Blackhat - Efficient Calculation of NLO amplitudes for Higgs + many jets

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Outline

1 Introduction

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2 Tree Level - BCFW Recursion Relation

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3 One Loop - Generalised Unitarity

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5 Conclusions

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- Why loops?
    - Amplitudes and effects that first appear at NLO
    - More precise theory predictions
    - ...

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  - Amplitudes and effects that first appear at NLO
  - More precise theory predictions
  - ...
- Why high multiplicity?
  - Important at LHC where lots of Jets is normal
  - Important background to other processes

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- BCFW Recursion Relations to build the tree amplitudes from smaller known amplitudes

Spinor Helicity Formalism

- Helicity Spinors
  - $u_{\pm}(k) = P^{\pm} u(k) = \frac{1 \pm \gamma_5}{2} u(k)$
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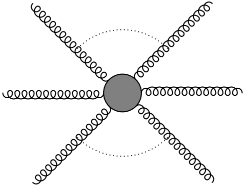
  - $\bar{u}^{+}(k_i) = \bar{v}^{-}(k_i) \equiv \left\langle k_i^{+} \right| \equiv [i|$
  - $\bar{u}^{-}(k_i) = \bar{v}^{+}(k_i) \equiv \left\langle k_i^{-} \right| \equiv \langle i|$

BCFW Recursion Relation



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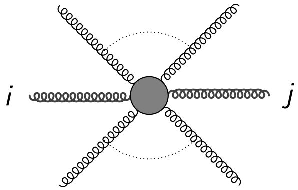

- $k_i^\mu \to k_i^\mu + z \cdot n^\mu$, $k_j^\mu \to k_j^\mu - z \cdot n^\mu$

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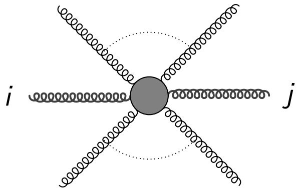

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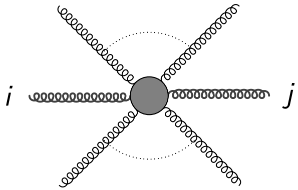

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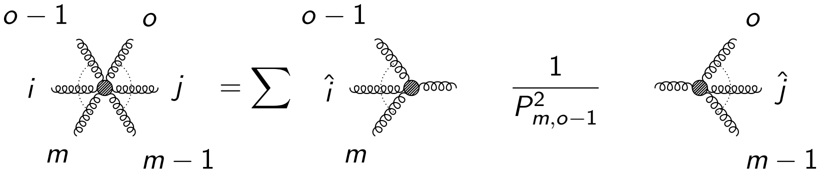
- $|i\rangle \to |i\rangle + z\,|j\rangle$, $|j] \to |j] - z\,|i]$

BCFW Recursion Relation

$$A = \sum_{\substack{m,o,h: \\ i \in [m,o), j \notin [m,o)}} \hat{A}^{h}_{m,o-1}(z_{m,o}) \frac{1}{P^2_{m,o-1}} \hat{A}^{-h}_{m,o-1}(z_{m,o}) \Bigg|_{\hat{P}^2_{o,m-1}(z_{m,o})=0}$$



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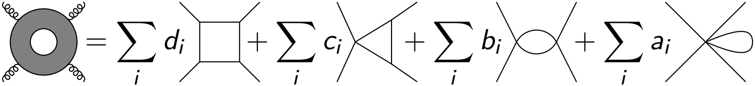

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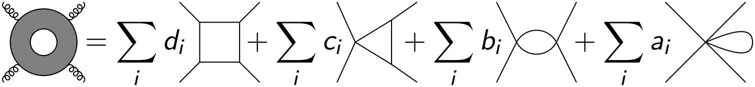

- Scalar basis functions: Boxes, Triangles, Bubbles & Tadpoles (I ignore tadpoles as I have massless internal particles)

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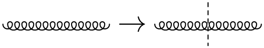
$$\frac{1}{l_i^2} \to \delta(l_i^2)$$



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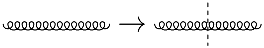

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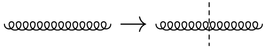

- Cuts select out a specific basis integral
- For more than two cuts need complex momenta

Box Coefficients

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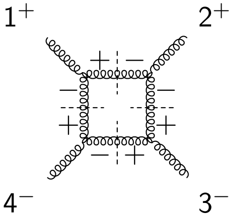

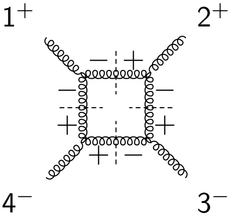

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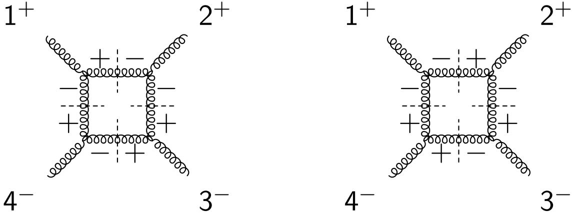

