Higher Dimensional Theories and the Banks-Zaks Fixed Point of QCD

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Overview

1. The Renormalization Group and Fixed Points
   - Concept of The Renormalization Group
   - Critical Exponents
   - Universality and Higher Dimensional Theories
2. $O(N) \times O(m)$ Landau-Ginzburg-Wilson Model in $6 - 2\epsilon$ Dimensions
3. Banks-Zaks Fixed Point in QCD and Renormalization Scheme Independence

The Renormalization Group

In perturbative QFT we have to regularise the divergences that occur in loop diagrams.

Dimensional regularisation $\Rightarrow$ Analytically continue the space-time to d-dimensions. However, still need the action to be dimensionless $[S] = 0$

Rescale coupling constant in such a way that it is dimensionless in d-dimensions $g \to g\mu^{\frac{\epsilon}{2}}$, by doing this we need to introduce an arbitrary energy scale μ

Renormalization group theory postulates that one can change the arbitrary scale of the theory in such a way that the physics on energy scales $< \mu$ remains constant.

The Renormalization Group

Problem: Physical quantities *cannot* depend on arbitrary scales.

Resolution is via the renormalization group equation.

Take the bare Green's function $\Gamma_0(n) = \langle \phi_0(p_1) \ldots \phi_0(p_n) \rangle$, clearly $\mu \frac{d}{d\mu} \Gamma_{0(n)} = 0$
But, $\Gamma_{(n)}$ are not unconnected as $\phi_0 = \sqrt{Z_\phi} \phi \quad \Rightarrow \quad \Gamma_{0(n)} = Z_\phi^{\frac{n}{2}} \Gamma_{(n)}$

$$\Rightarrow \boxed{\mu \frac{d}{d\mu} \left(Z_\phi^{\frac{n}{2}} \Gamma_{(n)} \right) = 0}$$

From this we can deduce

Callan-Symanzik Equation

$$0 = \left[\mu \frac{\partial}{\partial \mu} + \mu \frac{\partial g}{\partial \mu} \frac{\partial}{\partial g} + \mu \frac{\partial m}{\partial \mu} \frac{\partial}{\partial m} + \frac{n}{2} \frac{\mu}{Z_\phi} \frac{\partial Z_\phi}{\partial \mu} \right] \Gamma_{(n)}$$

The Renormalization Group

Callan-Symanzik Equation

$$0 = \left[\mu\frac{\partial}{\partial\mu} + \mu\frac{\partial g}{\partial\mu}\frac{\partial}{\partial g} + \mu\frac{\partial m}{\partial\mu}\frac{\partial}{\partial m} + \frac{n}{2}\frac{\mu}{Z_\phi}\frac{\partial Z_\phi}{\partial\mu}\right]\Gamma_{(n)}$$

$$\underbrace{\beta(g) = \mu\frac{\partial g}{\partial\mu}}_{\beta-\text{function}} \qquad \underbrace{\gamma_m(g) = \frac{\mu}{m}\frac{\partial m}{\partial\mu}}_{\text{mass anomalous dimension}} \qquad \underbrace{\gamma_\phi(g) = \mu\frac{\partial(\ln(Z_\phi))}{\partial\mu}}_{\text{wavefunction anomalous dimension}}$$

Every renormalization group function is scheme independent at its leading term. The β-function is scheme independent to two loops in mass independent schemes with only one coupling constant.

Renormalization Group Flows

In perturbation theory (In $4-$dimensions),

$$\beta(g) = (d-4)g + Ag^2 + Bg^3 + Cg^4 + \dots$$

($A < 0 \Leftarrow$ QCD)

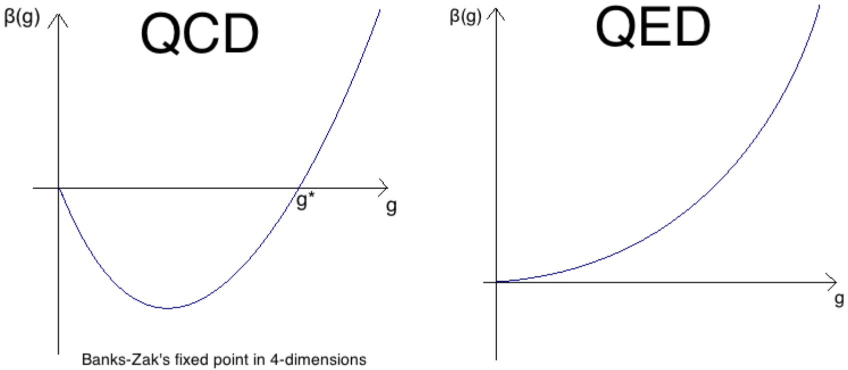
General property: There exists a value g^* for which $\beta(g^*) = 0$, which are known as **fixed points.** These underlie phase transitions.
The non-trivial fixed point in $d-$dimensions is known as the **Wilson-Fisher fixed point.**

The **conformal window** is the range of N_f values for which the non-trivial fixed point exists.

