Double Copy of Classical Bremsstrahlung Solutions

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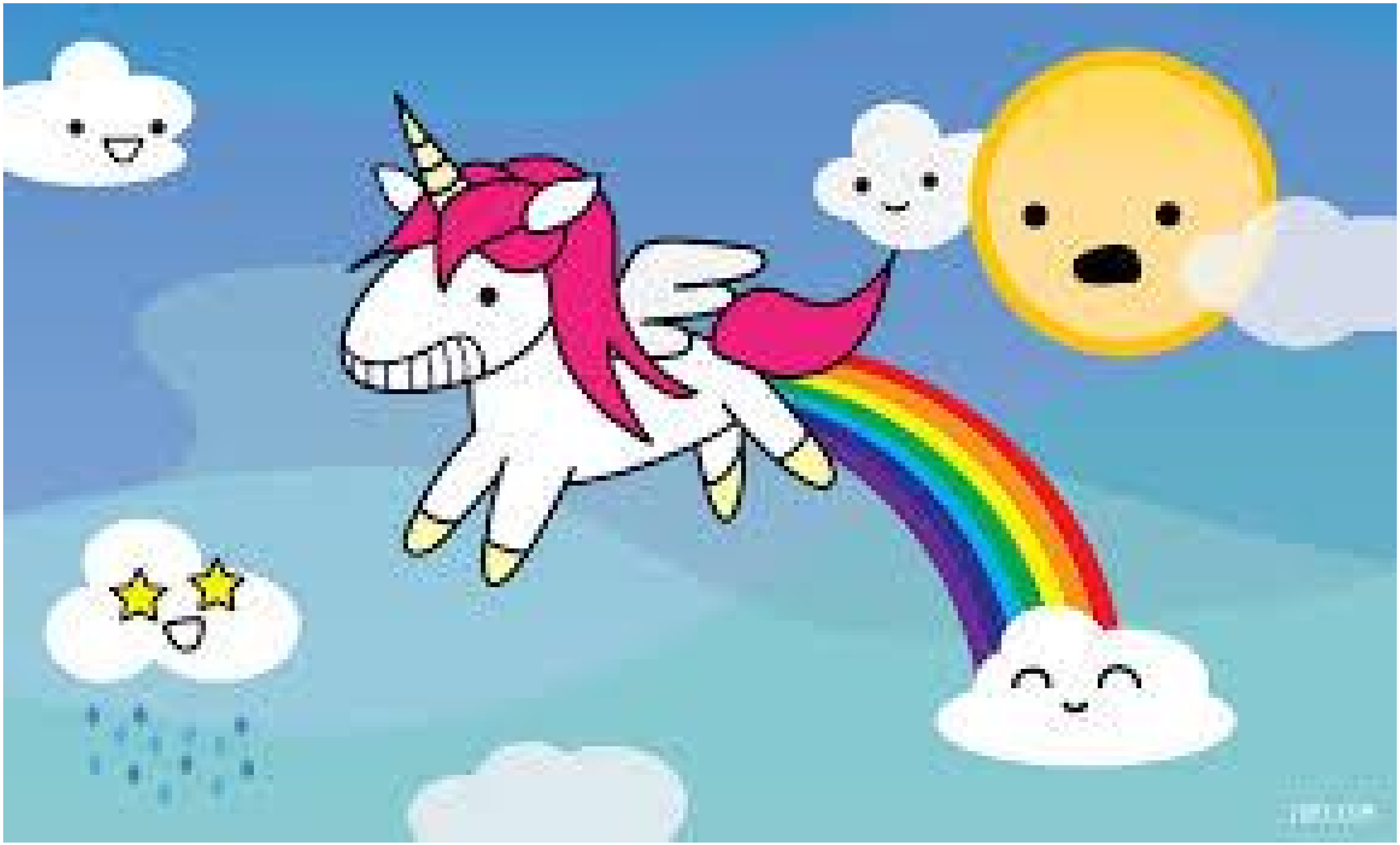
Based on work with Ricardo Monteiro, Isobel Nicholson, Donal O’Connell and Chris White

