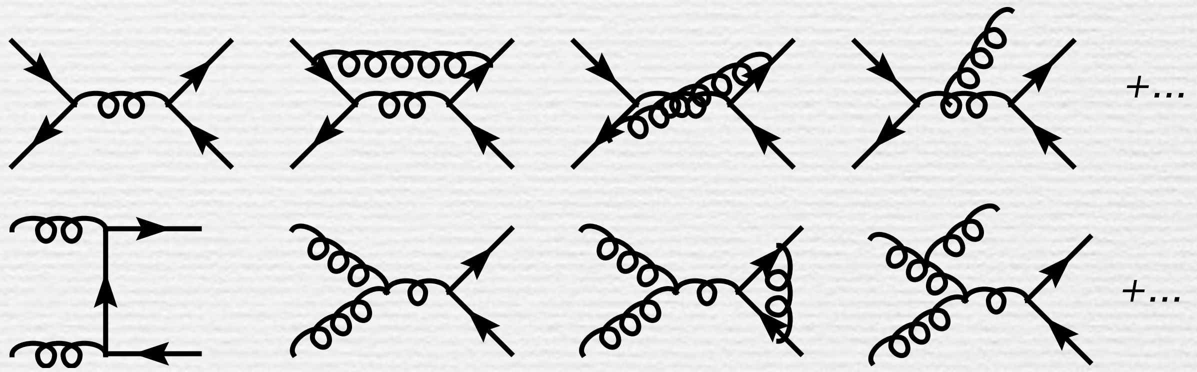
$$\langle |M_{qq}|^2 \rangle = \frac{4g_s^4}{9}\left[\frac{t_1^2}{s^2} + \frac{u_1^2}{s^2} + \frac{2m^2}{s}\right], \quad t_1 = -2p_1 \cdot p_3, \; u_1 = -2p_2 \cdot p_3$$

$$\langle |M_{gg}|^2 \rangle = g_s^4\left[\left(\frac{s^2}{6t_1 u_1} - \frac{3}{8}\right)\left(\frac{t_1^2}{s^2} + \frac{u_1^2}{s^2} + \frac{4m^2}{s} - \frac{4m^4}{t_1 u_1}\right)\right]$$

- To be combined with phase space measure (and flux factor) for partonic cross section
  - Notice: expressions are symmetric under $t_1 \leftrightarrow u_1$ interchange. This amounts to top $\leftrightarrow$ anti-top interchange at fixed kinematics

# Pair production cross section at NLO

Some of the diagrams involved at NLO




Beenakker, Kuijf, Smith, van Neerven, Meng, Schuler; Nason, Dawson, Ellis; Mangano, Nason, Ridolfi

- NLO since late 80's
  - single particle inclusive and fully differential. Codes (MNR) still available

# Beyond NLO

Moch, Vermaseren, Vogt; Mitov, Sterman, Sung
Ahrens Ferroglia, Neubert, Pecjak, Yang

- All-order resummation for cross sections

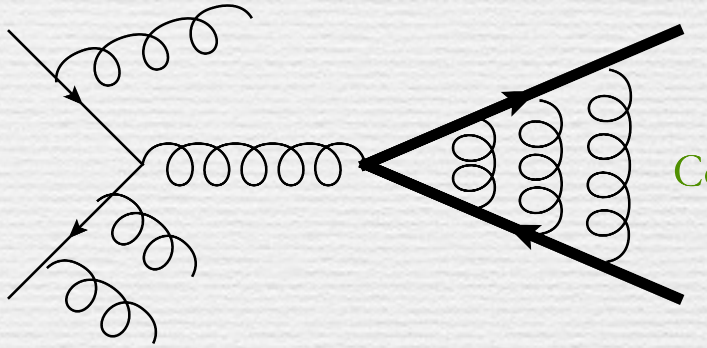
$$\sigma^{resum} = \Big\{ \underbrace{\alpha_s^2 C_0}_{\text{LL,NLL}} + \underbrace{\alpha_s^3 C_1}_{\text{NNLL}} \Big\} \times$$
$$\exp\Big[ \underbrace{L g_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \ldots \Big]$$

- There are two ways to use such a formula
  - all order predictions
    - Benefit: all-order, systematic, smaller scale uncertainty, but some ambiguities
  - after expanding resummed to second order, get $\text{NNLO}_{\text{approx}}$
    - Instructive, already less scale uncertainty than NLO, no all-order ambiguities

# Resummed cross sections

$$\sigma^{resum} = \Big\{ \underbrace{\alpha_s^2 C_0}_{\text{LL,NLL}} + \underbrace{\alpha_s^3 C_1}_{\text{NNLL}} \Big\} \times$$

$$\exp\Big[ \underbrace{L g_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \ldots \Big]$$




- Present status is NNLL
- L = ln(threshold condition)
- Caveat: different thresholds are used
  - e.g.

1. $\sum_n \alpha_s^n \ln^{2n}(s - 4m^2)$ $[\sigma(s)]$
2. $\sum_n \alpha_s^n \ln^{2n}(s - 4(m^2 + p_T^2))$ $[d\sigma(s)/dp_T]$
3. $\sum_n \alpha_s^n \ln^{2n}(s - 4(m^2 + p_T^2)\cosh y)$ $[d^2\sigma(s)/dp_T dy]$

# NNLO top cross section

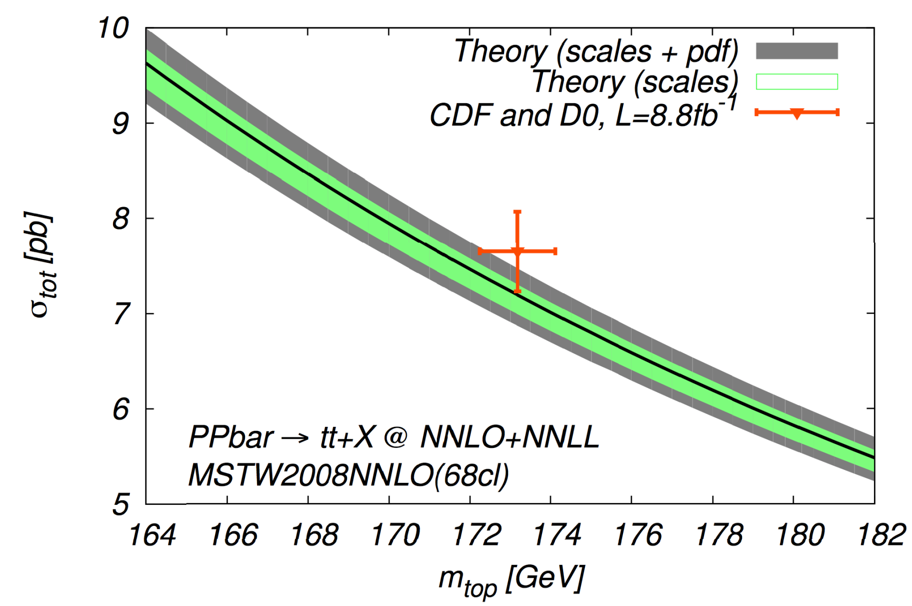
Baernreuther, Fiedler, Mitov, Czakon

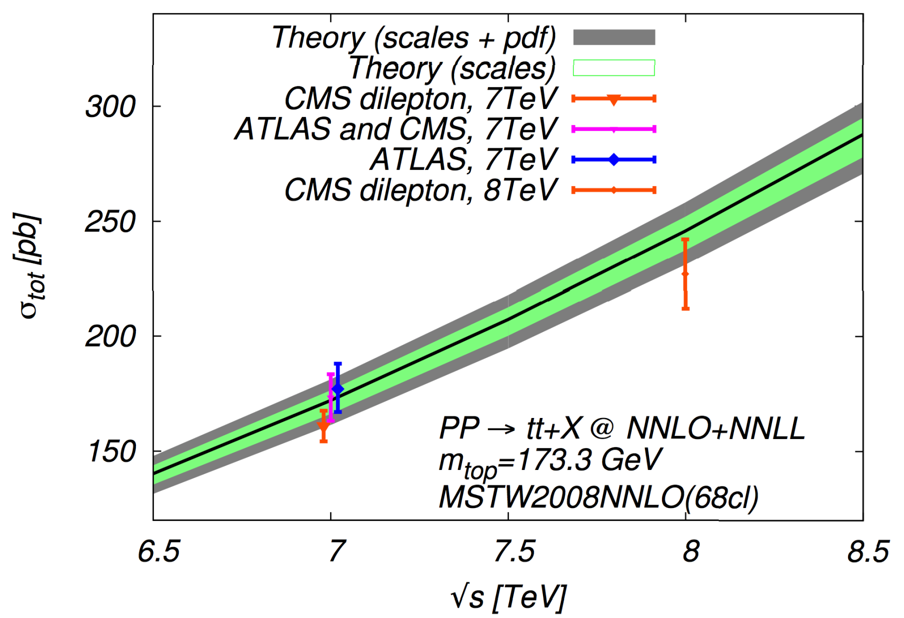
- **NNLO calculation with initial hadrons and full color structure completed**
    - One for the (QCD) history books
- Tools:
    - Highly involved computation and management of Feynman diagrams, Mellin-Barnes methods etc.
- At TOP2013 conference excitement of experimenters > theorists

