A smörgåsbord of flavour in GUTs

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Outline

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[arXiv: 1503.03306, 1505.05504, 1512.00850]

Setting the table

Unanswered questions in model-building

Why are there three generations of fermions?

Are neutrinos Majorana or Dirac fermions?

Why is there such a strong hierarchy in particle masses?

What is the origin of large lepton mixing?

How large is leptonic CP violation?

Why is there a Baryon Asymmetry of the Universe (BAU)?

Can the strong CP problem be resolved?

Why do the gauge couplings appear to converge at $\sim 10^{15-16}$ GeV?

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GUTs

Discrete flavour symmetry

Leptogenesis

SUSY

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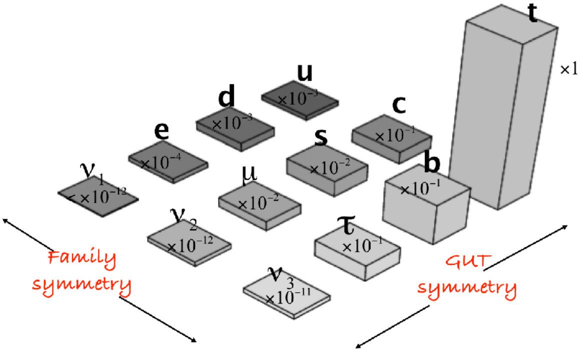
What is the lifetime of the proton?

How is doublet-triplet splitting achieved?

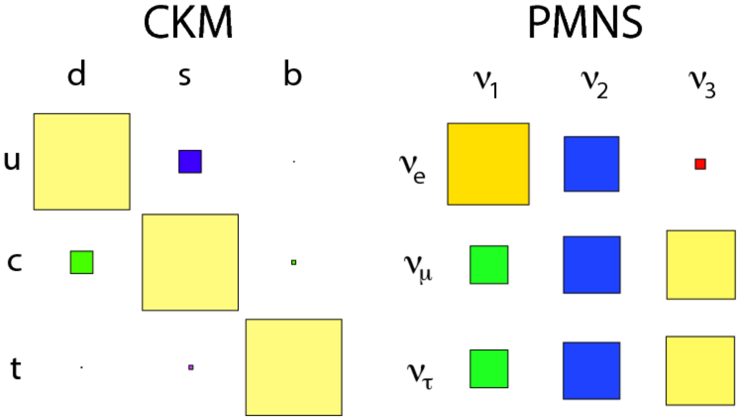
What is the scale of the MSSM μ-term?

Fermion masses



[King, 1301.1340]

Mixing matrices



[Stone, 1212.6374]

The strong CP problem

Why does QCD not break CP?

Source 1: topological term

$$\mathcal{L} \sim -\frac{g_s^2 \theta_{\mathrm{QCD}}}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Source 2: quark mass matrices

$$\theta_q = \arg\det(Y^u Y^d)$$

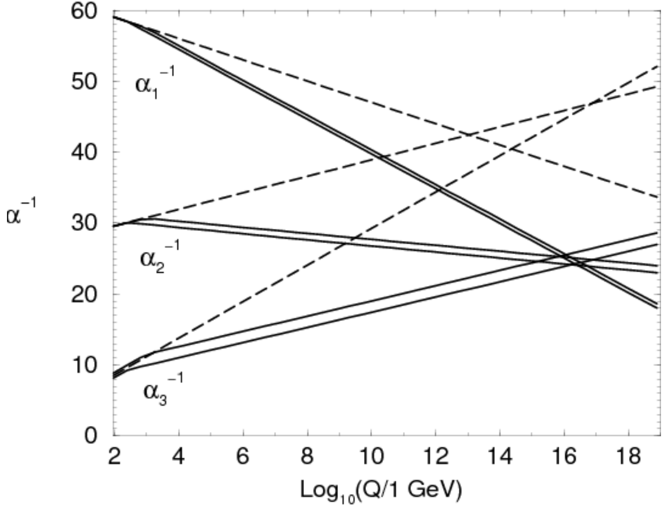
Physical parameter

$$\bar{\theta} = \theta_{\mathrm{QCD}} - \theta_q$$

must be very small due to non-observation of neutron EDM

Fine-tuning problem

Grand unification



[Bhattacharyya, 0807.3883]

Grand unification (2)

There are **15** known chiral states in the SM (per generation).

u_i, u_i^c (3 colours each)
d_i, d_i^c (3 colours each)
ℓ, ℓ^c
ν

Note: all states are left-handed.