Young Theorists’ Forum 8
Durham University
January 2016

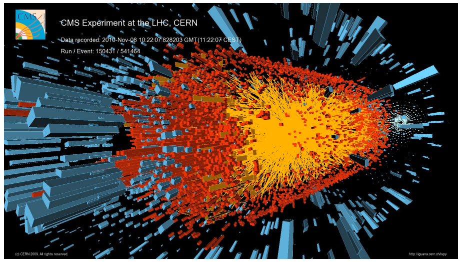
Vindicating the Unicorn Shit

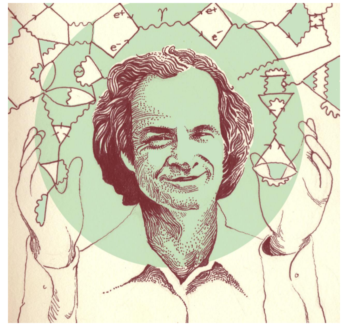


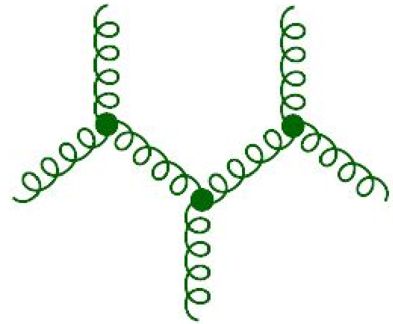
Outline

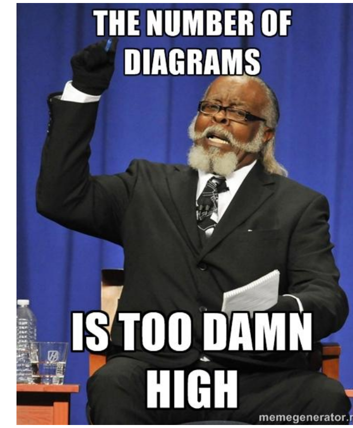
- Amplitudes and Color Kinematics duality
- Double Copy
- Classical Solutions
- Bremsstrahlung and back to Amplitudes

Amplitudes

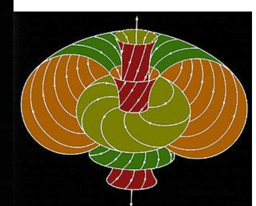


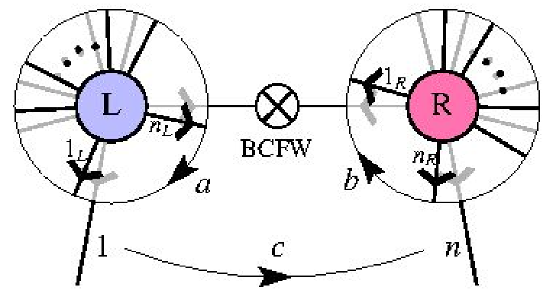






$n=$	4	5	6	7	8	9	10
	4	25	220	2485	34300	559, 405	10, 525, 900





Nuclear Physics B291 (1987) 392–428
North-Holland, Amsterdam

HELICITY AMPLITUDES FOR MULTIPLE BREMSSTRAHLUNG IN MASSLESS NON-ABELIAN GAUGE THEORIES*

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Department of Physics, Tsinghua University, Beijing, The People's Republic of China

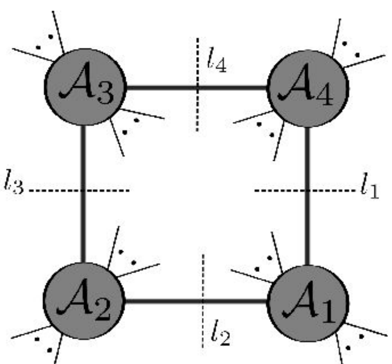
Received 22 January 1985
(Revised 10 December 1986)

We formulate the helicity amplitudes for multiple bremsstrahlung processes in massless gauge theories in terms of spinor inner-products. In the non-abelian case (QCD) we show that contributions from the diagrams with gluon self-coupling can be decomposed into parts to be included together with QED-like diagrams in gauge-invariant subsets, and the amplitudes of the parts are related to the amplitudes of QED-like diagrams. This enables us to make use of the simplifying power of the helicity amplitude method as far as possible.

