Baby Skyrmions in AdS

Matt Elliot-Ripley

Durham University

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Outline

- The baby Skyrme model in flat space
- The baby Skyrme model in AdS
- Point particle approximation for baby Skyrmions in AdS
- Conclusions

The baby Skyrme model

- Theory given in terms of a three-component unit vector $\boldsymbol{\phi}(x)$ with Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} + \frac{\kappa^2}{4}(\partial_\mu \boldsymbol{\phi} \times \partial_\nu \boldsymbol{\phi}) \cdot (\partial^\mu \boldsymbol{\phi} \times \partial^\nu \boldsymbol{\phi}) + m^2(1 - \boldsymbol{\phi} \cdot \boldsymbol{n})$$

- Finite (static) energy requires $\boldsymbol{\phi} \to \boldsymbol{n}$ as $|\boldsymbol{x}| \to \infty$. W.L.O.G. can take $\boldsymbol{n} = (0, 0, 1)$.
- Points at spatial infinity are identified, so $\boldsymbol{\phi} : S^2 \to S^2$. This gives rise to a winding number

$$B = -\frac{1}{4\pi} \int \boldsymbol{\phi} \cdot (\partial_x \boldsymbol{\phi} \times \partial_y \boldsymbol{\phi})\, dxdy$$

- Usual Bogomolny argument gives $E \geq 4\pi|B|$.
- Soliton solutions are permitted by Derrick's argument.

Radial baby Skyrmions, tail decays and mass

- Radial solutions can be obtained from the hedgehog ansatz

$$\boldsymbol{\phi} = (\sin f(r)\cos B\theta, \sin f(r)\sin B\theta, \cos f(r))$$

- The size of the baby Skyrmion μ scales as $\mu \sim \sqrt{\kappa/m}$
- Linearising the equations of motion for large radii give the asymptotic tail decays as

$$f(r) \sim \begin{cases} r^{-B}, & \text{if } m = 0 \\ \frac{1}{\sqrt{r}} e^{-mr}, & \text{if } m \neq 0 \,. \end{cases}$$

Motivating baby Skyrmions in AdS

- The Skyrme model has been derived as a low energy effective field theory for QCD in the large colour limit, and the baby Skyrme model can be viewed as a lower dimensional analogue of this.
- In holographic QCD, baryons are described by (extended) Skyrmions on the boundary of an "AdS-like" spacetime.
- Baby Skyrmions have been used to study a low-dimensional analogue of the Sakai-Sugimoto model of holographic QCD.
- Solitons in pure AdS spaces have also been of interest recently e.g. monopole and monopole walls in AdS, Skyrmions in hyperbolic space.

Coordinates in AdS

- Different coordinate systems will be useful for studying baby Skyrmions in AdS
- *Sausage coordinates* are useful numerically as they give a global coordinate system over a finite range:

$$ds^2 = -\left(\frac{1+r^2}{1-r^2}\right)^2 dt^2 + \frac{4L^2}{(1-r^2)^2}(dr^2 + r^2 d\theta^2)$$

- *Global coordinates* are useful analytically as the radial coordinate coincides with the geodesic distance from the origin:

$$ds^2 = -\cosh^2\frac{\rho}{L}dt^2 + d\rho^2 + L^2\sinh^2\frac{\rho}{L}d\theta^2$$

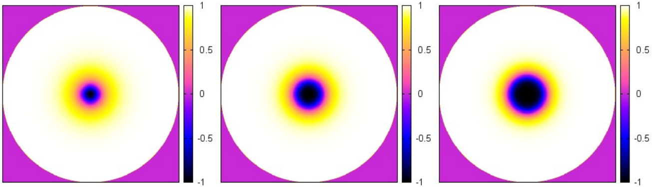
Radial solutions and tail decays

- In AdS radial solutions can still be generated using the hedgehog ansatz
- It can be shown numerically that the size of baby Skyrmions in AdS scales as $\mu \sim \sqrt{\kappa L}$
- The asymptotic tail decays of radial solutions are

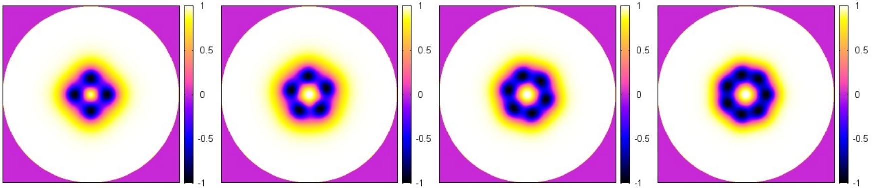
$$f(r) \sim e^{-(1+\sqrt{1+m^2L^2})\rho/L}$$

- The AdS curvature can be interpreted as adding an effective pion mass. Tails of solutions decay exponentially even in the massless ($m = 0$) case.
- The flat space tail decays can be obtained by taking the limit $L \to \infty$

- Numerical solutions were generated using a gradient flow algorithm using fourth-order accurate derivatives
- Initial conditions for solutions with $B > 1$ were generated using the product ansatz
- Minimal energy solutions for $1 \leq B \leq 3$ are radially symmetric