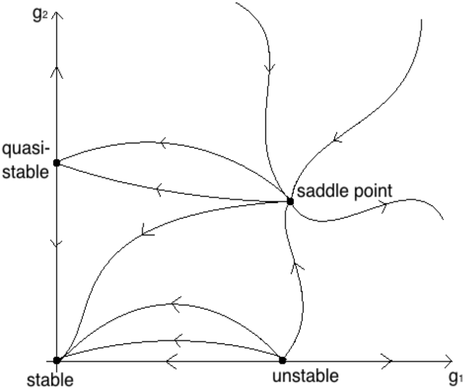
Example (QCD)

If we have $\quad \beta(g) = Ag^2 + Bg^3, \qquad g^* = -\frac{A}{B} > 0$
In QCD, A and B have opposite signs for $9 \leq N_f \leq 16$ (Conformal window)

Renormalization Group Flows



Renormalization Group Flows



UV/IR stability: sending $\mu \to \infty$ will send the RG flow to certain fixed points. These will be UV stable. The fixed points where the flow diverges away from will be UV unstable.
Reversing the flow direction ($\mu \to 0$) will show the IR stable and unstable fixed points.

Critical Exponents

If the $\beta-$function has a nontrivial fixed point at the value g^*, then the renormalization group functions evaluated at g^* are termed **critical exponents** which are thought to be renormalization group invariants.

Other critical exponents can also be found using scaling relations.

Critical exponents describe the behaviour of physical quantities near continuous phase transitions.

$$\underbrace{\omega = \beta'(g^*)}_{\text{measure of corrections to scaling}} \qquad \eta = \gamma_\phi(g^*)$$

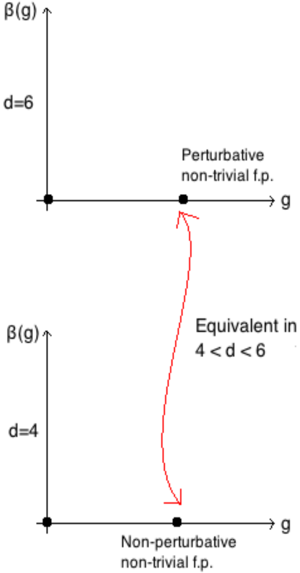
$$\underbrace{\rho = \gamma_{\bar{\psi}\psi}(g^*)}_{\substack{\textbf{Quark mass anomalous dimension exponent} \\ \text{is of primary interest because of its relation to the definition of conformal theory}}}$$

Universality

One defines a universality class as a set of theories which differ only by irrelevant parameters. In this case, the renormalization flows of these different theories all lead to the same IR physics, defined by the set of relevant parameters, because a modification to an irrelevant parameter will not have any consequence in the IR.

It ought to be possible to connect the fixed points in higher dimensional theories with CFTs in lower dimensions, including $d = 2$. In $d = 2$, CFTs have been classified. Since critical exponents are known exactly one could constrain the exponent estimated and allow us to extract the behaviour across several dimensions.

Higher Dimensional Theories



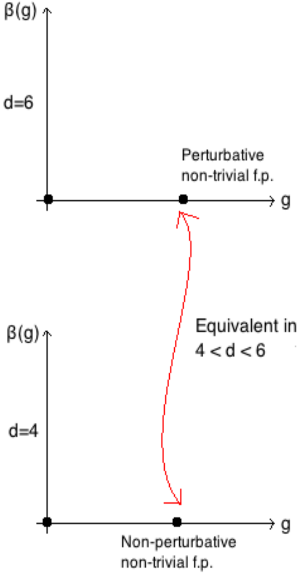
Why higher dimensional?

Perturbative fixed points in higher dimensions are connected to non-perturbative fixed points in lower dimensions. Possible to access these non-perturbative fixed points through higher dimensional theories perturbatively.

UV properties in one theory could be regarded as being driven by the IR behaviour of another.

Application to model building in higher dimensions.

