hep-ph/1512.06100

NNLO dijet soft functions and anomalous dimensions in SCET

Guido Bell || Rudi Rahn || **Jim Talbert**

14 January 2016 || YTF 2016, Durham IPPP, UK

Oxford Physics

Outline

1. ***Motivating automated soft functions***
   (a) Resummation in SCET
   (b) Thrust at N^3LL
   (c) Log-counting and the need for two-loop soft functions

2. ***Universal soft functions at NLO & NNLO***
   (d) Divergence structures and measurement functions
   (e) Sector decomposition via SecDec3.0

3. ***Renormalization and sample results***
   (f) Renormalization via RG evolution
   (g) Sample results at NNLO
   (h) Ongoing phenomenology with *Angularities*

Why an EFT for QCD?

- Collider phenomena often involve several momentum scales.
- Whenever these scales are disparate, large logarithms of their ratios are generated in perturbation theory—these must be *resummed*:

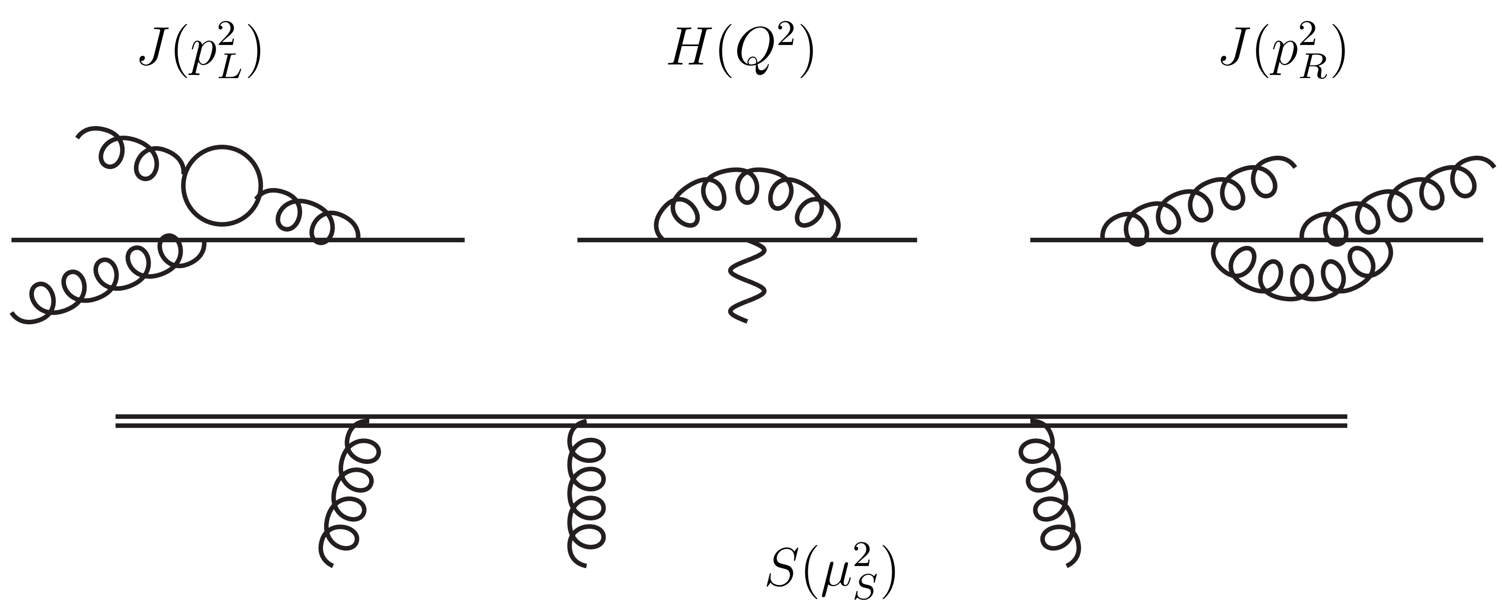
$$\alpha_s^n \ln^m\left(\frac{\mu_1}{\mu_2}\right)$$

- Effective field theories allow for analytic resummations using renormalization group techniques at the amplitude level.
- Hierarchy of scales implemented at the level of the Lagrangian…

Resummation in SCET

- SCET permits the derivation of all-order factorization theorems:

$$d\sigma \sim H \cdot \mathcal{J} \otimes \mathcal{J} \otimes \mathcal{S}$$



- Once factorized, we resum logs via RG Equations:

$$\frac{dH(Q^2,\mu)}{d\ln\mu} = \left[2\Gamma_{cusp}\ln(\frac{Q^2}{\mu^2}) + 4\gamma_H(\alpha_s)\right] H(Q^2,\mu)$$

- To increase the accuracy of the resummations one needs the anomalous dimensions and the matching corrections to higher orders.

Automated resummations

- In the traditional approach NLL resummations have been fully automated in *CAESAR* (*Banfi, Salam, Zanderighi* / 0407286)

- The procedure has recently been extended to NNLL (*Banfi, McAslan, Monni, Zanderighi* / 1412.2126).

- To date, SCET resummations have been performed on a case-by case basis:

- **NNLL Resummations:**

  Jet Broadening: *Becher, Bell* / 1210.0580

  Jet veto: *Becher, Neubert, Rothen* / 1307.0025, *Stewart, Tackmann, Walsh, Zuberi* / 1307.1808

  + ….