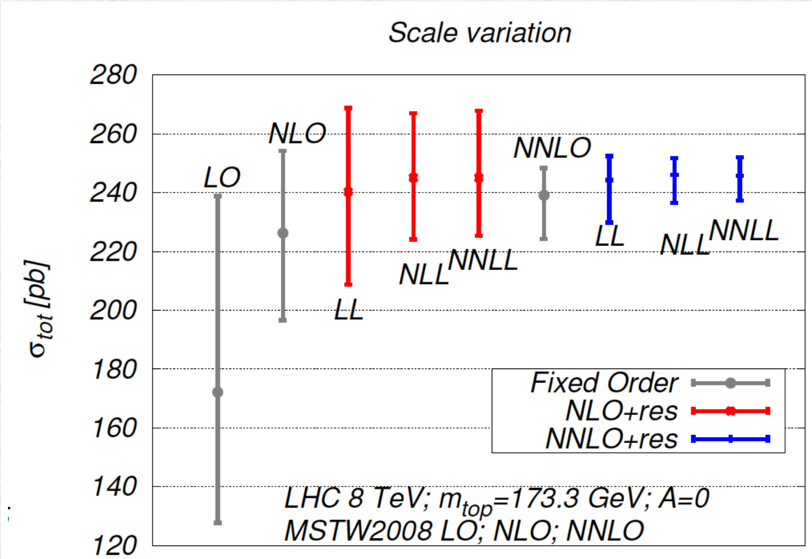
# NNLO top cross section

Baernreuther, Fiedler, Mitov, Czakon

- **Pay-off excellent [fame, fortune, convergence, agreement with data]**



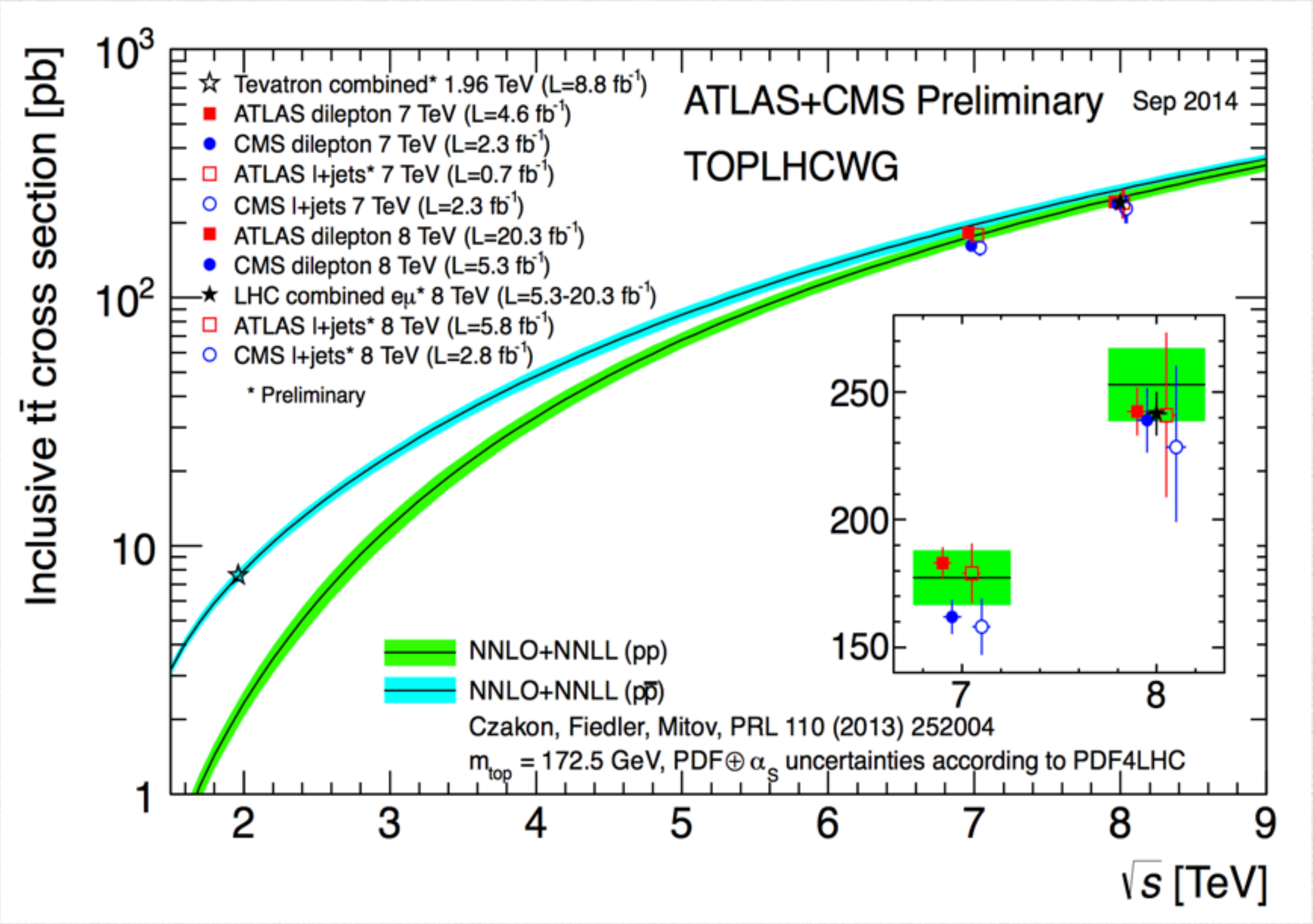

Concurrent uncertainties:

| | |
|---|---|
| Scales | ~ 3% |
| pdf (at 68%cl) | ~ 2-3% |
| $\alpha_s$ (parametric) | ~ 1.5% |
| $m_{top}$ (parametric) | ~ 3% |

Soft gluon resummation makes a differenc

5% -> 3%

# Beautiful agreement at LHC

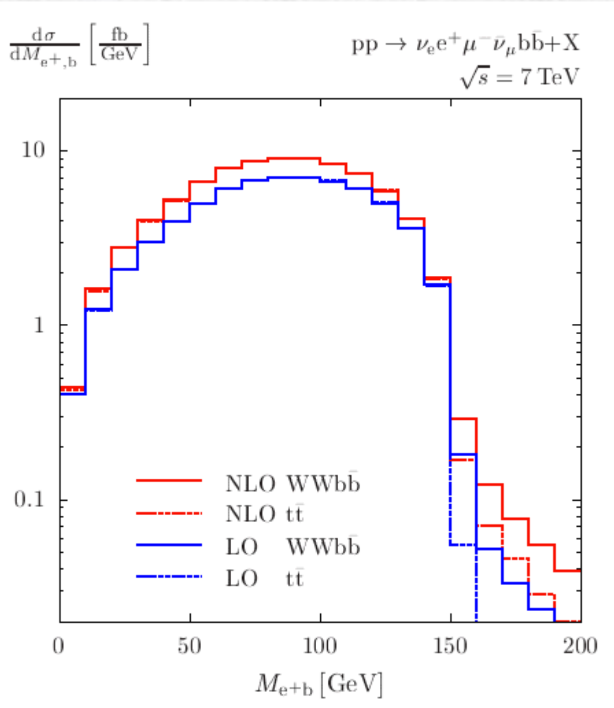


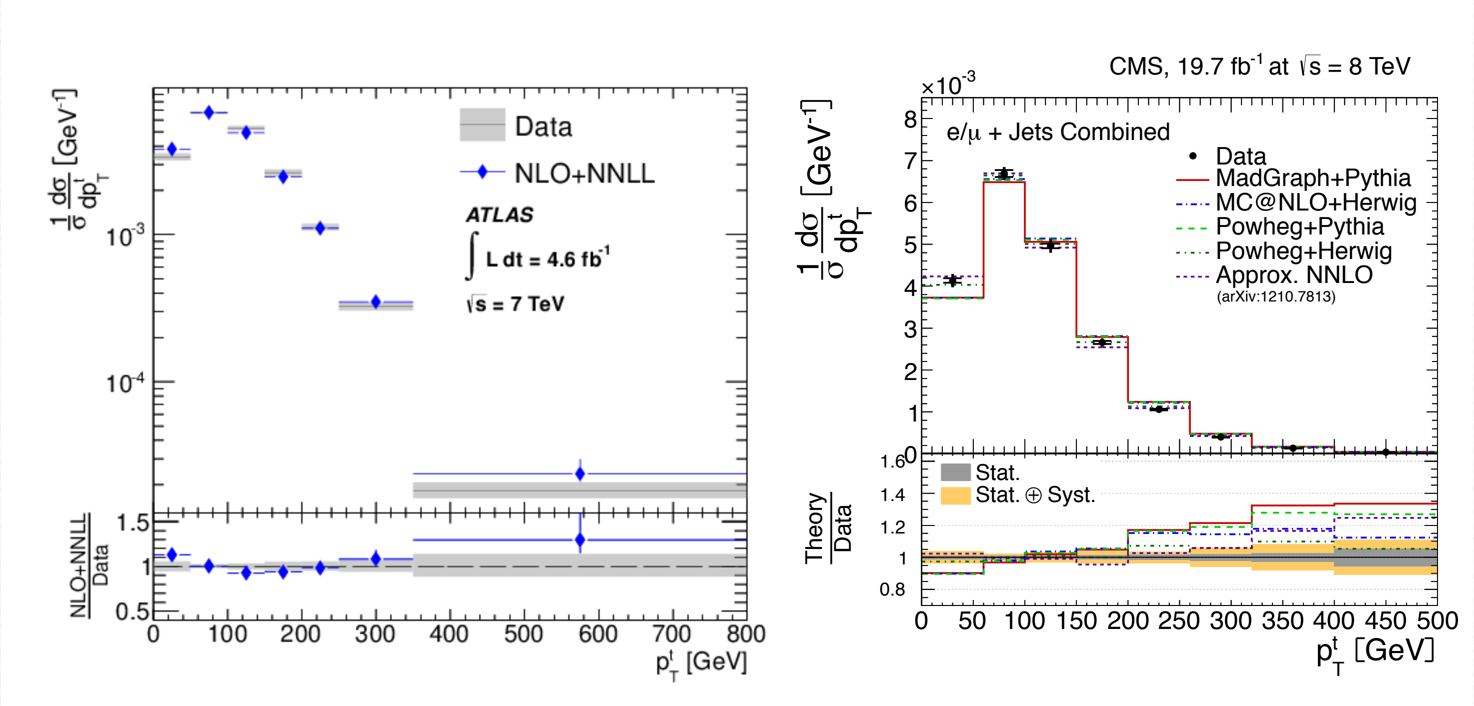

Theory error> exp. error

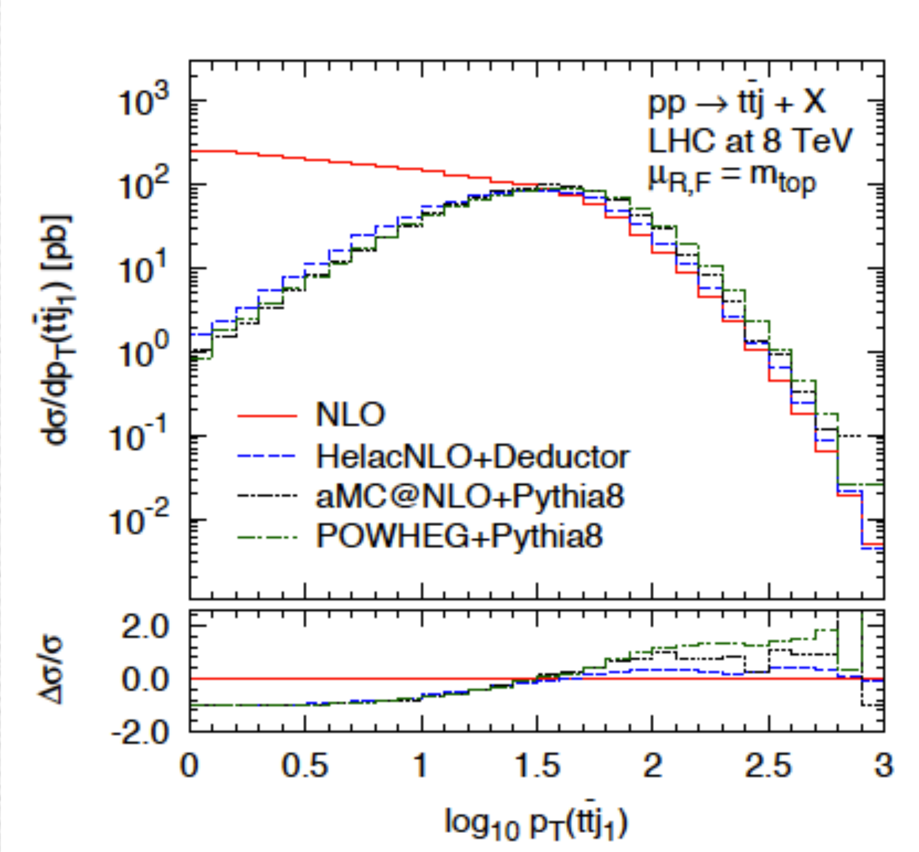
- More naturalness:
  - $\sigma_{tt}$(7 TeV) / $m_t$ = 172.3/173.2 = 0.99
  - $\sigma_{tt}$(8 TeV) / $\sqrt{2}$ x $m_t$ = 238/245 = 0.97

