Applying Colour-Kinematics Duality to the Unitarity Method

based on work done with Badger, Ochirov & O'Connell
[arXiv:1507.08797]

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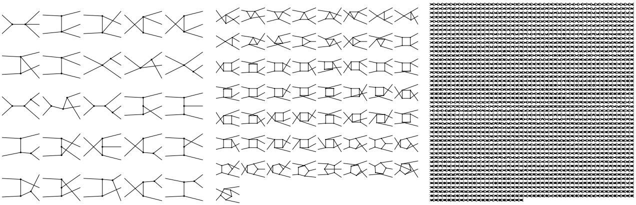
Complexity of Feynman Diagrams

- Scattering amplitudes are traditionally formed as sums of Feynman diagrams.
- These grow rapidly in number and complexity with increasing numbers of scattered particles and internal loops.



(a) LO (b) NLO (c) NNLO

Figure: Three gluon jet production events.

- Individual Feynman diagrams are unphysical quantities.

Hidden Structure in Yang-Mills Amplitudes

- The Parke-Taylor formula for tree-level Yang-Mills scattering takes the form

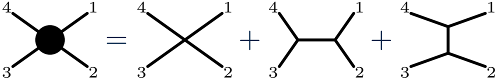
$$\mathcal{A}_n^{(0)} = \sum_{\sigma \in S_n} \frac{\langle \sigma(i)\sigma(j) \rangle^4}{\langle \sigma(1)\sigma(2) \rangle \cdots \langle \sigma(n)\sigma(1) \rangle} \mathrm{Tr}\left[T^{a_{\sigma(1)}} T^{a_{\sigma(2)}} \dots T^{a_{\sigma(n)}}\right].$$

- Recent years have seen huge advances in understanding of Yang-Mills amplitudes.
- Yangian symmetry, grassmannia, dual coordinates, the amplituhedron, etc. are all Lagrangian free.
- Most recently: colour-kinematics duality!

Colour-Ordered Yang-Mills Amplitudes

- One can define colour-ordered amplitudes using planar Feynman diagrams with ordered external legs only, e.g.

$A^{(0)}(1,2,3,4) =$



- Constructible using colour-ordered Feynman rules,

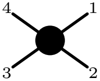
$$V^{\mu\nu\rho}(1,2,3) = -\sqrt{2}\,(\eta^{\mu\nu} p_1^{\rho} + \eta^{\nu\rho} p_2^{\mu} + \eta^{\rho\mu} p_3^{\nu})\,,$$
$$V^{\mu\nu\rho\sigma}(1,2,3,4) = \eta^{\mu\rho}\eta^{\nu\sigma}.$$

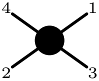
- Tree-level Yang-Mills amplitudes are reconstructed, with $[T^a, T^b] = if^{abc}T^c$, using

$$\mathcal{A}_n^{(0)} = \sum_{\sigma\in S_n} A^{(0)}(\sigma(1),\sigma(2),\ldots,\sigma(n))\mathrm{Tr}\left[T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}\ldots T^{a_{\sigma(n)}}\right].$$

Colour-Kinematics Duality

- New colour-kinematic identities due to Bern, Carrasco & Johansson [arXiv:0805.3993], e.g.

$(p_1 + p_2)^2$ 

$= (p_1 + p_3)^2$ 

.

- Further relationships exist for more external legs.
- Equivalent to the statement for colour-dressed Yang-Mills amplitudes,

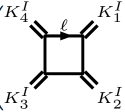
$$\mathcal{A}_n^{(0)} = \sum_{\text{diagrams } i} \frac{c_i n_i}{D_i},$$

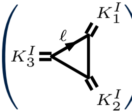
$\exists$ kinematic numerators n_i satisfying

$$c_i \pm c_j \pm c_k = 0 \iff n_i \pm n_j \pm n_k = 0.$$

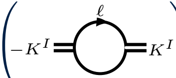
One-Loop Master Integrals

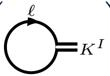
- One-loop colour-ordered amplitudes in $D = 4 - 2\epsilon$ live on a basis of scalar integrals:

$A^{(1)}(1, 2, \ldots, n) = \sum_I a_I \mathcal{I}$ (

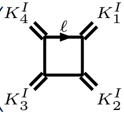
) $+ \sum_I b_I \mathcal{I}$ (

)

$+ \sum_I c_I \mathcal{I}$ (

) $+ \sum_I d_I \mathcal{I}$ (

) + rational.