Higher Dimensional Theories

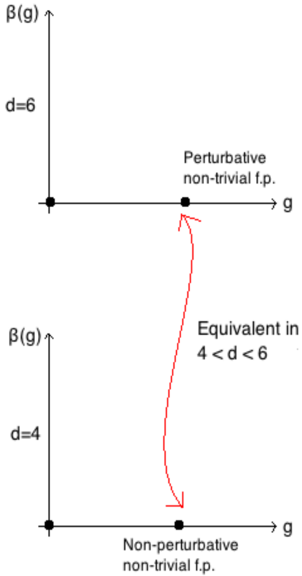


Can build a tower of theories in the same universality class

- Same symmetry group
- Renormalizable
- One connecting interaction in all theories (all other interactions are called spectator interactions)
- Check using large-N expansion of critical exponents.

Similar towers of gauge theories should be feasible, based on what has been found in the scalar theory case. $\Rightarrow$ Relevant to possible directions of BSM physics.

Higher Dimensional Theories



Examples:

Scalar - Connection of ϕ^3 theory with $O(N)$ ϕ^4 theory in the dimension range $4 < d < 6$ at the Wilson-Fisher fixed point via the large N expansion. Connection of ϕ^4 theory with NLσM in $2 < d < 4$

In $6 - 2\epsilon$ and $4 - 2\epsilon$ dimensions: Weakly coupled IR fixed points of the cubic and quartic scalar theories. In $4 + 2\epsilon$ and $2 + 2\epsilon$ dimenions: Weakly coupled UV fixed points of the $O(N)$ ϕ^4 and NLσM theories. *Fei, Giombi, Klebanov (2014) and Gracey (2015)*

Gauge - May be able to access non-perturbative fixed point in QCD from 6−dimensional QCD. Colour confinement comes from non-perturbative regime. Similar work in higher dimensional QED. *Giombi, Klebanov, Tarnopolsky (2015) and Gracey (2015)*

Higher Dimensional Theories

Operators which are UV irrelevant may become IR relevant and dominate IR dynamics.

$$\Delta_{\mathcal{O}} = A - B\epsilon - C\epsilon^2 - D\epsilon^4 - \dots$$

(A, B, C, D are some constants)

$\mathcal{O}$ continues to be relevant as ϵ is increased i.e. $\epsilon = 1$ where $d = 6 - 2\epsilon$. Strongly suggests that $\mathcal{O}$ is relevant for the entire range $4 \leq d < 6$
We may find operators in higher dimensions that continue to be relevant in lower dimensions, thus potentially affecting the dynamics of that lower dimensional theory.

Looking at higher dimensional gauge theories could give an insight into physics beyond the standard model.

$O(N) \times O(m)$ Landau-Ginzburg-Wilson Model in $d = 6 - 2\epsilon$

$$\mathcal{L} = \frac{1}{2}(\partial\phi^{ai})^2 + \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}(\partial T^{ij})^2 + \frac{1}{2}g_1\sigma\phi^{a\alpha}\phi^{ai} + \frac{1}{2}g_3 T^{ij}\phi^{ai}\phi^{aj} + \frac{1}{6}g_2\sigma^3 + \frac{1}{2}g_4\sigma T^{ij}T^{ij} + \frac{1}{6}g_5 T^{ij}T^{jk}T^{ki}$$

Renormalizable 6−dimensional scalar theory. Same symmetries as 4−dimensional LGW model.
g_1, g_3 **common interactions** in both models, g_2, g_4, g_5 are **spectator interactions.**
T is an anti-symmetric (in $O(m)$) traceless field.

Number of Feynman Diagrams Calculated

	$\phi\phi$	$\sigma\sigma$	TT	$\phi\phi\sigma$	$\phi\phi T$	$\sigma\sigma\sigma$	$TT\sigma$	TTT
1-loop	2	3	3	4	5	3	4	5
2-loop	23	19	27	106	137	68	112	155
3-loop	514	343	589	-	4984	-	-	5857

Total 12963

$O(N) \times O(m)$ Landau-Ginzburg-Wilson Model in $d = 6 - 2\epsilon$

Non-perturbative results: $4-$dimensional non-trivial IR fixed point accessed non-perturbatively using conformal bootstrap. *Y. Nakayama, T. Ohtsuki (2014)*

Motivation:

- Conformal bootstrap proves there is an IR non-trivial fixed point we can access from $6-$dimensions.
- Setting $m = 1$ gives us the cubic ϕ^3 theory which connects to $O(N)$ ϕ^4 theory in $4 < d < 6$.
- To get a better understanding of the UV/IR duality.
- Connects to $4-$dim LGW theory, get a better picture of the whole universality class.

