Constraining theories of gravity using CMB observations

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- Key facts will be possible detection of primordial gravitational waves. Therefore is important to understand them better.
- We can look at the progress done on inflation to get some insight.

EFT of inflation

- In inflation a particularly useful method is by studying it as an EFT(Cheung, Weinberg) Inflation is thought as a broken de Sitter symmetry phase.
- The Goldstone boson associated to this broken symmetry is the inflaton. By doing so we can constraint the theory by studying its simmetries.
- For example, considering extra symmetries (galileons) we can constrain all the correlators without solving the theory.

- Allowed elements on an EFT would have to respect the symmetries of the problem.
- It is convenient to describe the manifold in terms of the constant time hypersurce Σ_t,and the unit vector orthogonal to it n_μ.
- The only elements presents invariant under spatial diffs are g^{00}, R^{00} and the extrinsic curvature tensor $K_{\mu\nu} = h^\rho_\mu \nabla_\rho n_\nu$, the induced metric $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ and the Riemman curvature $\hat{R}_{\alpha\beta\gamma\delta}$ on the induced metric Σ_t.
- By considering broken time diffs, the most general action constructed with these elements is

$$S = \int \mathrm{d}^4x \sqrt{-g}\mathcal{L}\left[R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \hat{R}_{\mu\nu}, \nabla_\mu; t\right] \tag{1}$$

To get an EFT for inflation we assume a FRW background

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2 \tag{2}$$

Then we calculate the tadpole expand

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{{M_{\mathrm{Pl}}}^2}{2}R + M_{\mathrm{Pl}}^2(3H^2 + \dot{H}) + M_{\mathrm{Pl}}^2\dot{H}g^{00}\right] \tag{3}$$

The high order terms can be organized as an expansion in powers of fluctuations to the background,

$$
\begin{aligned}
\Delta S \;=\; & \int \mathrm{d}^4x\sqrt{-g} \\
& \Big[\frac{M_2^4(t)}{2}(\delta g^{00})^2+\frac{M_3^4(t)}{3!}(\delta g^{00})^3+\frac{M_4^4(t)}{4!}(\delta g^{00})^4+\ldots \\
& -\frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K-\frac{\bar{M}_2^3(t)}{2}(\delta K)^2-\frac{\bar{M}_3^3(t)}{2}(\delta K^{\mu}_{\ \nu})^2+.. \qquad (4) \\
& -\frac{\hat{M}_1^2(t)}{2}\delta g^{00}R+\ldots\Big]
\end{aligned}
$$

where $\delta g^{00}=g^{00}+1$ and $\delta K_{\mu\nu}=K_{\mu\nu}-Hh_{\mu\nu}$

EFT of inflation

Focusing on the Goldstone boson which shifts under time diffs
$\pi(t,\ \vec{x}) \to \pi(t,\ \vec{x}) - \xi^0(t,\ \vec{x})$

$$
\begin{aligned}
S &= \int \mathrm{d}^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - M_{\rm Pl}^2 \dot{H}\left(\dot{\pi}^2 - \frac{(\partial\pi)^2}{a^2}\right)\right. \\
&\left. +2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 \ldots\right]
\end{aligned}
\tag{5}
$$

- This setup unifies most of the models of single field inflation. The parameters involved are the speed of sound for the scalar modes,

$$c_s^{-2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2 \dot{H}} \tag{6}$$

- This term parameterises deviation of a Gaussian theory. For a particular limit the amplitude of the deviations is proportional to c_s^{-2}. This is very constrained by CMB observations
- For example for DBI the only relevant parameters is c_s. The relation between its parameters is $M_3^4 \sim M_{\rm Pl}^2 \dot{H} c_s^{-4}$
- It can be written to incude several fields (Senatore and Zaldarriaga), but it is more useful to understand models of one inflating degree of freedom.

Gravitational waves

Expanding in term of the tensor perturbations γ_{ij}, the background action for the graviton is

$$S = \frac{{M_{\mathrm{Pl}}}^2}{8} \int \mathrm{d}^4 x a^3 \left[\dot{\gamma}_{ij}^2 - (\partial \gamma_{kj})^2 \right] . \tag{7}$$

There is an observable r, measuring ratio of the amplitude of the tensor against the scalr fluctuations

$$r \equiv \frac{A_t}{A_s} \tag{8}$$

In the case of single field inflation this paratemer measures directly the energy scale of inflation H

- Furthermore we have that the only terms that give second order equation of motion for the tensor modes are $M_2^4(t)$, $\bar{M}_1^3(t)$, $\bar{M}_2^3(t)$ and $\bar{M}_3^3(t)$

$$(\delta K^{\mu}_{\nu})^2 - (\delta K)^2 = \frac{1}{4}(\dot{\gamma}_{ij})^2 \tag{9}$$

- And thus only the combination $\bar{M}_3^3(t) - \bar{M}_2^3(t)$, can contribute to modifications of the speed of sound up to second order.
- The action as,

$$S = \frac{{M_{\mathrm{Pl}}}^2}{8}\int \mathrm{d}^4x a^3 c_T^{-2}\left[\dot{\gamma}_{ij}^2 - c_T^2\frac{(\partial_k\gamma_{ij})^2}{a^2}\right]. \tag{10}$$

- r is sensitive to c_T different from one

Disformal transformation

In a theory where there is an speed of sound for the tensor modes c_T we can make the transformation

$$g_{\mu\nu} \mapsto \frac{1}{c_T}\left(g_{\mu\nu} + (1 - c_T^2(t))n_\mu n_\nu\right) \tag{11}$$

which removes c_T from the tensor modes action. By doing so the scalar action get reescaled and becomes,

$$S = \int \mathrm{d}^4x \frac{a^3}{c_T^2} {M_{\mathrm{Pl}}}^2 \dot{H} \left\{ \dot{\pi}^2 - \frac{c_T^2}{a^2} (\nabla\pi)^2 \right\} \tag{12}$$

(Creminelli *et al.*)

- This can be thought as a disformal and a conformal transformation. The disformal transformation changes the slope of the lightcone.
- In the EFT lenguage this will rescaale all the parameters resulting in a effective speed for the scalar modes.
- This lead to a $c_S = c_T^{-1}$ but since all the parameters are supressed the interactive theory stays the same. Thus this is an approximate symmetry for inflation.

