Supergravity Couplings from CFT Correlation Functions

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Overview

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Introduction

Introduction

What do we want to achieve?

- Holographic principle offers a unique tool
- Crucial to understand the precise correspondence
- Need understanding of:
  1. The CFT field/operator content
  2. The gravity side fields and couplings/scalar potential
  3. What corresponds to what?
- Many of these aspects still need to be tested or established

$\Rightarrow$ My ongoing project

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Introduction

Setup:

- CFT: 4d $\mathcal{N} = 4$ $SU(N)$ SYM
- Gravity: 5d supergravity on $AdS_5(\times S^5)$

Starting point:

- A symmetry argument fixes the value of some correlators $\langle \mathcal{O} \ldots \mathcal{O} \rangle$ for a certain $\mathcal{O}$.

To do:

- Establish correspondence
- Understand gravity couplings

The CFT Correlator

The CFT Correlator

CFT Symmetries and Fields

- The 4d $\mathcal{N} = 4$ $SU(N)$ SYM field content comes from a dimensional reduction of a 10d $\mathcal{N} = 1$ SYM on $\mathbb{T}^6$.
- **Symmetries:** Gauge $SU(N)$ and global $SU(2, 2|4)$, which, in particular, contains the R-symmetry $SO(6) \sim SU(4)_R$
- **Scalar fields:** 6 real scalars ϕ_i^a, in the SU(N) adjoint and in the $\mathbf{6}$ of $SO(6)$ global

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The CFT Correlator

A symmetry argument

- Focus on the 6 real scalar fields ϕ_i in the $\mathbf{6}$ of $SO(6)$
- Define $Z \coloneqq \phi_1 + i\phi_2$
- There is a $U(1)$ subgroup of the global $SO(6)$ under which $Z \to e^{i\alpha} Z$.
- Define $\mathcal{O}_k \coloneqq \mathrm{Tr}_{SU(N)}(Z^k)$, it has conformal dimension $\Delta = k$
- $\mathcal{O}_k \to e^{ik\alpha}\mathcal{O}_k$ under the $U(1)$
- $\langle \mathcal{O}_k \mathcal{O}_k \mathcal{O}_k \mathcal{O}_k \rangle = 0$ because it has a $U(1)$ charge.

  $\langle \mathcal{O}_k \mathcal{O}_k \bar{\mathcal{O}}_k \bar{\mathcal{O}}_k \rangle \neq 0$ because it's neutral.
- With respect to the $SO(6)$ indices $\mathcal{O}_k$ are **symmetric** and **traceless**

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Holographic computation of the correlator

Holographic computation of the correlator

Corresponding fields on the gravity side

- Symmetric traceless tensor fields in CFT holographically correspond to KK states on gravity side
- We are looking at CFT operators $\mathcal{O}_k$ with scaling dimension $\Delta = k$
- The corresponding fields in the bulk are scalars with mass $m^2 = \Delta(\Delta - 4) = k(k-4)$
- Schematic bulk Lagrangian:

$$S_k \sim \frac{4N^2}{(2\pi)^5} \int d^5x \sqrt{-g} \left(-\frac{1}{2} \nabla_a s_k \nabla^a s_k - \frac{1}{2} m_s^2 s_k^2 \right) + \text{interactions}$$

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Holographic computation of the correlator

Holographic computation

- Remember that the mapping between the SYM correlators and bulk fields:

$$\exp(-\Gamma[\phi_{0,\Delta}]) = \langle \exp\left(\int_{\partial AdS} \phi_{0,\Delta}\mathcal{O}_\Delta\right)\rangle \stackrel{!}{=} \exp(-S^{AdS_5}_{\text{on-shell}}[\phi_{0,\Delta}])$$

  where $\Gamma[\phi_{0,\Delta}]$ is the generating functional for all correlators of single trace operators $\mathcal{O}_\Delta$.