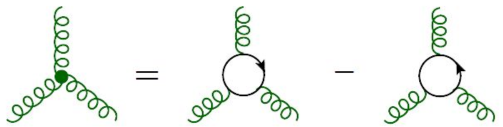
1. Introduction

The cross section of multiple bremsstrahlung processes at high energies in abelian (and especially in non-abelian) gauge theories are of interest for both experimentalists and theoreticians, because of the development of experiments on colliders. In these processes, as far as the electromagnetic and strong interactions are concerned, electrons, muons and light quarks can be considered massless. When perturbation theory is applicable, the problem is reduced to the calculation of the amplitudes of Feynman diagrams. The state of a massless particle can be specified by its momentum and helicity, thus the amplitudes for the above processes depend on the momenta and helicities of all particles involved.

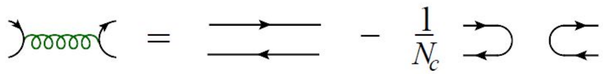
(a)



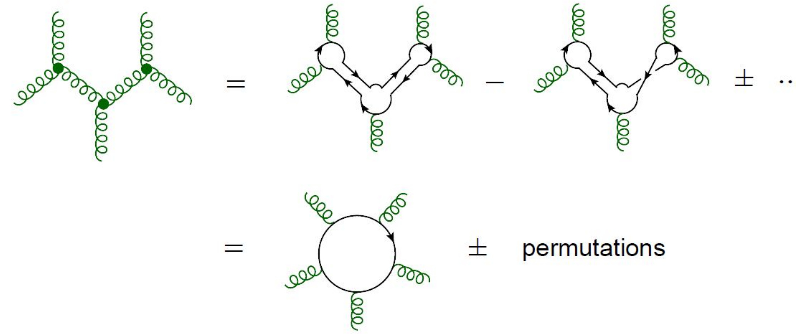
Color Ordered Amplitudes



$$\tilde{f}^{abc} = \mathrm{Tr}(T^a T^b T^c) - \mathrm{Tr}(T^a T^c T^b)$$



$$(T^a)_{i_1}^{\bar{j}_1} (T^a)_{i_2}^{\bar{j}_2} = \delta_{i_1}^{\bar{j}_2} \delta_{i_2}^{\bar{j}_1} - \frac{1}{N_c} \delta_{i_1}^{\bar{j}_1} \delta_{i_2}^{\bar{j}_2}$$



$$\mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n) = g^{n-2} \sum_{\mathcal{P}(2,3,\ldots,n)} \mathrm{Tr}[T^{a_1} T^{a_2} T^{a_3} \cdots T^{a_n}]\, A_n^{\text{tree}}(1,2,3,\ldots,n)$$

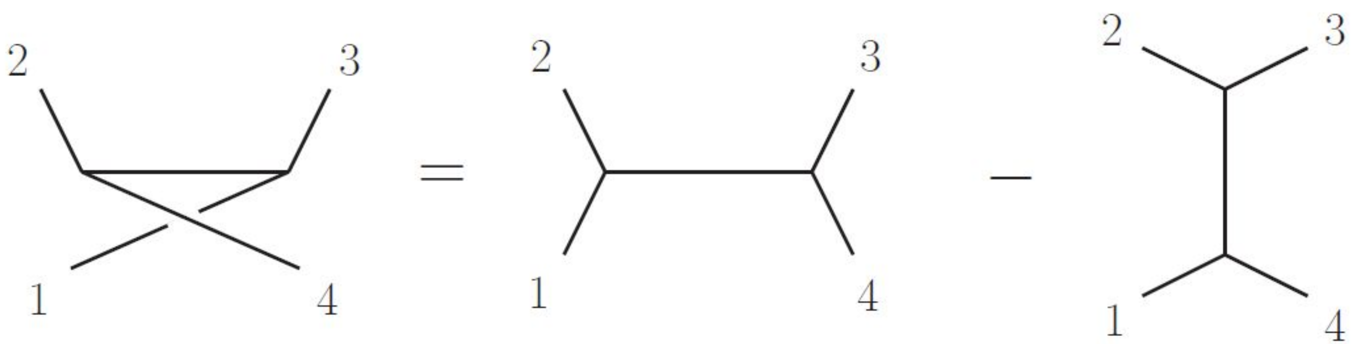
Cyclic Relation →

$$\sum_{\sigma \in \text{cyclic}} A_n^{\text{tree}}(1, \sigma(2,3,\ldots,n)) = 0$$

