Shift-symmetry at one-loop

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Outline

- Low-energy effective theories
- Shift-symmetry
- One-loop calculations
- Results

Supergravity

- Supergravity couplings can be described in terms of three functions:
  - Superpotential, $W(\phi^i)$.
  - Kähler potential, $K(\phi^i, \bar{\phi}^i)$.
  - Gauge kinetic energy function, $f_{\alpha\beta}(\phi^i, \bar{\phi}^i)$.
- Kähler metric, $K_{i\bar{j}} = \frac{\partial^2 K}{\partial \phi^i \partial \bar{\phi}^{\bar{j}}}$.
- Scalar potential,

$$V_F = e^K((D_i W)K^{i\bar{k}}(\bar{D}_{\bar{k}}\bar{W}) - 3|W|^2),$$

where $D_i W = \partial_i W + (\partial_i K)W$.

- Kähler potential can receive quantum corrections.

Tree-level Kähler potential

- Compactify Heterotic string theory on a $\mathbb{T}_2$ torus with Kähler and complex structure moduli T and U.
- Include two real continuous Wilson lines $\mathcal{A}_1^a$, $\mathcal{A}_2^a$, $\quad a = 1, 2$.
- Tree level Kähler potential

$$K^{(0)} = -\log\left[-(T - \bar{T})(U - \bar{U}) - (B + \bar{C})(\bar{B} + C)\right]$$

where,

$$B = -\frac{1}{\sqrt{2}}(iA^1 + A^2), \quad C = -\frac{1}{\sqrt{2}}(iA^1 - A^2).$$

$$A^a = U\mathcal{A}_1^a - \mathcal{A}_2^a.$$

Modular Invariance

- Supergravity theory will be invariant under $SL(2,\mathbb{Z})$ transformations of the moduli:

$$\text{As } T \to \frac{aT+b}{cT+d}, \qquad a,b,c,d \in \mathbb{Z}, \quad ad-bc=1.$$

$$U \to U - \frac{c}{cU+d}BC, \quad B \to \frac{B}{cT+d}, \quad C \to \frac{C}{cT+d}.$$

- The Kähler potential must be invariant up to some Kähler transformation:

$$K \to K + f(z) + \bar{f}(\bar{z}).$$

- For $K^{(0)}$ on the previous slide we have

$$K^{(0)} \to K^{(0)} + \log(cT+d) + \log(c\bar{T}+d).$$

Shift-symmetry

- Shift-symmetry: Symmetry under which two fields, B and C, transform as

$$B \to B + c, \qquad C \to C - \bar{c}.$$

- Kähler potential of a theory with such a symmetry takes the form

$$K = G + \left|B + \bar{C}\right|^2 f + \dots$$

- We can see that the tree-level Kähler potential considered earlier clearly exhibits this shift symmetry,

$$K^{(0)} = -\log\left[-(T - \bar{T})(U - \bar{U}) - (B + \bar{C})(\bar{B} + C)\right].$$

- Does this shift-symmetry still hold beyond tree-level?

Shift-symmetry

- Shift-symmetry not expected to be preserved to all orders in general.
- However, there is a particular limit when we do expect it to still hold.
- Recall,

$$B = -\frac{1}{\sqrt{2}}(iA^1 + A^2), \quad C = -\frac{1}{\sqrt{2}}(iA^1 - A^2).$$

$$A^a = U\mathcal{A}_1^a - \mathcal{A}_2^a.$$

- In the limit $U_2 \gg 1$, $A^a \sim iU_2\mathcal{A}_1^a$ and the fields $\bar{B}$ and C become degenerate,

$$\bar{B}, C \sim \frac{U_2}{\sqrt{2}}(\mathcal{A}_1^1 + i\mathcal{A}_1^2).$$

One-loop calculation

$$K = G + Z_{C\bar{C}} C\bar{C} + Z_{B\bar{B}} B\bar{B} + (H_{BC} BC + \text{c.c.}) + \dots$$

- Compute the CP even part of one-loop two-point functions involving moduli T or U as external states.
- Extract the $\mathcal{O}(k^2)$ piece and compare with the corresponding kinetic term in the effective supergravity Lagrangian,

$$K_{i\bar{j}} \partial \phi^i \partial \phi^{\bar{j}}.$$

- This gives the Kähler metric $K^{(1)}_{i\bar{j}}$ from which we could then hope to determine the Kähler potetial.
- Want to find corrections to coefficients Z and H to check for any breaking of shift-symmetry.