In the SM, we collect various states in representations of $SU(3) \times SU(2) \times U(1)$:

$$\{u_i, d_i\} \to Q = (\mathbf{3}, \mathbf{2})_{1/6}, \qquad \{\ell, \nu\} \to L = (\mathbf{1}, \mathbf{2})_{-1/2}$$

In GUTs, we further collect these states into representations of a higher-dimensional gauge group.[1]

Should there be a ν^c?

[1] First to do this was Georgi and Glashow.

Grand unification (3)

Example: in $SU(5)$, the SM Higgs doublet typically resides in a $\mathbf{5}$.
  This contains **coloured Higgses**.
  They must be **heavy**, otherwise they'd lead to rapid proton decay.

Explaining why doublets are light, while triplets are heavy, is the doublet-triplet splitting problem.

Analogous scale splitting problems are ubiquitous: any good GUT should resolve them.

Naturalness problem

An $A_4 \times SU(5)$ model

The model

Symmetries of the model

$$SU(5) \times A_4 \times \mathbb{Z}_9 \times \mathbb{Z}_6 \times \mathbb{Z}_4^R \times \mathrm{CP}$$
$$\Downarrow$$
$$SU(3) \times SU(2) \times U(1) \times \mathbb{Z}_2^R$$

MSSM fields

$$\begin{array}{rcll}
F & = & (\bar{\mathbf{5}}, 3) & \to \; d_i^c, \ell, \nu \;\; (\times 3 \text{ gens}) \\
(3\times)\; T & = & (\mathbf{10}, 1) & \to \; u_i, u_i^c, d_i, \ell^c \\
(3\times)\; N^c & = & (\mathbf{1}, 1) & \to \; \nu^c \\
H_5, H_{45} & = & (\mathbf{5}, 1), (\mathbf{45}, 1) & \to \; H_u \\
H_{\bar{5}}, H_{\overline{45}} & = & (\bar{\mathbf{5}}, 1), (\overline{\mathbf{45}}, 1) & \to \; H_d
\end{array}$$

Full model in 1503.03306

Yukawa structure from A_4 triplet flavons

Down-quark and charged lepton mass matrices come from terms that **couple F and $\mathbf{T}_{\mathrm{j}}$**:

$$m_{ij}\overline{L_i}\ell_j^c \overset{\mathrm{EWSB}}{\longleftarrow} y_{ij}H\overline{L_i}\ell_j^c \overset{\mathrm{SSB}}{\longleftarrow} y_{ij}H_d L_i \ell_j^c$$
$$\overset{\mathrm{FSB}}{\longleftarrow} \frac{\phi_j}{M}H_d L\ell_j^c \overset{\mathrm{GUTB}}{\longleftarrow} \frac{\phi_j}{M}H_5 F T_j$$

The flavons that couple to FT_j get VEVs with alignments

$$\phi_e \propto \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \phi_\mu \propto \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \phi_\tau \propto \begin{pmatrix}0\\0\\1\end{pmatrix}$$

This gives

$$Y^e \sim (Y^d)^T \sim \mathrm{diag}(y_e, y_\mu, y_\tau)$$

up to Clebsch-Gordan coefficients.

Yukawa structure from A_4 triplet flavons (2)

More precisely:

$$Y^d = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{4} d_{11} \frac{|v_\xi v_e|}{|v_{\Lambda_{24}}|^2} & d_{12} \frac{|v_\xi v_\mu|}{|v_{\Lambda_{24}} v_{H_{24}}|} e^{i\zeta} & 0 \\ 0 & 2 d_{22} \frac{|v_{H_{24}} v_\mu|}{M^2} & 0 \\ 0 & 0 & d_{33} \frac{|v_\tau|}{M} \end{pmatrix}$$

Y^u is real (not shown here!) $\Rightarrow$ *only* source of CP violation is phase ζ.

$\Rightarrow \theta_q = \arg\det(Y^u Y^d) = 0$

CP symmetry at high scale $\Rightarrow$ topological term **disallowed**

$\Rightarrow \theta_{\mathrm{QCD}} = 0$.