Kleiss-Kuijf Relation →

$$A_n^{\text{tree}}(1,\{\alpha\},n,\{\beta\}) = (-1)^{n_\beta} \sum_{\{\sigma\}_i \in \mathrm{OP}(\{\alpha\},\{\beta^T\})} A_n^{\text{tree}}(1,\{\sigma\}_i,n)$$

BCJ Duality



c_u $\qquad$ c_s $\qquad$ c_t

$$c_u \equiv \tilde{f}^{a_4 a_2 b} \tilde{f}^{b a_3 a_1}\,, \qquad c_s \equiv \tilde{f}^{a_1 a_2 b} \tilde{f}^{b a_3 a_4}\,, \qquad c_t \equiv \tilde{f}^{a_2 a_3 b} \tilde{f}^{b a_4 a_1}\,,$$

$$c_u = c_s - c_t\,.$$

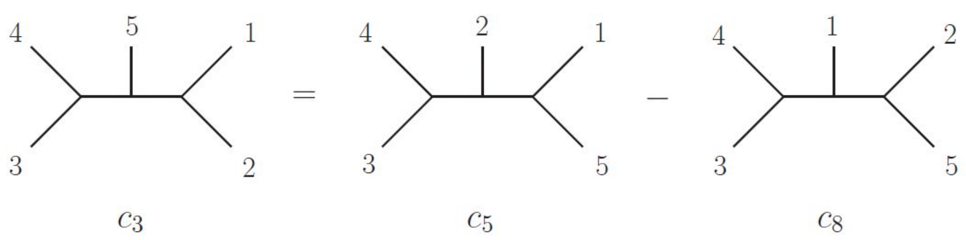
$$n_u = n_s - n_t$$

$$\frac{1}{g^{n-2}} \mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{n_i c_i}{(\prod_j p_j^2)_i}$$

BCJ Duality

$$\frac{1}{g^{n-2}}\mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{n_i c_i}{(\prod_j p_j^2)_i}$$

$$c_\alpha - c_\beta + c_\gamma = 0\,, \qquad \Rightarrow \qquad n_\alpha - n_\beta + n_\gamma = 0$$