One-loop calculation

- Vertex operator for T:

$$V_T = -\frac{i}{T_2 + \frac{(A-\bar{A})^2}{8U_2}}(\partial Z - ik\cdot\psi\Psi)\bar{\partial}\bar{Z}e^{ik\cdot X}.$$

- Internal partition function associated with the torus:

$$\mathcal{Z}_{\vec{m},\vec{n}} = \frac{T_2 + \frac{(A-\bar{A})^2}{8U_2}}{\tau_2}\sum_{\vec{m},\vec{n}\in\mathbb{Z}} e^{-S(\vec{m},\vec{n})}\sum_{Q^a} q^{(Q^a+\vec{\mathcal{A}}^a\cdot\vec{n})^2/2}e^{-2\pi i\vec{\mathcal{A}}^a\cdot\vec{m}(Q^a+\vec{\mathcal{A}}^a\cdot\vec{n}/2)},$$

where,

$$S(\vec{m},\vec{n}) = \frac{\pi}{\tau_2}(G_{IJ}+B_{IJ})(m_I+n_I\tau)(m_J+n_J\bar{\tau}).$$

One-loop calculation

- Amplitude of interest is,

$$\mathcal{A} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \int d^2 z \langle V_T(k,z) V_{\bar{T}}(-k,0) \rangle \mathcal{Z}_{\vec{m},\vec{n}} \mathcal{Z}_{\text{rest}}.$$

- Extract $\mathcal{O}(k^2)$ and spin dependent piece.
- Evaluate correlation functions and integrate over z, leaving,

$$\mathcal{A} = \frac{-\pi^2 k^2}{4\left(T_2 + \frac{(A-\bar{A})^2}{8U_2}\right) U_2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} \sum_{\vec{m},\vec{n}} F(U,\tau) \mathcal{Z}_{\vec{m},\vec{n}} \tilde{\mathcal{Z}}_{\text{rest}},$$

where,

$$F(U,\tau) = [m_1 + n_1\bar{\tau} + U(m_2 + n_2\bar{\tau})][m_1 + n_1\bar{\tau} + \bar{U}(m_2 + n_2\bar{\tau})].$$

Kähler potential corrections

$$\text{Recall } K = G + Z_{C\bar{C}} C\bar{C} + Z_{B\bar{B}} B\bar{B} + (H_{BC} BC + \text{c.c.}) + \ldots$$

$$\begin{aligned} Z^{(1)} =& \frac{-ic_1}{4\pi(S-\bar{S})T_2U_2}\left\{ \left(\frac{E(U,2)}{T_2} + \frac{\mathcal{P}(T)}{U_2} \right) \right\} \\ &+ \frac{\pi^2 c_2}{(S-\bar{S})T_2U_2} \log\left[4e^{-2\gamma}\pi T_2 U_2 |\eta(T)\eta(U)|^4 \right], \end{aligned}$$

$$\begin{aligned} H^{(1)}_{BC} =& \frac{-ic_1}{4\pi(S-\bar{S})T_2U_2}\left\{ \left(\frac{E(U,2)}{T_2} + \frac{\mathcal{P}(T)}{U_2} \right) \right\} \\ &- \frac{4\pi^2 c_2}{(S-\bar{S})}\left\{ \frac{\pi^2}{36} + \left[2\partial_U \log\eta(U) - \frac{i}{2U_2} \right] \left[2\partial_T \log\eta(T) - \frac{i}{2T_2} \right] \right\}, \end{aligned}$$

$$\text{where } \eta(T) = e^{\frac{\pi i T}{12}} \prod_{k=1}^{\infty}(1-q^k).$$

Breaking of shift-symmetry

- Analyse breaking of shift-symmetry by considering shifts in the typical induced soft-terms of the form

$$\frac{\delta m^2}{m^2} = \frac{\mathrm{Re}(H_{BC}^{(1)} - Z^{(1)})}{Z^{(1)}}.$$

- $U_2 \gg 1$:

$$\frac{\delta m^2}{m^2} \sim \frac{3 \log[4\pi e^{-2\gamma} T_2 U_2]}{\pi(T_2 + U_2)}.$$

- $U_2 \ll 1$:

$$\frac{\delta m^2}{m^2} \sim \frac{4\pi U_2}{3} \sum_{k>0} \frac{k q_U^k}{1 - q_U^k}.$$

Conclusions

- By explicit computation of one-loop corrections to the Kähler potential, we are able to show that shift-symmetries do not generally hold beyond tree-level.
- However, we do see that the symmetry is restored in the limit of large U_2 as was expected.