- **N^3LL Resummations:**

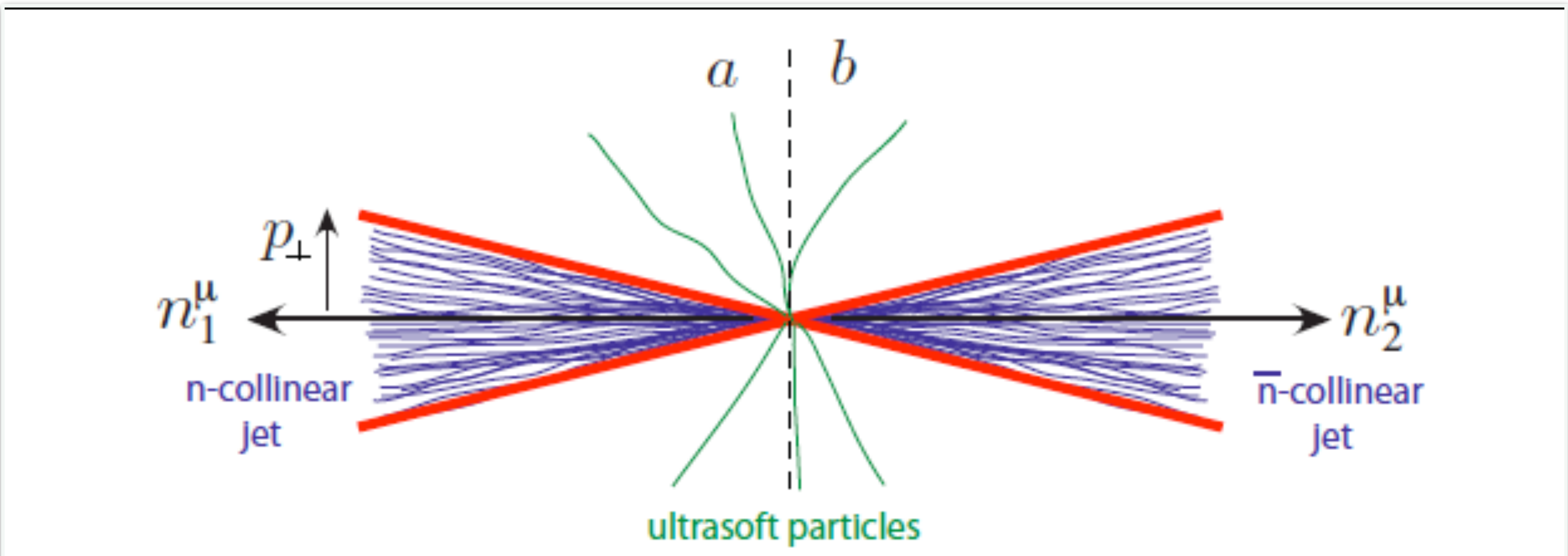
  Thrust: *Becher, Schwartz* / 0803.0342, *Abbate, Fickinger, Hoang, Mateu, Stewart* / 1006.3080

  W / Higgs @ large p_T: *Becher, Bell, Lorentzen, Marti* / *1309.3245* / 1407.4111

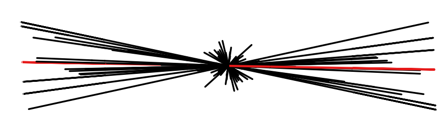
  + ….

Resummation example: thrust

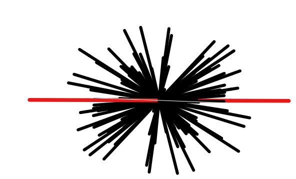
Thrust is an e^+e^- **event shape**—a geometric, dimensionless physical observable characterizing the momentum distribution of particles



