Unitarity Bound In Composite Two Higgs Doublet Models

Emine Yildirim
University of Southampton
Young Theorists' Forum, Durham

Work (in progress) in Collaboration with:
Stefania De Curtis, Stefano Moretti, and Kei Yagyu

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Outline

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The idea of composite two Higgs doublet model

- Higgs boson emerges as a pseudo-Nambu-Goldstone Boson (pNGB) from a new strong interaction.
- The pNGB associated to a spontaneously broken global symmetry(G $\rightarrow$ H) at compositeness f.
- The Composite 2 Higgs Doublet Model (C2HDM) based on $SO(6)/SO(4) \times SO(2)$ coset.
- It develops dim(G)−dim(H)=$15 - 7 = 8$ GBs, which are identified with the (composite) two Higgs doublet fields.
- Symmetry breaking occurs in two steps
  1. Spontaneously global symmetry breaking $SO(6) \xrightarrow{f} SO(4) \times SO(2)$ at scale f.
  2. Electroweak symmetry breaking is triggered by coupling of the SM particles to the composite sector.
- The Higgs potential is generated through the Coleman-Weinberg (CW) mechanism at loop levels.

Effective Lagrangian approach for C2HDM

$\Rightarrow$The invariant kinetic Lagrangian under the $SO(6)$ symmetry can be constructed by the analogue of the construction in non-linear sigma models developed by Callan-Coleman-Wess-Zumino (CCWZ) as

$$\mathcal{L}_{kin} = \frac{f^2}{4}(d^{\hat{a}}_{\alpha})_{\mu}(d^{\hat{a}}_{\alpha})^{\mu} \qquad (d^{\hat{a}}_{\alpha})_{\mu} = i\ tr(U^{\dagger}D_{\mu}UT^{\hat{a}}_{\alpha})$$

$$U = \exp(i\frac{\Pi}{f}) \quad \Pi \equiv \sqrt{2}h^{\hat{a}}_{\alpha}T^{\hat{a}}_{\alpha} = -i\begin{pmatrix} O_{4\times 4} & h^{\hat{a}}_1 & h^{\hat{a}}_2 \\ -h^{\hat{a}}_1 & 0 & 0 \\ -h^{\hat{a}}_2 & 0 & 0 \end{pmatrix}.$$

$$i(d^{\hat{1}}_{\alpha})_{\mu} + (d^{\hat{2}}_{\alpha})_{\mu} = -\frac{2}{f}\left[\partial_{\mu}\phi^{+}_{\alpha} - i\frac{g}{\sqrt{2}}\phi^{0}_{\alpha}W^{+}_{\mu} - ig_Z\left(\frac{1}{2} - s^2_W\right)\phi^{+}_{\alpha}\right] + \mathcal{O}(1/f^3),$$

$$-i(d^{\hat{3}}_{\alpha})_{\mu} + (d^{\hat{4}}_{\alpha})_{\mu} = \frac{2}{f}\left[\partial_{\mu}\phi^{0}_{\alpha} - i\frac{g}{\sqrt{2}}\phi^{+}_{\alpha}W^{-}_{\mu} + i\frac{g_Z}{2}\phi^{0}_{\alpha}Z_{\mu}\right] + \mathcal{O}(1/f^3).$$

$\Rightarrow$ Modified Higgs to gauge boson coupling from SM prediction:

$$\frac{\lambda^{C2HDM}_{hW^+W^-}}{\lambda^{SM}_{hW^+W^-}} = \sqrt{1-\xi}$$

$$\frac{\lambda^{C2HDM}_{HW^+W^-}}{\lambda^{2HDM}_{HW^+W^-}} = \sqrt{1-\xi}$$

$$\frac{\lambda^{C2HDM}_{hH^+W^-}}{\lambda^{2HDM}_{hH^+W^-}} = \sqrt{1-\frac{1}{6}\xi\tan\theta}$$

where $\xi = \frac{v^2}{f^2}$ with $v \simeq 246\,GeV$.

Perturbative Unitarity

- Perturbative unitarity gives a bound on the parameters of the model.
- $$\mathcal{M} = 16\pi\Sigma_{J=0}^{\infty}(2J+1)P_J(\cos\theta)a_J(s)$$

  Perturbative unitarity bound $\boxed{|a_J|^2 \leq \frac{1}{2}}$
- We calculate the S-wave amplitude for all the possible two body to two body elastic scalar boson scatterings.
- Equivalence theorem $W_L^{\pm}, Z_L \Rightarrow G^{\pm}, G$ in the high energy limit.
  $M(G^+G^- \to G^+G^-)$ is the same as $M(W^+W^- \to W^+W^-)$.
- Coefficient of the quartic scalar interactions from the potential and that of scalar quartic interaction terms with two derivatives from kinetic Lagrangian contribute to S-wave amplitude matrix.