# Differential distributions

- With more data, more relevant
- Theory calculations very good now
  - NNLO! Czakon, Heymes, Mitov
- Important for testing top physics, and many QCD aspects of it

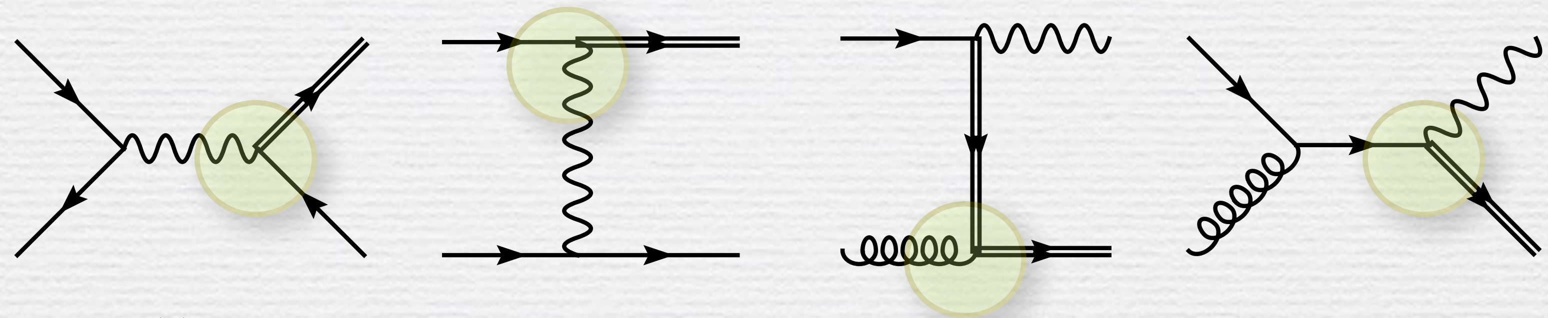


# Single top

# Single top production



(1) (2) (3)

s-channel: timelike W

4 pb @ LHC7

t-channel: spacelike W

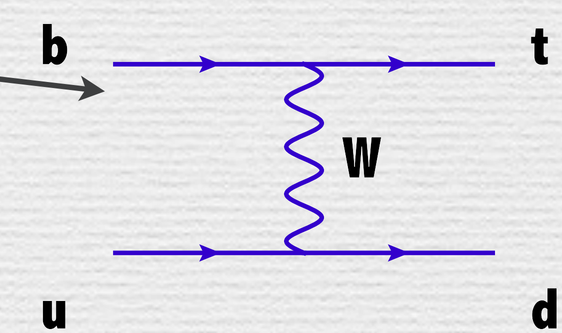
62 pb @ LHC7

Wt channel: real W

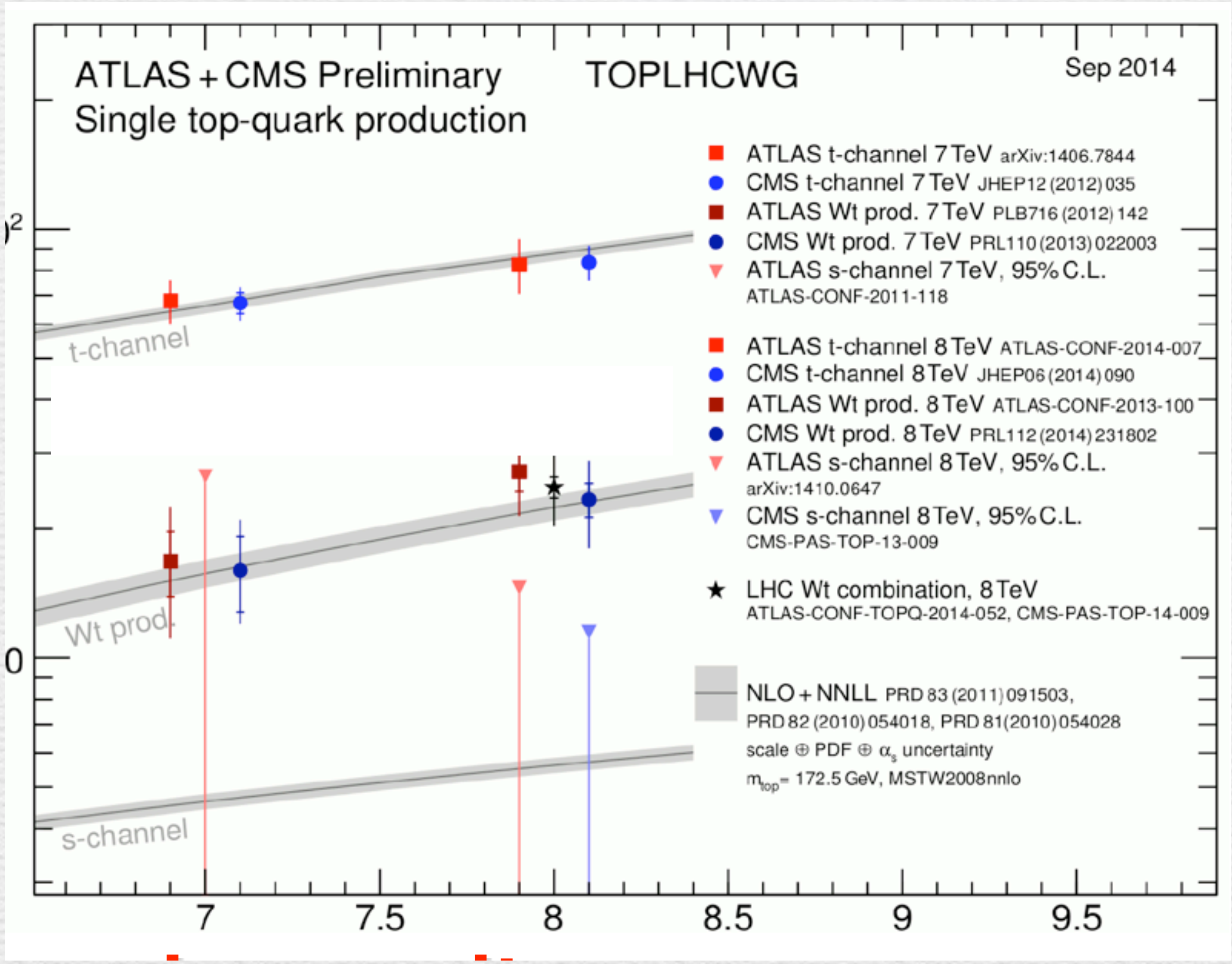
10 pb @ LHC7

## Things you can do with single top production

- process is sensitive to different New Physics/channel
- It helpt determine (t-channel) the high-scale b-quark PDF
- It tests electroweak production of top, through left-handed coupling
    - It allows measurement of mixing coefficient $V_{tb}$ per channel.

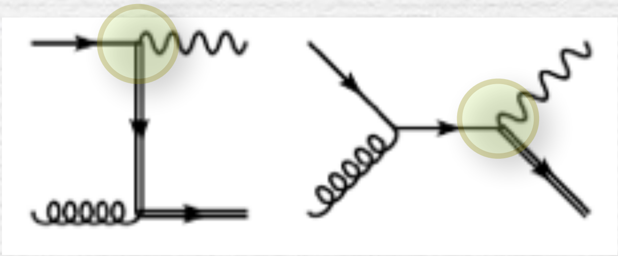


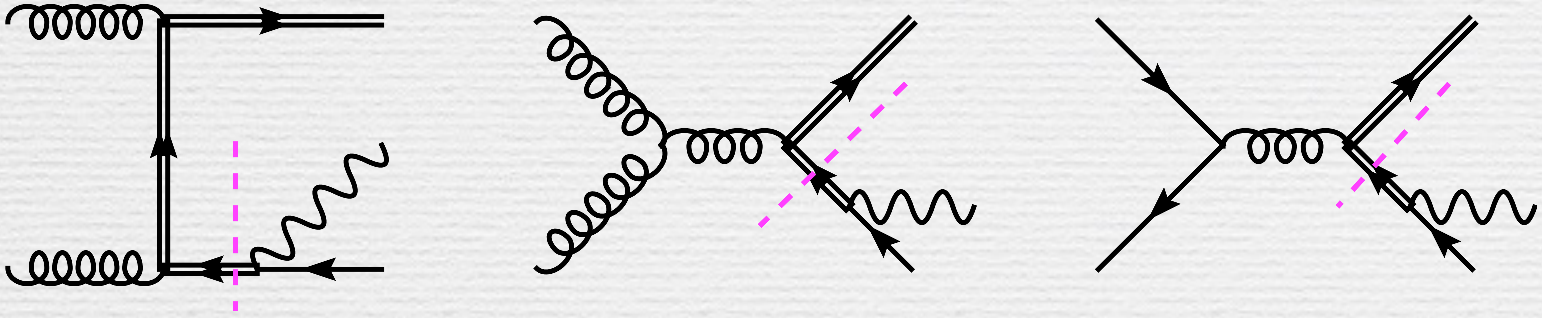
# Single top: theory vs expt.




SM works well, again..