$$\textbf{thrust} \quad T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$



two-jet like: $T \simeq 1$



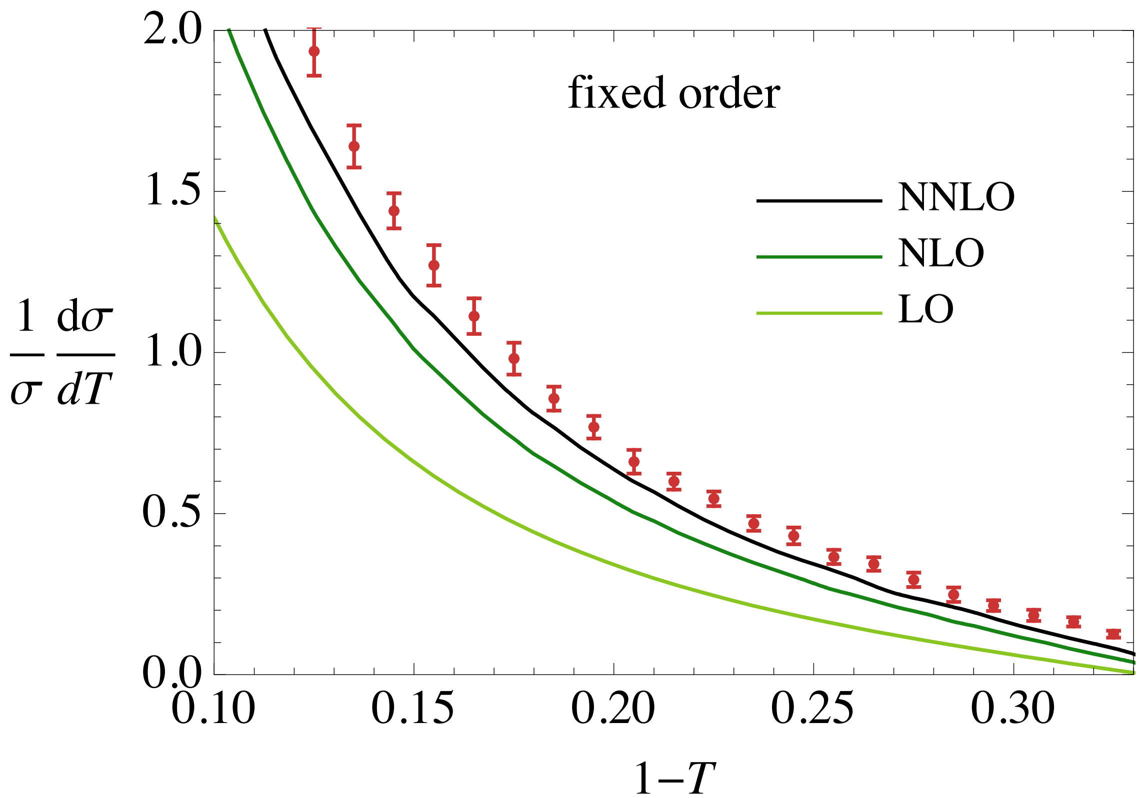
spherical: $T \simeq 1/2$

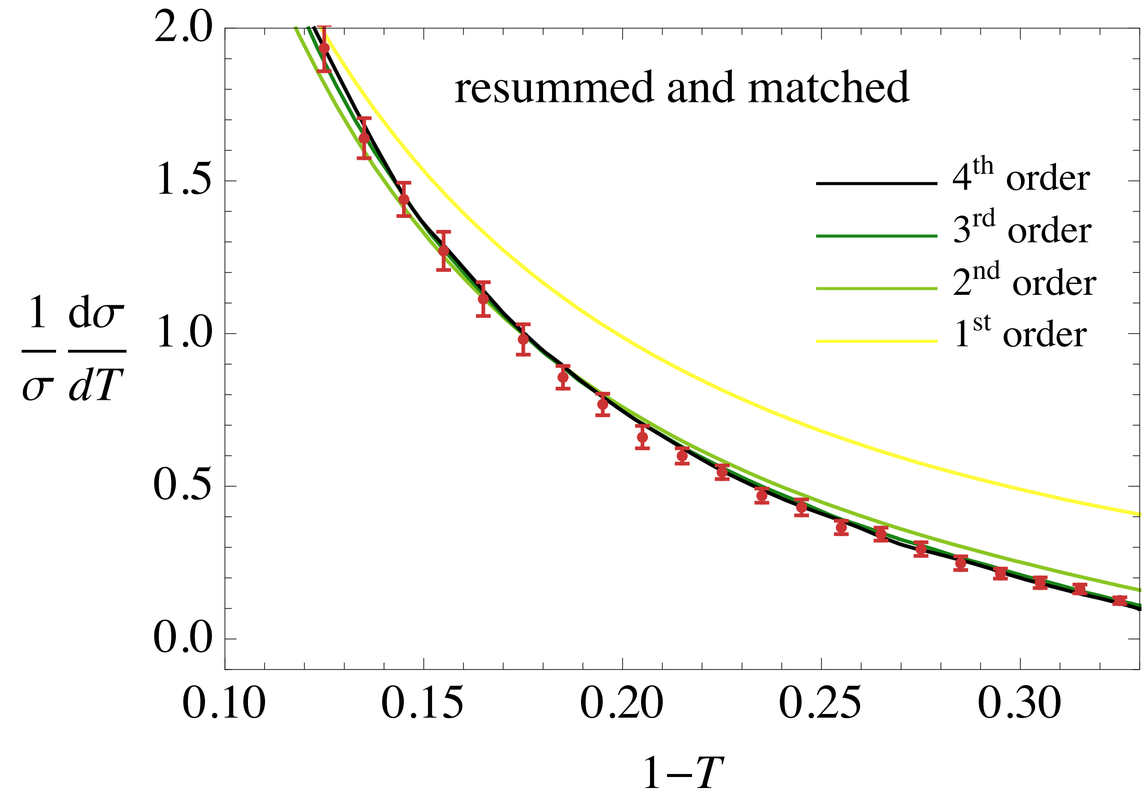
Resummation example: thrust

0803.0342v2(Becher/Schwartz)

- Traditional QCD resummation achieved via CTTW @ NLL
- Extended to NNLL (Monni, Gehrmann, Luisoni)
- Becher, Schwartz achieve N³LL resummation using SCET methods

$$\tau = 1 - T \qquad \alpha_s^n \ln^{2n} \tau$$





Red = Aleph data

Resummation ingredients

Logarithmic Accuracy	Γ_{Cusp}	$\gamma_H,\ \gamma_J,\ \gamma_S$	$C_H,\ C_J,\ C_S$
LL	1-loop	tree	tree
NLL	2-loop	1-loop	tree
NNLL	3-loop	2-loop	1-loop
N3LL	4-loop	3-loop	2-loop

To achieve NNLL resummation, we need the soft anomalous dimension to two-loop accuracy

SCET soft functions @ NNLO

- **e^+e^- observables:**

  Hemisphere masses: *Kelley, Schabinger, Schwartz, Zhu* / 1105.3676 & *Hornig, Lee, Stewart, Walsh, Zuberi* / 1105.4628

  Jet-mass w/ veto: *Kelley, Schwartz, Schabinger, Zhu* / 1112.3343

  Jet-broadening: *Becher, Bell* / 1210.0580

- **LHC observables:**

  Exclusive Drell-Yan: *Li, Mantry, Petriello* / 1105.5171

  W/Higgs @ large p_T: *Becher, Bell, Marti* / 1201.5572

Motivation: Can these computations be achieved more systematically?

Universal dijet soft functions

- We can write down a universal dijet soft function as the vacuum matrix element of a product of Wilson lines along the direction of energetic quarks.

$$S(\omega,\mu)=\sum_{X,reg.}\mathcal{M}(\omega,\{k_i\})|\langle X|S_n^\dagger(0)S_{\bar{n}}(0)|0\rangle|^2 \qquad S_n(x)=Pexp(ig_s\int_{-\infty}^{0}n\cdot A_s(x+sn)ds)$$

- The **matrix element** of soft wilson lines is *independent of the observable*. It contains the universal (implicit) UV/IR-divergences of the function.
- The **measurement function** (M) encodes all of the information of the particular observable at hand. It is *independent of the singularity structure*. Take thrust as an example:

$$\boxed{\mathcal{M}_{thrust}(\omega,\{k_i\})=\delta(\omega-\sum_{i\in L}k_i^+-\sum_{i\in R}k_i^-)}$$

- **Idea**: isolate singularities at each order and calculate the associated coefficient numerically:

$$\boxed{\bar{S}(\tau)\sim 1+\alpha_s\{\frac{c_2}{\epsilon^2}+\frac{c_1}{\epsilon^1}+c_0\}+\mathcal{O}(\alpha_s^2)}$$

- The coefficients depend on the observable. We are working in Laplace Space.

Universal soft functions: NLO

- We work in **Laplace space**, so that our functions are not distribution valued. At 1-loop the virtual corrections are scaleless in DR and we can write the NLO soft function as:

$$\bar{S}^{(1)}(\tau,\mu) = \frac{\mu^{2\epsilon}}{(2\pi)^{d-1}} \int \delta(k^2)\ \theta(k^0)\ \mathcal{R}_\alpha(\nu; k_+, k_-)\ \left(\frac{16\pi\alpha_s C_F}{k_+ k_-}\right)\ \bar{\mathcal{M}}(\tau,k)\ d^d k$$

- Where we use a symmetric version of the analytic $\mathrm{SCET_{II}}$ regulator (*Becher, Bell* / 1112.3907):

$$\mathcal{R}_\alpha(\nu, k_+, k_-) = \theta(k_- - k_+)(\nu/k_-)^\alpha + \theta(k_+ - k_-)(\nu/k_+)^\alpha$$