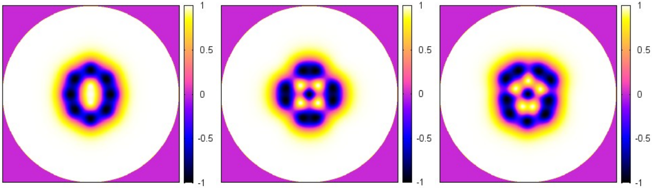
- Solutions for $4 \leq B \leq 7$ form regular polygons



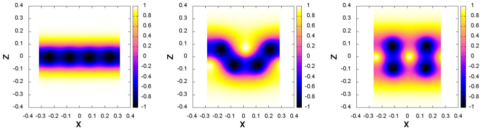
- The relative phase difference between neighbouring constituents is π ($\pi \pm \pi/B$) for even (odd) charges

Numerical solutions: $8 \leq B \leq 10$ - the first pop

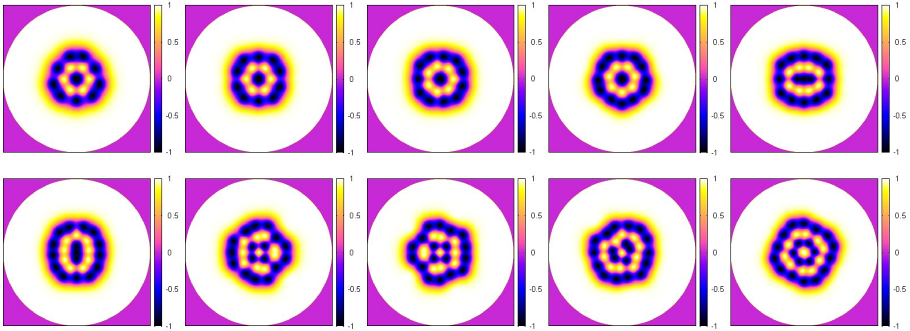
- A popcorn-like transition occurs at $B = 9$



- Just before the pop (at charge $B = 8$) the ring structure deforms
- The transition is reminiscent of the popcorn phenomena predicted in the Sakai-Sugimoto model, and observed in toy models of the Sakai-Sugimoto model



- No further pops are observed for charges $B \leq 20$



A point particle approximation

- Numerical solutions found so far resemble ring-like formations of $B = 1$ baby Skyrmions
- Can we predict higher charge structures and popcorn transitions using a point-particle approximation for the constituent solitons?
- To do this we must find ways of approximating the gravitational potential induced by the metric and the inter-soliton interaction

The gravitational potential (1)

- In the non-relativistic limit the geodesic equations from the metric give

$$\ddot{x} \approx -\frac{x(1+r^2)}{L^2(1-r^2)} \equiv -\frac{x}{r}\partial_r \Phi$$

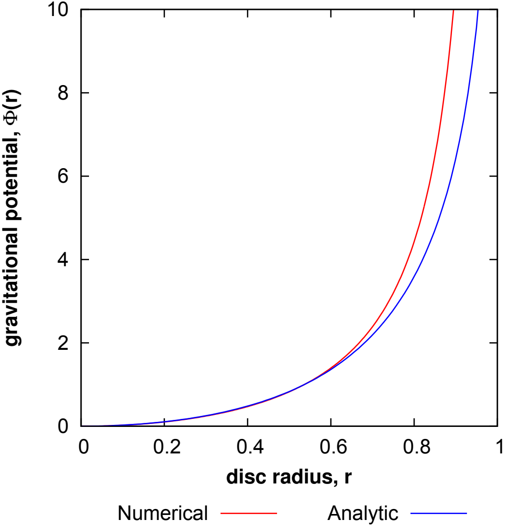
- This gives the potential

$$\Phi(r) = -\frac{\alpha}{2L^2}\left(r^2 + 2\log\left(1-r^2\right)\right)$$

where α is a constant to account for differences in units.

- A numerical approximation is obtained by evaluating the energy of the $B = 1$ soliton translated around the grid
- α can then be fixed by performing a least-squares fit to the numerical data

The gravitation potential (2)



The inter-soliton interaction (1)

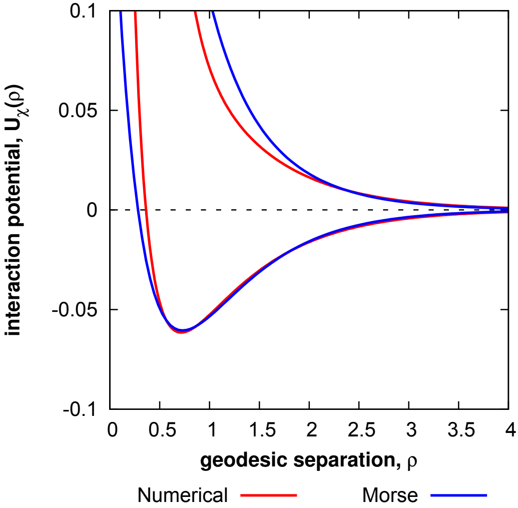
- The inter-soliton interaction can be numerically approximated by translating pairs of $B = 1$ solutions around the grid
- We find solitons are maximally attractive (repulsive) when they have a relative phase shift of 0 (π)
- The out-of phase interaction potential can be fit to a Morse potential of the form

$$U_\pi(\rho) = D\left(e^{2a(1-\rho/\rho_e)} - 2e^{a(1-\rho/\rho_e)}\right)$$

- Dependence on the relative phase difference between neighbours can be introduced by assuming the solitons are most attractive (repulsive) when they are out of (in) phase

$$U_\chi(\rho) = D\left(e^{2a(1-\rho/\rho_e)} + 2\cos(\chi) e^{a(1-\rho/\rho_e)}\right)$$

