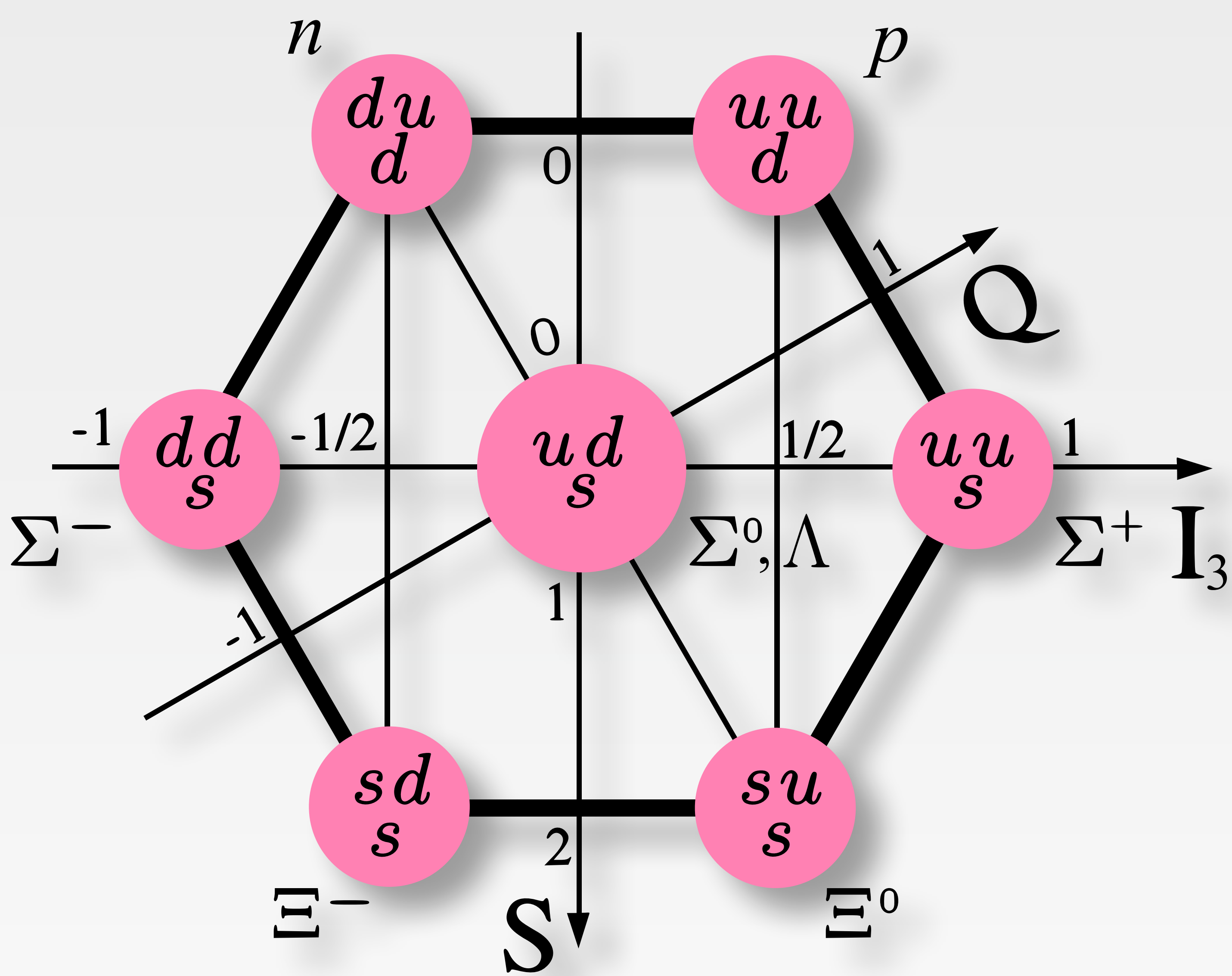

Inclusion of isospin breaking effects in lattice simulations

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Annual Theory Meeting, Durham

CENTRE FOR THEORETICAL PHYSICS Higgs
THE UNIVERSITY of EDINBURGH

- Motivations
- Lattice QCD+QED
- Isospin splitting in the hadron spectrum
- Summary & outlook

Motivations

Isospin symmetry breaking

- Isospin symmetric world: up and down quarks are particles with identical physical properties.
- Isospin symmetry is explicitly broken by:
  - the up and down **quark mass difference**

$$|m_u - m_d|/\Lambda_{\rm QCD} \simeq 0.01$$

  - the up and down **electric charge difference**

$$\alpha \simeq 0.0073$$

	up	down
Mass (MeV)	$2.3(^{+0.7}_{-0.5})$	$4.8(^{+0.5}_{-0.3})$
Charge (e)	2/3	-1/3

[PDG, 2015]

Nucleon mass splitting

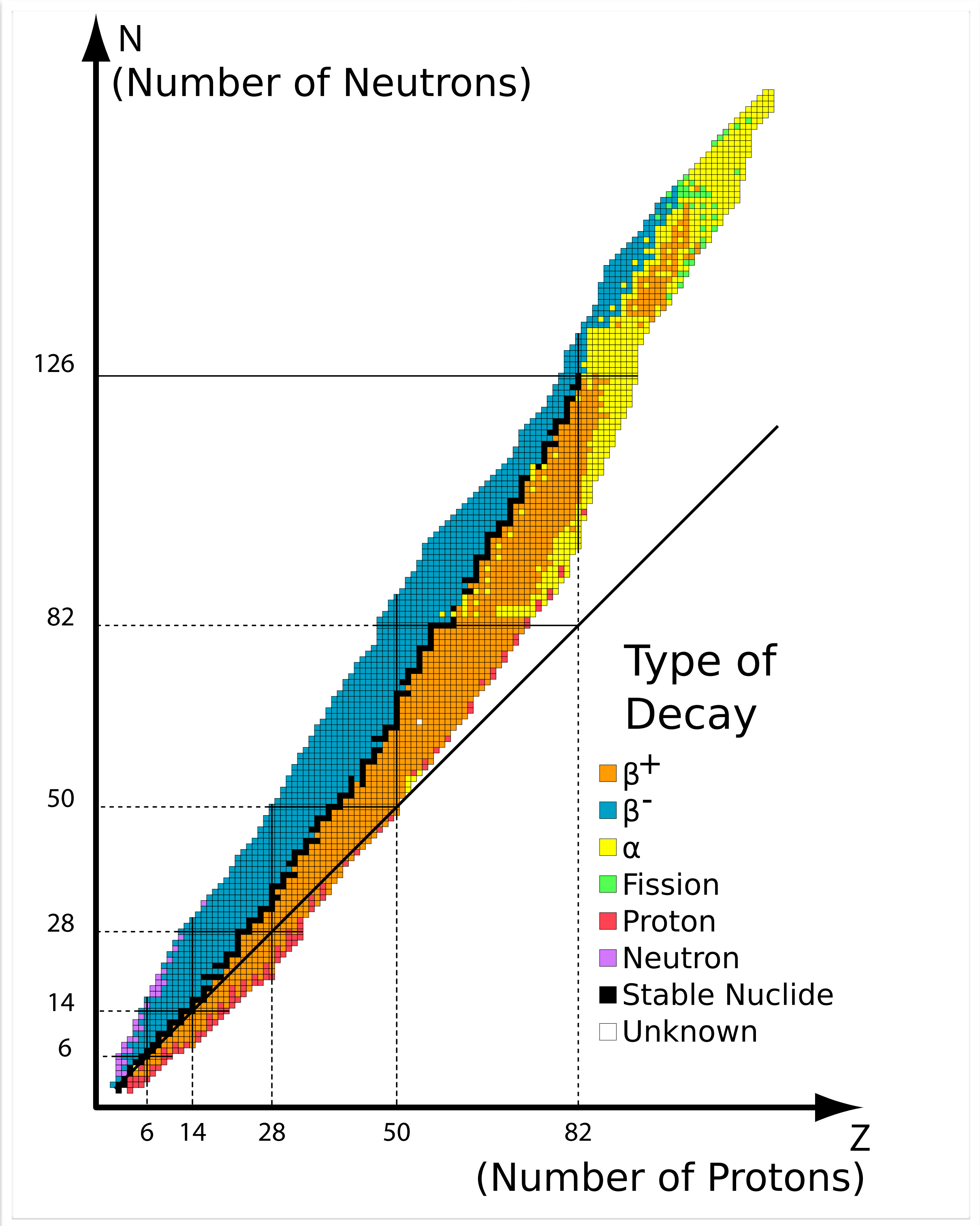
- Well known experimentally:

$$M_n - M_p = 1.2933322(4)\ \mathrm{MeV}$$

- Needed for **proton stability**

- Determines through β-decay the stable nuclide chart

- Initial condition for **Big-Bang nucleosynthesis**



Dashen's theorem

- In the SU(3) chiral limit [Dashen, 1969]:

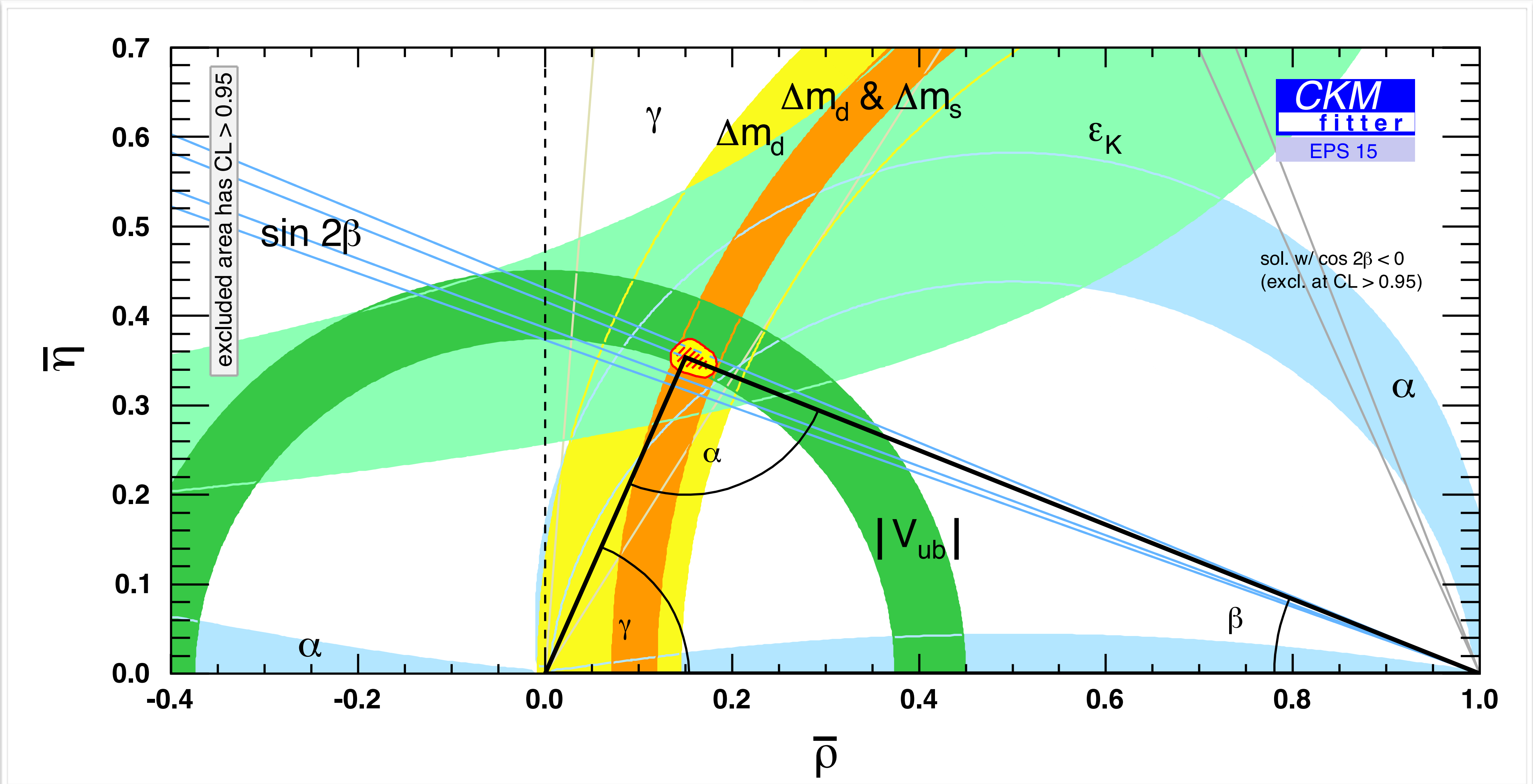
$$\Delta_{\mathrm{QED}} M_K^2 = \Delta_{\mathrm{QED}} M_\pi^2 + \mathrm{O}(\alpha m_s)$$

- **How large are the corrections?**
  FLAG parametrisation:

$$\varepsilon = \frac{\Delta_{\mathrm{QED}} M_K^2 - \Delta_{\mathrm{QED}} M_\pi^2}{\Delta M_\pi^2}$$

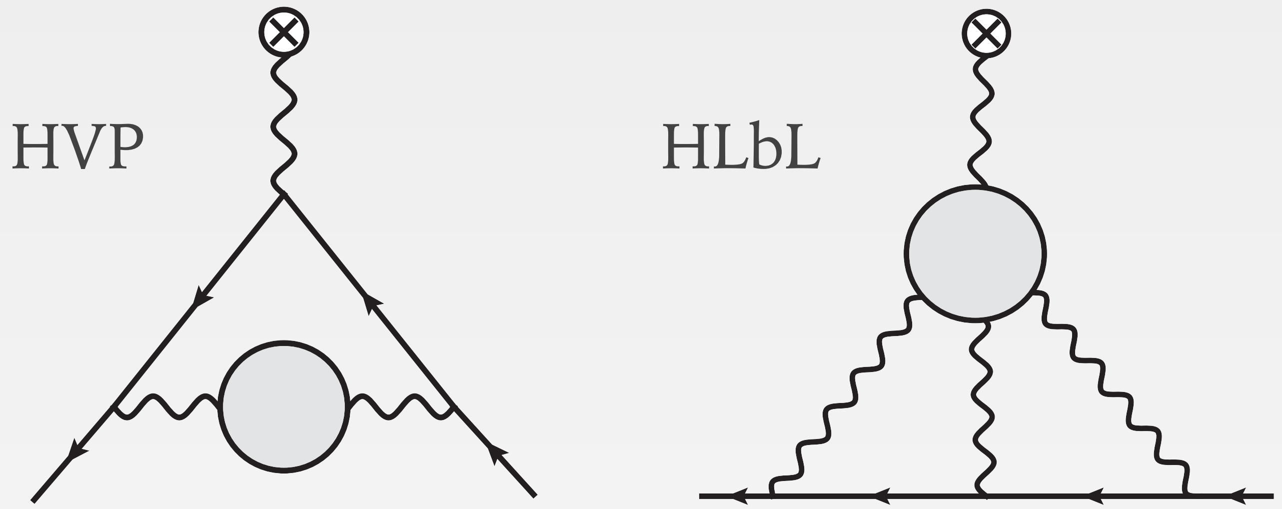
- ε is important to determine **light quark mass ratios**

Precision flavour physics



Inclusion of isospin breaking corrections is needed to obtain sub-percent theoretical constraints on the Standard Model flavour sector.