- We want to disentangle all of the UV and IR divergences. We thus split the integration region into two hemispheres and make the following physical substitutions:

$$k_- \to \frac{k_T}{\sqrt{y}} \qquad k_+ \to k_T\sqrt{y}$$

- We can now specify the measurement function *M*. We assume it can be written in terms of two dimensionless functions f & g:

$$\bar{\mathcal{M}}(\tau,k) = g(\tau k_T, y, \theta)\ exp(-\tau k_T f(y,\theta))$$

Universal soft functions: examples

$$\bar{\mathcal{M}}(\tau, k) = g(\tau k_T, y, \theta)\ exp(-\tau k_T f(y,\theta))$$

Obs.	$\boldsymbol{g(\tau k_T, y, \theta)}$	$\boldsymbol{f(y,\theta)}$
Thrust	1	$\sqrt{y}$
Angularities	1	$y^{(1-A)/2}$
C-Parameter	1	$\sqrt{y}/(1+y)$
Broadening	$\Gamma(1-\epsilon)\left(\frac{z\tau k_T}{4}\right)^{\epsilon}\mathcal{J}_{-\epsilon}\left(\frac{z\tau k_T}{2}\right)$	$1/2$
W/H @ large p_T	1	$\frac{1+y-2\sqrt{y}\cos\theta}{\sqrt{y}}$
Transverse Thrust	1	$\frac{1}{\lvert s\rvert}\{\sqrt{1+\frac{1}{4}\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)^2 s^2+\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)cs\cos\theta - s^2\cos^2\theta}$ $-\lvert c\cos\theta+\frac{1}{2}\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)s\rvert\}$

Universal soft functions: NLO master formula

- We switch to a dimensionless variable (x) and extract the scaling of the observables in the collinear limit y ⇒ 0:

$$\tau k_T f(y,\theta) \to x \qquad f(y,\theta) \to y^{\frac{n}{2}} \hat{f}(y,\theta)$$

- We are now in a position to write a master formula for the calculation of NLO dijet soft functions:

$$\boxed{\bar{S}^{(1)}(\tau,\mu) \sim \int_{-1}^{1} \sin^{-1-2\epsilon}\theta \; d\cos\theta \int_0^\infty dx \int_0^1 dy \; x^{-1-2\epsilon-\alpha} \; y^{-1+n\epsilon+(n-1)\alpha/2} \; \hat{g}(x,y,\theta) \; [\hat{f}(y,\theta)]^{2\epsilon+\alpha} \; e^{-x}}$$

- Note that n=0 corresponds to a $\mathrm{SCET_{II}}$ observable.

- We are in a position to apply a subtraction technique to extract the singularities. Consider a simple 1-D example:

$$\underbrace{\int_0^1 dx \; x^{-1-n\epsilon} f(x)}_{\text{divergent}} = \int_0^1 dx \; x^{-1-n\epsilon}\{\underbrace{f(x) - f(0)}_{\text{finite}/O(x)} + f(0)\}$$

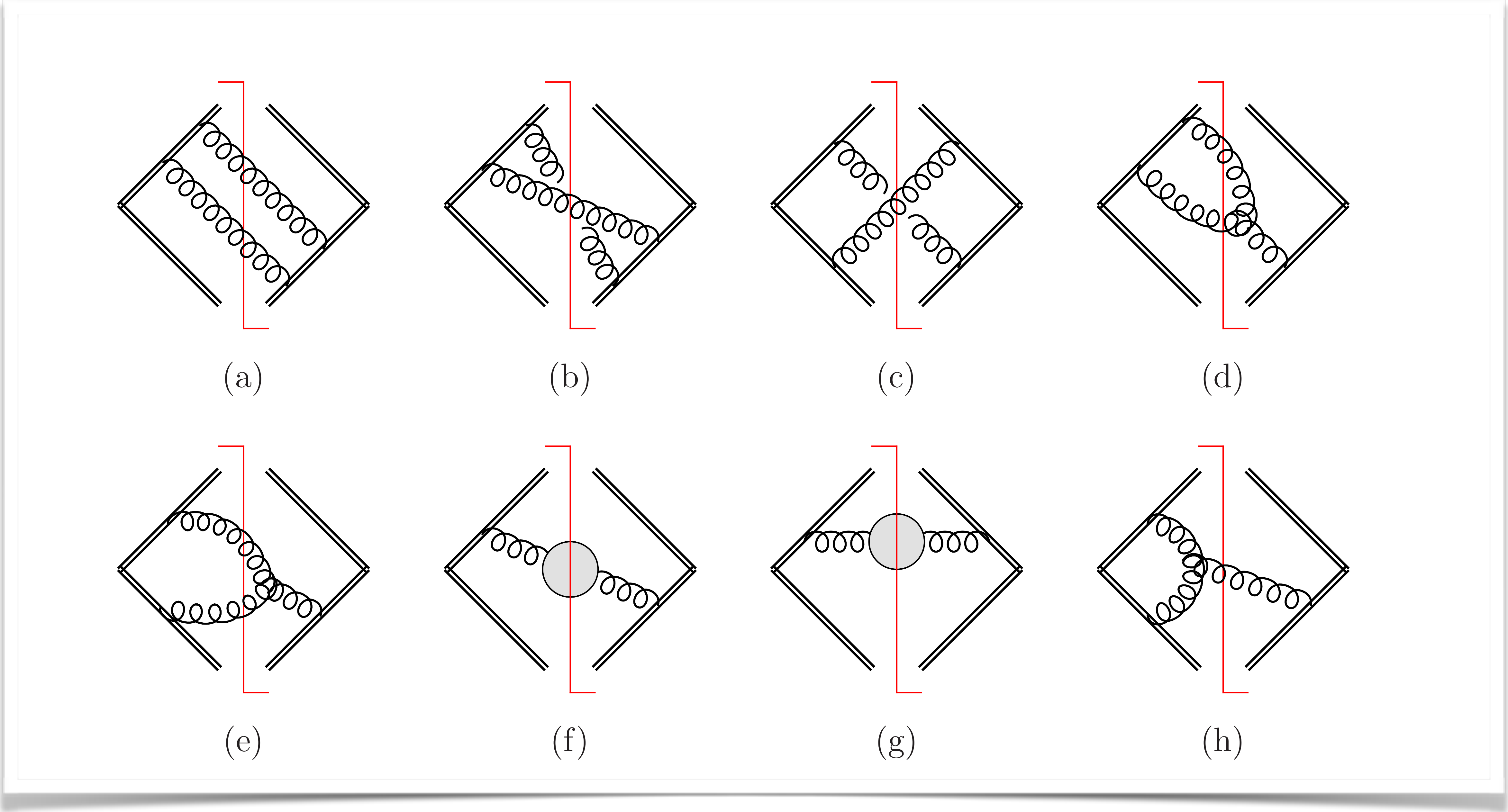
divergent

- finite/O(x)
- expand in ϵ
- integrate numerically

$\sim -\dfrac{1}{n\epsilon}$

- singularity isolated

NNLO diagrams



- Three color structures are present: $C_F^2, \quad C_F C_A, \quad C_F T_F n_f$
- We use analytic results for the C_F^2 terms and the one-particle cuts.