- Momentum Solutions

$$(l_1^+)^\mu = \frac{\langle 1|2\,3\,4\,\gamma^\mu|1\rangle}{2\,\langle 1|2\,4|1\rangle} \qquad (l_1^-)^\mu = \frac{[1|2\,3\,4\,\gamma^\mu|1]}{2\,[1|2\,4|1]}$$

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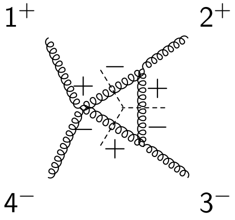

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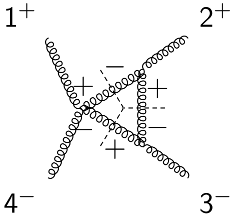

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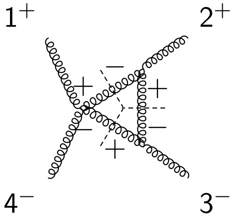

- One free degree of freedom in loop momentum, t
- Solve by choosing a basis of $\tilde{k}_1^\mu$, $\tilde{k}_3^\mu$, $n_1^\mu = \frac{1}{2}\left\langle \tilde{k}_1 \middle| \gamma^\mu \middle| \tilde{k}_3 \right]$ and $n_3^\mu = \frac{1}{2}\left\langle \tilde{k}_3 \middle| \gamma^\mu \middle| \tilde{k}_1 \right]$

Triangle Coefficients

- Solutions for with massless corners

$$\left(l_1^+\right)^\mu = -\frac{K_3^2}{K_3 \cdot k_1} k_1^\mu + t n_1^\mu \qquad \left(l_1^-\right)^\mu = -\frac{K_3^2}{K_3 \cdot k_1} k_1^\mu + \frac{1}{t} n_3^\mu$$

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- Triangle coefficient is c_0
- Extract using either contour integration or discrete Fourier projection

$$c_0 = \frac{1}{2\pi i}\oint \frac{dt}{t} C_3(t) = \frac{1}{2p+1}\sum_{j=-p}^{p} C_3(t_0 e^{2\pi i j/(2p+1)})$$

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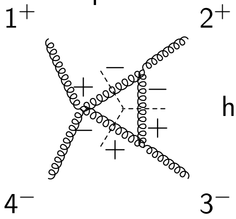
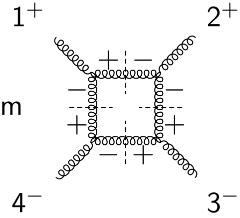
- For example



has contributions from



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- Want to remain numeric so need well defined spinors - 6,8,10,... dimensions
- Only actually need one fixed dimension then can extrapolate down with known forms of dependence on d.
- Can simply and write in terms of massive 4 dimensional spinors.
- Can show that tree amplitudes depend on the phase of the mass in a very well defined way
  - Loop amplitudes can't depend on phase
  - Can remove a degree of freedom

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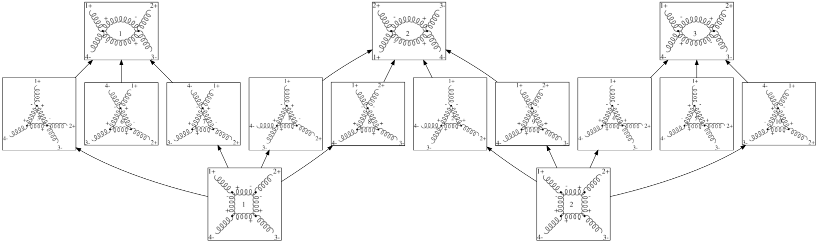
- Tree and cut parts extend to interactions with the Higgs Boson easily
- Not fully implemented rational parts for Higgs Boson but progressing well

- L. J. Dixon, "Calculating scattering amplitudes efficiently," `arXiv:hep-ph/9601359 [hep-ph]`.
- R. Britto, F. Cachazo, B. Feng, and E. Witten, "Direct proof of tree-level recursion relation in Yang-Mills theory," *Phys.Rev.Lett.* **94** (2005) 181602, `arXiv:hep-th/0501052 [hep-th]`.
- C. F. Berger, Z. Bern, L. J. Dixon, D. Maître, F. Febres Cordero, D. Forde, H. Ita, and D. A. Kosower, "Automated implementation of on-shell methods for one-loop amplitudes," *Physical Review D* **78** no. 3, (Aug., 2008) 036003, `arXiv:0803.4180 [hep-ph]`.

Cut Part Dependancies



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