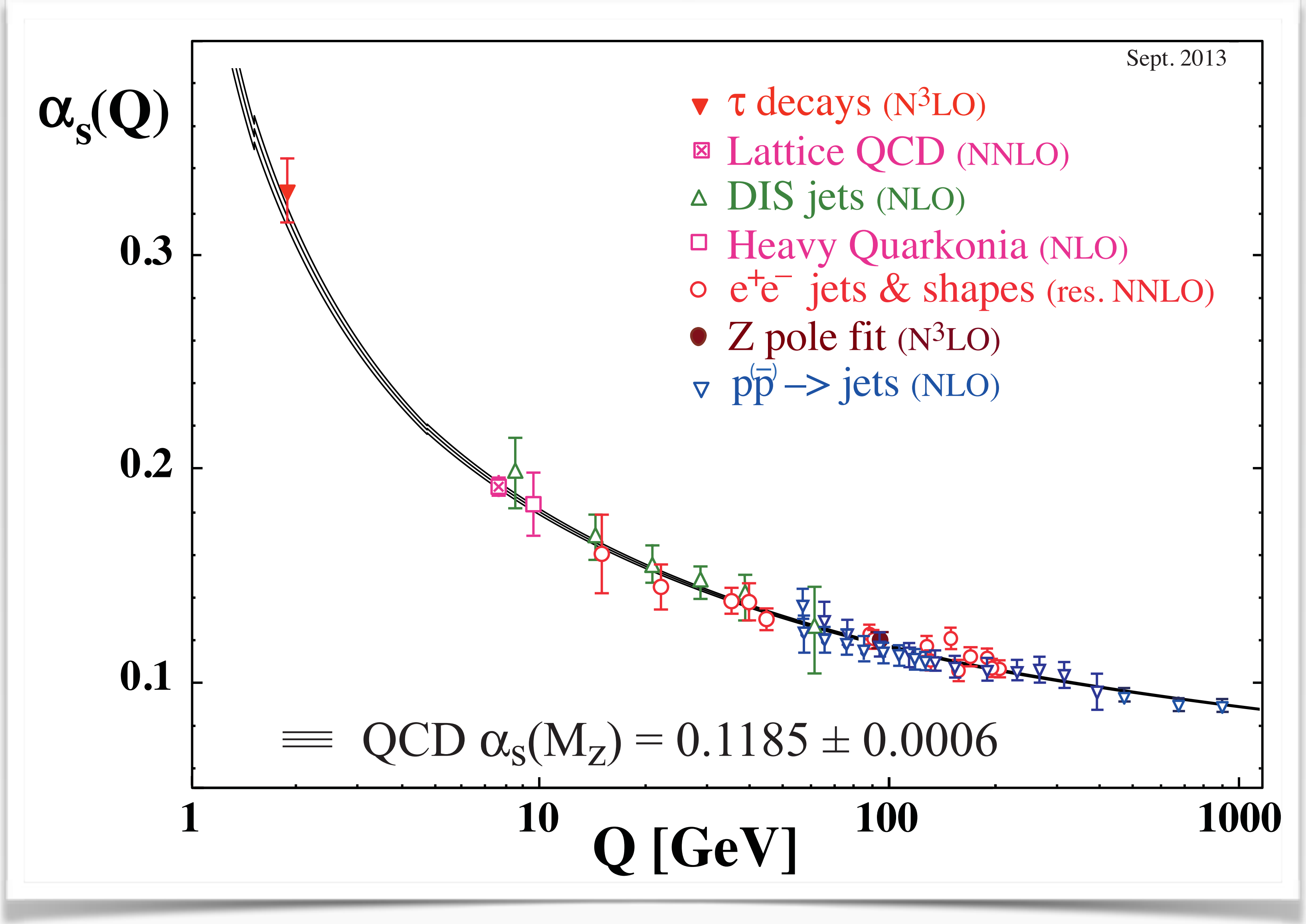
Anomalous magnetic moment of the muon (g-2)



- $\sim 3.6\sigma$ discrepancy between experiment and the Standard Model.
- Theoretical uncertainty dominated by hadronic errors.
- Inclusion of isospin breaking effects in lattice determinations is necessary to reduce the theoretical uncertainty.

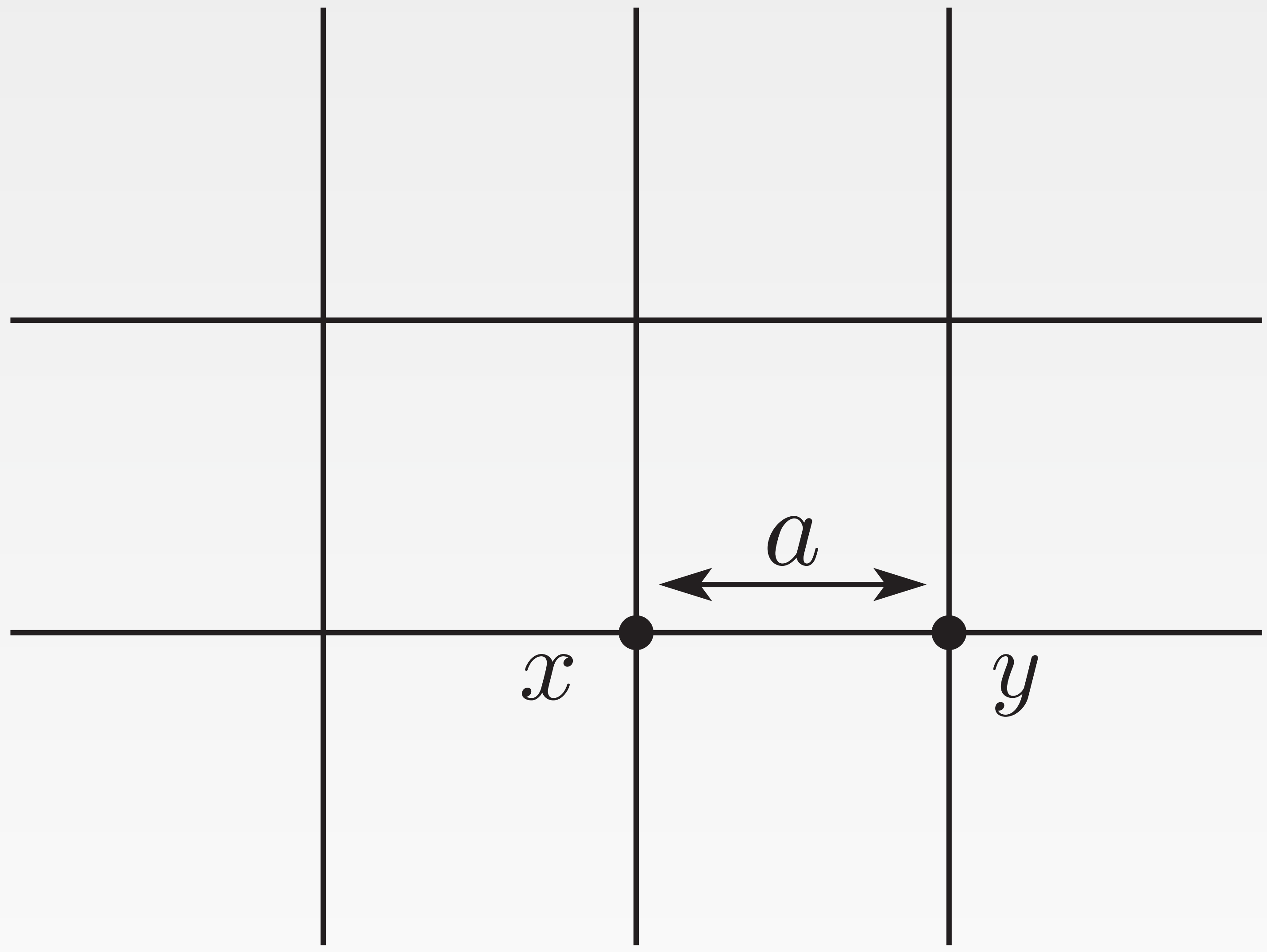
Lattice QCD + QED

QCD at low energies



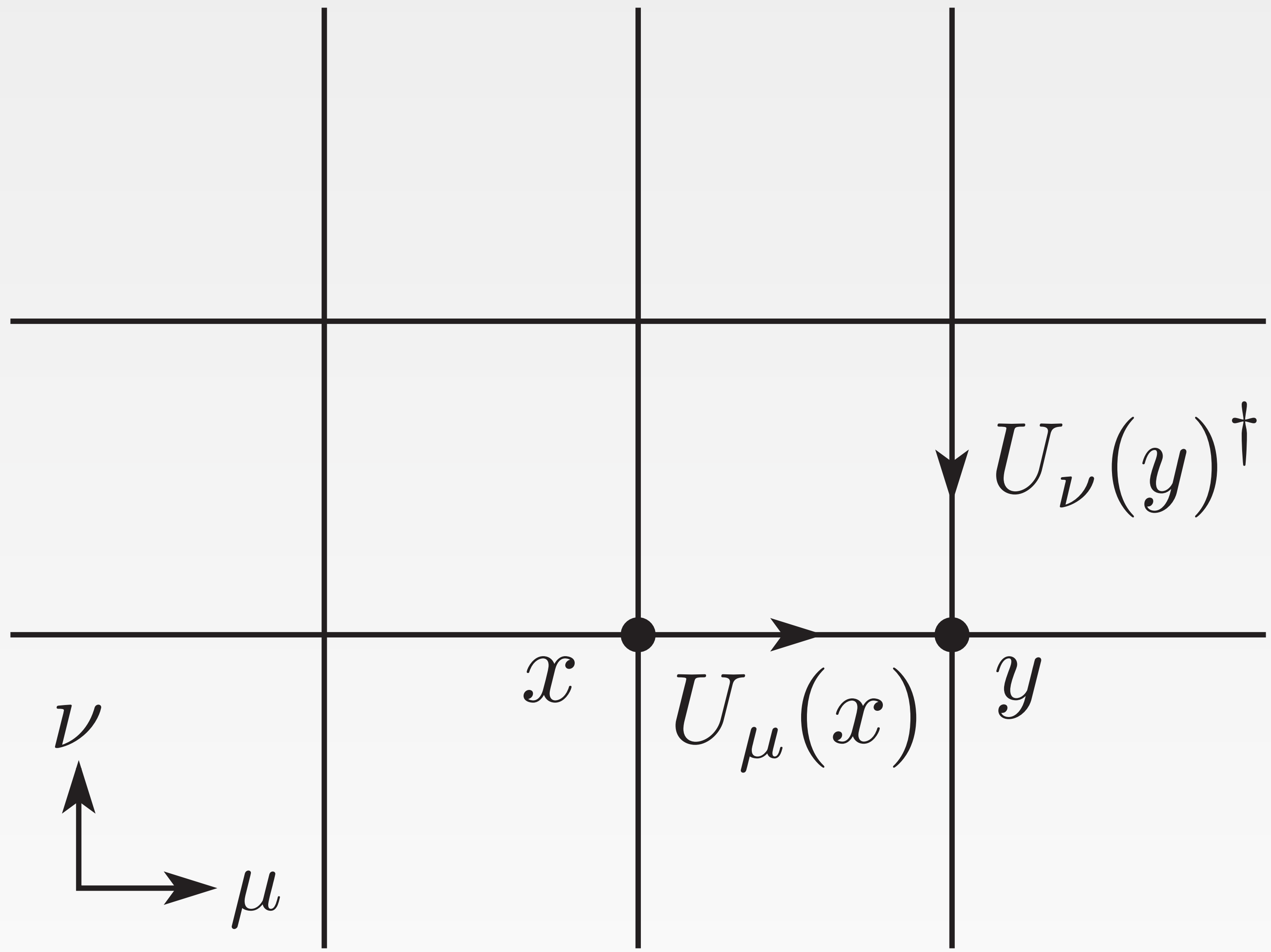
- QCD is **strongly non-perturbative** at GeV scale.
- Non-perturbative phenomena like **colour confinement** dominate.