NNLO soft functions

- Consider the double real emission (drop additional regulator):

$$\bar{S}^{(2)}_{RR}(\tau) = \frac{\mu^{4\epsilon}}{(2\pi)^{2d-2}} \int d^d k \ \delta(k^2) \ \theta(k^0) \int d^d l \ \delta(l^2) \ \theta(l^0) \ |\mathcal{A}(k,l)|^2 \ \bar{\mathcal{M}}(\tau,k,l)$$

- Decompose into light-cone coordinates and perform trivial integrations:

$$\begin{aligned} \bar{S}^{(2)}_{RR}(\tau) &\sim \Omega_{d-3}\Omega_{d-4} \int_0^\infty dk_+ \int_0^\infty dk_- \int_0^\infty dl_+ \int_0^\infty dl_- \int_{-1}^1 d\cos\theta_k \ \sin^{d-5}\theta_k \\ &\times \int_{-1}^1 d\cos\theta_l \ \sin^{d-5}\theta_l \int_{-1}^1 d\cos\theta_1 \ \sin^{d-6}\theta_1 \ (k_+k_-l_+l_-)^{-\epsilon} |\mathcal{A}(k,l)|^2 \ \bar{\mathcal{M}}(\tau,k,l) \end{aligned}$$

- Consider, e.g., the $C_F T_F n_f$ color structure:

$$|\mathcal{A}(k,l)|^2 = 128\pi^2\alpha_s^2 C_F T_F n_f \frac{2k\cdot l(k_-+l_-)(k_++l_+) - (k_-l_+ - k_+l_-)^2}{(k_-+l_-)^2(k_++l_+)^2(2k\cdot l)^2}$$

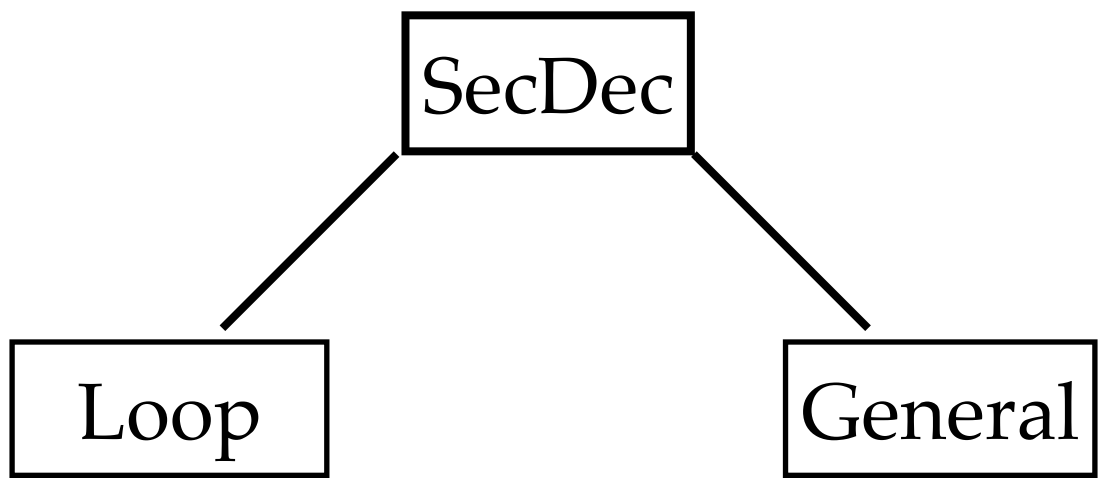
- It is clear the singularity structure is non-trivial, and that the singularities are overlapping…

Automation: *SecDec*

https://secdec.hepforge.org

Borowka, Heinrich, Jones, Kerner, Schlenk, Zirke

- A tool is already on the market that exploits a sector decomposition algorithm: ***SecDec***
- "A program to evaluate dimensionally regularized parameter integrals numerically"



- We utilize the 'general' mode of the program. Simple interface to our NLO and NNLO master formulas (✔), multiple numerical integrators for crosschecks (✔)
- Active collaboration with *SecDec* team to implement special features of our algorithm, e.g. 'epsilon-dependent' functions and additional regulator for $\mathrm{SCET}_{\mathrm{II}}$.
- Currently limited to $\mathrm{SCET}_{\mathrm{I}}$ observables, though additional rapidity regulator in development.

Renormalization

- We calculate the bare soft function with SecDec. We have to renormalize, where the renormalized soft function obeys an RG equation:

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\,S(\tau,\mu) = -\frac{1}{n}\left[4\,\Gamma_{\text{cusp}}(\alpha_s)\,\ln(\mu\bar{\tau}) - 2\gamma^S(\alpha_s)\right]S(\tau,\mu)}$$

- The cusp and non-cusp anomalous dimensions can be expanded perturbatively in a power series:

$$\Gamma_{\text{cusp}}(\alpha_s) = \textstyle\sum_{n=0}^{\infty}\Gamma_n\left(\frac{\alpha_s}{4\pi}\right)^{n+1} \qquad \gamma^S(\alpha_s) = \textstyle\sum_{n=0}^{\infty}\gamma_n^S\left(\frac{\alpha_s}{4\pi}\right)^{n+1}$$

- With these forms, the RG equation can be solved explicitly to any given order (the same is true for the renormalization Z factor…):

$$S(\tau,\mu) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\left\{-\frac{2\Gamma_0}{n}L^2 + \frac{2\gamma_0^S}{n}L + c_1^S\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{2\Gamma_0^2}{n^2}L^4 - 4\Gamma_0\left(\frac{\gamma_0^S}{n^2} + \frac{\beta_0}{3n}\right)L^3 - 2\left(\frac{\Gamma_1}{n} - \frac{(\gamma_0^S)^2}{n^2} - \frac{\beta_0\gamma_0^S}{n} + \frac{\Gamma_0 c_1^S}{n}\right)L^2 + 2\left(\frac{\gamma_1^S}{n} + \frac{\gamma_0^S c_1^S}{n} + \beta_0 c_1^S\right)L + c_2^S\right\}$$

Thrust

$$\mathcal{M}_{thrust}(\omega, \{k_i\}) = \delta(\omega - \sum_{i\in L} k_i^+ - \sum_{i\in R} k_i^-)$$

- We use *SecDec* to calculate the double emission contribution. To obtain the renormalized soft function we have to add the counterterms, which are known analytically at the required order.