$\therefore \bar{\theta} = \theta_{\mathrm{QCD}} - \theta_q = 0$ (at lowest order)

Strong CP problem solved

Yukawa structure from A_4 triplet flavons (3)

Neutrinos couple to different flavons, ϕ_{atm} and ϕ_{sol}, where

$$\langle\phi_{\mathrm{atm}}\rangle \propto \begin{pmatrix}0, 1, 1\end{pmatrix}, \quad \langle\phi_{\mathrm{sol}}\rangle \propto \begin{pmatrix}1, 3, 1\end{pmatrix}$$

This gives neutrino Dirac matrix Y^ν and RH Majorana mass matrix M_R:

$$Y^\nu = \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}, \qquad M_R = \begin{pmatrix} M_{\mathrm{atm}} & 0 \\ 0 & M_{\mathrm{sol}} \end{pmatrix}$$

This alignment is called **CSD(3)** and explains large lepton mixing in good agreement with experiment.

Best fit $\delta \approx -90^\circ$ suggests leptonic CP violation.

Numerical analysis of CSD(n) framework: 1412.6996

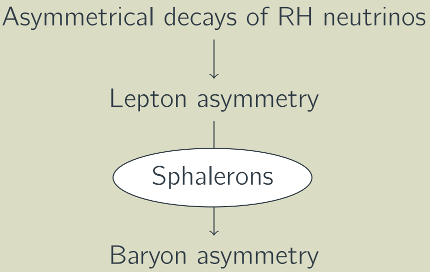
BAU from leptogenesis

Sakharov conditions for producing a baryon asymmetry:

1. B **violation** **2.** C **and** CP **violation**

3. Out-of-equilibrium interactions

Leptogenesis



BAU from leptogenesis (2)

Washout factors → $\eta_{1,\alpha}$; Decay asymmetries → $\epsilon_{1,\alpha}$

$$Y_B = \frac{10}{31} \sum_\alpha \eta_{1,\alpha} \left[Y_{N_1} + Y_{\tilde{N}_1} \right] \epsilon_{1,\alpha}$$

$$= (0.87 \pm 0.01) \times 10^{-10} \quad \text{[obs]}$$

- $Y_{N_1} \approx Y_{\tilde{N}_1} \approx 45/(\pi^4 g_*) \approx 0.002$
- $\eta_{1,\alpha}$ given by solutions to Boltzmann equations[2]
- $\epsilon_{1,\alpha}$ calculated from lepton Yukawa matrices

CSD(n) models give:

$$Y_B = \frac{675}{31\pi^5 g_*} \frac{M_1}{M_2} \eta_{1,\mu} \, (n-1)^2 |b|^2 \sin\eta$$

$SU(5)$ + CSD(3) model fits $M_1 \approx 4 \times 10^{10}$ GeV

[2] S. Antusch, S. F. King and A. Riotto, 2006

A $\Delta(27) \times SO(10)$ model

The model

Symmetries of the model

$$SO(10) \times \Delta(27) \times \mathbb{Z}_9 \times \mathbb{Z}_{12} \times \mathbb{Z}_4^R \times \mathrm{CP}$$
$$||$$
$$[SU(5)]$$
$$\Downarrow$$
$$SU(3) \times SU(2) \times U(1) \times \mathbb{Z}_2^R$$

MSSM fields

$$\begin{aligned} \Psi &= (\mathbf{16}, 3) && \rightarrow \text{fermions} \\ H_{10}^u, H_{10}^d &= (\mathbf{10}, 1) && \\ H_{16}, H_{\overline{16}} &= (\mathbf{16}, 1), (\overline{\mathbf{16}}, 1) && \rightarrow H_u, H_d \end{aligned}$$

Full model in 1512.00850

Mass matrices

Schematically, Yukawa superpotential looks like

$$\mathcal{W} \sim \Psi\Psi H_{10} \left(\bar{\phi}_{\mathrm{dec}} \bar{\phi}_{\mathrm{dec}} + \bar{\phi}_{\mathrm{atm}} \bar{\phi}_{\mathrm{atm}} \xi + \bar{\phi}_{\mathrm{sol}} \bar{\phi}_{\mathrm{sol}} \xi^2 \right) + \dots$$

Introduce a GUT scalar field ξ, with a VEV below the GUT scale[3], i.e.

$$\langle \xi \rangle \sim 0.1 M_{\mathrm{GUT}}$$

Coupling of flavons ϕ to ξ^n explains the existence of mass hierarchies.