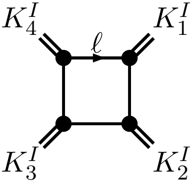
$\mathcal{I}$ (

) $= \int \frac{d^D \ell}{(2\pi)^D} \frac{1}{\ell^2 (\ell - K_1^I)^2 (\ell - K_1^I - K_2^I)^2 (\ell + K_4^I)^2}$

- Sums are over ordered partitions of external legs, e.g. $\{K_1, K_2, K_3, K_4\} = \{p_1 + p_2, p_3 + p_4, p_5, p_6\}$ or $\{K_1, K_2, K_3, K_4\} = \{p_1, p_2, p_3 + p_4 + p_5, p_6\}$ at 6 points.

One-Loop Unitarity Cuts

- The box coefficient a_I is a unitarity cut, evaluated through products of tree-level colour-ordered amplitudes:

$a_I =$ 

$$= \frac{1}{2}\sum_{\ell\in\mathcal{S}} A^{(0)}(K_1^I, \ell - K_1^I, -\ell) A^{(0)}(K_2^I, \ell - K_1^I - K_2^I, -\ell + K_1^I) \times A^{(0)}(K_3^I, \ell + K_4^I, -\ell + K_1^I + K_2^I) A^{(0)}(K_4^I, \ell, -\ell - K_4^I)$$

- The cut momentum ℓ is taken on shell through this channel:

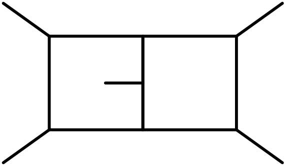
$$\mathcal{S} = \{\ell \mid \ell^2 = 0, (\ell - K_1)^2 = 0, (\ell - K_1 - K_2)^2, (\ell + K_4)^2 = 0\}.$$

The Five-Gluon, Two-Loop All-Plus QCD Amplitude

- At two loops, there are subleading colour contributions:

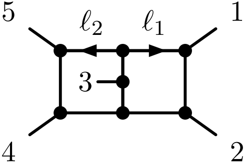
$$\mathcal{A}^{(2)}_{+++++} = \sum_{\sigma\in S_5} A^{(2)}(\sigma(1),\ldots,\sigma(5))\,\mathrm{Tr}\left[T^{a_{\sigma(1)}}\ldots T^{a_{\sigma(5)}}\right] + \sum_{c=2}^{4}\sum_{\sigma\in S_5/S_{5;c}} B^{(2)}_c(\sigma(1),\sigma(2),\ldots,\sigma(5))\times \mathrm{Tr}\left[T^{a_{\sigma(1)}}\ldots T^{a_{\sigma(c-1)}}\right]\mathrm{Tr}\left[T^{a_{\sigma(c)}}\ldots T^{a_{\sigma(5)}}\right].$$

- Diagrammatic interpretation through appearance of nonplanar diagrams, e.g.

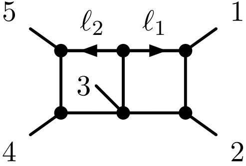


Extracting Nonplanar Cuts

- Nonplanar cuts are constructible from planar cuts, e.g.

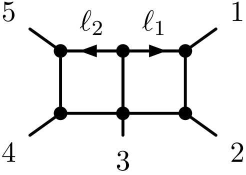


$= (\ell_1 + \ell_2 + p_3)^2$



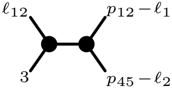
$\Big|_{(\ell_1+\ell_2+p_3)^2=0}$

$= (\ell_1 + p_{45})^2$

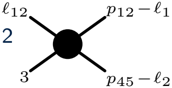


$\Big|_{(\ell_1+\ell_2+p_3)^2=0}$

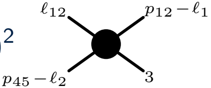
- Here we have used, while keeping $(\ell_1 + \ell_2 + p_3)^2 = 0$,



$= (\ell_1 + \ell_2 + p_3)^2$



$= (\ell_1 + p_{45})^2$



Further Challenges at Two Loops

- Lack of an “obvious” basis of master integrals on which to fit cut solutions: we cannot only use scalar integrals!
- The solution is not unique: terms can be exchanged between master integrals.
- Need for a multi-peripheral colour decomposition:

$$\mathcal{A}_n^{(0)} = -ig^{n-2} \sum_{\sigma \in S_{n-2}} f^{\,a_1 a_{\sigma(2)} b_1} f^{\,b_1 a_{\sigma(3)} b_2} \ldots f^{\,b_{n-3} a_{\sigma(n-1)} a_n} \times A^{(0)}(1, \sigma(2), \ldots, \sigma(n-1), n).$$

Summary and Outlook

- Five-gluon, two-loop all-plus QCD amplitude now fully computed by Badger, GM, Ochirov & O'Connell [arXiv:1507.08797].
- Master integrals have now been performed by Gehrmann, Henn & Presti [arXiv:1511.05409].
- Colour-kinematic techniques are applicable to higher points, alternative helicity configurations, other kinds of particles, etc.

Thanks for listening!