- We show the cancellation of the divergences for thrust, setting $\ln(\mu\bar{\tau}) \to 0$

$$\tilde{S}^{(2)}_{ren} = \frac{\alpha_s^2(\mu)}{(4\pi)^2}\{C_A C_F\left(\frac{0}{\epsilon^4} - \frac{5.07333\times 10^{-9}}{\epsilon^3} + \frac{1.07523\times 10^{-6}}{\epsilon^2} + \frac{.0000102661}{\epsilon}\right) + C_F T_F n_f\left(-\frac{1.40667\times 10^{-8}}{\epsilon^3} + \frac{6.83778\times 10^{-8}}{\epsilon^2} - \frac{1.44697\times 10^{-8}}{\epsilon}\right)\} + \tilde{S}^{(2)}_0$$

- We thus also have an indication of our numerical precision…

- For the finite portion, we find (setting again $\ln(\mu\bar{\tau}) \to 0$):

$$\boxed{\tilde{S}^{(2)}_0 = \frac{\alpha_s^2(\mu)}{(4\pi)^2}\left(48.7045 C_F^2 - 56.4992 C_A C_F + 43.3902 C_F T_F n_f\right)}$$

- Versus the analytic expression calculated by *Kelley, Schabinger, Schwartz, Zhu* / 1105.3676 (see also *Monni, Gehrmann, Luisoni* / 1105.4560):

$$\tilde{S}^{(2)}_0 = \frac{\alpha_s^2(\mu)}{(4\pi)^2}\left(48.7045 C_F^2 - 56.4990 C_A C_F + 43.3905 C_F T_F n_f\right)$$

Table of sample NNLO results

Soft function	γ_0^S/C_F	c_1^S/C_F	$\gamma_1^{C_A}$	$\gamma_1^{n_f}$	$c_2^{C_A}$	$c_2^{n_f}$
Thrust [5, 6]	0	$-\pi^2$	15.7945 (15.7945)	3.90981 (3.90981)	-56.4992 (-56.4990)	43.3902 (43.3905)
C-parameter [24]	0	$-\pi^2/3$	15.7947 (15.7945)	3.90980 (3.90981)	-57.9754 ($-$)	43.8179 ($-$)
Threshold Drell-Yan [4]	0	$\pi^2/3$	15.7946 (15.7945)	3.90982 (3.90981)	6.81281 (6.81287)	-10.6857 (-10.6857)
W@large p_T [9]	0	π^2	15.88 (15.7945)	3.905 (3.90981)	-2.78 (-2.65010)	-25.28 (-25.3073)

Table 2: Anomalous dimensions and finite terms of the renormalised soft function for sample SCET-1 observables. The upper numbers are the numerical results that we obtain with the `SecDec` implementation of our algorithm, and the lower ones correspond to the known analytic expressions.

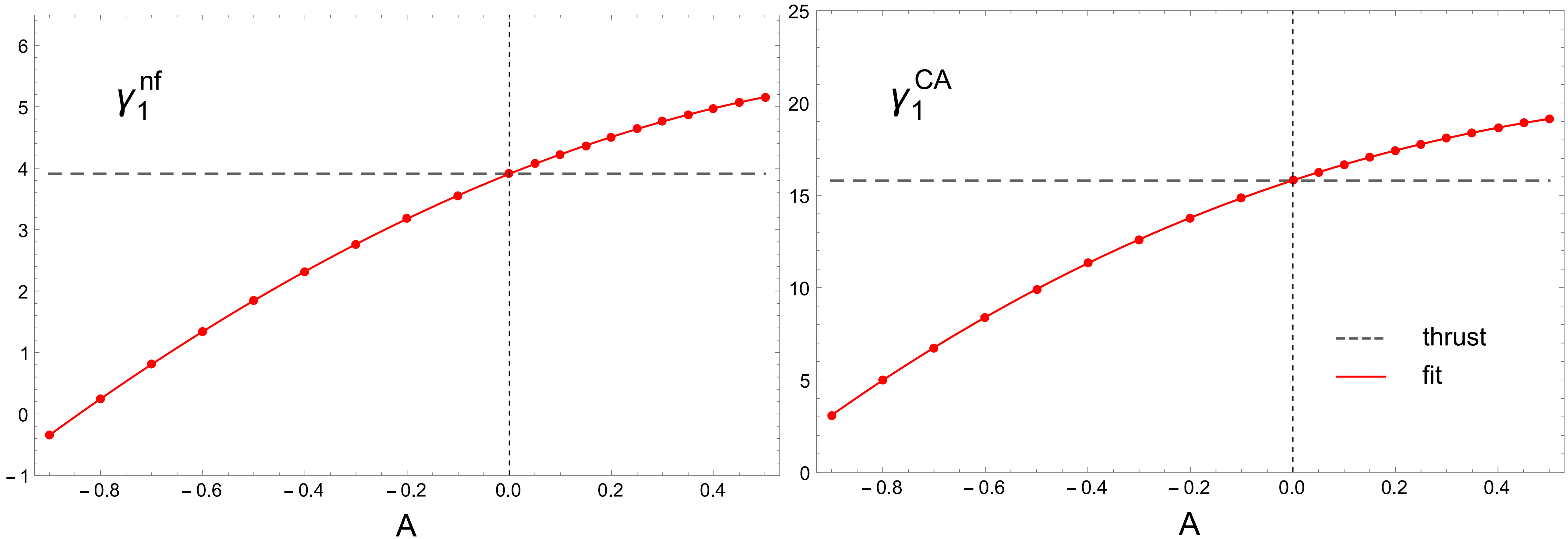
Angularities

- *Angularities* measurement function:

$$\mathcal{M}_{Ang}(\omega,\{k_i\}) = \delta(\omega - \sum_{i\in L} k_{i,+}^{1-A/2}\ k_{i,-}^{A/2} - \sum_{i\in R} k_{i,+}^{A/2}\ k_{i,-}^{1-A/2})$$