Unification into $\Psi \Rightarrow$ all mass matrices have the **same structure**:

$$m \sim m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix} + m_c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[3] A variation of the Froggatt-Nielsen mechanism.

Proton decay

Proton decay is mediated by (SUSY) **dim-5 operators** like $\Psi\Psi\Psi\Psi$.

Such terms are **forbidden at the GUT scale** by the symmetries and messenger sector of the theory.

They may, however, be produced by operators suppressed by the Planck mass M_P. The lowest-order non-zero term:

$$\Psi\Psi\Psi\Psi \frac{Z\bar{\phi}_{\rm dec}\xi^3}{M_P^6}$$

As a result, proton decay is highly suppressed.

Doublet-triplet splitting

In $SO(10)$, DT splitting may be achieved by the Dimoupoulos-Wilzcek mechanism:

- Introduce a field H_{DW} (a **45** of $SO(10)$), with VEV

$$\langle H_{DW} \rangle = \begin{pmatrix} 0 & \langle H_{24} \rangle \\ -\langle H_{24} \rangle & 0 \end{pmatrix}.$$

- Take $\langle H_{24} \rangle \propto \mathrm{diag}(1,1,1,0,0)$
  $\Rightarrow$ only terms coupling triplets survive.

H_{10}^{u}, H_{10}^{d} and $H_{16,\overline{16}}$ all contain $SU(3)$ triplets. We write down all terms allowed by the symmetries (there are quite many!). After GUT breaking, we find that all Higgs triplets have GUT scale masses.

Doublet-doublet splitting and the μ term

H_{10}^u, H_{10}^d and $H_{16,\overline{16}}$ all contain $SU(2)$ doublets.

We only expect **two at the MSSM level**.

All others should be at least unification scale.

Introducing specific **messenger fields** Z_i, Σ_i that couple pairs of H fields to powers of ξ, we arrive at a superpotential

$$\mathcal{W}_\mu \sim Z H_{10}^u H_{10}^u \frac{\xi^6}{M_Z^6} + Z H_{10}^u H_{10}^d \frac{\xi^7}{M_Z^7} + Z H_{10}^d H_{10}^d \frac{\xi^8}{M_Z^8} + \xi H_{16} H_{\overline{16}} + \frac{Z}{M_\Sigma}\left(H_{16} H_{16} H_{10}^d + \frac{\xi^8}{M_\Sigma^8} H_{16} H_{16} H_{10}^u + H_{\overline{16}} H_{\overline{16}} H_{10}^u + \frac{\xi}{M_\Sigma} H_{\overline{16}} H_{\overline{16}} H_{10}^d \right).$$

Doublet-doublet splitting and the μ term (2)

From that superpotential, may write the $SU(2)$ doublet mass matrix as:

$$M_D \sim \begin{array}{c} \\ H_d^u \\ H_d^d \\ H_d^{16} \end{array} \begin{array}{c} \begin{array}{ccc} H_u^u & H_u^d & H_u^{\overline{16}} \end{array} \\ \begin{pmatrix} \tilde{\xi}^6 & \tilde{\xi}^7 & \tilde{H}_{\overline{16}} \\ \tilde{\xi}^7 & \tilde{\xi}^8 & \tilde{\xi}\tilde{H}_{\overline{16}} \\ \tilde{H}_{16}\tilde{\xi}^8 & \tilde{H}_{16} & \xi/M_{\text{GUT}} \end{pmatrix} \end{array} M_{\text{GUT}}$$

where $\tilde{\xi} \equiv \dfrac{\langle \xi \rangle}{M_{\text{GUT}}} \sim 0.1.$

Eigenvalues: $m_D \sim \tilde{\xi} M_{\text{GUT}}, \quad \tilde{\xi} M_{\text{GUT}}, \quad \tilde{\xi}^8 M_{\text{GUT}}.$

MSSM μ term: $\dfrac{\langle \xi \rangle^8}{M_{\text{GUT}}^7} H_d^d H_u^u \ll M_{\text{GUT}}$

$\Rightarrow$ explains the smallness of the μ term.

Summary

Notes

Many open questions in HEP and model-building.

- Flavour GUTs can answer many of these questions!