$O(N) \times O(m)$ Landau-Ginzburg-Wilson Model in $d = 6 - 2\epsilon$

For $N_f = 1000$

$$\eta_\phi = 0.00522536888758\epsilon - 0.01866980955\epsilon^2 - 0.00135288754531\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\eta_\sigma = 1.23233220028\epsilon - 0.784699586777\epsilon^2 - 0.297110840137\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\eta_T = 2.08594109981\epsilon - 0.258030086086\epsilon^2 + 0.191513285439\epsilon^3 + \mathcal{O}(\epsilon^4)$$

For $N_f = 1038$

$$\eta_\phi = 0.00502566263355\epsilon - 0.0178970261282\epsilon^2 - 0.00153158065316\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\eta_\sigma = 1.22189357673\epsilon - 0.743934128955\epsilon^2 - 0.287120134262\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\eta_T = 2.0819067641\epsilon - 0.242855351048\epsilon^2 + 0.179516633652\epsilon^3 + \mathcal{O}(\epsilon^4)$$

For $N_f = 1500$

$$\eta_\phi = 0.00343126422593\epsilon - 0.0119102603797\epsilon^2 - 0.00220603234381\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\eta_\sigma = 1.14508881058\epsilon - 0.468203077985\epsilon^2 - 0.175398286746\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\eta_T = 2.05243914251\epsilon - 0.142412681167\epsilon^2 + 0.105431668913\epsilon^3 + \mathcal{O}(\epsilon^4)$$

Banks-Zaks Fixed Point in QCD and Renormalization Scheme Independence

We can also look at exact dimensions. Important for SUSY and toy models.

When one computes the critical exponents the expressions ought to be the same at the Banks-Zaks fixed point for different renormalization schemes.
Critical exponents are physical quantities and hence renormalization scheme invariant. Want to see if this is indicated in our calculations.

Landau gauge $\beta-$function in the scheme S defined by

$$\beta_n^S(a,0) = \sum_{r=1}^{n} \beta_r^S a^{r+1} \tag{1}$$

Banks-Zaks fixed point a_L at the Lth loop order defined as the first non-trivial zero of

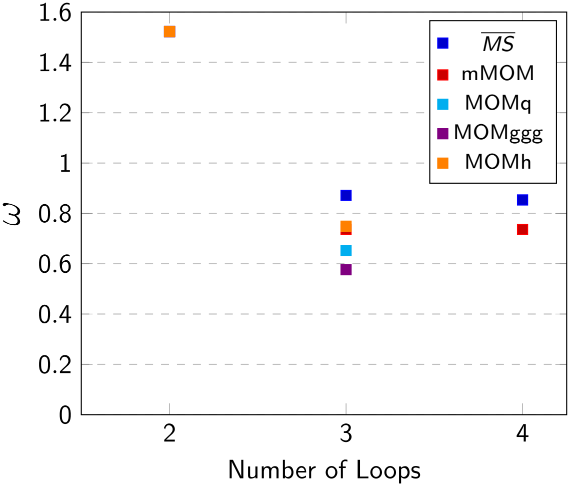
$$\beta_L^S(a_L,0) = 0 \tag{2}$$

J.A. Gracey, R.M. Simms (2015) Phys. Rev. D91 085037

Critical exponent ω at the Banks-Zaks fixed point

$$\omega_L = 2\beta_L'(a_L, 0)$$

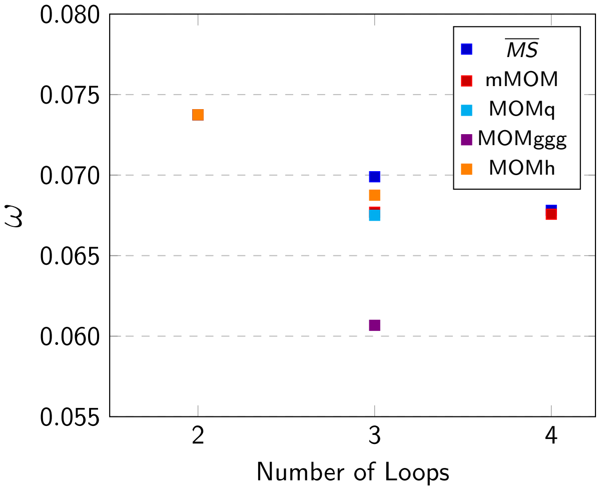
Value of the critical exponent ω in $SU(3)$ for $N_f = 10$