# Single top in Wt mode meets tt..





+ non-resonant diagrams

Serious interference with top pair production (15 times bigger)

- Can one actually define this process?
  - Yes: one can separate the resonant tt background, using cuts, and testing for interference

Frixione, EL, Motylinski, Webber, White

- Much recent work on proper description of production + decay

Papanasthasiou,

Cascioli, Kallweit, Maierhoefer, Pozzorini

# Testing for interference

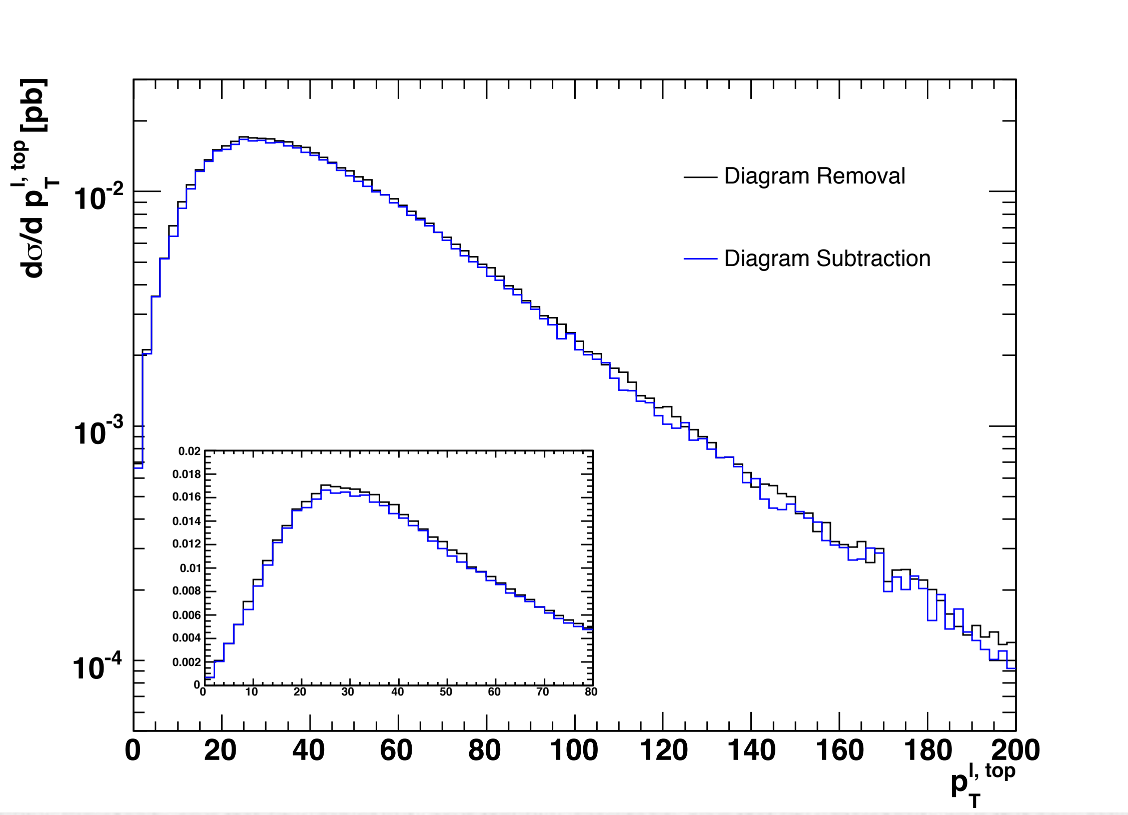
Frixione, EL, Motylinski, Webber, White

Two approaches in MC@NLO (now also in POWHEG (Re))

- I. Remove resonant diagrams (DR)
- II. Construct a gauge invariant, local counterterm: diagram subtraction (DS)
- DS - DR is measure of interference

$$d\sigma^{(2)} + \sum_{\alpha\beta} \int \frac{dx_1 dx_2}{2x_1 x_2 S} \mathcal{L}_{\alpha\beta} \left( \hat{\mathcal{S}}_{\alpha\beta} + \mathcal{I}_{\alpha\beta} + \mathcal{D}_{\alpha\beta} - \widetilde{\mathcal{D}}_{\alpha\beta} \right) d\phi_3$$

Interference effects quite small, in general, but not always



# Top and new physics

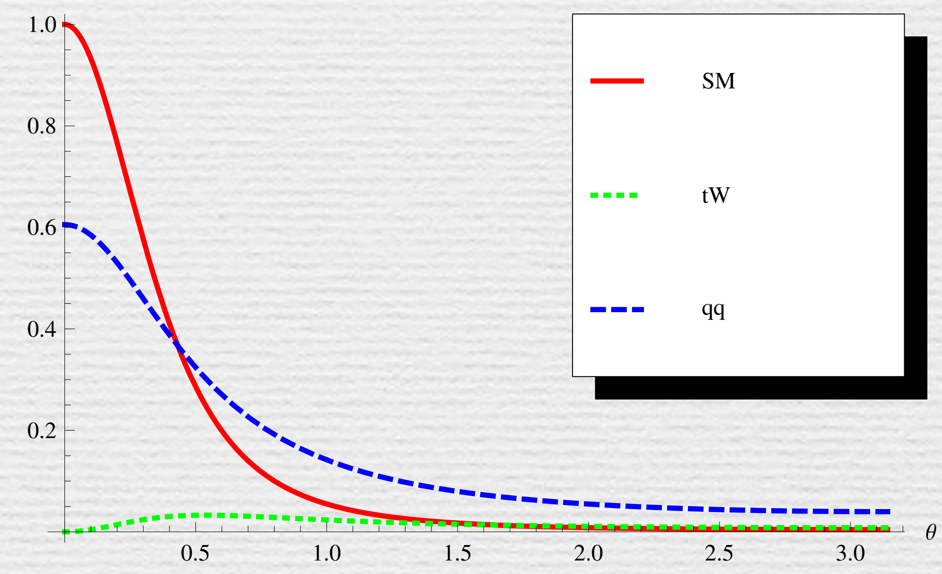
Buckley, Englert, Ferrando, Miller, Moore, Russell, White; Durieux, Maltoni, Zhang

Test for New Physics in top production with effective dimension 6 operators

$$\mathcal{L}_{\rm SM} + \sum_i \frac{C_i}{\Lambda^2} O_i^{[6]} \, .$$

- and then fit/constrain $C_i$ using NLO theory
  - E.g. in single top t-channel
  - or in global fits to many observables

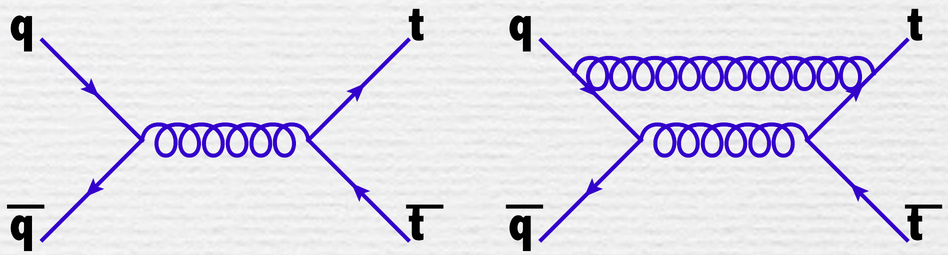
$$\frac{d\sigma(ub \to td)}{d\cos\theta}$$



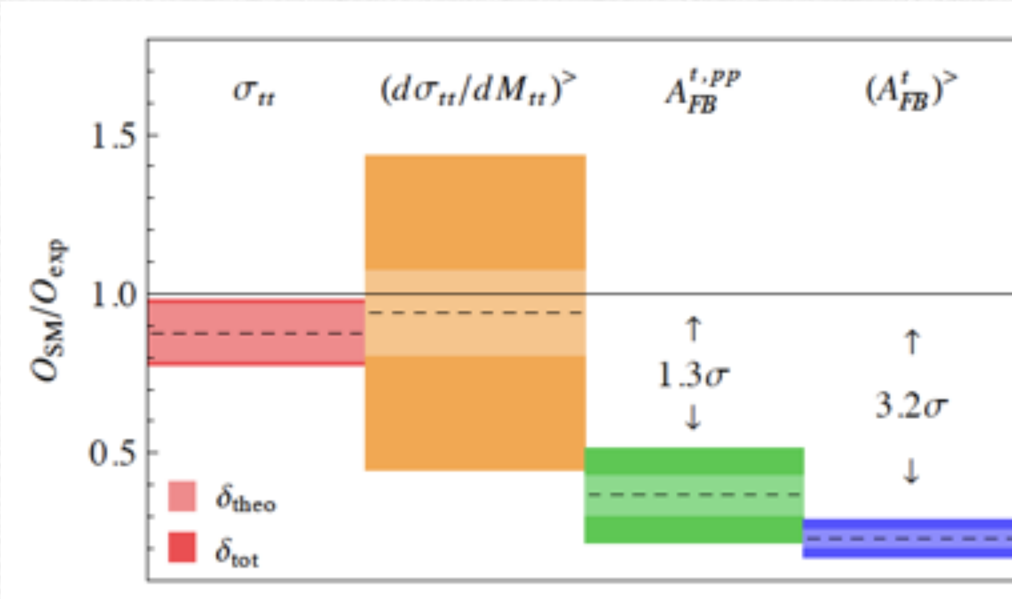
# Charge asymmetry:
# Standard Model trouble?

# Charge (aka. forward-backward) asymmetry

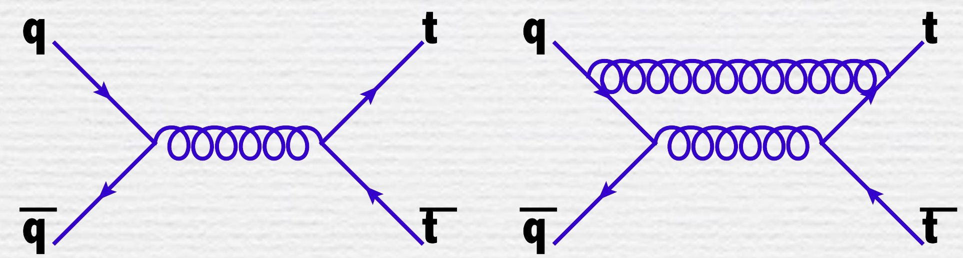
$$A_t(y) = \frac{N_t(y) - N_{\bar{t}}(y)}{N_t(y) + N_{\bar{t}}(y)}$$



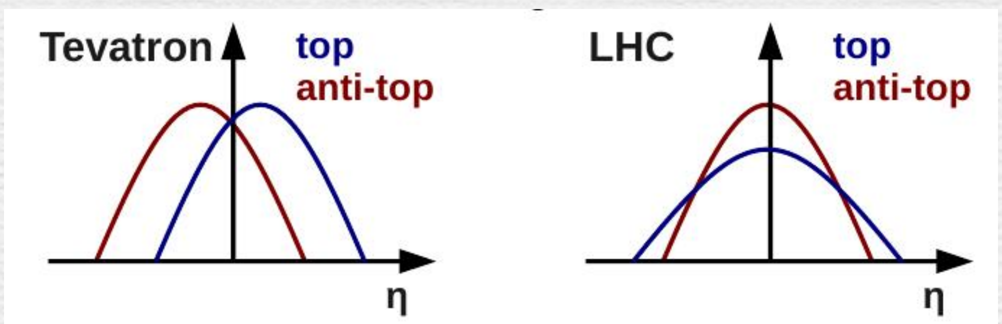

- Check if top quarks are distributed just as antitops
- A small difference expected from QCD effects,
  - in QCD, proportional to SU(3) $d_{abc}$ symbol
- Serious discrepancies in some measurements with SM at Tevatron



# Some intuition about $A_{FB}$




Quark "repels" top via second gluon, leading to "preferred" situations:




Recall: top-antitop exchange = t, u exchange.