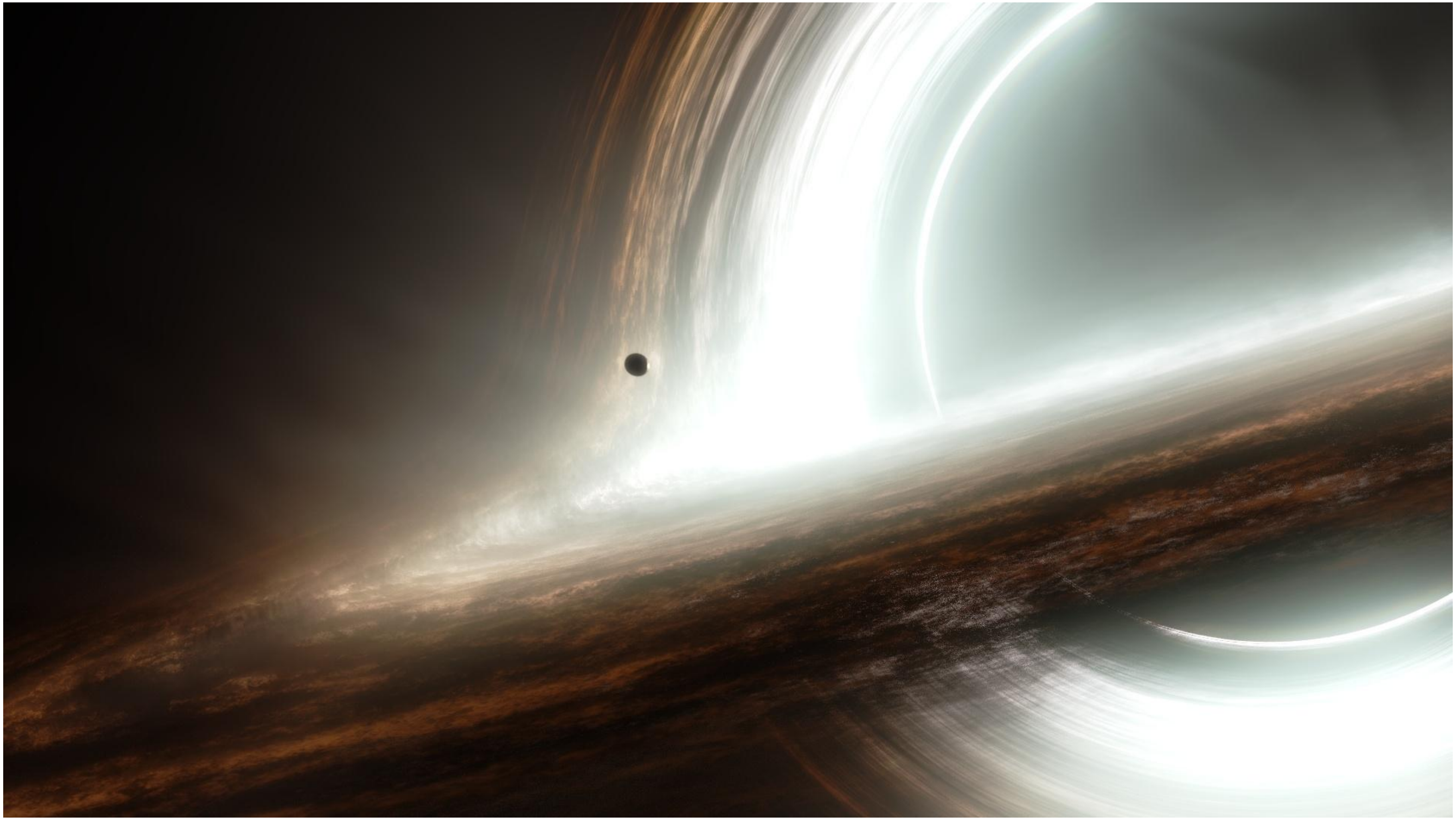
BCJ Relations

$$0 = I_4 = A(2,4,3,1)(s_{43}+s_{41}) + A(2,3,4,1)s_{41}$$

$$\begin{aligned} 0 = I_5 = {} & A(2,4,3,5,1)(s_{43}+s_{45}+s_{41}) \\ & + A(2,3,4,5,1)(s_{45}+s_{41}) + A(2,3,5,4,1)s_{41} \end{aligned}$$

$$\begin{aligned} 0 = I_6 = {} & A(2,4,3,5,6,1)(s_{43}+s_{45}+s_{46}+s_{41}) \\ & + A(2,3,4,5,6,1)(s_{45}+s_{46}+s_{41}) \\ & + A(2,3,5,4,6,1)(s_{46}+s_{41}) + A(2,3,5,6,4,1)s_{41} \end{aligned}$$

Gravity



KLT

$$
\begin{aligned}
-iM_5^{\text{tree}}(1,2,3,4,5) \;=\; & \frac{s_{12}s_{45}\big(s_{12}s_{14}s_{23}+s_{34}(s_{12}+s_{13})(s_{23}+s_{25})\big)}{s_{13}s_{24}s_{35}} \\
& \times A_5^{\text{tree}}(1,2,3,4,5)\widetilde{A}_5^{\text{tree}}(1,2,3,4,5) \\
& - \frac{s_{12}s_{14}s_{25}(s_{13}+s_{35})s_{45}}{s_{13}s_{24}s_{35}}\Big(A_5^{\text{tree}}(1,2,3,4,5)\widetilde{A}_5^{\text{tree}}(1,4,3,2,5) \\
& \qquad + A_5^{\text{tree}}(1,4,3,2,5)\widetilde{A}_5^{\text{tree}}(1,2,3,4,5)\Big) \\
& + \frac{s_{14}s_{25}\big(s_{12}s_{14}s_{34}+s_{23}(s_{13}+s_{14})(s_{34}+s_{45})\big)}{s_{13}s_{24}s_{35}} \\
& \times A_5^{\text{tree}}(1,4,3,2,5)\widetilde{A}_5^{\text{tree}}(1,4,3,2,5)\,.
\end{aligned}
$$