Cosmological gravitational waves

We now want to apply this formalism to a general theory for gravity. Let us consider a tensor modes with high interactions producing an effective speed of sound

$$E_{ij}'' - c_T^2 \nabla^2 E_{ij} + 2\mathcal{H} E_{ij}' = 0. \tag{13}$$

This equation is identical to the case for inflation and therefore we can remove c_T by doing the same transformation as before. We then go from a frame where E_{ij} moves at a speed different from one to one in which the scalar modes move at a speed different from 1

Doing so (13) can be written as,

$$\tilde{E}_{ij}'' - \nabla^2 \tilde{E}_{ij} + 2\tilde{\mathcal{H}}\tilde{E}_{ij}' = 0. \tag{14}$$

Where we have expressed the functions in the new frame as tilded. This transformation will have an effect on the scalar modes, by inducing a tensor speed $c_S = c_T^{-1}$. which will be reflected in the Einstein eqs.

CMB

We can calculate the CMB using this modified eqs.. We will simplify the calculation by using the two fluid approximations. The ingredients for this will be the photon velocity,

- Baryons are tightly coupled to photons. The interaction ratio is given by R. We will consider as functions the photon density, $delta_\gamma$ and the gravitation potential ϕ

$$\delta_\gamma'' + \frac{\mathcal{H}R}{1+R}\delta_\gamma' - \frac{c_T^{-2}}{3(1+R)}\nabla^2\delta_\gamma = 4\phi'' + \frac{\mathcal{H}R}{1+R}\phi' + \frac{4}{3}c_T^{-2}\nabla^2\phi \quad (15)$$

- A crucial point will be that there are two timescales which are independent on the reescaling. These are the Silk damping and the final damping scales. This is because they are related to the scattering process, not to a cosmological process.
- This effect will be significant because the Hubble factor will be reescaled.
- Furthermore the effect of a speed of sound modifies the dispersion relation and the CMB peaks are shifted.

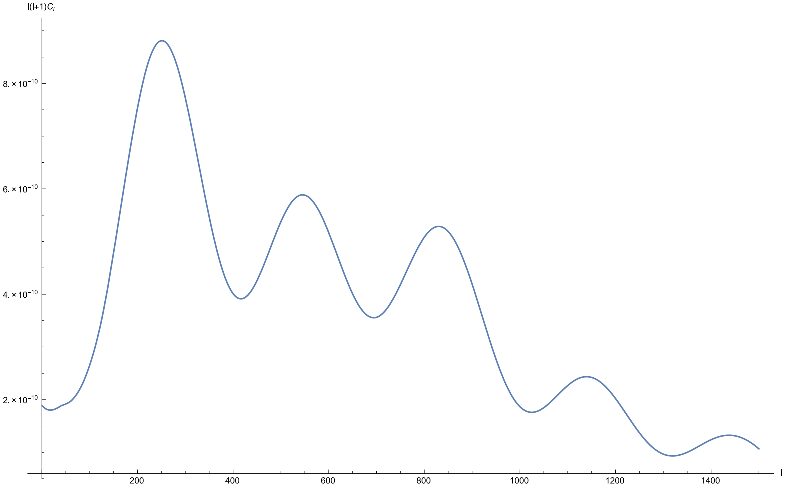
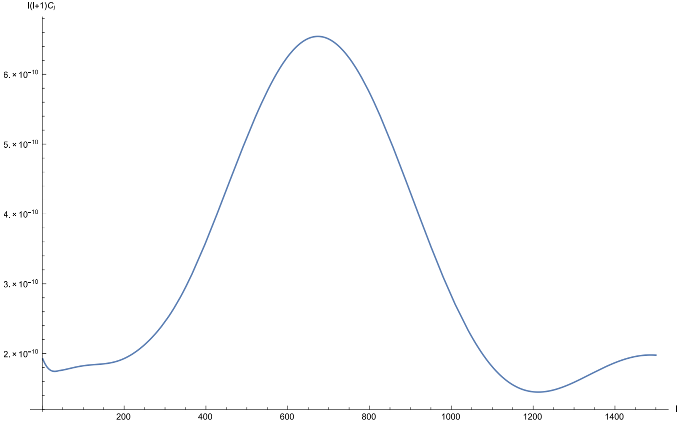

l(l+1)C_l
8. × 10⁻¹⁰
6. × 10⁻¹⁰
4. × 10⁻¹⁰
2. × 10⁻¹⁰
200
400
600
800
1000
1200
1400
l

Figure: $c_T = 1$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l

Figure: $c_T = 2$

- It was expected that a modification of c_T would have a dramatic effect. This can be seen on the scalar power spectrum.
- The difference between this case and the inflationary one is that the first is coupled to matter.
- It is still needed to calculate observables from the gravitational waves. But it is expectable that they might have a similar effect

Conclusions

- EFT are very powerful to study simmetries during cosmological epoch.
- During inflation there is always a frame where there the speed of sound for the tensor modes is equal to one. This is given because there is a tree level symmetry of the Einstein eqs.
- A differente c_T at later times works differently and it is possible to constrain c_T bettter.