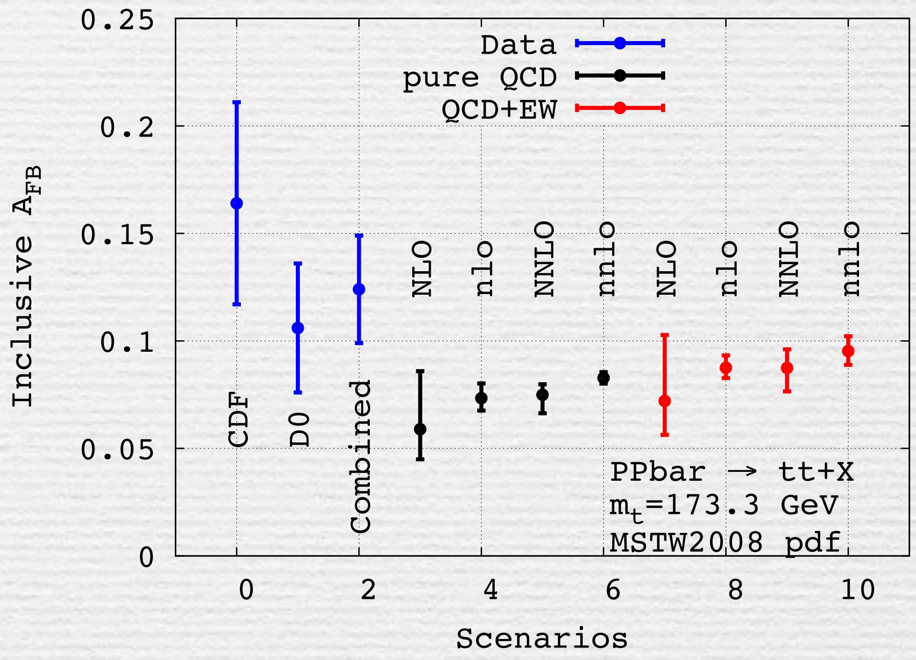
From resummation:

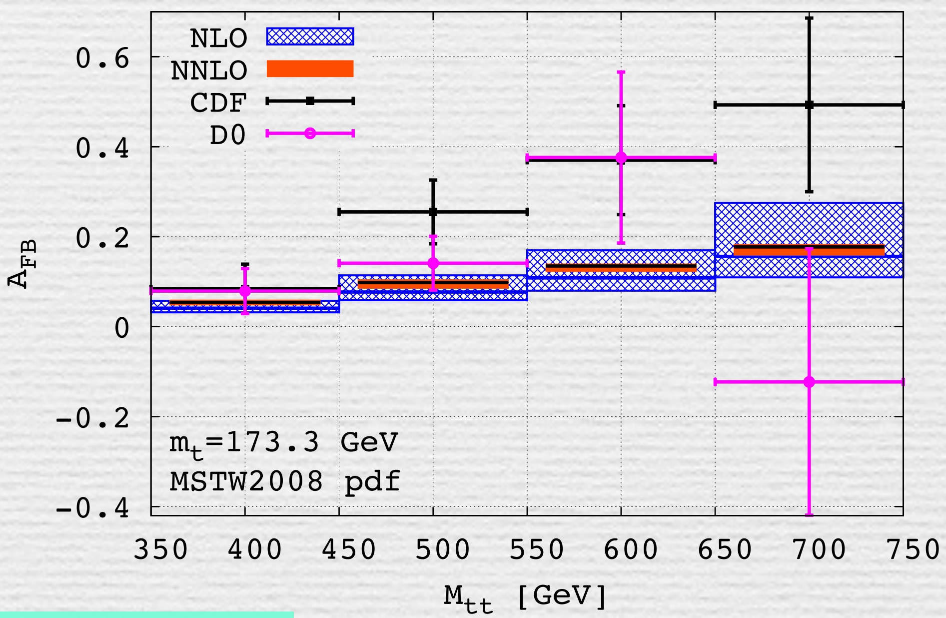
$$\frac{\Delta\sigma}{\sigma} \simeq \exp\left\{\alpha_s L \left[\frac{32}{6} - \frac{27}{6}\right] \ln\frac{u}{t}\right\}$$

# New: NNLO and $A_{FB}$

Czakon, Fiedler, Mitov
1411.3007

- First complete differential calculation in NNLO QCD for 2->2 with all colored partons
- Unexpectedly large correction: 27% w.r.t. NLO
  - Inclusive $A_{FB}$ = 0.095 ± 0.007
  - Now agreement with D0, not far from CDF
- Differential in $M_{tt}$: similar to inclusive case




Standard Model re-asserts itself..

# Top and friends

# $t\bar{t}$+W,Z,$\gamma$

- Photon

  Kardos, Trocsanyi

  - NLO + PS calculation
  - dominated by gluon fusion
  - Control sample/background for ttH, $H \to \gamma\gamma$
- Z

  Garzelli, Kardos, Papadopoulos, Trocsanyi

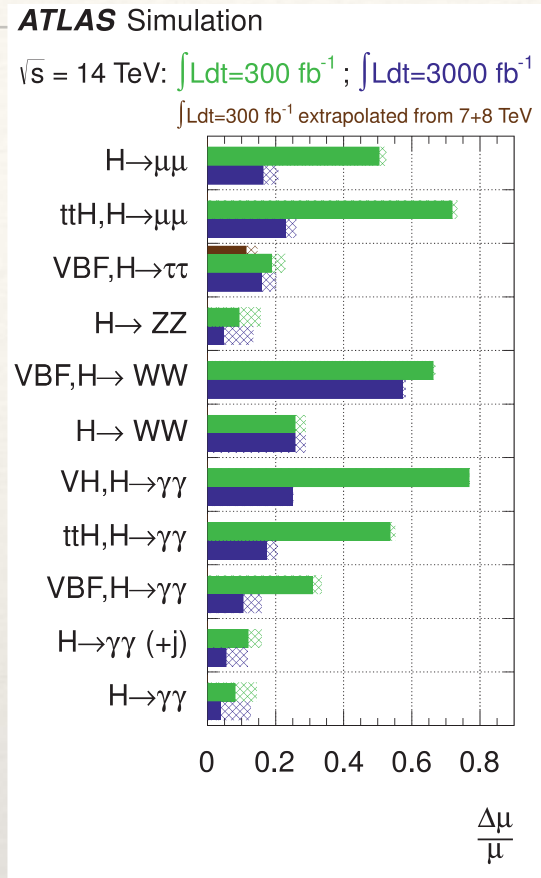
  - NLO + PS calculation
  - not yet “seen”
- W
  - NLO + PS calculation
  - ttW at LHC has little sensitivity to tWb coupling

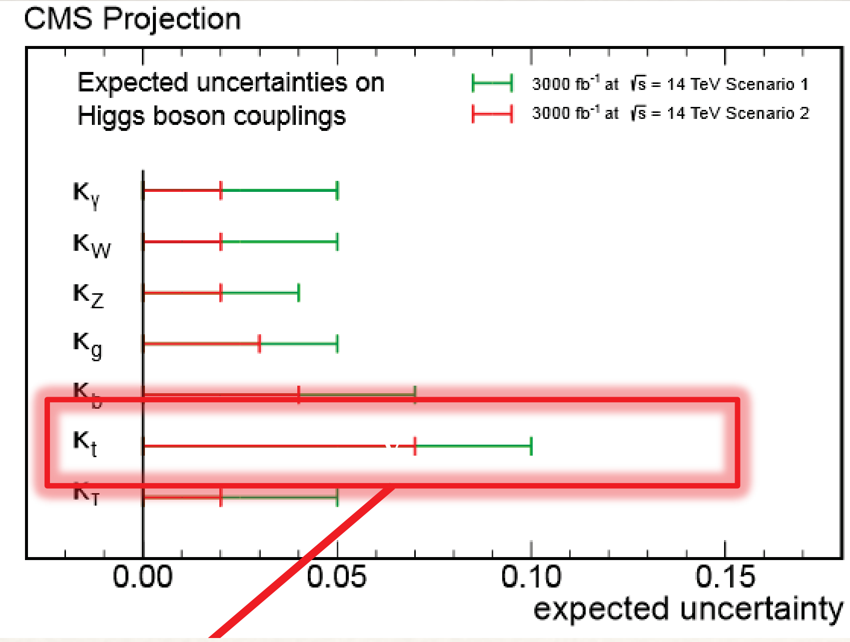
    Garzelli, Kardos, Papadopoulos, Trocsanyi