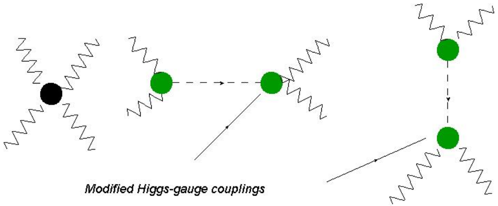
Generic formulae for the 2 body scalar boson scattering amplitude

$$\mathcal{M}_c(AB \to CD) = -(g_{AB,CD}P_{AB} + g_{CD,AB}P_{CD})$$
$$+ g_{AC,BD}P_{AC} + g_{BD,AC}P_{BD} + g_{AD,BC}P_{AD} + g_{BC,AD}P_{BC} + \lambda_{ABCD},$$
$$\mathcal{M}_s(AB \to X \to CD) = -\frac{1}{s - m_X^2}(g_{AB,X}P_{AB} - g_{XA,B}P_{XA} - g_{BX,A}P_{BX} - \lambda_{ABX})$$
$$\times (g_{CD,X}P_{CD} - g_{XC,D}P_{XC} - g_{DX,C}P_{DX} - \lambda_{CDX}),$$
$$\mathcal{M}_t(AB \to X \to CD) = -\frac{1}{t - m_X^2}(-g_{AC,X}P_{AC} - g_{XA,C}P_{XA} + g_{CX,A}P_{CX} - \lambda_{ACX})$$
$$\times (-g_{BD,X}P_{BD} + g_{XB,D}P_{XB} - g_{DX,B}P_{DX} - \lambda_{BDX}),$$
$$\mathcal{M}_u(AB \to X \to CD) = -\frac{1}{u - m_X^2}(-g_{AD,X}P_{AD} - g_{XA,D}P_{XA} + g_{DX,A}P_{DX} - \lambda_{ADX}$$
$$\times (-g_{BC,X}P_{BC} + g_{XB,C}P_{XB} - g_{CX,B}P_{CX} - \lambda_{BCX},$$
$$g_{ab,cd} \equiv \frac{\partial^4 \mathcal{L}_{kin}}{\partial(\partial_\mu a)\partial(\partial_\mu b)\partial(c)\partial(d)}, \quad g_{ab,c} \equiv \frac{\partial^3 \mathcal{L}_{kin}}{\partial(\partial_\mu a)\partial(\partial_\mu b)\partial(c)},$$
$$\lambda_{abcd} \equiv -\frac{\partial^4 V}{\partial a \partial b \partial c \partial d} \quad \lambda_{abc} \equiv -\frac{\partial^3 V}{\partial a \partial b \partial c}$$

Perturbative Unitarity in $W_L W_L$ scattering I

$\Rightarrow A(V_L V_L \to V_L V_L)$ grows with energy due to modified $hV_L V_L$, unitarity is lost in the C2HDM.

$$W_L W_L \to W_L W_L$$



Perturbative Unitarity in $W_L W_L$ scattering II

$\Rightarrow$ S-wave amplitude a_0 for $W_L W_L$ scattering:

$$a_0(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s}{32\pi v^2}\xi - \frac{1}{8\pi v^2}(m_h^2 \cos^2\theta + m_H^2 \sin^2\theta)(1-\xi) \leq \frac{1}{2}$$

- Amplitude does not have energy dependence in the SM and 2HDM, in the C2HDM energy dependence survives.

Where, $\theta \to$ angle between CP-even scalar states.

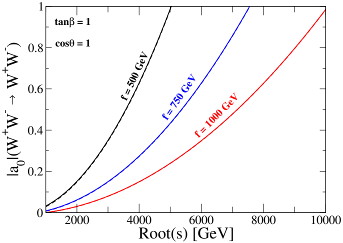


FIG. (right hand side): S-wave amplitude for the $W^+W^- \to W^+W^-$ process as a function of $\sqrt{s}$. The solid (dashed) curve is the result with (without) $\mathcal{O}(\xi s^0)$ term.