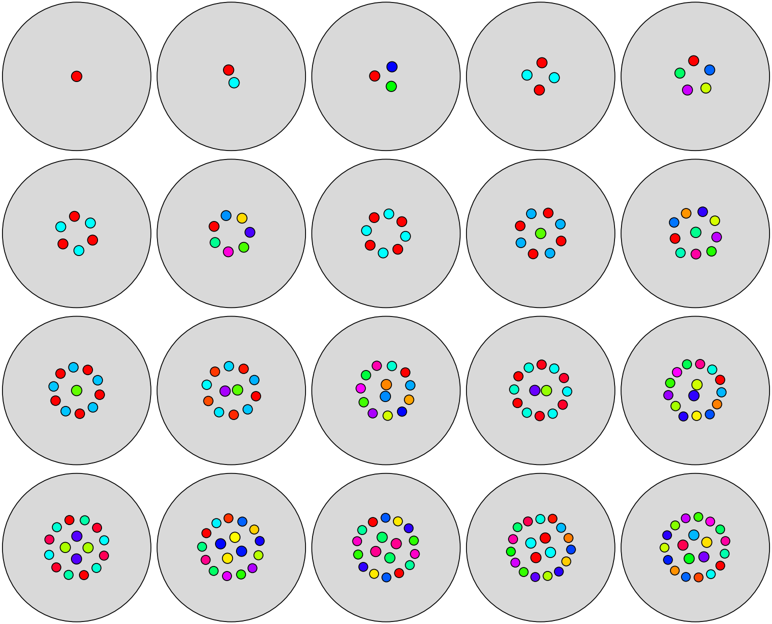
The inter-soliton interaction (2)



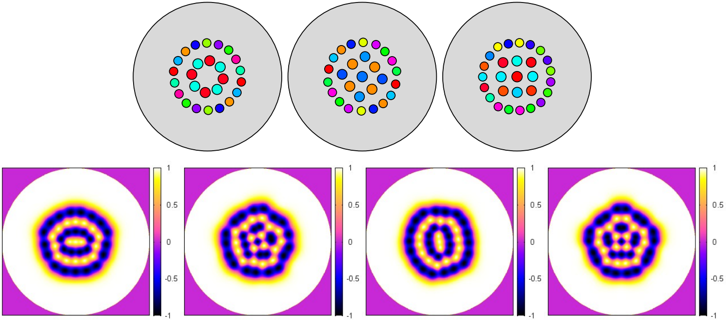
Point particle approximation results (1)

- The resulting point-particle energy can be minimised using a multi-start stochastic hill-climbing algorithm
- Randomly generated initial configurations are allowed to relax iteratively
- Solutions were also verified using a finite temperature annealing method

Point particle approximation results (2)

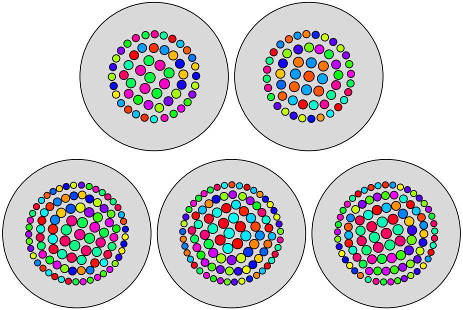


- The point particle approximation is qualitatively successful for charges $1 \leq B \leq 20$
- It also accurately predicts the second popcorn transition around $B = 27$



Point particle approximation results (4)

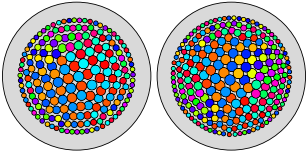
- Third and fourth transitions are predicted around $B = 54$ and $B = 95$



- It is difficult to numerically test these predictions due to the large number of initial configurations

Point particle approximation results (5)

- Very high charge approximations can be found with the point particle approximation



- Ring-like structure is still clearly present in the outer layers
- Deformations at the centre of the structures may indicate an emerging lattice structure as $B \to \infty$

Summary

- Baby Skyrmions in AdS spacetime have been investigated
- The curvature of AdS acts as an effective pion mass when compared with baby Skyrmions in flat space
- Soliton solutions for a range of topological charges have been found numerically and exhibit ring-like structures
- At certain critical charges the rings pop out into multiple layers, reminiscent of baryonic popcorn in the Sakai-Sugimoto model
- A point particle approximation has been derived which is able to qualitatively estimate the forms of minimal energy solutions
- The point particle approximation has also been able to accurately predict the charges at which popcorn transitions occur

Thank you for listening.