    - ✓ Use single top production here

# ttH

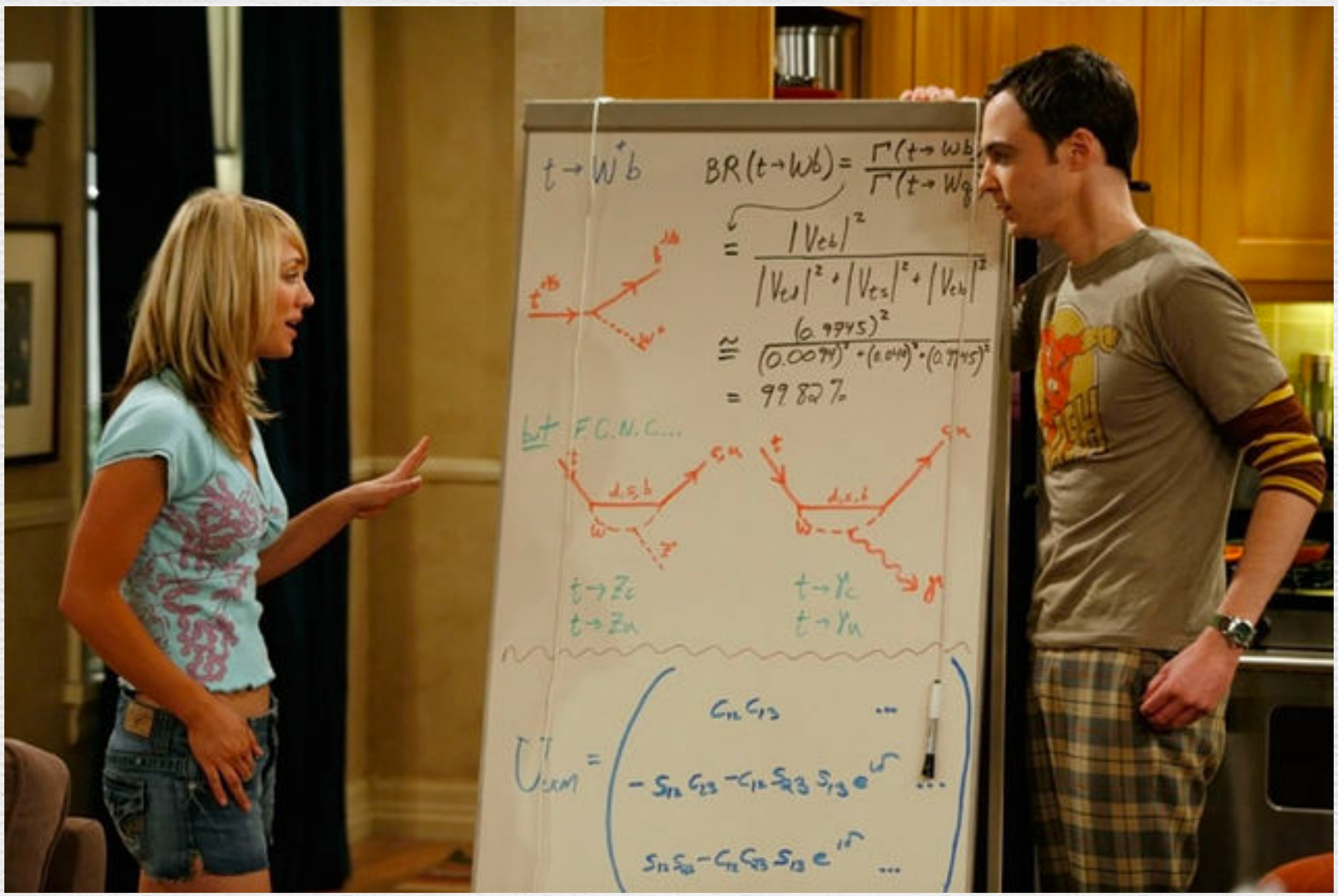
- Should become very interesting for the new run
- $\sigma_{tth}$ (14) ≃ 4.6 x $\sigma_{tth}$ (8)
- NLO calculations for signal — Beenakker, Dittmaier, Kraemer, Plumper, Spira, Zerwas; Dawson, Orr, Reina, Wackeroth
  - plus PS — Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrieli; Garzelli, Kardos, Papadopoulos, Trocsanyi; Hartanto, Jaeger, Reina, Wackeroth
    - ✓ and spin correlations
  - plus EW — Frixione, Hirschi, Pagani, Shao, Zaro
- and e.g. ttbb backgrounds to NLO(+PS)



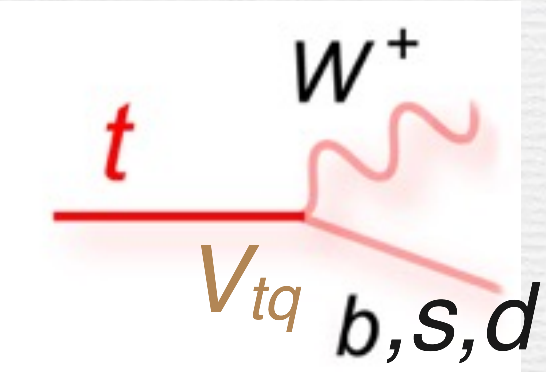

Backgrounds difficult, but expect
10% accuracy in Yukawa coupling by 2030

# Top decay and spin



# Top decay in SM

- Given a large sample of tops, polarized or unpolarized, produced, what are the detail of its decay?




- In SM, 99% via bottom quark. Does this mean $V_{tb}$=1?
  - Not necessarily. Number of events:

$$N_{events} = \epsilon \times \mathcal{L} \times \sigma \times \underbrace{\frac{\Gamma(t \to W + b)}{\sum_q \Gamma(t \to W + b)}}_{\text{Branching ratio } R}$$

  - Branching ratio

$$R = \frac{\Gamma(t \to W + b)}{\sum_{light\ quarks} \Gamma(t \to W + b)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}$$

  - being almost 1 just means $|V_{tb}|^2 >> |V_{ts,td}|^2$. There could be heavy 4th generation quark
    - Cannot measure $V_{tb}$ this way, R is basically independent of it.

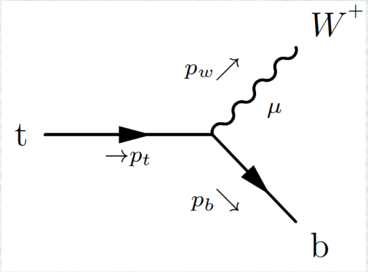
# Top-Higgs coupling and W polarization

- Yukawa interaction of top and Higgs

$$y_t h\bar{t}t$$

- leads to much longitudinal W polarization. Top width:

$$\Gamma(t \to Wb) \propto g^2 m_t \, a \, (1 + \mathcal{O}(1/a)), \qquad a = \frac{m_t^2}{2m_W^2} = \frac{y_t^2}{g^2}$$




- enhanced by “a” (about 2.3) compared to naive expectation. Width to $W_L$ only

$$\Gamma(t \to W_L b) \propto g^2 m_t \, a$$

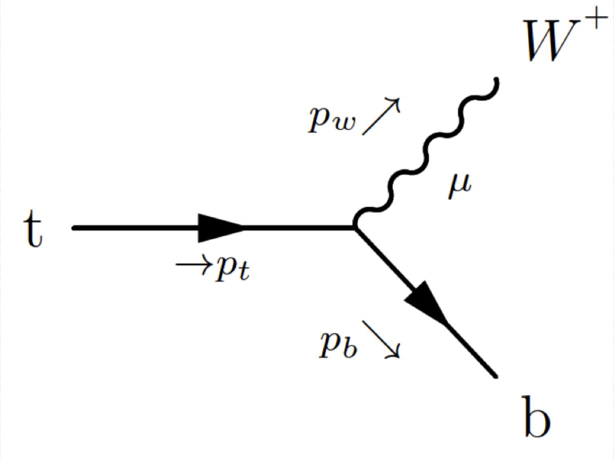
- Ratio

$$\frac{\Gamma(t \to W_L b)}{\Gamma(t \to Wb)} \simeq \frac{a}{1+a} \simeq 70\%$$

# Top decay to W and b

- Let's examine the decay to W and b a bit closer

$$-i\frac{g_W}{2\sqrt{2}}V_{tb}\gamma_\mu(1-\gamma_5)$$




- Momenta

$$p_t^\mu = (m_t, 0, 0, 0)\ ,\ p_b^\mu = (E_b, 0, 0 - p)\ ,\ p_w^\mu = (E_w, 0, 0, p)$$

- Polarization vectors for W

$$\varepsilon_0^\mu = \frac{1}{m_w}(p, 0, 0, E_w)\ ,\ \varepsilon_\pm^\mu = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$$

- Spin-averaged squared matrix element

$$\langle |M|^2 \rangle = \frac{4G_f}{\sqrt{2}}|V_{tb}|^2 m_w^2 [p_t^\mu p_b^\nu + p_t^\nu p_b^\mu - g^{\mu\nu}(p_t \cdot p_b) + i\epsilon^{\mu\nu\rho\sigma} p_{t\rho} p_{b\sigma}] \sum_\lambda \varepsilon_\mu^{\lambda *} \varepsilon_\nu^\lambda$$

- Evaluate per case λ

# Contributions from each polarization state

$$\langle |M|^2 \rangle = \frac{4G_f}{\sqrt{2}} |V_{tb}|^2 m_w^2 [p_t^\mu p_b^\nu + p_t^\nu p_b^\mu - g^{\mu\nu}(p_t \cdot p_b) + i\epsilon^{\mu\nu\rho\sigma} p_{t\rho} p_{b\sigma}] \sum_\lambda \varepsilon_\mu^{\lambda *} \varepsilon_\nu^\lambda$$

$$x = \frac{m_W}{m_t}$$

$$y = \frac{m_b}{m_t}$$

- λ = 0 : No contribution from Levi Civita tensor. Use mass shell conditions to find E's and p's. Result

$$\boxed{\langle |M_0|^2 \rangle = \frac{2G_f}{\sqrt{2}} |V_{tb}|^2 m_t^4 \left[1 - x^2 - y^2(2 + x^2 - y^2)\right]}$$

- λ = -: (left-handed). Now Levi-Civita does contribute

$$\boxed{\langle |M_-|^2 \rangle = \frac{2G_f}{\sqrt{2}} |V_{tb}|^2 m_t^4 \left[2x^2\left(1 - x^2 + y^2\right)\right]}$$

- λ = +: cancellations take place

$$\boxed{\langle |M_+|^2 \rangle = 0}$$

- As before

$$\frac{"0"}{"0" + "-" + "+"} = \frac{m_t^2}{2m_W^2 + m_t^2} \simeq 70\%$$

Longitudinal polarization from Higgs doublet!!

# Unpolarized top decays to polarized W's

- Sum the three and combine with phase space measure

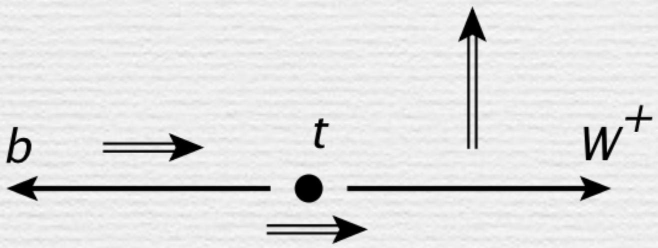
$$\boxed{\Gamma_{(t\to bW^+)} = \frac{G_f m_t^3 |V_{tb}|^2}{8\pi\sqrt{2}} I(x,y)}$$

$$x = \frac{m_W}{m_t}$$

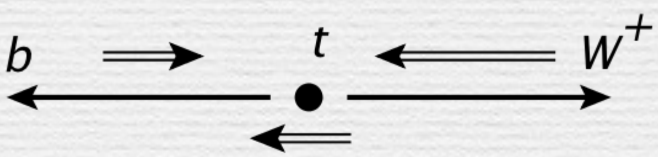
$$y = \frac{m_b}{m_t}$$

  - I(x,y) can be approximated by 1

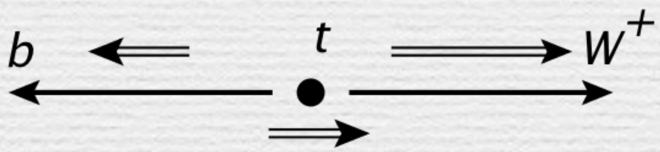
- Some intuition: for λ = + , the b-quark would have to emerge right-handed. But that is not allowed by the chiral coupling




(A) $\lambda_w = 0$ (longitudinal)