Critical exponent ω at the Banks-Zaks fixed point

$$\omega_L = 2\beta'_L(a_L, 0)$$

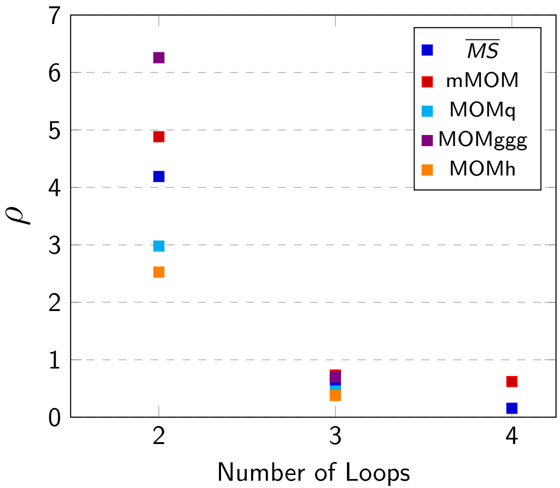
Value of the critical exponent ω in $SU(3)$ for $N_f = 14$



Critical exponent ρ at the Banks-Zaks fixed point

$$\rho_L = -2\gamma_{\bar{\psi}\psi L}(a_L, 0)$$

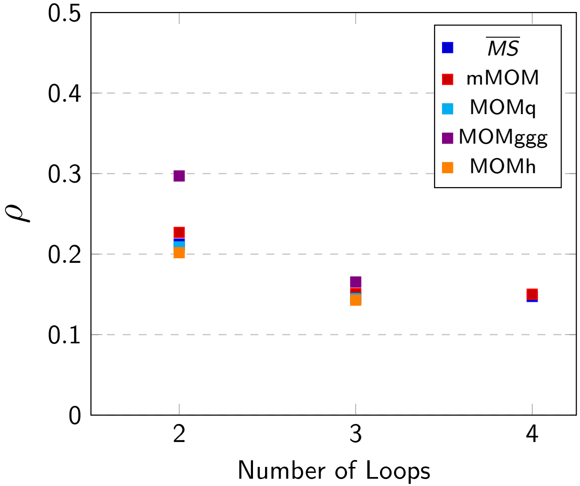
Value of the critical exponent ρ in $SU(3)$ for $N_f = 10$



Critical exponent ρ at the Banks-Zaks fixed point

$$\rho_L = -2\gamma_{\bar{\psi}\psi L}(a_L, 0)$$

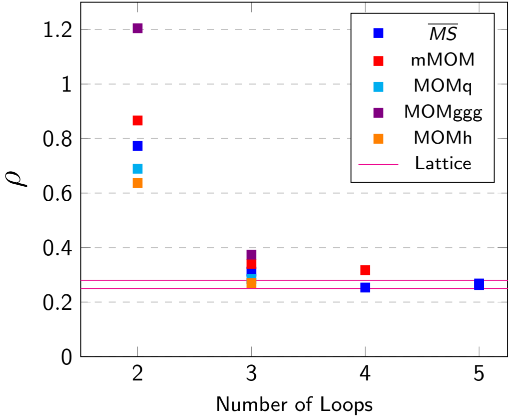
Value of the critical exponent ρ in $SU(3)$ for $N_f = 14$



Critical exponent ρ at the Banks-Zaks fixed point

$$\rho_L = -2\gamma_{\bar{\psi}\psi L}(a_L, 0)$$

Value of the critical exponent ρ in $SU(3)$ for $N_f = 12$



Five loop: P.A. Baikov, K.G. Chetyrkin & J.H. Kühn (2014)

Lattice: Cheng, Hasenfratz, Liu, Petropoulos & Schaich (2014). Lombardo, Miura, Nunes da Silva & Pallante (2014)

Summary

- Interest in higher dimensional QFT's due in part to non-trivial fixed points in 4-dimensional theories.
- UV properties in one theory could be regarded as being driven by the IR behaviour of another.
- Gauge theories with different symmetry groups could give an insight into the theory believed to lie beyond the standard model. For instance, $SU(3) \times SU(2) \times U(1)$ may have a non-trivial fixed point which connects with a unified theory.
- Understanding the low energy Yang-Mills theory and QCD is a major goal.

Thank you!