Perturbative Unitarity In $(H^+H^- \to H^+H^-)$ Scattering I

$$a_0(H^+H^- \to H^+H^-) = \left[\boxed{\frac{s}{32\pi \upsilon^2}\xi - \frac{2}{3}\frac{m^2_{H^\pm}}{\upsilon^2}\xi} + \boxed{\lambda_{H^+H^-H^+H^-}} \right]$$

Emerges From Kinetic Term

From Potential Term

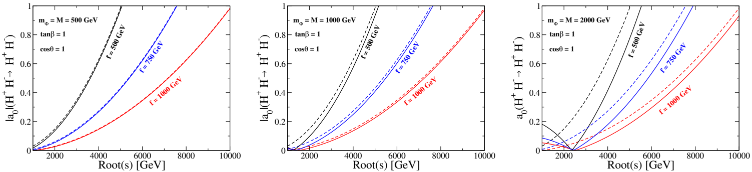


Figure : S-wave amplitude for the $H^+H^- \to H^+H^-$ process as a function of $\sqrt{s}$ in the case of $\cos\theta = 1$ and $\tan\beta = 1$ with each fixed value of $f = 500$ (black),750 (blue) and 1000 GeV (red). The solid (dashed) curve is the result with (without) $\mathcal{O}(\xi s^0)$ term. The left, center and right panels show the result for $m_\Phi = M = 500, 1000$ and 2000 GeV, respectively.

$$\lambda_{H^+H^-H^+H^-} = [\frac{2}{\upsilon^2}4M^2\cot^2 2\beta - m_h^2(c_\theta + 2\cot 2\beta \sin\theta)^2) - m_H^2(s_\theta - 2\cot 2\beta c_\theta)^2](1 - \frac{\xi}{3}) + \frac{4c_{2\beta}}{3\upsilon^2 s^2_{2\beta}}[m_h^2(c_\theta s_{2\beta} + 2s_\theta c_{2\beta})s_\theta + m_H^2(2c_\theta c_{2\beta} - s_\theta s_{2\beta})c_\theta]\xi.$$

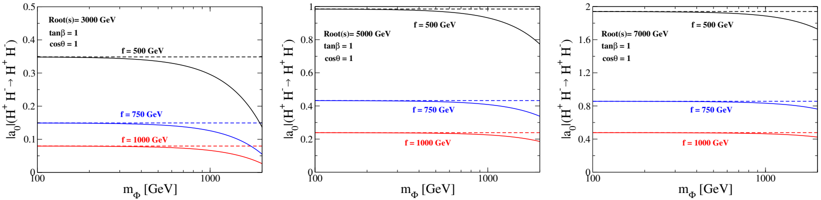


Figure : S-wave amplitude for the $H^+H^- \to H^+H^-$ process as a function of $m_\phi = m_A (= m_H = m_H^\pm)$ in the case of $\cos\theta = 1, \tan\beta = 1$ and $M = m_\phi$ with each fixed value of $f = 500$ (black),750 (blue) and 1000 GeV (red). The left, center and right panels respectively show the case with $\sqrt{s} = 3000, 5000$ and 7000 GeV. The solid (dashed) curve is the result with (without) $\mathcal{O}(\xi s^0)$ term.

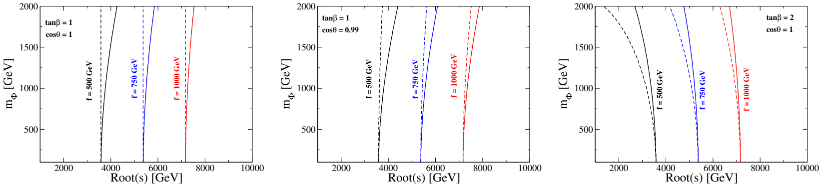


Figure : Bound on the parameter space on the $\sqrt{s} - m_\Phi$ plane from the requirement of $|a_0|(H^+H^- \to H^+H^-) < 1/2$ in the case of $M = m_\Phi$. In the left, center and right panels, we take $(\cos\theta, \tan\beta) = (1,1), (0.99,1)$ and $(1,2)$, respectively. The solid (dashed) curve is the result with (without)$\mathcal{O}(\xi s^0)$ term.

S-wave amplitude matrix for all scattering channels

$\Rightarrow$ We calculate all the two body scalar boson scattering amplitudes.

14 neutral channels are expressed by

$$\boxed{G^+G^-, \frac{GG}{\sqrt{2}}, \frac{hh}{\sqrt{2}}, hG, H^+H^-, \frac{AA}{\sqrt{2}}, \frac{HH}{\sqrt{2}}, HA, hH, GA, hA, HG, G^+H^-, H^+G^-}$$

8 singly charged channels are expressed by

$$\boxed{G^+Z, H^+A, G^+h, H^+h, G^+A, H^+Z, G^+H, H^+h}$$

3-doubly charged channels are expressed by

$$\boxed{\frac{G^+G^+}{\sqrt{2}}, \frac{H^+H^+}{\sqrt{2}}, G^+H^+}$$

- Although each of neutral, singly-charged and doubly-charged states respectively gives the $14 \times 14, 8 \times 8$ and 3×3 S-wave amplitude matrix, they can be simplified to the be block diagonalized 2×2 sub-matrices according to their quantum numbers hyper-charge Y, isospin number I and its third component I_3.