Lattice gauge action



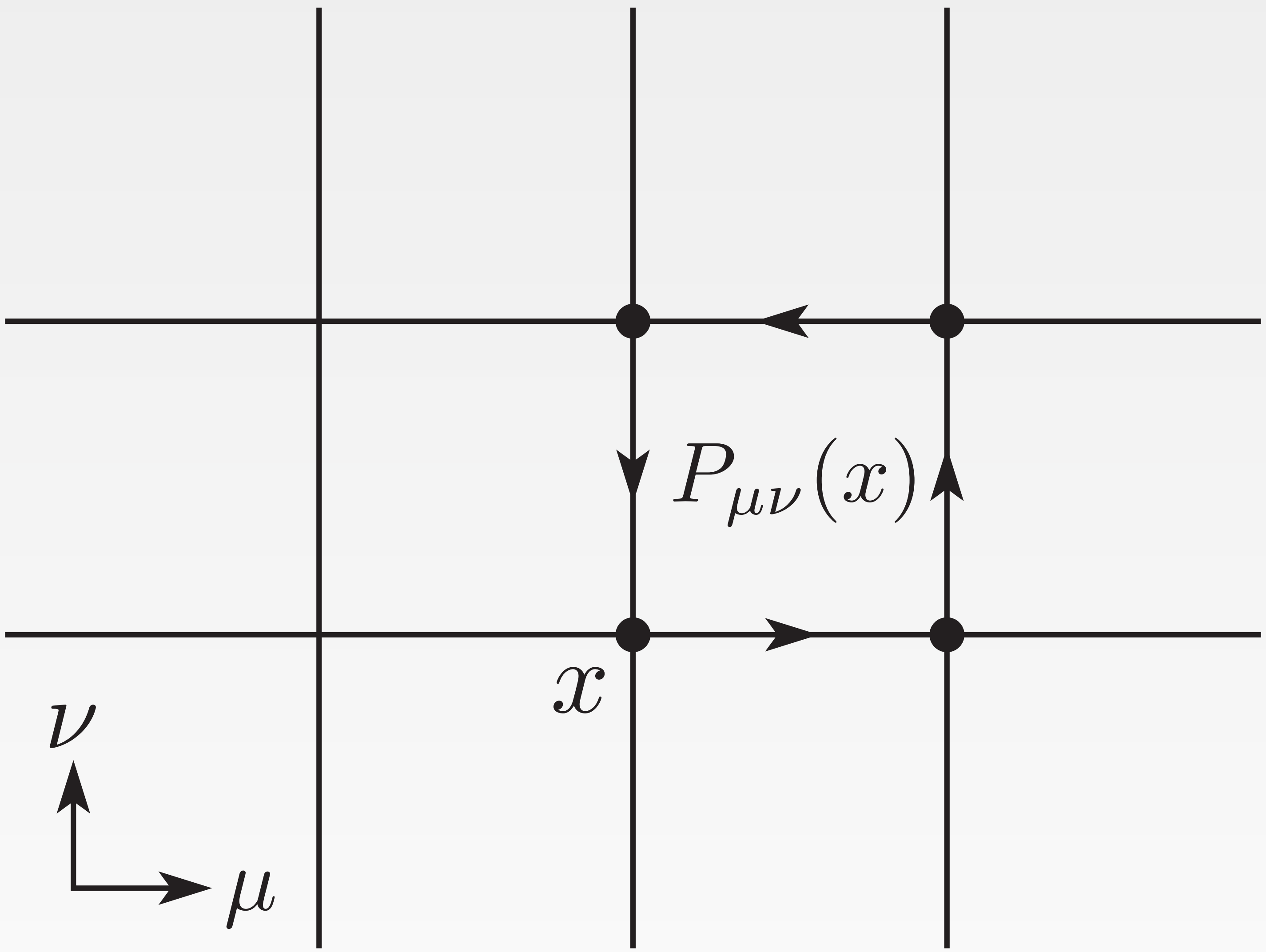
Euclidean space-time: 4D periodic hypercubic lattice

Lattice gauge action



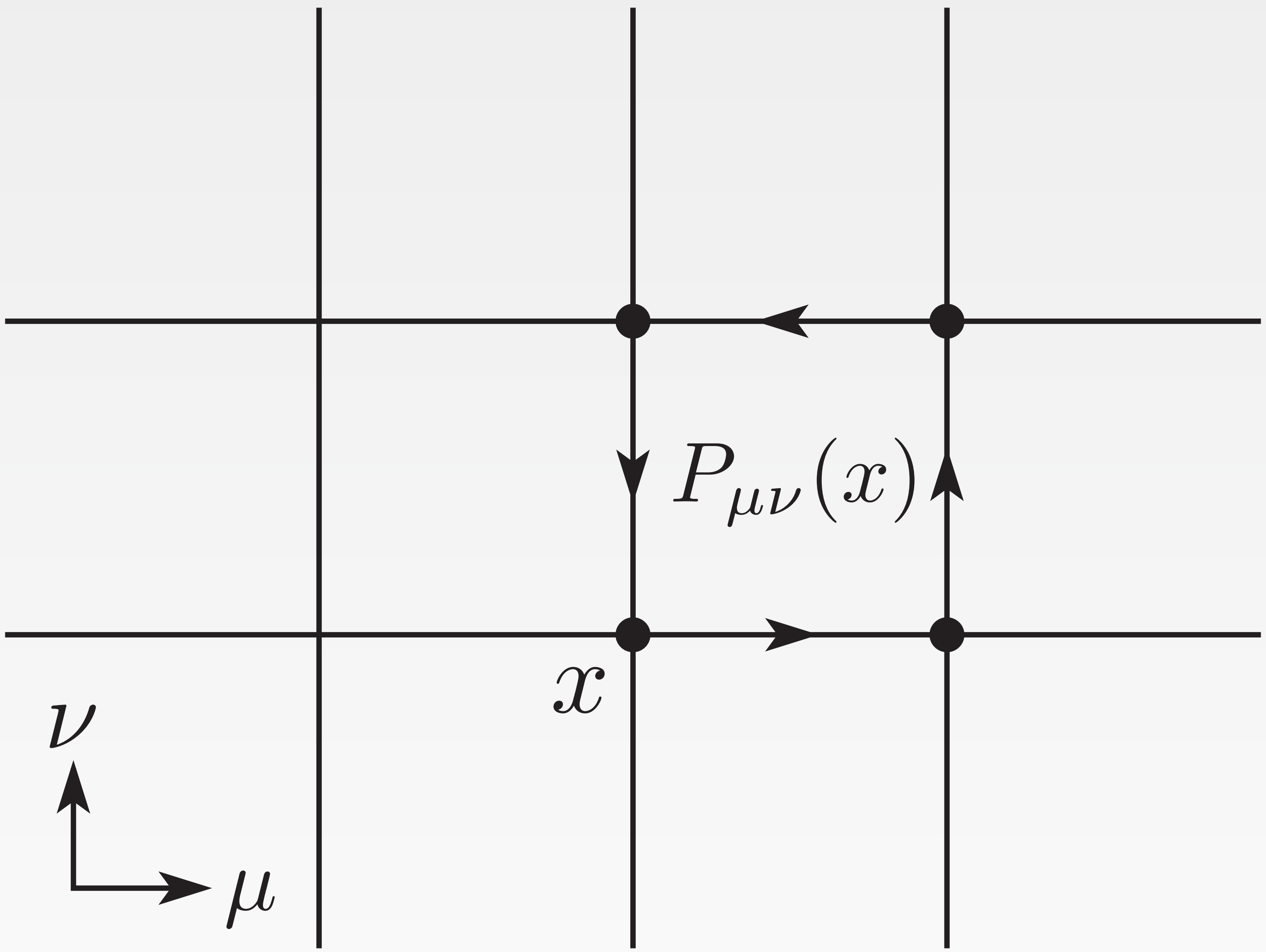
Gauge variables: straight Wilson lines over one lattice spacing

Lattice gauge action



Most elementary gauge invariant quantity: the plaquette $P_{\mu\nu}$

Lattice gauge action



Wilson gauge action:

$$S_{\text{gauge}} = \frac{\beta}{N_c} \sum_{x,\nu>\mu} \Re\, \mathrm{tr}[1 - P_{\mu\nu}(x)] = \frac{\beta g^2}{8N_c} a^4 \sum_{x,\mu,\nu} F_{\mu\nu} F_{\mu\nu} + \mathrm{O}(a^2)$$

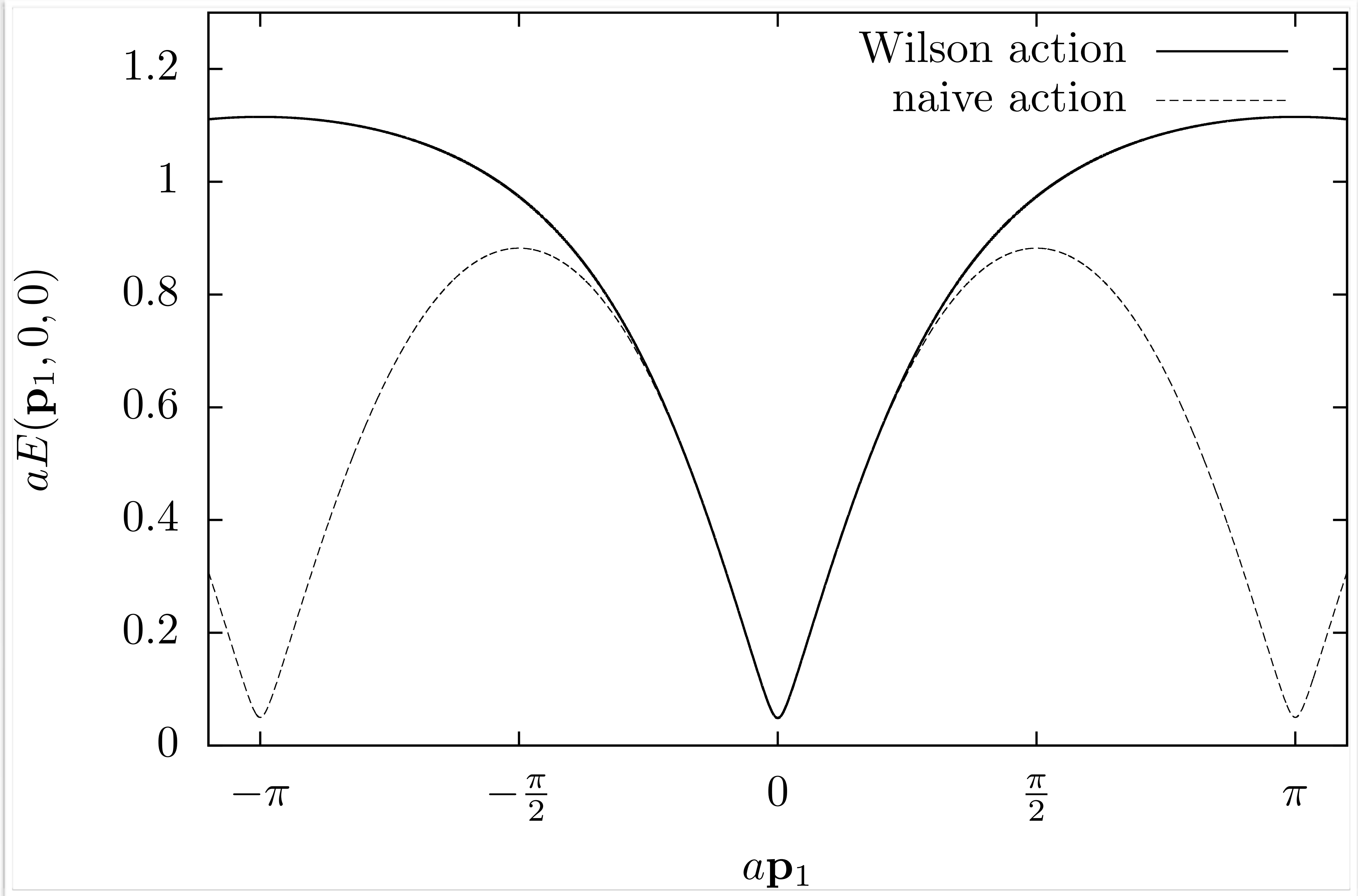
Lattice fermion action

- The naive discrete Dirac action contains 16 degenerate fermion “flavours” which survive in the continuum limit. It is the so-called **doublers problem.**
- One cannot have a local action, no doublers and chiral symmetry. *(Nielsen-Ninomiya theorem)*
- One possible choice: **Wilson fermions**

$$S_{\mathrm{fermion},W} = S_{\mathrm{fermion,naive}} - \frac{a^5}{2} \sum_{x,\mu} \overline{\psi}(x) \partial_\mu^* \partial_\mu \psi(x)$$

- Wilson fermions are local and free of doublers, but chiral symmetry is explicitly broken.

Wilson fermions



Path integral calculation

$$\langle O \rangle = \frac{1}{\mathscr{Z}} \int \mathrm{d}\psi \mathrm{d}\overline{\psi} \mathrm{d}U_\mu \, O[\psi, \overline{\psi}] \exp(-S_{\mathrm{LQCD}}[\psi, \overline{\psi}, U_\mu])$$

Path integral calculation

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Lattice action is quadratic in the quark fields: integration can be done through Wick's theorem

Path integral calculation

$$\langle O \rangle = \frac{1}{\mathscr{Z}} \int \mathrm{d}U_\mu \, O[(D_\mathrm{W} + M)^{-1}] \det(D_\mathrm{W} + M) \exp(-S_\mathrm{gauge}[U_\mu])$$

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Probability weight in the integral

Path integral calculation

$$\langle O \rangle = \frac{1}{\mathscr{Z}} \int \mathrm{d}U_\mu \, O[(D_\mathrm{W} + M)^{-1}] \, \det(D_\mathrm{W} + M) \exp(-S_\mathrm{gauge}[U_\mu])$$

Probability weight in the integral

Monte-Carlo computation

Non-compact lattice QED

- Naively discretised **Maxwell action:**

$$S[A_\mu] = \frac{1}{4}\sum_{\mu,\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

- Pure gauge theory is free, it can be solved exactly
- **Gauge invariance is preserved**
- No mass gap: **large finite volume effects expected**

Zero-mode subtraction

Finite volume: **momentum quantisation**

$$\alpha \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2} \cdots \qquad \longmapsto \qquad \frac{\alpha}{V} \sum_k \frac{1}{k^2} \cdots$$

Zero-mode subtraction

Finite volume: **momentum quantisation**

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Possibly IR divergent, but not for physical quantities

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Possibly IR divergent, but not for physical quantities

Contains a straight 1/0 !

Zero-mode subtraction

- This problem can be solved by **removing zero modes**.
- **Many possible schemes:**
  modification of $A_\mu(k)$ on a set of measure 0.
- Different schemes:
  different finite volume behaviours.
- Some more interesting that others.

QED$_{\mathrm{TL}}$ zero-mode subtraction

- QED$_{\mathrm{TL}}$: $A_\mu(0) = 0$

  [A. Duncan, E. Eichten, and H. Thacker, PRL 76(21), pp. 3894–3897, 1996]

- With QED$_{\mathrm{TL}}$, the $T \to \infty\,, \quad L = \mathrm{cst.}$ limit **can diverge**:

$$\frac{\alpha}{V} \sum_{k \neq 0} \frac{1}{k^2} \cdots \qquad \longmapsto \qquad \frac{\alpha}{L^3} \int \frac{\mathrm{d}k_0}{2\pi} \sum_{\mathbf{k}} \frac{1}{k^2} \cdots$$

- QED$_{\mathrm{TL}}$ **does not have reflection positivity**.