Nuclear Physics B269 (1986) 1–23

A RELATION BETWEEN TREE AMPLITUDES OF CLOSED AND OPEN STRINGS*

H. KAWAI, D.C. LEWELLEN and S.-H.H. TYE

Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853, USA

Received 11 October 1985

We derive a formula which expresses any closed string tree amplitude in terms of a sum of the products of appropriate open string tree amplitudes. This formula is applicable to the heterotic string as well as to the closed bosonic string and type II superstrings. In particular, we demonstrate its use by showing how to write down, without any direct calculation, all four-point heterotic string tree amplitudes with massless external particles.

Double Copy

$$c_\alpha - c_\beta + c_\gamma = 0\,, \qquad \Rightarrow \qquad n_\alpha - n_\beta + n_\gamma = 0$$

$$\mathcal{A}_m^{(L)} = i^L g^{m-2+2L} \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^d p_l}{(2\pi)^d} \frac{1}{S_i} \frac{\boxed{n_i}\, c_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

$$\mathcal{M}_m^{(L)} = i^{L+1} \left(\frac{\kappa}{2}\right)^{m-2+2L} \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^d p_l}{(2\pi)^d} \frac{1}{S_i} \frac{\boxed{n_i\, \tilde{n}_i}}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

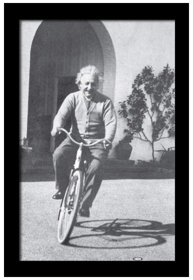
Black Holes and Double Copy

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$\equiv \eta_{\mu\nu} + k_\mu k_\nu \phi$$

$$\eta^{\mu\nu} k_\mu k_\nu = 0 = g^{\mu\nu} k_\mu k_\nu \qquad g^{\mu\nu} = \eta^{\mu\nu} - \phi k^\mu k^\nu$$

Kerr-Schild Solutions

$$R^\mu{}_\nu = \frac{1}{2}\left(\partial^\mu \partial_\alpha \left(\phi k^\alpha k_\nu\right) + \partial_\nu \partial^\alpha \left(\phi k_\alpha k^\mu\right) - \partial^2 \left(\phi k^\mu k_\nu\right)\right)$$



$$R^0{}_0 = \frac{1}{2}\nabla^2 \phi;$$
$$R^i{}_0 = -\frac{1}{2}\partial_j \left[\partial^i \left(\phi k^j\right) - \partial^j \left(\phi k^i\right)\right]$$

$$A_\mu = \phi k_\mu$$
$$\partial_\mu F^{\mu\nu} = \partial_\mu(\partial^\mu(\phi k^\nu) - \partial^\nu(\phi k^\mu)) = 0$$

Black Holes and Double Copy

Kerr-Schild Solutions

$$A^{\mu}_{a} = c_a \phi k^{\mu}$$

$$g^{\mu\nu} = \eta^{\mu\nu} + \phi k^{\mu} k^{\nu}$$

Amplitudes

$$\mathcal{A}^{(L)}_{m} = i^{L} g^{m-2+2L} \sum_{i \in \Gamma} \int \prod_{l=1}^{L} \frac{d^d p_l}{(2\pi)^d} \frac{1}{S_i} \frac{n_i c_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

$$\mathcal{M}^{(L)}_{m} = i^{L+1} \left(\frac{\kappa}{2}\right)^{m-2+2L} \sum_{i \in \Gamma} \int \prod_{l=1}^{L} \frac{d^d p_l}{(2\pi)^d} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

R.Monteiro, D. O'Connell, C. D. White [arXiv:1410.0239]

Taub-NUT and Double Copy

$$\begin{aligned} g_{\mu\nu} &= \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \\ &= \bar{g}_{\mu\nu} + \kappa\,(\phi\, k_\mu\, k_\nu + \psi\, l_\mu\, l_\nu) \end{aligned}$$