- The two-loop soft anomalous dimension is not known. We define in Laplace space:

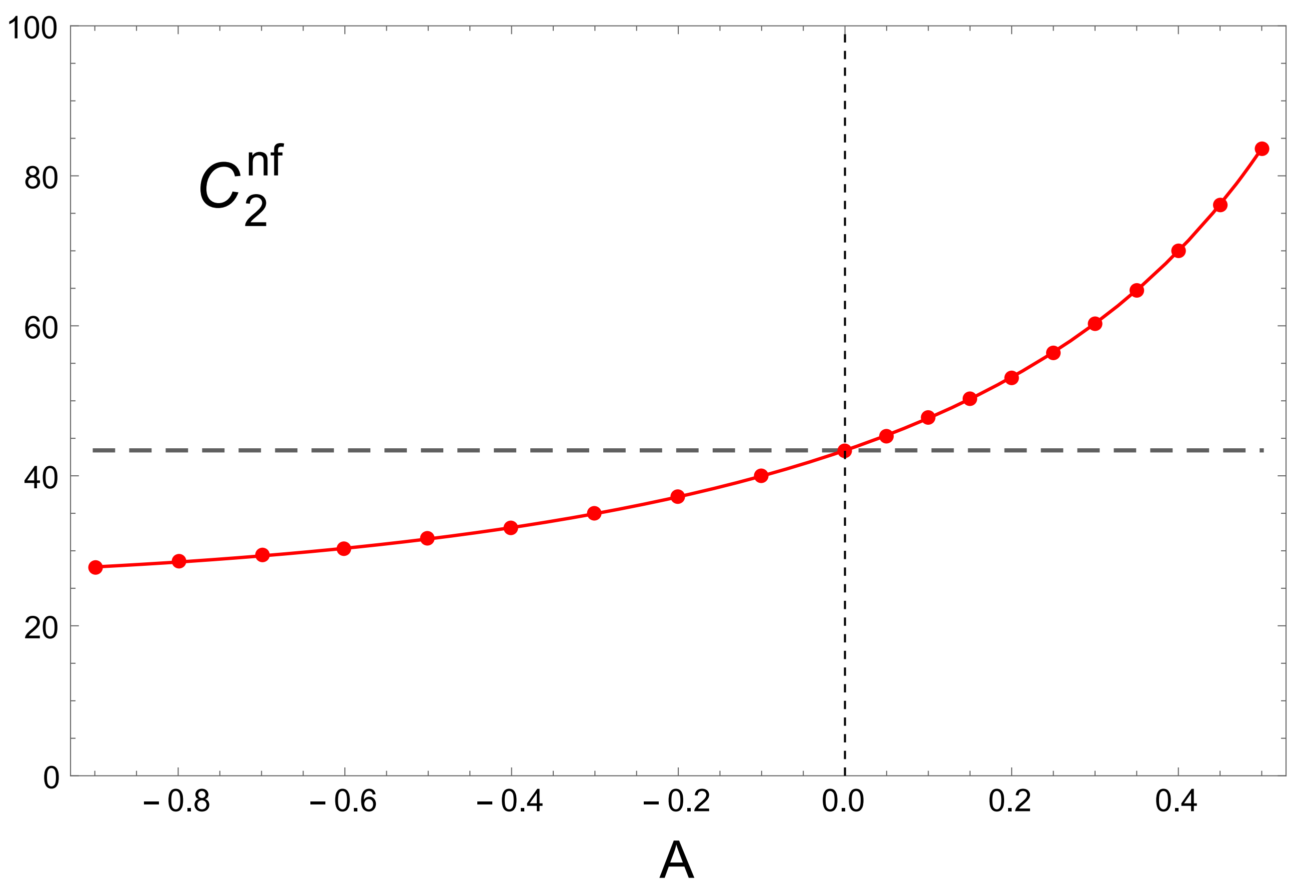
$$\frac{d\tilde{S}(\tau)}{d\ln\mu} = -\frac{1}{(1-A)}\left[4\Gamma_{cusp}\ln(\mu\bar{\tau}) - 2\gamma_S\right]\tilde{S}(\tau)$$