  [BMWc 2015, Science 347, pp. 1452–1455]

QED_{TL} finite-volume effects

[BMWc 2015, Science 347, pp. 1452–1455]

- Example — 1-loop QED_{TL}:

$$m(T,L) \underset{T,L\to+\infty}{\sim} m\left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\left[1-\frac{\pi}{2\kappa}\frac{T}{L}\right]\right)\right.\right.$$
$$\left.\left. -\frac{3\pi}{(mL)^3}\left[1-\frac{\coth(mT)}{2}\right]-\frac{3\pi}{2(mL)^4}\frac{L}{T}\right]\right\}$$

up to exponential corrections, with $\kappa = 2.83729\ldots$

- **Divergent finite volume effects** with $T \to \infty$, $L = \text{cst.}$

$\mathrm{QED_L}$ zero-mode subtraction

- $\mathrm{QED_L}$: $A_\mu(k_0, \mathbf{0}) = 0$

  [M. Hayakawa and S. Uno, Prog. Theor. Phys. 120(3), pp. 413–441, 2008]

- $\mathrm{QED_L}$ maintains reflection positivity

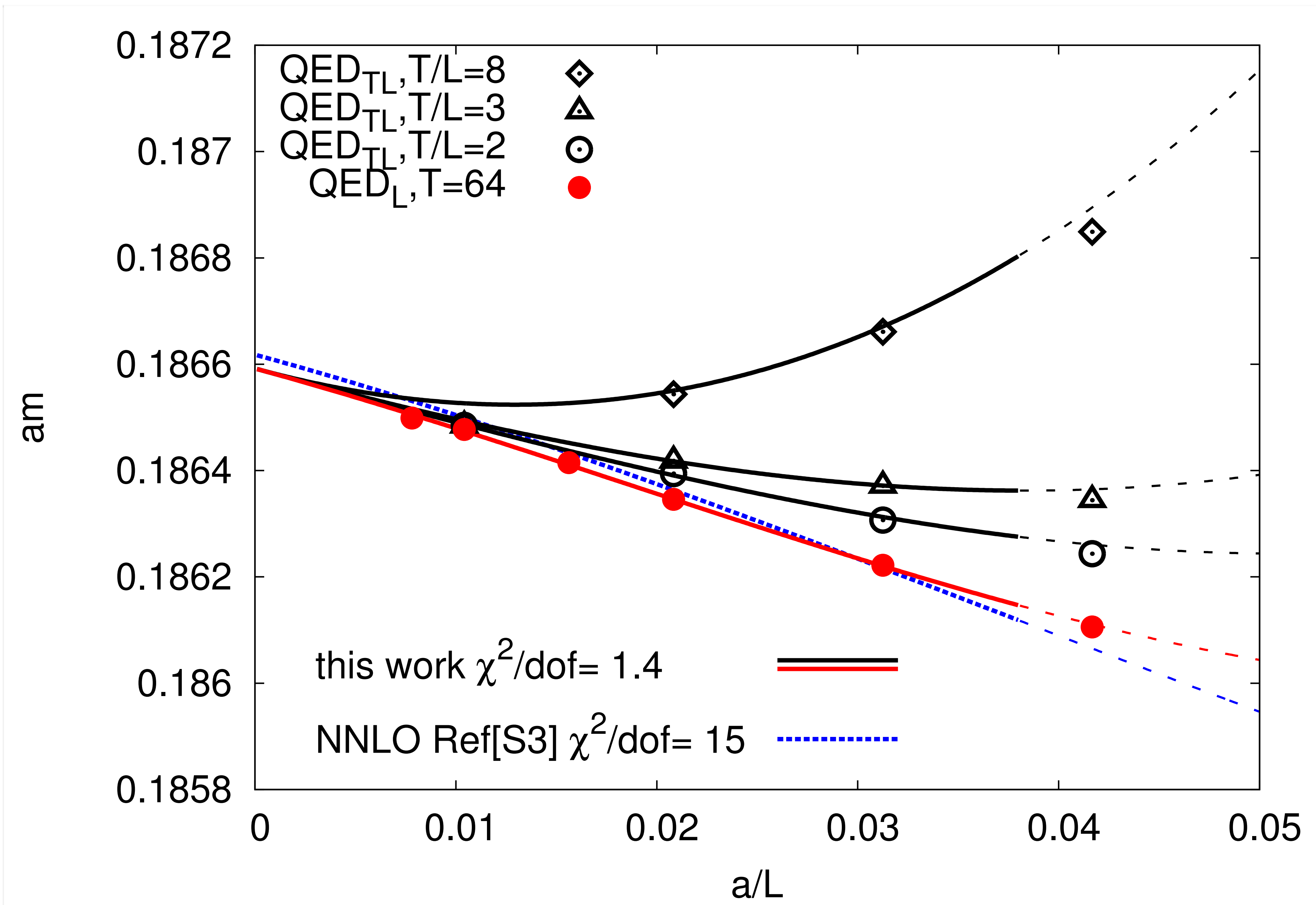
  [BMWc 2015, Science 347, pp. 1452–1455]

- $\mathrm{QED_L}$ finite volume effects:

$$m(T,L) \underset{T,L\to+\infty}{\sim} m\left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\right) - \frac{3\pi}{(mL)^3}\right]\right\}$$

inverse powers of L, independent of T

Pure QED test



Pure QED simulations (quenched) from [BMWc 2015, Science 347, pp. 1452–1455]
[S3] = [Z. Davoudi and M. J. Savage, PRD 90(5), p. 054503, 2014] (more on that later)

QCD+QED$_\mathrm{L}$ finite-volume effects

- What about **composite particles** (QCD + QED)?
- 2008: SU(3) PQChPT
  [M. Hayakawa and S. Uno, Prog. Theor. Phys. 120(3), pp. 413–441, 2008]
- 2010: SU(2) PQChPT + heavy kaons
  [RBC-UKQCD, 82(9), p. 094508, 2010]
- 2014: NREFTs mesons, baryons, nuclei and HVP

$$m(L) \underset{L\to+\infty}{\sim} m\left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\right) + \mathrm{O}\left(\frac{1}{L^3}\right)\right]\right\}$$

  [Z. Davoudi and M. J. Savage, PRD 90(5), p. 054503, 2014]
- 2015: Ward identities: **NLO is universal**
  [BMWc 2015, Science 347, pp. 1452–1455, 2015]
- 2015: non-local effects in NREFTs
  [Fodor *et al.*, arXiv:1502.06921, 2015]
  [J.-W. Lee and B.C. Tiburzi, arXiv:1508.04165, 2015]

Non-local effects in NREFTs

[Fodor *et al.*, arXiv:1502.06921, 2015]
[J.-W. Lee and B.C. Tiburzi, arXiv:1508.04165, 2015]

- $\mathrm{QED}_{\mathrm{L}}$ (and $\mathrm{QED}_{\mathrm{TL}}$) are **non-local theories**.

- **No hard coupling between IR and UV**. Checked at 2 loop in QED.
  [BMWc, private communication, 2015]

- NRQED: non-local particle-antiparticle vertex.

- Not taken into account in [D. & S., 2014], explaining the observed discrepancy.

Alternatives to zero-mode subtraction

- **Massive photons** [M.G. Endres *et al.*, arXiv:1507.08916]
  - Local, additional photon mass extrapolation necessary.
- **Zero-mode bounding** [QCDSF, arXiv:1508.06401]
  - Use of shift symmetry to impose $-\frac{\pi}{L_\mu} \leq A_\mu(0) < \frac{\pi}{L_\mu}$.
  - Finite-volume effects unknown.
- **C* boundary conditions ($\mathrm{QED_C}$)** [B. Lucini *et al.*, arXiv:1509.01636]
  - Charge conjugation at the volume boundary, local.
  - Finite-volume effects smaller than $\mathrm{QED_L}$.
  - Not implemented in QCD+QED so far.

    Used for $K \to \pi\pi$ decays [RBC-UKQCD, PRL, 115(2), p. 212001, 2015]

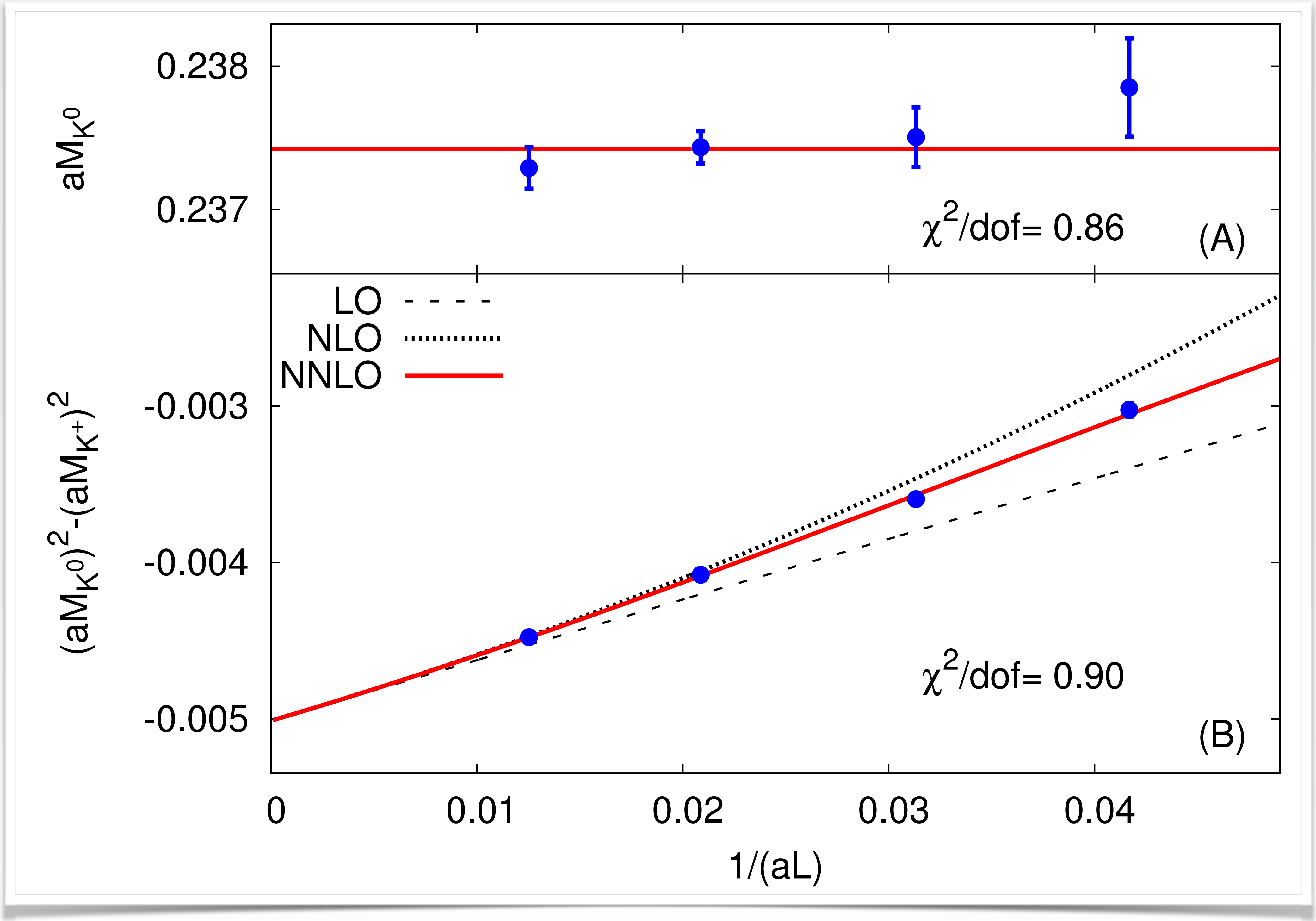
Electro-quenched approximation

- Electro-quenched approximation: **charged valence quarks**, but **neutral sea quarks**
- **Non-unitary** theory (partially quenched)
- **Greatly reduce** the computational cost
- Missing contributions are large-N_c and $SU(3)$ flavour suppressed: $O(10\%)$ of EM effects
- In agreement with PQChPT estimates
  [J. Bijnens & N. Danielsson, PRD 75(1), p. 014505, 2007]