- In other words

$$\Gamma[\phi_{0,\Delta}] = S^{AdS_5}_{\text{on-shell}}[\phi_{0,\Delta}]$$

  and

$$\langle \mathcal{O}_\Delta \ldots \mathcal{O}_\Delta\rangle \sim \frac{\partial}{\partial\phi_{0,\Delta}} \cdots \frac{\partial}{\partial\phi_{0,\Delta}} S^{AdS_5}_{\text{on-shell}}[\phi_{0,\Delta}]$$

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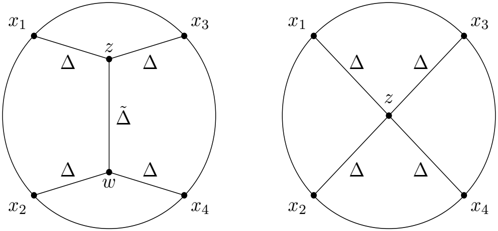
Holographic computation of the correlator

- Thus we need the on-shell supergravity Lagrangian $S^{AdS_5}_{\text{on-shell}}[\phi_{0,\Delta}]$ in terms of the boundary conditions $\phi_{0,\Delta}$
- The bulk equations of motion are solved perturbatively, where the Greens-functions are usually represented as Witten-diagrams, e. g.:



Holographic computation of the correlator

Holographic computation

Challenges:

1. Identify and take into account the correct scalar/vector/graviton exchange diagrams

   $\Rightarrow$ study the interaction structure of the bulk gravity theory

2. Bulk couplings

   $\Rightarrow$ Study the scalar potential of gauged supergravity

3. Evaluate bulk integrals

   $\Rightarrow$ Simplify integral expressions, use symmetries

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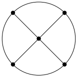
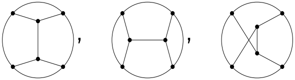
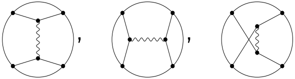
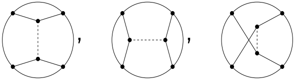
Holographic computation of the correlator

Contributing Witten diagrams

- Contact: 
- Scalar exchange: 
- Vector exchange: 
- Graviton exchange: 

$\Rightarrow$ What are the exchanged particles?

$\Rightarrow$ The correlator is proportional to the sum of all diagrams **weighted by the correct couplings**

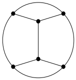
Holographic computation of the correlator

Evaluate the exchange diagrams

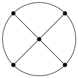
⇒ **Problem:** Exchange diagrams lead to **two** bulk integrals

By using inversion and Poincare symmetries one bulk integral can be written as a solution of a differential equation and thus can be eliminated.

[D'Hoker, Freedman, Rastelli]



$$= \sum_n a_n \times$$



Holographic computation of the correlator

Evaluate the exchange diagrams

Current status

- All exchange diagrams can be brought to a similar form
- In particular we can reduce everything to a sum over similar contact diagrams
- Finish re-summation of all diagrams
- Check the gravity couplings

Understand the Gravity Couplings

Understand the Gravity Couplings

Evaluate the exchange diagrams

- Want to understand bulk couplings in general
- Several approaches:
  - Dimensionally reduce 10d supergravity on S^5 by hand, get couplings order by order from Einstein and self-duality equations [Seiberg et al.; Skenderis,Taylor; Arutyunov,Frolov]

    $\Rightarrow$ Keep the full fields spectrum, but get couplings order by order
  - Gauged 5d supergravity on AdS_5, derive emergent full scalar potential [Günaydin,Romans,Warner]

    $\Rightarrow$ Consistently truncate field content, get the full scalar potential
- Currently looking at gauged supergravity

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Conclusion and Summary

Conclusion and Summary

Evaluate the exchange diagrams

⇒ It is still an ongoing project.

To do

- Get proficient with gauged supergravity and develop an efficient procedure to derive the scalar potential
- Simplify maths in the Witten diagram expansion and combine with couplings from scalar potential to re-sum
- Compare the correlator at strong coupling so obtained with the results at weak coupling obtained directly from the CFT

Expected results

- Possibly new non-renormalization theorems
- Interaction structure of supergravity is confirmed by the correlator structure of the CFT

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