The two models presented here are among the most **complete** and **realistic** models:

- Renormalisable!
- Good fits to data, with some tension that may allow for future tests of the models.
- But: they require a large GUT-scale field content, as well as SUSY (which has not yet been found!)

Conclusion

	$SU(5)$	$SO(10)$
Why are there three generations of fermions?	✓	✓
Are neutrinos Majorana or Dirac fermions?	✓	✓
Why is there such a strong mass hierarchy?	✓	✓
What is the origin of large lepton mixing?	✓	✓
How large is leptonic CP violation?	✓	✓
Why is there a BAU?	✓	?
Can the strong CP problem be resolved?	✓	✗
Why do the gauge couplings appear to converge?	✓	✓
+		
What is the lifetime of the proton?	✓	✓
How is doublet-triplet splitting achieved?	✓	✓
What is the scale of the MSSM μ-term?	✓	✓

Thank you!

Backup slides

Numerics - SU(5)

Input		Output			
u_{ij} :	$\begin{pmatrix} 0.9566 & 0.7346 & 0.7198 \\ \cdot & 0.5961 & 0.3224 \\ \cdot & \cdot & 0.5435 \end{pmatrix}$	θ^q_{12}	13.027°	θ^l_{12}	34.3°
		θ^q_{13}	0.1802°	θ^l_{13}	8.67°
		θ^q_{23}	2.054°	θ^l_{23}	45.8°
		δ^q	69.18°	δ^l	-86.7°
d_{ij} :	$\begin{pmatrix} 2.133 & 0.8363 & \\ & 1.108 & \\ & & 1.021 \end{pmatrix}$	y_u	2.92×10^{-6}	Δm^2_{21}	7.38×10^{-5} eV^2
		y_c	1.43×10^{-3}	Δm^2_{31}	2.48×10^{-3} eV^2
		y_t	5.34×10^{-1}		
		y_d	4.30×10^{-6}	y_e	1.97×10^{-6}
m_a : 26.57 meV		y_s	9.51×10^{-5}	y_μ	4.16×10^{-4}
m_b : 2.684 meV		y_b	7.05×10^{-3}	y_τ	7.05×10^{-3}

Table: $\tan\beta = 5$. $\chi^2 \approx 8$. Majorana phase: $\alpha_{21} = 72^\circ$.

Numerics - SO(10)

Observables	Model	Data fit 1σ range
θ^q_{12} /°	13.024	12.985 $\rightarrow$ 13.067
θ^q_{13} /°	0.1984	0.1866 $\rightarrow$ 0.2005
θ^q_{23} /°	2.238	2.202 $\rightarrow$ 2.273
δ^q /°	69.32	66.12 $\rightarrow$ 72.31
m_u /MeV	0.575	0.351 $\rightarrow$ 0.666
m_c /MeV	248.4	240.1 $\rightarrow$ 257.5
m_t /GeV	92.79	89.84 $\rightarrow$ 95.77
m_d /MeV	0.824	0.744 $\rightarrow$ 0.929
m_s /MeV	15.55	15.66 $\rightarrow$ 17.47
m_b /GeV	0.939	0.925 $\rightarrow$ 0.948
θ^l_{12} /°	33.83	32.83 $\rightarrow$ 34.27
θ^l_{13} /°	8.35	8.29 $\rightarrow$ 8.68
θ^l_{23} /°	42.88	40.63 $\rightarrow$ 43.85
δ^l /°	270	192 $\rightarrow$ 318

Numerics - SO(10)

Observables		Model	Data fit 1σ range
m_e	/MeV	0.342	$0.340 \rightarrow 0.344$
m_μ	/MeV	72.25	$71.81 \rightarrow 72.68$
m_τ	/GeV	1.229	$1.223 \rightarrow 1.236$
Δm_{21}^2	/eV^2	7.58×10^{-5}	$(7.33 \rightarrow 7.69) \times 10^{-5}$
Δm_{31}^2	/eV^2	2.44×10^{-3}	$(2.41 \rightarrow 2.50) \times 10^{-3}$
m_1	/meV	0.13	–
m_2	/meV	8.71	–
m_3	/meV	49.4	–
$\sum m_i$	/meV	58.2	< 230
α_{21}	/°	24	–
α_{31}	/°	105	–
$\lvert m_{ee} \rvert$	/meV	2.78	–