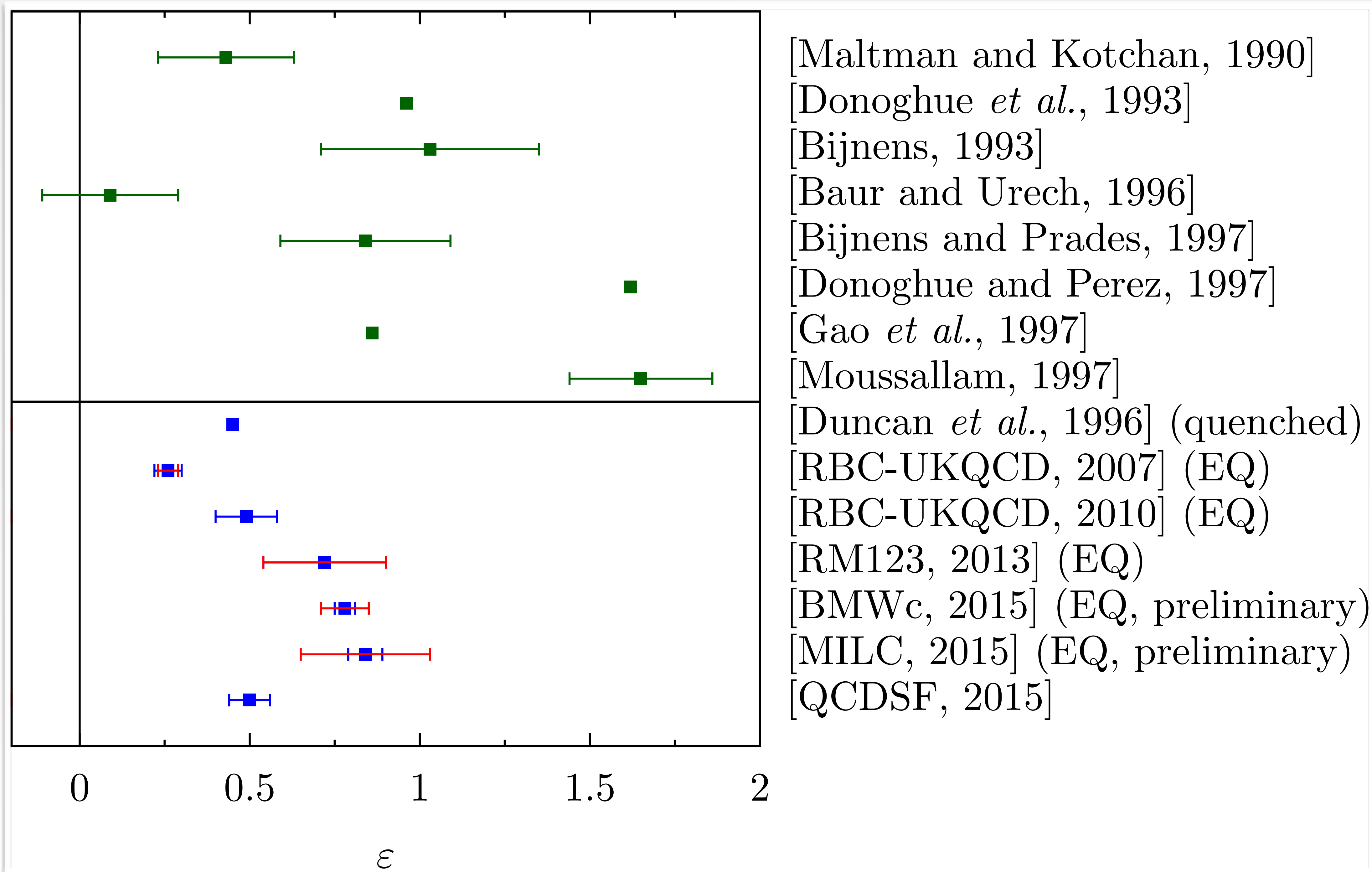
Isospin splittings in the hadron spectrum

Finite volume effects

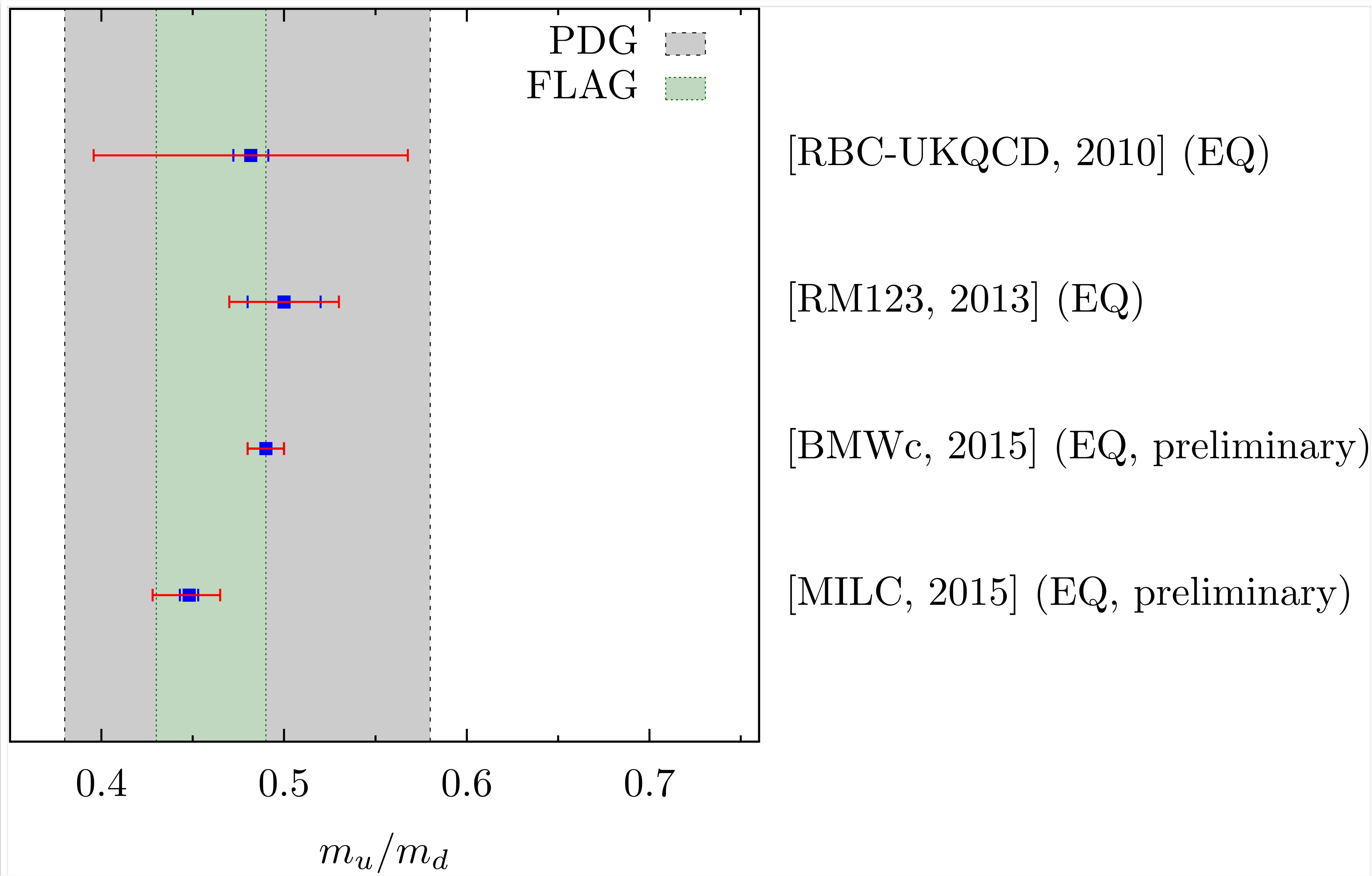
[BMWc 2015, Science 347, pp. 1452–1455, 2015]



Dashen's theorem corrections

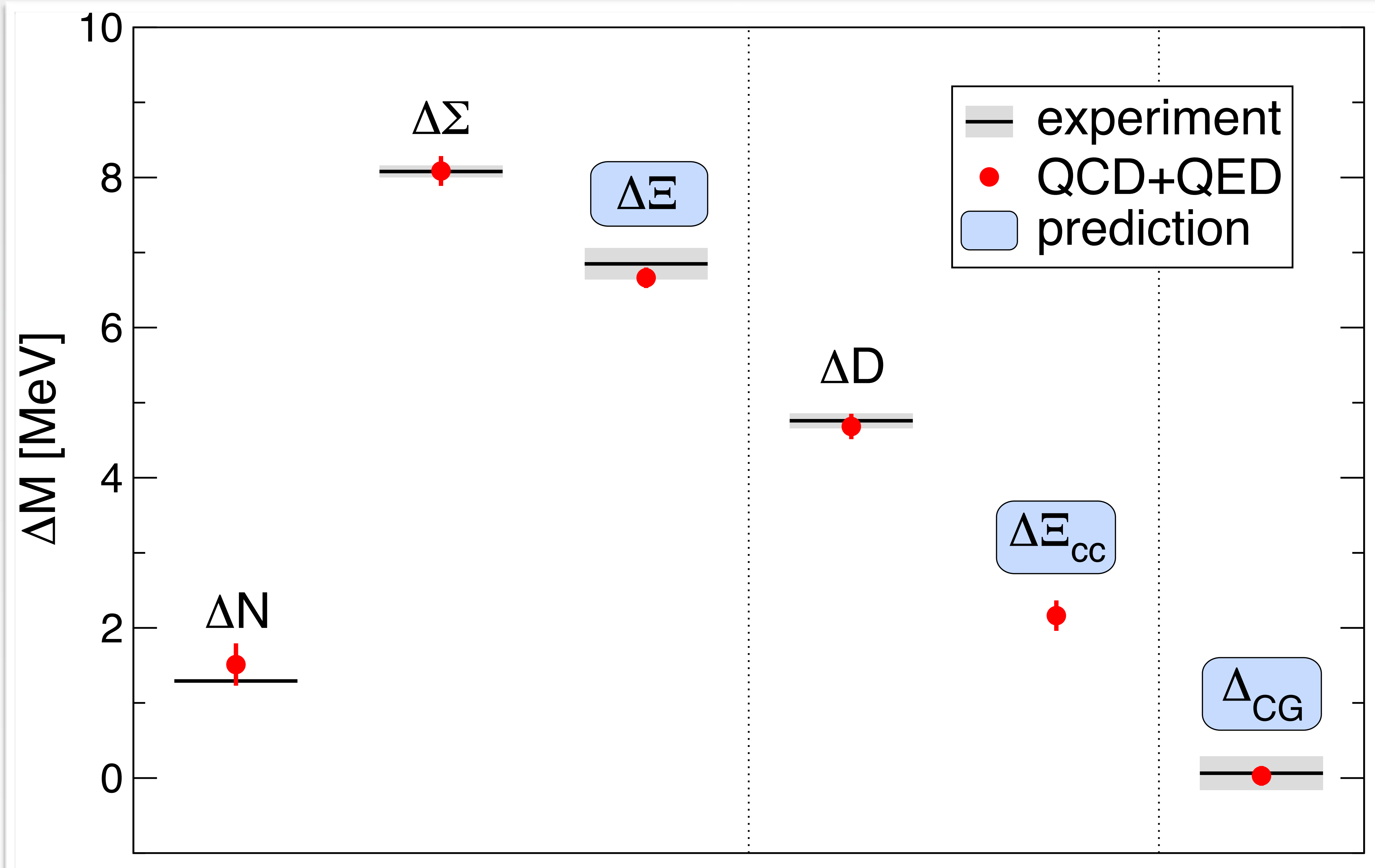


Light quark mass ratio



Hadron spectrum splittings

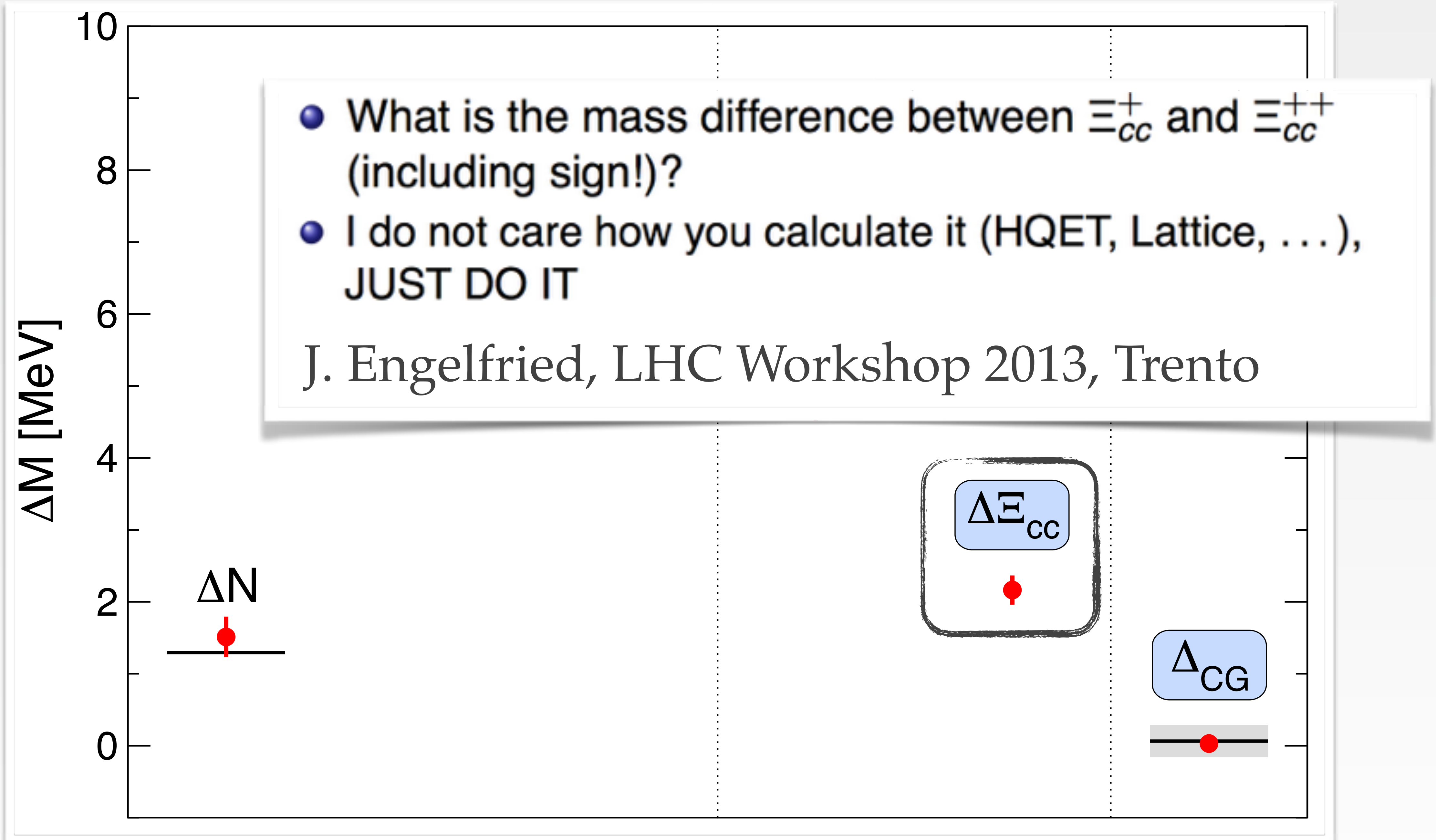
[BMWc 2015, Science 347, pp. 1452–1455, 2015]



$\Delta_{\mathrm{CG}} = \Delta M_N - \Delta M_\Sigma + \Delta M_\Xi$ (Coleman-Glashow relation)

Hadron spectrum splittings

[BMWc 2015, Science 347, pp. 1452–1455, 2015]