All independent eigenvalues values

$$16\pi a_1^{\pm} = \frac{3}{2}\frac{\xi s}{\upsilon_{SM}} - \frac{1}{2}[3(\lambda_1+\lambda_2) \pm \frac{1}{2}\sqrt{9(\lambda_1-\lambda_2)^2 + (\frac{\xi s}{\upsilon_{SM}} + 4\lambda_3 - 2\lambda_4)^2}],$$

$$16\pi a_2^{\pm} = -\frac{1}{2}\frac{\xi s}{\upsilon_{SM}} - \frac{1}{2}[(\lambda_1+\lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1-\lambda_2)^2 + (\frac{\xi s}{\upsilon_{SM}} - 2\lambda_4)^2}],$$

$$16\pi a_3^{\pm} = \pm\frac{1}{2}\frac{\xi s}{\upsilon_{SM}} - \frac{1}{2}[(\lambda_1+\lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1-\lambda_2)^2 + (\frac{\xi s}{\upsilon_{SM}} - 2\lambda_5)^2}],$$

$$16\pi a_4^{\pm} = \frac{\xi s}{\upsilon_{SM}} - (\lambda_3 + 2\lambda_4 \pm 3\lambda_5), \quad 16\pi a_5^{\pm} = \pm\frac{\xi s}{\upsilon_{SM}} - (\lambda_3 \mp \lambda_5),$$

$$16\pi a_6^{\pm} = \pm\frac{\xi s}{\upsilon_{SM}} - (\lambda_3 \mp \lambda_5).$$

$\Rightarrow$The eigenvalues listed above give the constraints $|a_i^{\pm}| \leq 8\pi$

- All λ terms are defined in terms of physical parameters, as a result perturbative unitarity gives upper limit on the masses of Higgs boson.

Unitarity constraints by all the channels

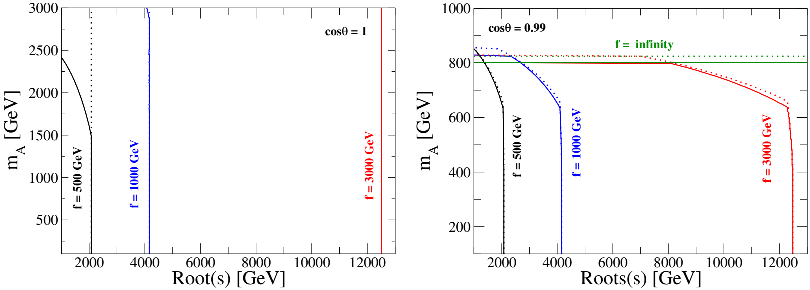


Figure : Constraint on the parameter space on the $\sqrt{s} - m_A$ plane from the unitarity and the vacuum stability in the case of $\tan\beta = 1$ and $m_{H^\pm} = m_A$ for several fixed values of f.The left and right panels show the case with $\cos\theta = 1$ and $\cos\theta = 0.99$ respectively.The lower left region from each curve is allowed. We take the value of $m_H =$ to be equal to be m_A for the solid curves, while we scan it within the region of $m_A \pm 100 GeV$ for the dashed curves. For all the plots, M^2 is scanned.

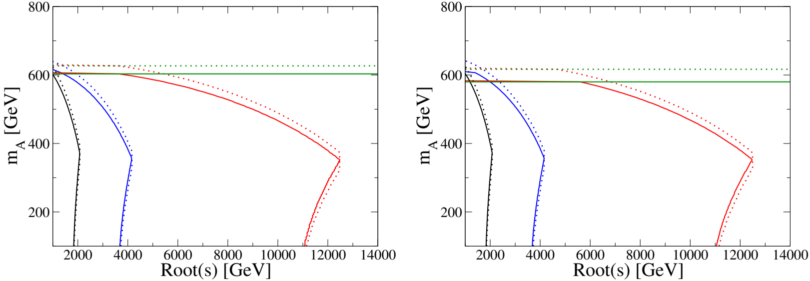


Figure : Constraint on the parameter space on the $\sqrt{s} - m_A$ plane from the unitarity and the vacuum stability in the case of $\tan\beta = 1$ and $m_{H^\pm} = m_A$ for several fixed values of f.The left and right panels show the case with $\cos\theta = 1$ and $\cos\theta = 0.99$ respectively.The lower left region from each curve is allowed. We take the value of $m_H =$ to be equal to be m_A for the solid curves, while we scan it within the region of $m_A \pm 100 GeV$ for the dashed curves. For all the plots, $M^2 = 0$.