Double Kerr-Schild

$$d\bar{s}^2 = -\frac{1}{q^2-p^2}\left[\bar{\Delta}_p(d\tilde{\tau} + q^2 d\tilde{\sigma})^2 + \bar{\Delta}_q(d\tilde{\tau} + p^2 d\tilde{\sigma})^2\right] - 2(d\tilde{\tau} + q^2 d\tilde{\sigma})dp - 2(d\tilde{\tau} + p^2 d\sigma)dq$$

De Sitter

$$A^a_\mu = c^a\,(\phi k_\mu + \psi l_\mu)$$

$$F_{\mu\nu} = \frac{(c_a T^a)}{4\pi}\left[\begin{pmatrix} 0 & -\frac{g_s}{r^2} & 0 & 0 \\ \frac{g_s}{r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\tilde{g}_s \csc\theta}{r^4} \\ 0 & 0 & \frac{\tilde{g}_s \csc\theta}{r^4} & 0 \end{pmatrix}\right]$$

Dyon-like Object

A.Luna, R.Monteiro, D. O'Connell, C. D. White [arXiv:1507.01869]

Photon Rockets

$$g_{\mu\nu}=\eta_{\mu\nu}-2mr^{-3}\sigma_\mu\sigma_\nu\,, \qquad (9)$$

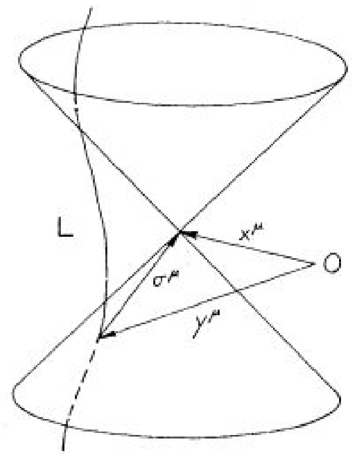
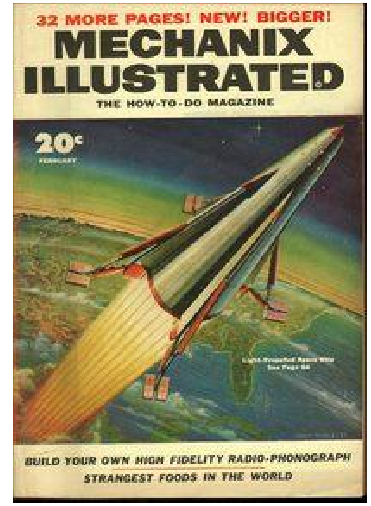
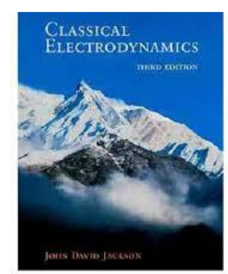


FIG. 1. Unique retarded null vector σ^μ connecting an arbitary point x to the timelike world line L.

$$h_{\mu\nu} = \frac{m}{r} k_\mu k_\nu$$

$$T^{\mu\nu} = -3m \frac{k\cdot\dot{u}}{r^2} k^\mu k^\nu \Big|_{\tau=\tau_{ret}}$$