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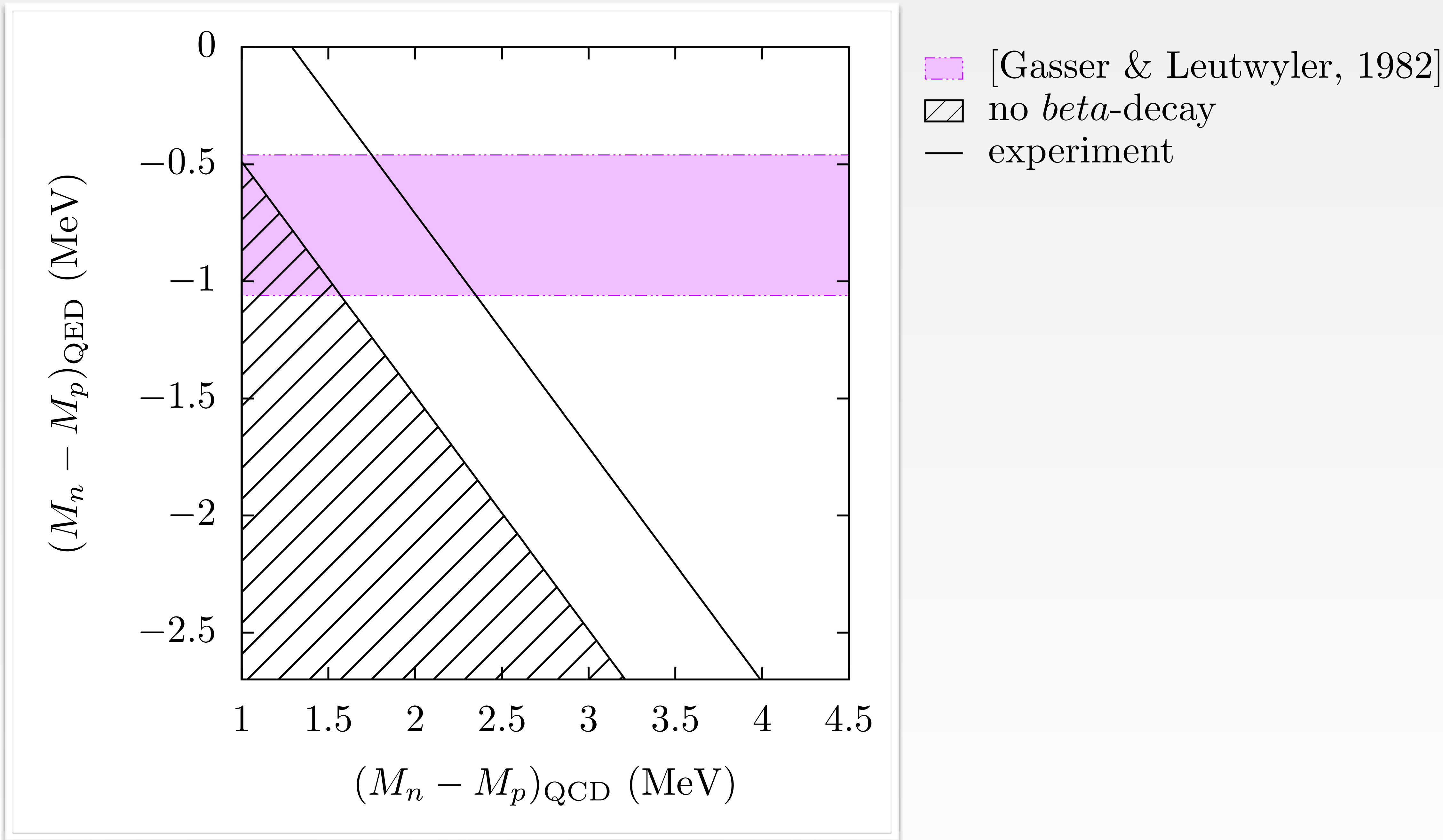
Nucleon mass splitting



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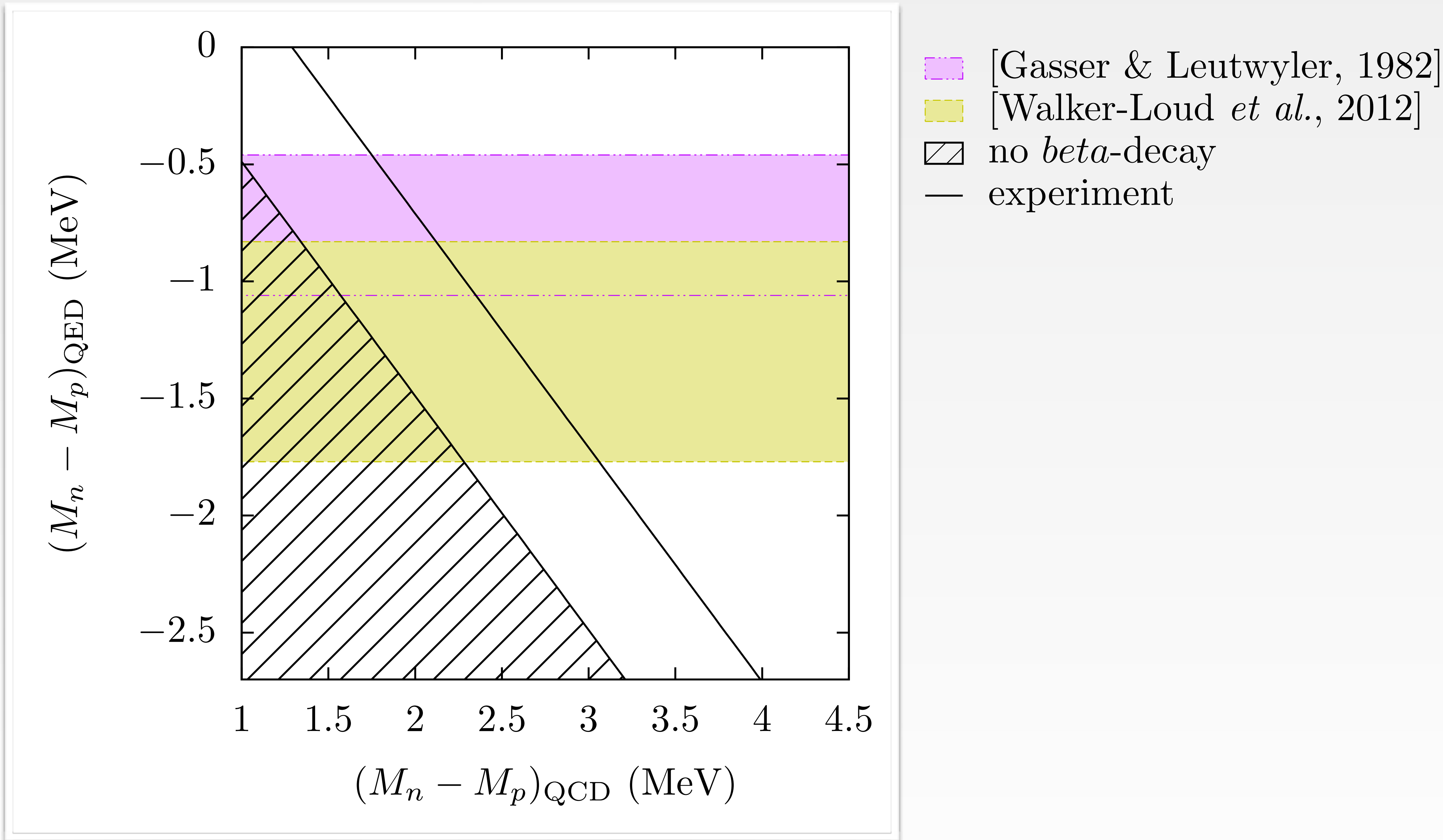

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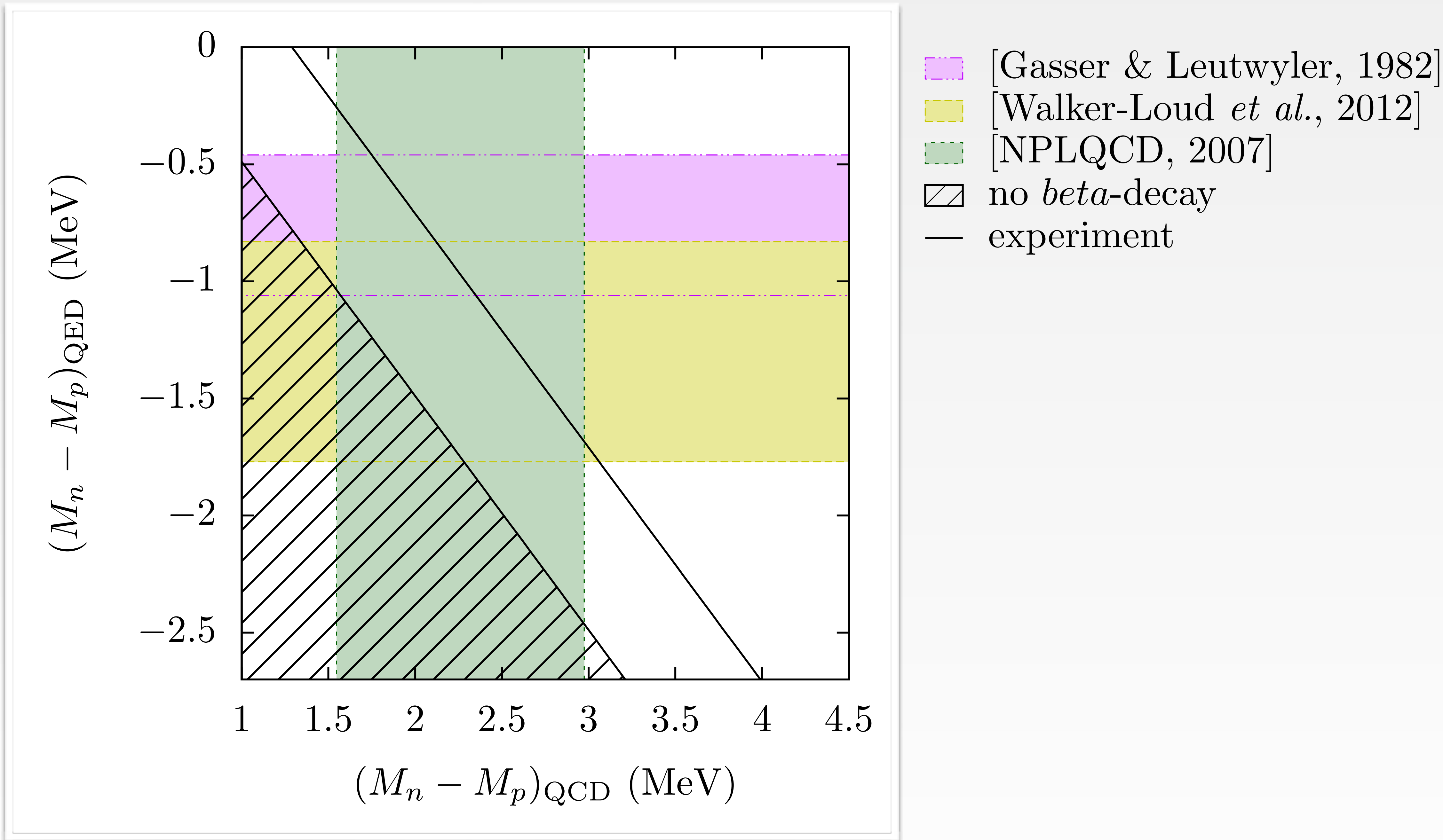

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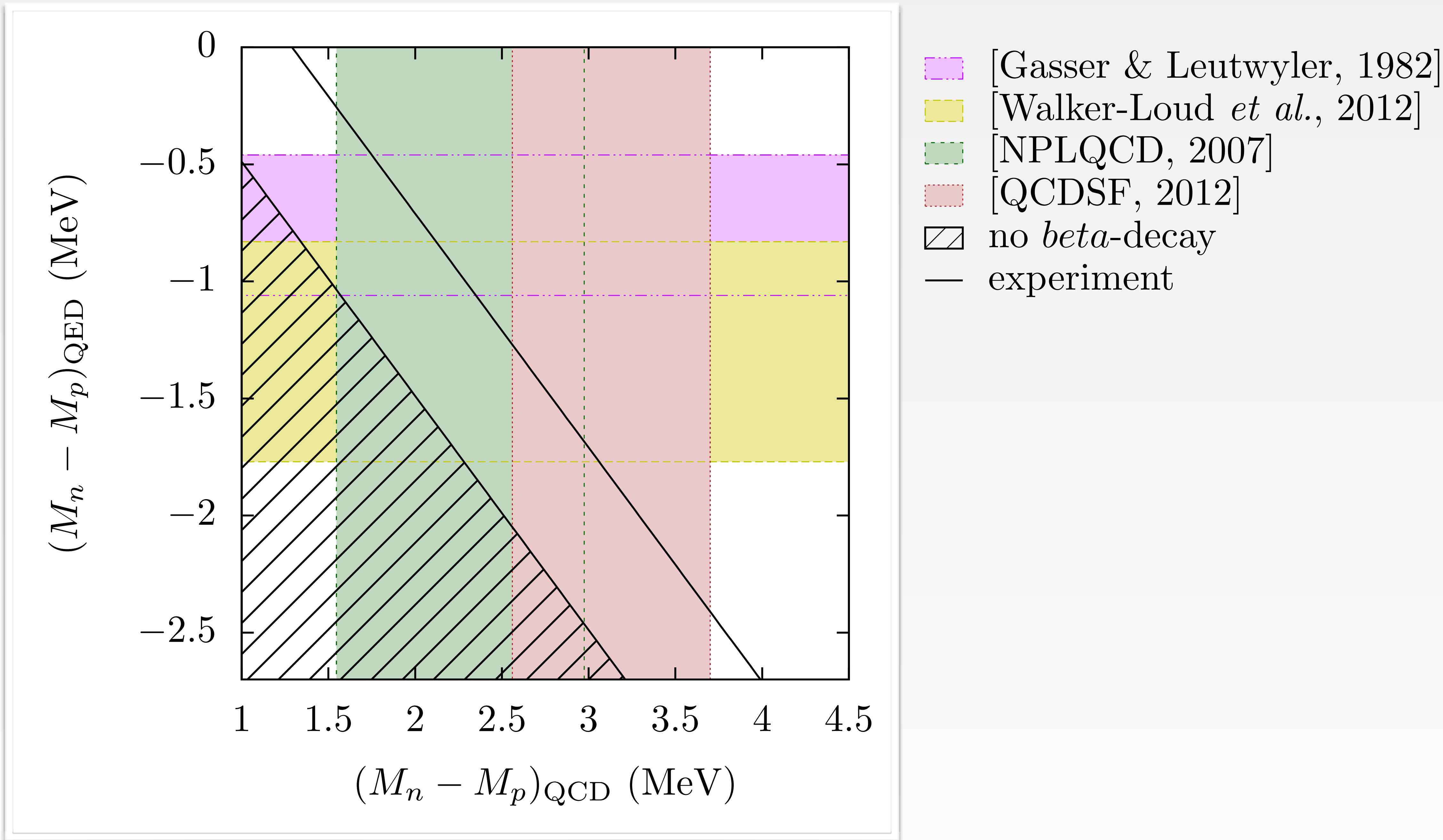