(B) $\lambda_w = -$ (left handed)

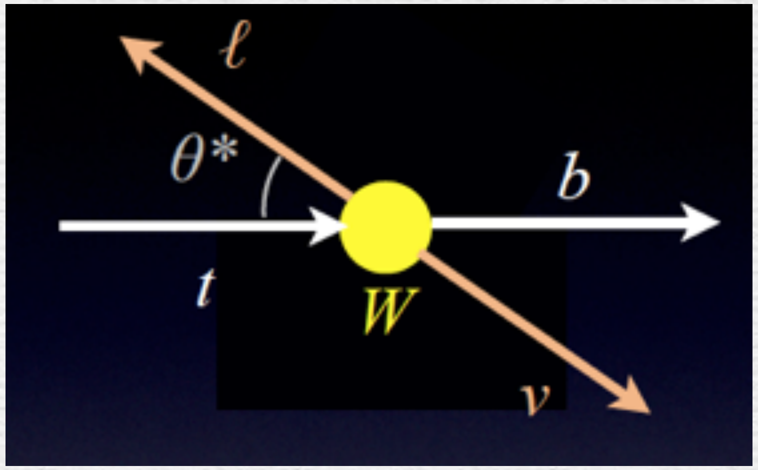



(C) $\lambda_w = +$ (right handed)

# Unpolarized top decays to polarized W's

- (With QCD and EW corrections, there are tiny changes to these decay rates)
- Can we detect this ratio? Consider with further decay of W

$$t \to Wb \to bl^+\nu_l$$




- Define ψ to be angle between
  - direction of lepton in W rest frame
  - direction of W in t rest frame
  - then one finds, after some work

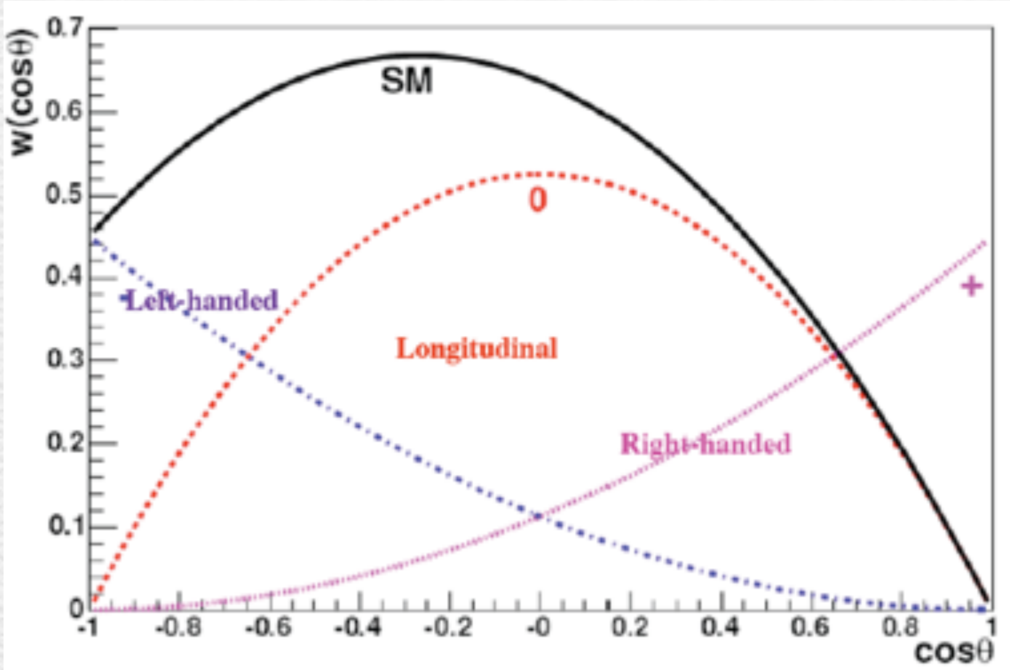
$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\psi} = \frac{3}{4}F_0\sin^2\psi + \frac{3}{8}F_-(1-\cos\psi)^2 + \frac{3}{8}F_+(1+\cos\psi)^2$$

- Note: no interference of different amplitudes for this distribution

# Unpolarized top decays to polarized W's

- Formula

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\psi} = \frac{3}{4}F_0\sin^2\psi + \frac{3}{8}F_-(1-\cos\psi)^2 + \frac{3}{8}F_+(1+\cos\psi)^2$$

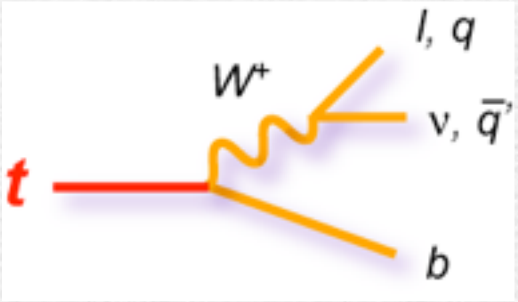



- Experimentally, now well confirmed by Tevatron and LHC experiments, e.g.:

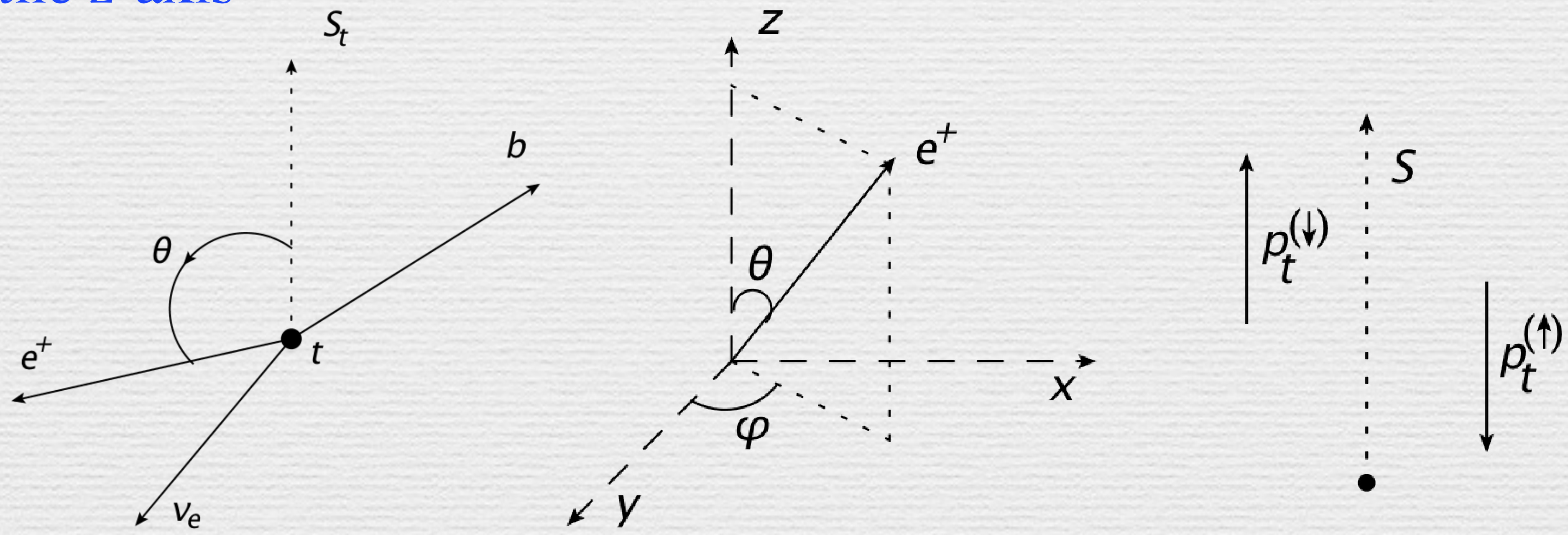
$$\text{Tevatron ICHEP 2012}: F_0 = 0.722 \pm 0.062 \pm 0.052$$

# Polarized top decay

- Let us now assume we can somehow polarize the top quark sample. Can we detect the top quark spin?
  - Take now full decay




  - We must take into account the top spin vector, conveniently along the z-axis




(A) Spin axis - products

(B) $e^+$ momentum decomposition

(C) Top quark rest frame

# Polarized top decay

- The result for spin-up top is

$$\langle |M(t^{(\uparrow)} \to b e^+ \nu_e)|^2 \rangle = 64 G_f^2 |V_{tb}|^2 \frac{M_w^4}{(q^2 - M_w^2)^2 + M_w^2 \Gamma_w^2} m_t |\vec{p}_{\bar{e}}| (1 + \cos\theta_{e^+})(p_b \cdot p_n)$$

- For spin-down top

$$\langle |M(t^{(\downarrow)} \to b e^+ \nu_e)|^2 \rangle = 64 G_f^2 |V_{tb}|^2 \frac{M_w^4}{(q^2 - M_w^2)^2 + M_w^2 \Gamma_w^2} m_t |\vec{p}_{\bar{e}}| (1 - \cos\theta_{e^+})(p_b \cdot p_n)$$

- For spin-up differential decay width

$$\frac{1}{\Gamma_T} \frac{d\Gamma_{(\uparrow)}}{d(\cos\theta_{e^+})} = \frac{1}{2}(1 + \cos\theta_{e^+})$$

- What is the implication, given that the a-priori form is (p=polarization degree of top, c spin-analyzing power of f)?

$$\frac{1}{\Gamma_f} \frac{d\Gamma_{f,(\uparrow)}}{d(\cos\theta_f)} = \frac{1}{2}(1 + p c_f \cos\theta_f)$$