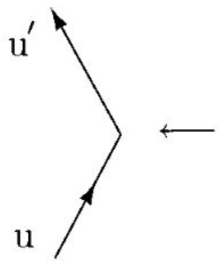



KEEP
CALM
AND INTERPRET
~~EAT~~ UNİCORN
SHİT

Back to Scattering Amplitudes

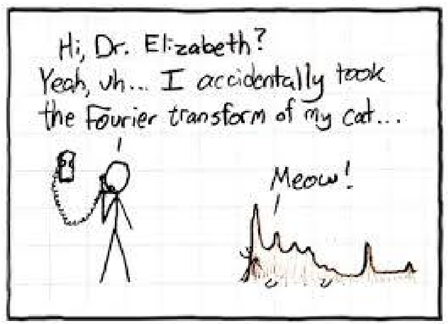
$$A_\mu = \frac{e}{r} k_\mu$$

$$j^\nu = -2e\frac{k\cdot\dot{u}}{r^2}k^\nu\Big|_{\tau=\tau_{ret}}$$



← sudden kick at time $t = 0$, when particle is at $\mathbf{x} = 0$

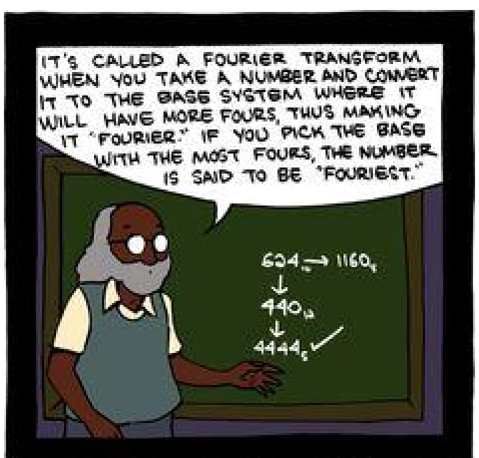
$$j^\nu(x) = -2e\theta(x^0)\delta(x^2)\left[\frac{\partial}{\partial u'_\nu}\frac{1}{x\cdot u'} - (u \leftrightarrow u')\right]$$



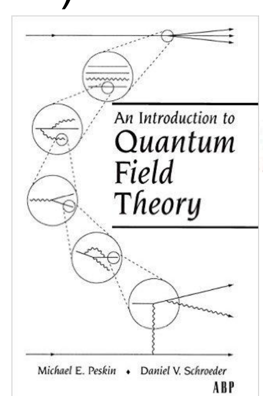
(There you go, Euan)

+ $k^2 \to 0$

$$\tilde{j}^\nu(k) = 2ie\left(\frac{u'^\nu}{k\cdot u'} - \frac{u^\nu}{k\cdot u}\right)$$



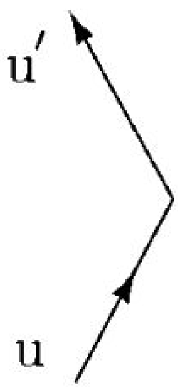
Teaching math was way more fun after tenure.



$$SA = ie\left[\epsilon^*\cdot\left(\frac{u'}{u'\cdot k} - \frac{u}{u\cdot k}\right)\right]$$

Double Copy

Gauge	Gravity
$A_\mu = \frac{e}{r} k_\mu$	$h_{\mu\nu} = \frac{m}{r} k_\mu k_\nu$
$j^\nu = -2e \frac{k \cdot \dot{u}}{r^2} k^\nu \Big\rvert_{\tau=\tau_{ret}}$	$T^{\mu\nu} = -3m \frac{k \cdot \dot{u}}{r^2} k^\mu k^\nu \Big\rvert_{\tau=\tau_{ret}}$



$\longleftarrow$ sudden kick at time $t = 0$, when particle is at $\mathbf{x} = 0$

\+ 

\+ $k^2 \to 0$

$$\tilde{j}^\nu(k) = 2ie\left(\frac{u'^\nu}{k \cdot u'} - \frac{u^\nu}{k \cdot u}\right)$$

$$\tilde{T}^{\mu\nu}(k) = 2im\left(\frac{u'^\mu u'^\nu}{k \cdot u'} - \frac{u^\mu u^\nu}{k \cdot u}\right)$$

Double Copy of (Classical) Scattering Amplitudes for Bremsstrahlung

Further Work

Wilson Lines


THANK
YOU
GRACIAS
ARIGATO
SHUKURIA
DANKSCHEEN
BİYAN
SHUKRIA
TINGKI
YAQHANYELAY
TASHAKKUR ATU
SUKSAMA
EKHMET
GRAZIE
MEHRBANI
PALDIES
BOLZÏN
MERCI
KOMAPSUMNIDA
MAAKE
GOZAIMASHITA
EFCHARISTO
JUSPAXAR
SPASSIBO
SNACHALHUYA
NUHUN
CHALTU
WABEEJA
MAITEKA
SPASIBO
DENKAUJA
NENACHALHYA
SIKOMO
MINMONCHAR
TAVTAPUCH
MEDAWAGSE
MERASTAWHY
GAEJTHO
AGUYJE
FAKAAUE