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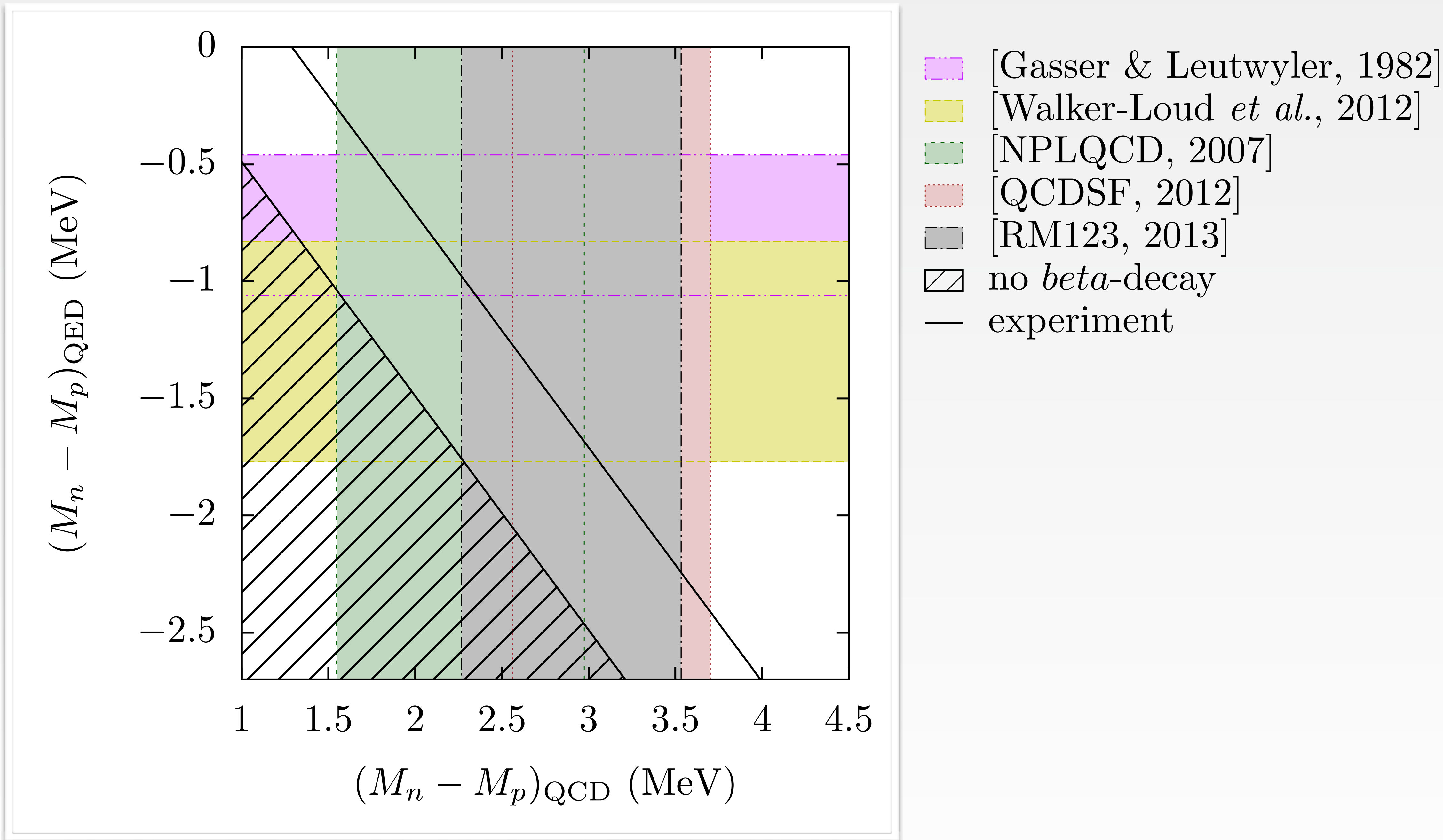

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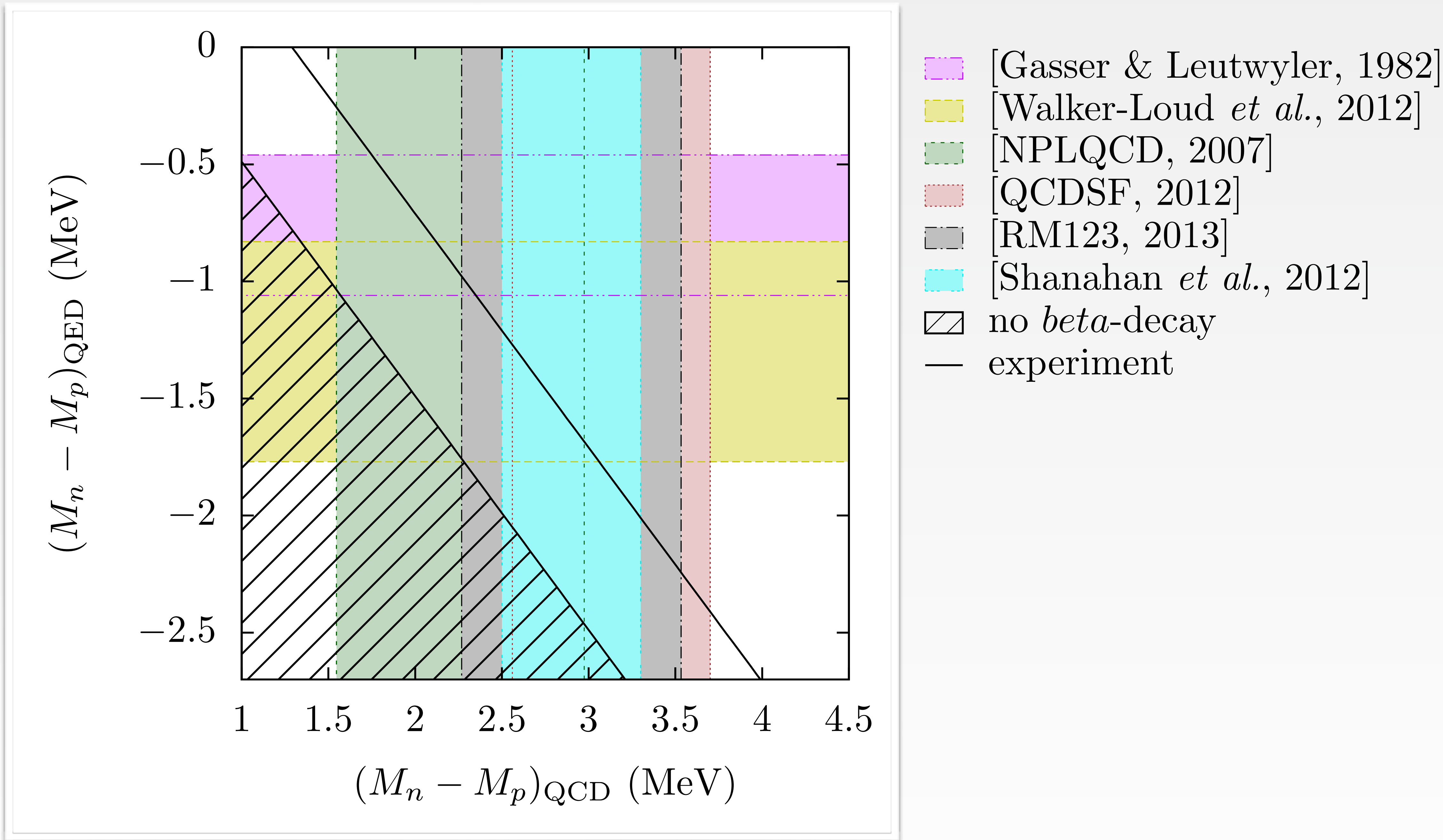

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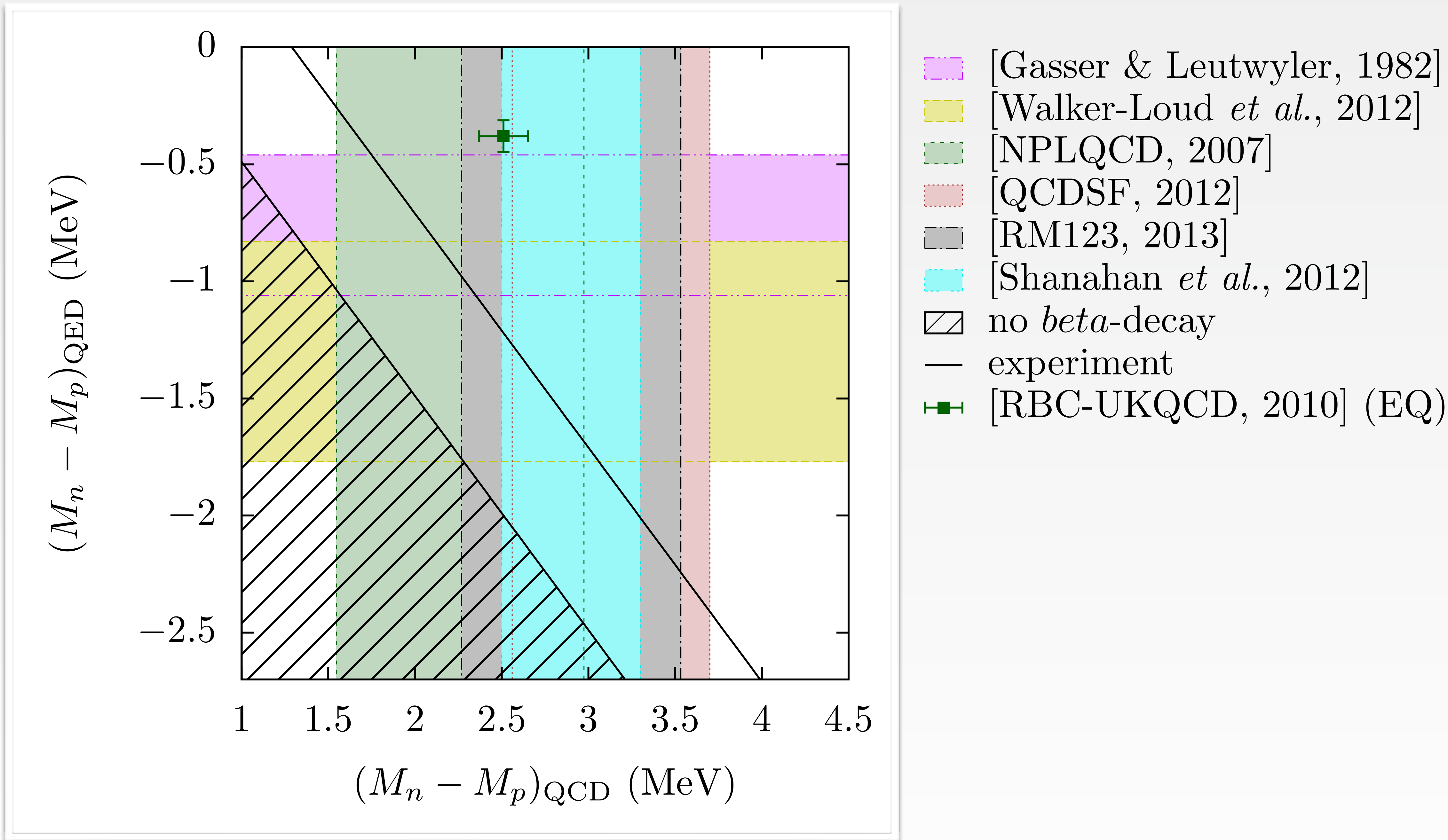