Tanβ Dependence

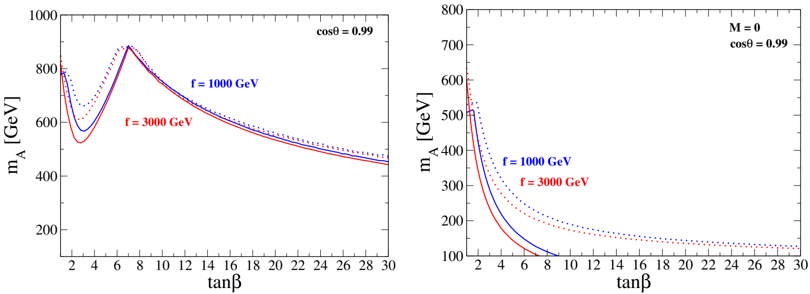


Figure : Constraint on the parameter space on the $\tan\beta - m_A$ plane from the unitarity and the vacuum stability in the case of $\cos\theta = 0.99, \sqrt{s} = 3000$ GeV and $m_{H^\pm} = m_A$ for several fixed values of f . The lower left region from each curve is allowed. The left panel shows the case with M^2 to be scanned, while the right one does the case with $M^2 = 0$. We take the value of m_H to be equal to m_A for the solid curves, while we scan it within the region of $m_A \pm 100$ GeV for the dashed curves.

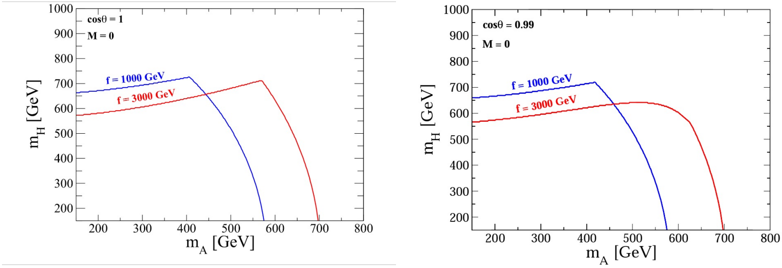


Figure : Constraint on the parameter space on the $m_A - m_H$ plane by unitarity and vacuum stability in the case with $m_{H^\pm} = m_A$. The right-left panels show the case of $(\cos\theta, M) = (1, 0)$ and $(0.99, 0)$ respectively.

Unitarity Bound for Inert Case

- In the absence of VEV in the second doublet (Φ_2) provides four inert scalars , active doublet (Φ_1) provides SM-like Higgs boson.

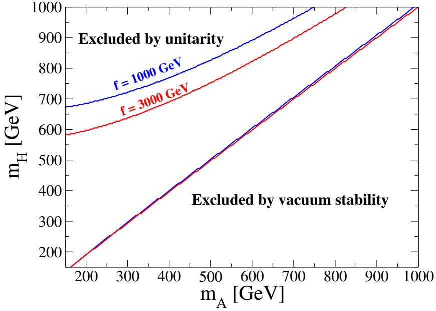


FIG. Constraint on the parameter space on the $m_A - m_H$ plane by unitarity and vacuum stability in the case of $\lambda_2 = 0.1$ and $\sqrt{s} = 3000$ GeV . We take $m_{H\pm} = m_A = m_2$.

Conclusion and Outlook

- Quartic scalar interaction terms with two derivatives in the kinetic Lagrangian give important difference between E2HDM and C2HDM.
- Because of s dependence in the amplitude, unitarity is violated at a certain energy scale.
- Smaller value of compositeness scale f gives stronger upper bound on $\sqrt{s}$.

Future Work

- We will discuss strong EWSB for C2HDM via Coleman-Weinberg (CW) mechanism. In this project, however, we do not calculate the CW potential, because we do not explicitly specify fermion representations. Instead of performing the explicit calculation, we assume the same form of the Higgs potential as that in the elementary 2HDM, but each of the parameters is expressed in terms of those in a strong sector.

Thank You

Backup Slides

$$V(\Phi_1, \Phi_2) = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - m_3^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \quad + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 +$$
$$\frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)$$
$$+ \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{1}{2}\lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}].$$