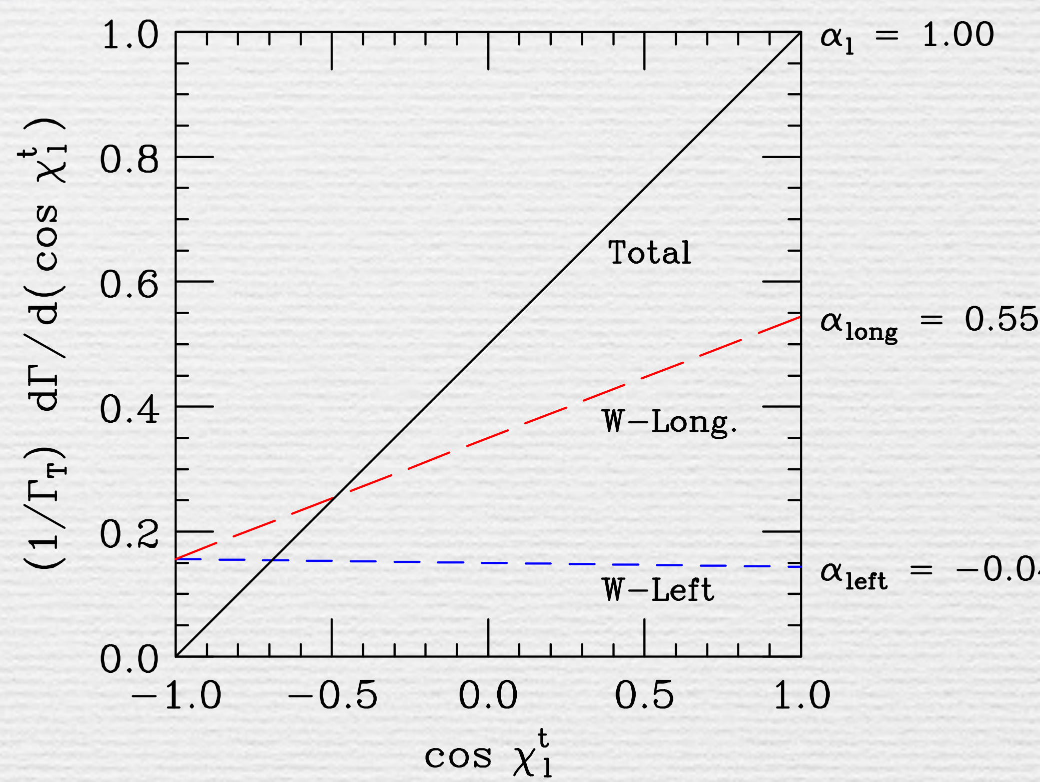
# Polarized top decay

- For f = charged lepton: c=1 $\Rightarrow$ 100% correlation !
    - Top self-analyzes its spin
    - Charged leptons easy to measure, good handle on top spin
        - if they can be produced in a polarized fashion
- For spin-up top the polar angle distribution is

$$\frac{1}{\Gamma_T}\frac{d\Gamma_{(\uparrow)}}{d(\cos\theta_{e^+})} = \frac{1}{2}(1+\cos\theta_{e^+})$$

- Note: charged lepton has larger "spin-analyzing power" than its parent W!
    - Reason: for this distribution intermediate λ = 0 and λ = - amplitudes interference.
- There is a lot to check in the new run for top spin behavior

$$\frac{d\ln\Gamma_f}{d\cos\chi_f} = \frac{1}{2}(1+\alpha_f\cos\chi_f)$$



# Spin/angular correlations for single top

Frixione, EL, Motylinski, Webber

- Let us first be very general. A process with an intermediate "resonant" particle P (e.g. W, Z, top..) reads

$$a + b \longrightarrow P(\longrightarrow d_1 + \cdots + d_n) + X$$

- has "production" spin/angular correlations if it depends on $d_i.a$, $d_i.b$ or $d_i.X$
  - Can be introduced even after the Monte Carlo has been written..
- Let P be an intermediate W, which will be nearly on-shell. We can approximate the intermediate W propagator through the Narrow Width approximation

$$\frac{1}{(q^2 - m_V^2)^2 + (m_V \Gamma_V)^2} \longrightarrow \frac{\pi}{m_V \Gamma_V} \delta\left(q^2 - m_V^2\right) .$$

- Resulting expression

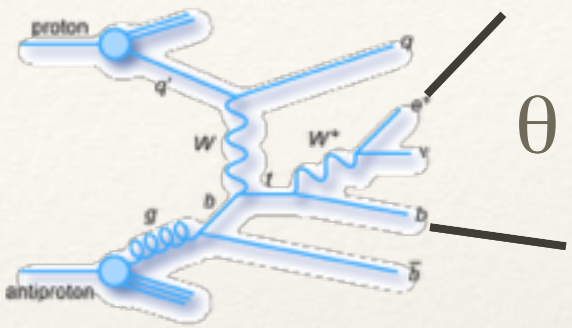
$$\sum_{spin} |A|^2 = \frac{\pi}{m_V \Gamma_V} \sum_{\lambda\lambda'} \tilde{M}_\lambda \rho_{\lambda\lambda'} \tilde{M}^*_{\lambda'} \, \delta\left(q^2 - m_V^2\right)$$

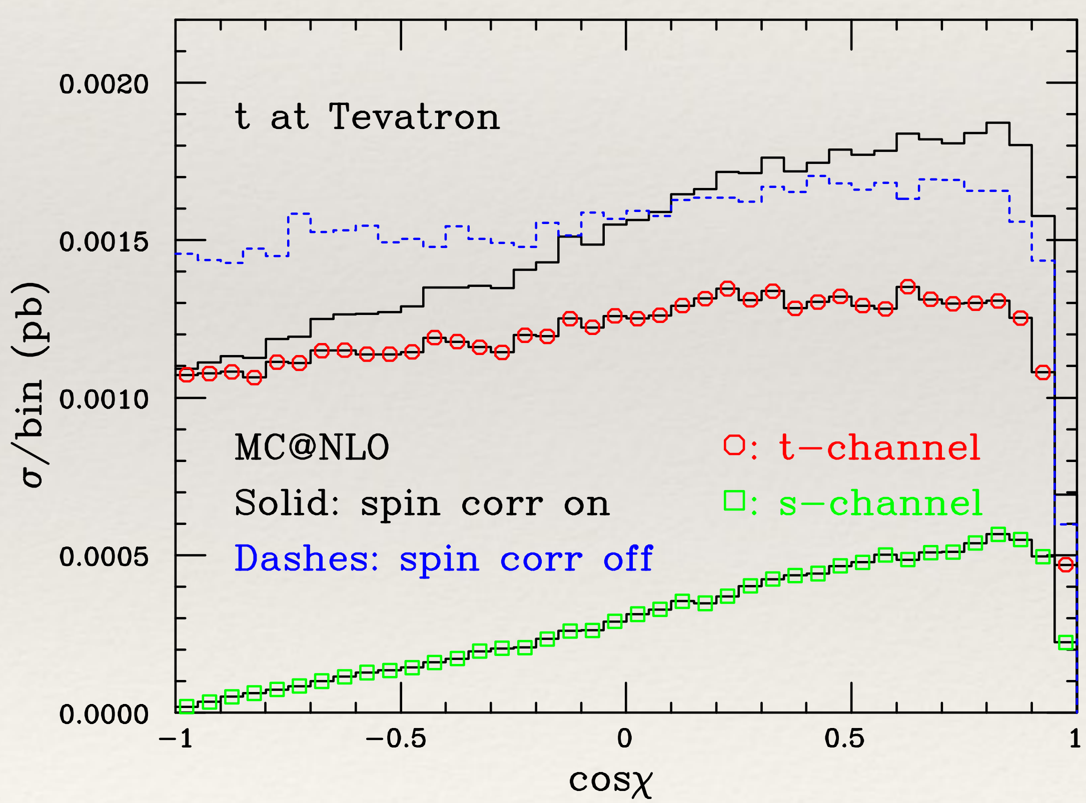
- with $\rho_{\lambda\lambda'}$ the spin-density matrix for W-decay. Can do this also for top decay.
- We used this to implement spin correlations in MC@NLO

# Spin correlations for single top in MC@NLO

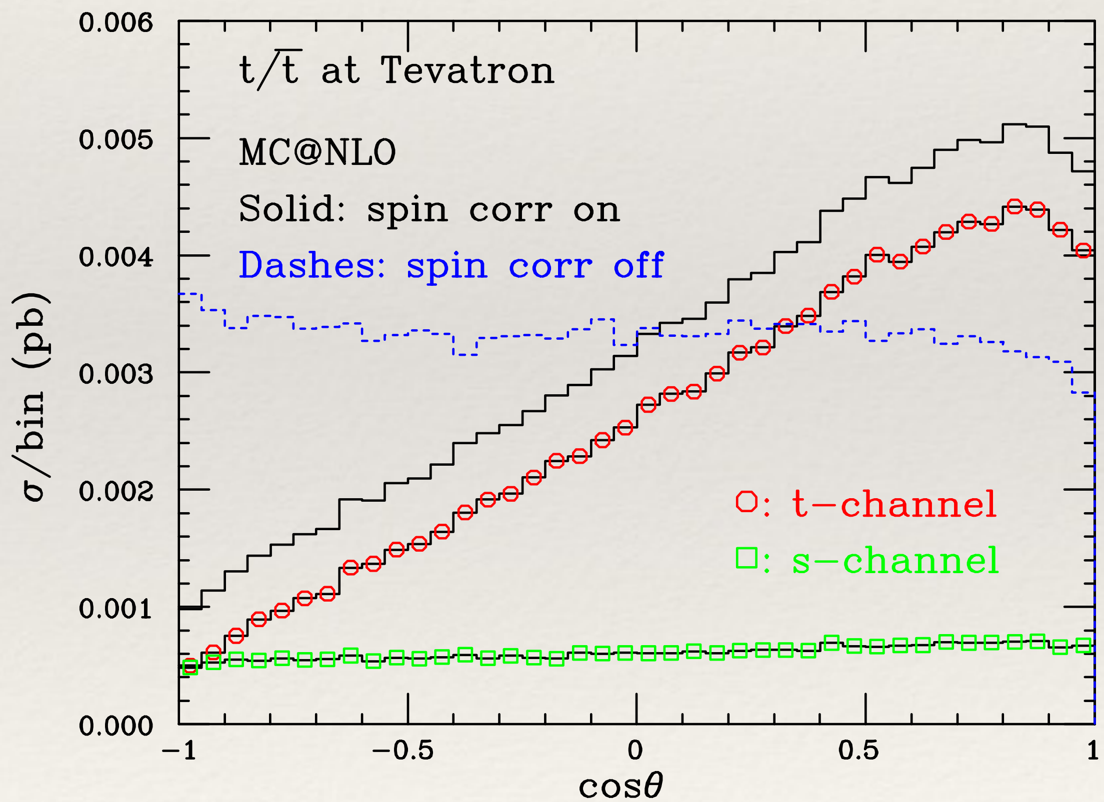
Frixione, EL, Motylinski, Webber

- Top is produced polarized by EW interaction
  - 100% correlation between top spin and charged lepton direction
- Angle of lepton with appropriate axis is different per channel
- Method included “a posteriori”. Also used in POWHEG  Aioli, Nason, Oleari, Re
- Implemented in MadSpin  Artoisenet, Frederix, Rietkerk




Beam direction




Hardest, non-b jet

# Top vision

- Mass: linear e+e- collider estimate
  - 20-30 MeV statistical uncertainty, total about 100 MeV
  - 2-4 % uncertainty for Top-Higgs coupling
- Future circular collider: 100 TeV hadron collider near Geneva
  - Rich top physics, rates about 100 times the present LHC
    - ttWW, tttt, ttjjj, etc.
- Theory: aMC@NNLO? Another N?

# But first: Top in Run II at LHC

- LHC has restarted v
  - First interesting re
  - Analyses requiring                                  asible
    - Close collaborat
- LHC: T-factory. Top
- With luck and perse                                  ing of the Particles