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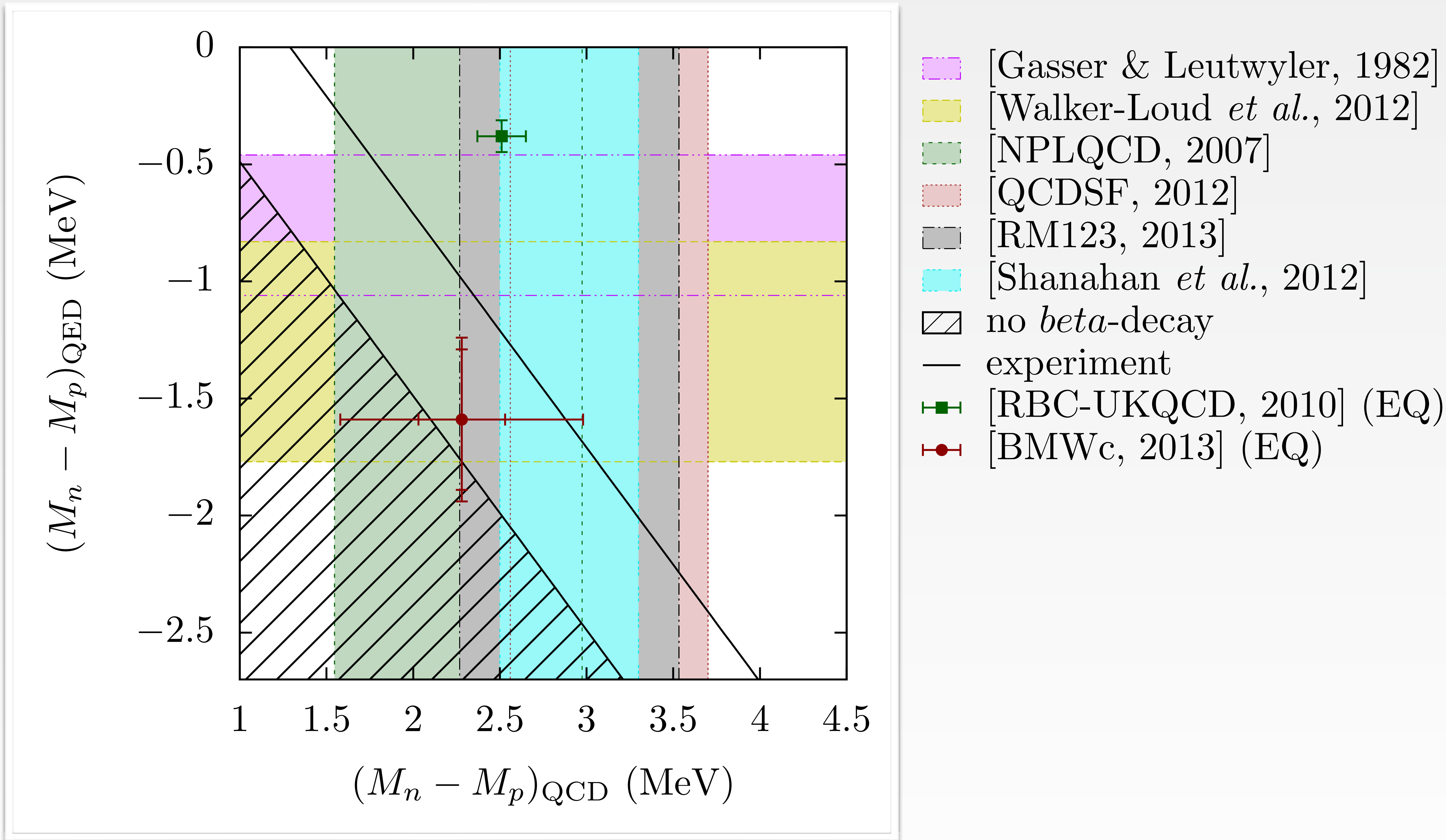

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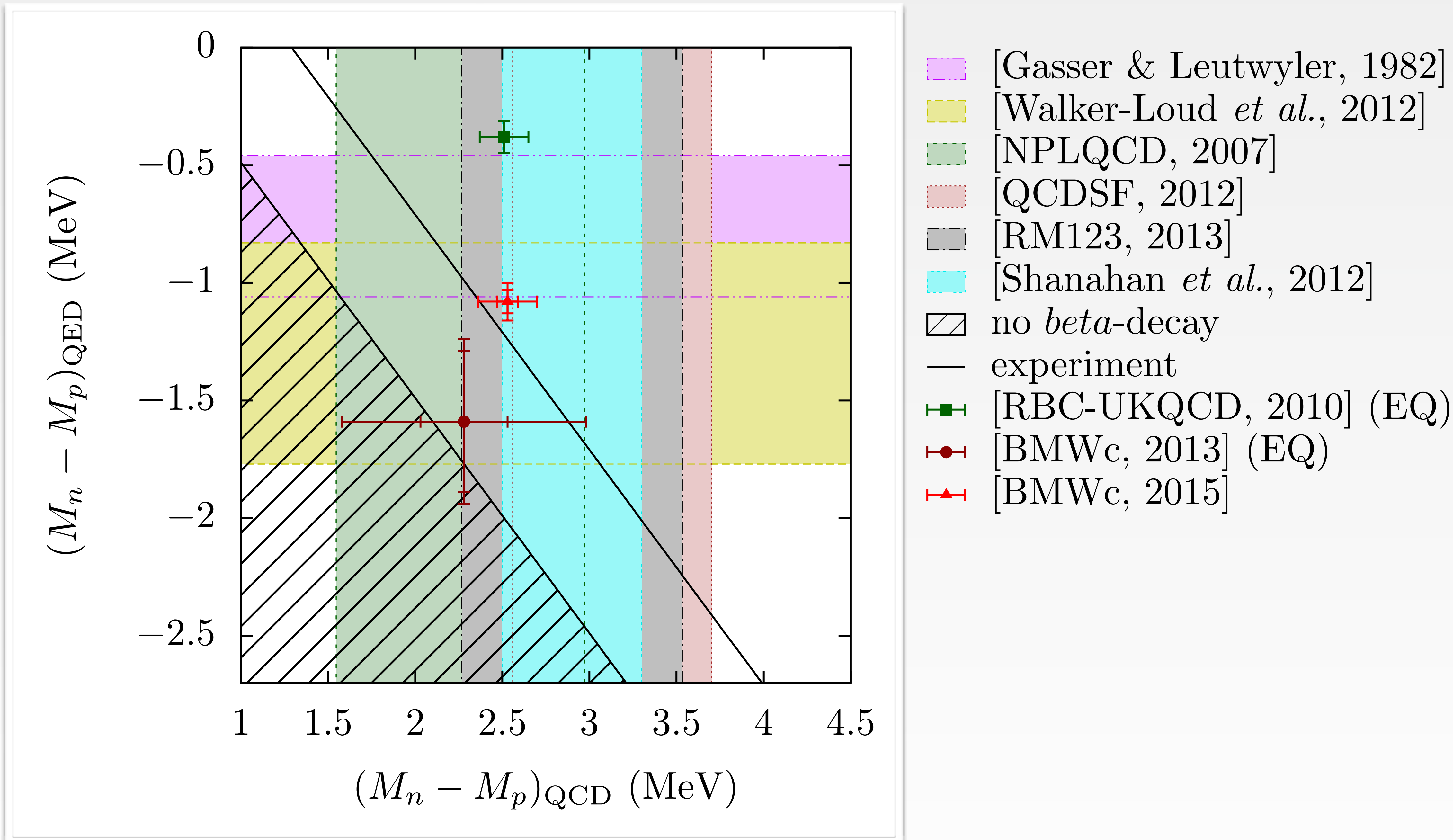

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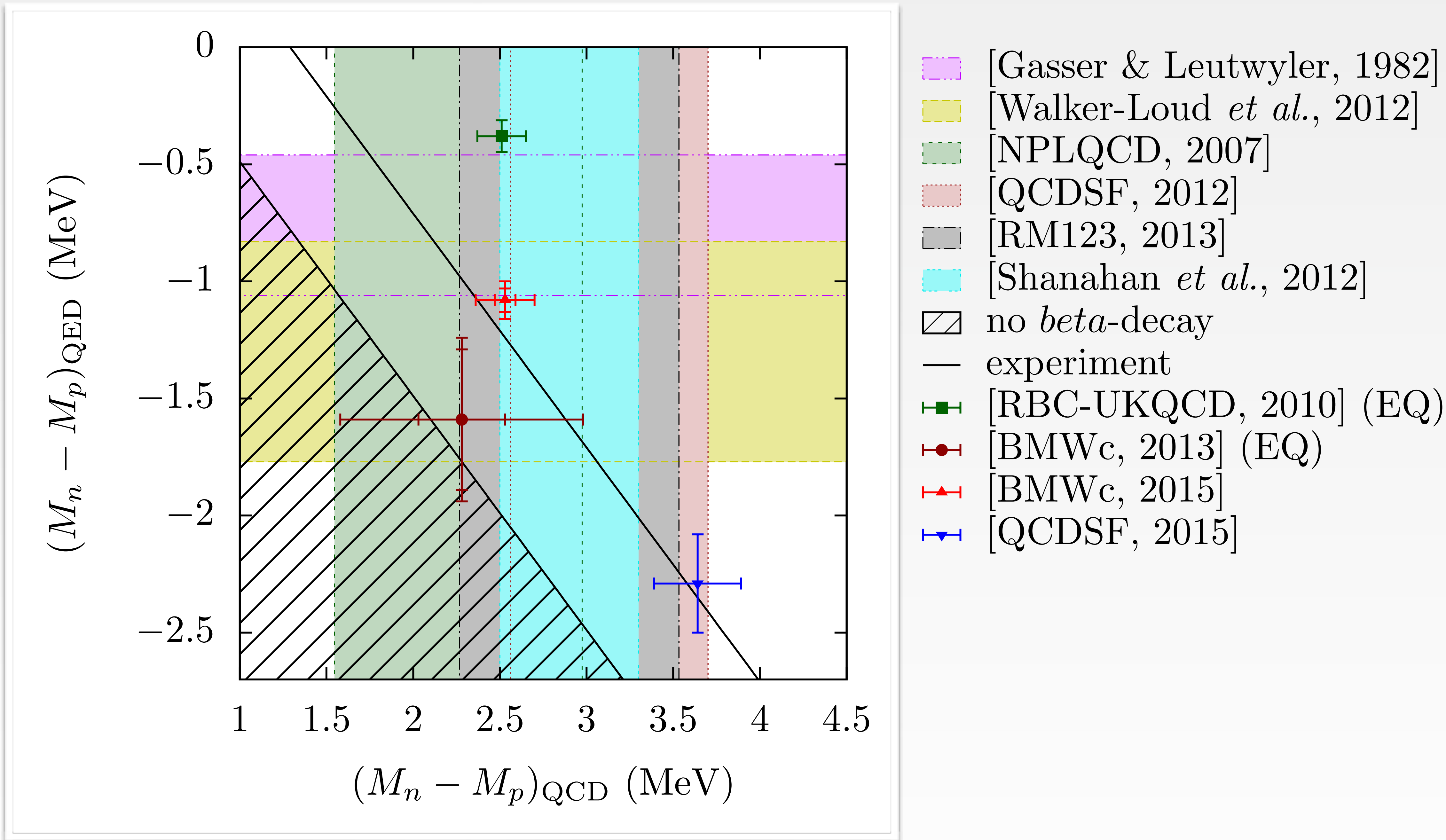
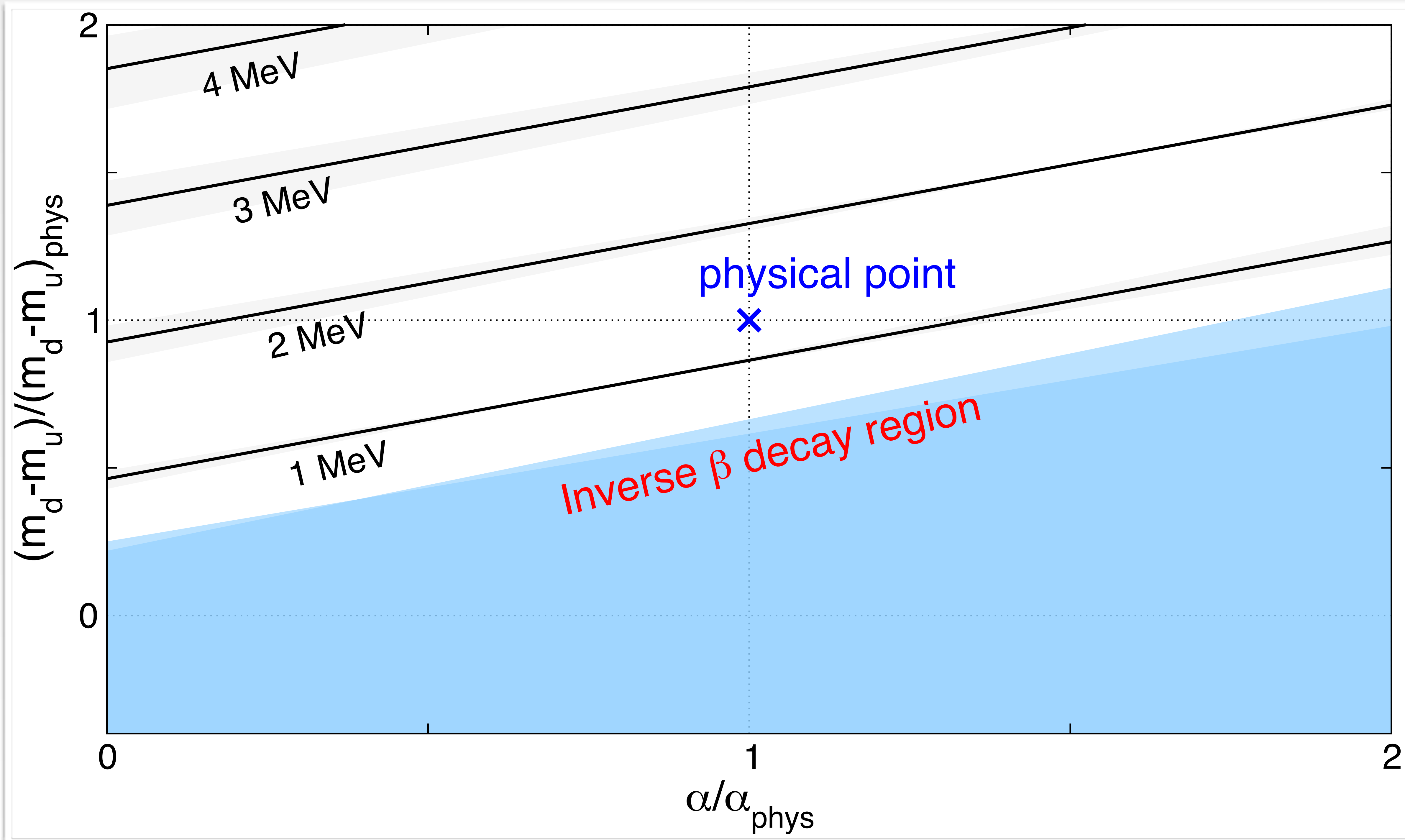


Constraint on SM parameters

[BMWc 2015, Science 347, pp. 1452–1455, 2015]



Summary & outlook

Summary

- We now have a good understanding of QCD+QED on a finite lattice.
- Finite-size effects on masses are now **well controlled**.
- First **full simulations of the low-energy SM** with a potential precision of $\mathrm{O}[(N_c m_b^2)^{-1}, \alpha^2] \sim 10^{-4}$.
- The isospin splittings in the hadron spectrum are determined with a **high accuracy and full control of uncertainties**.
- The nucleon mass splitting is determined as a $> 5\sigma$ effect.

Outlook

- Unquenched computations of the light quark masses and Dashen's theorem corrections.
- QED corrections to g-2.
- Corrections to matrix elements ($K_{\ell 3}$, $K \to \pi\pi$)
  cf. [N. Carrasco *et al.*, PRD 91(7), p. 074506, 2015]

Thank you!