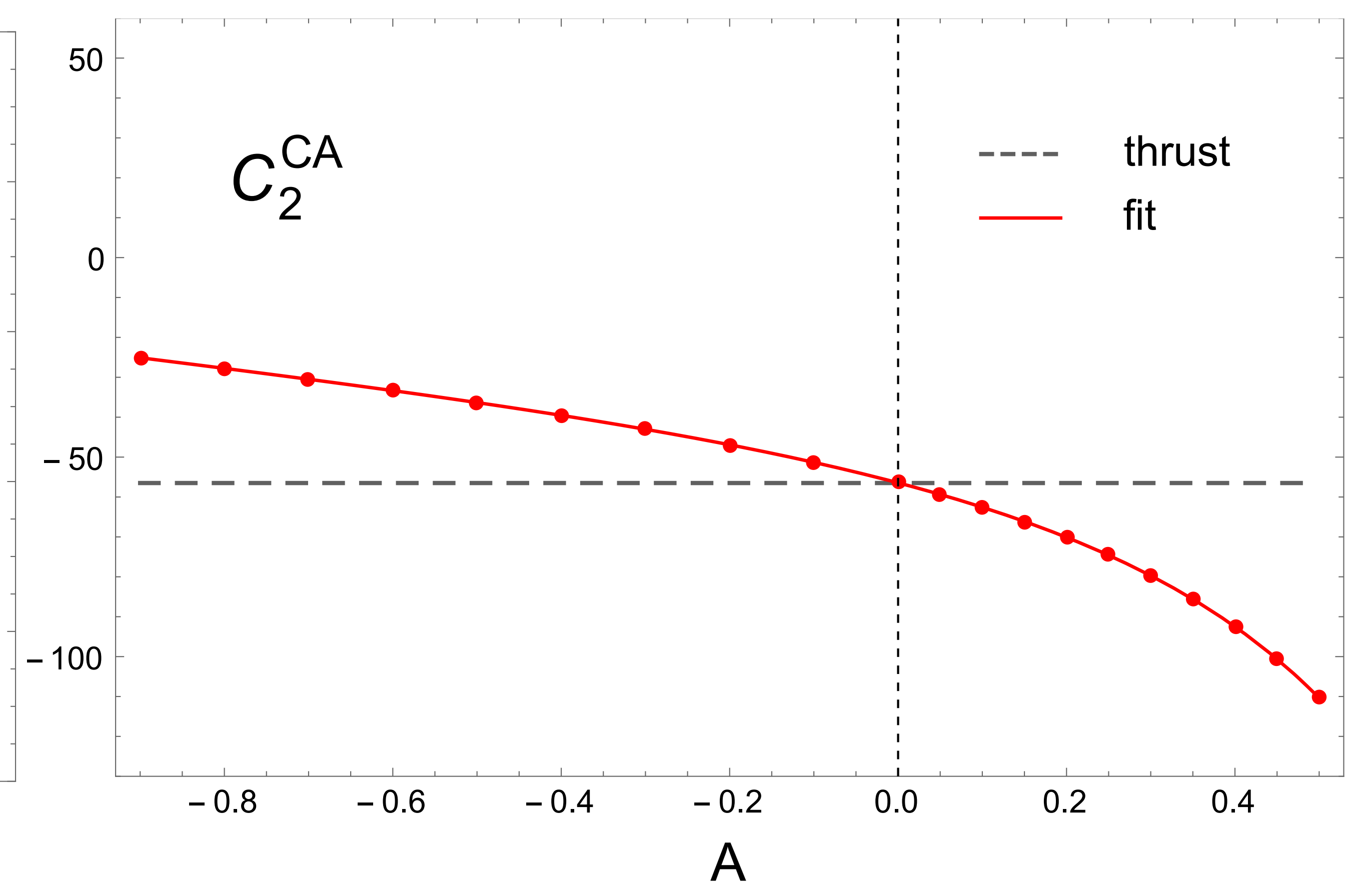


Angularities

$$\mathcal{M}_{Ang}(\omega, \{k_i\}) = \delta(\omega - \sum_{i\in L} k_{i,+}^{1-A/2}\ k_{i,-}^{A/2} - \sum_{i\in R} k_{i,+}^{A/2}\ k_{i,-}^{1-A/2})$$

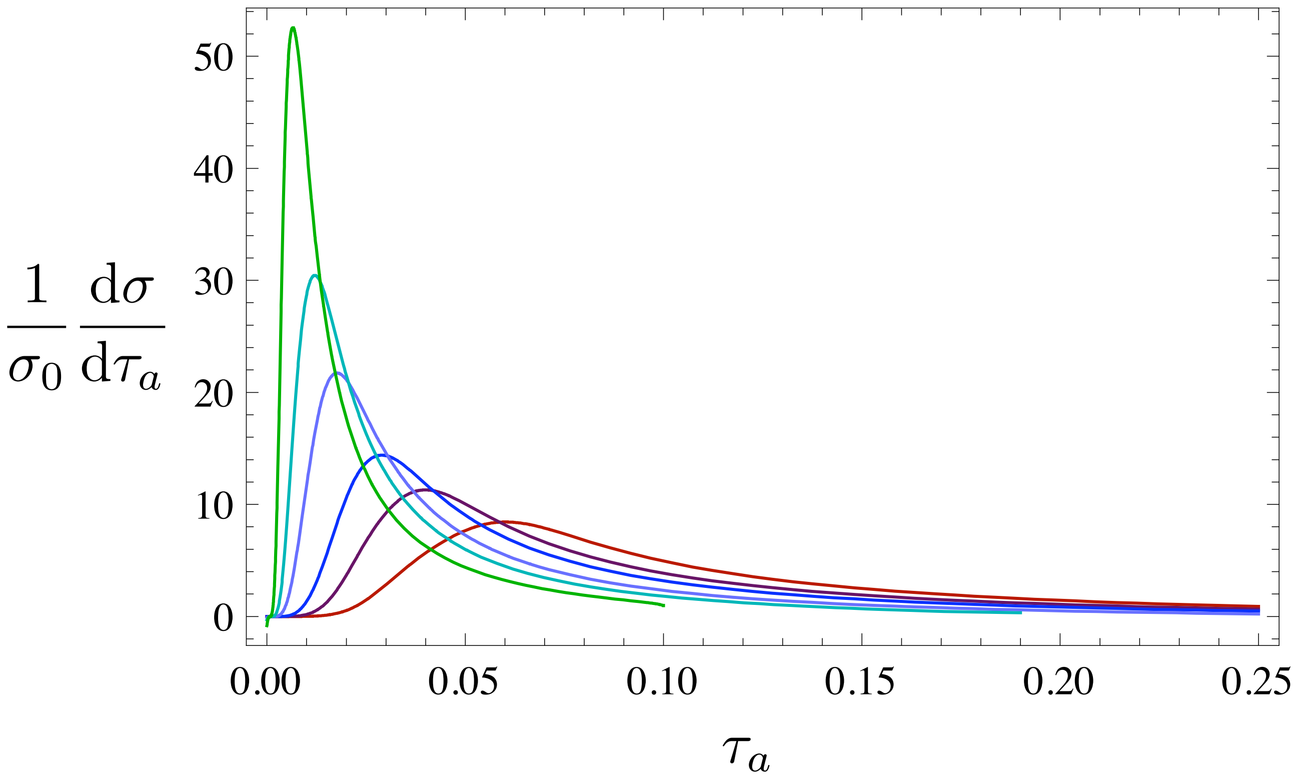
- For the finite terms we find:





Angularities @ N(*)LL

hep-ph/0901.3780



Currently, *Angularities* have been resummed and matched to NLL/NLO accuracy: **Hornig/Lee/Ovanesyan**

With the NNLO soft anomalous dimension, we can resum to NNLL', match to NNLO (with some EVENT2 legwork), and do a first-ever theory comparison to [LEP] data...

Logarithmic Accuracy	Γ_{Cusp}	$\gamma_H,\ \gamma_J,\ \gamma_S$	$C_H,\ C_J,\ C_S$
LL	1-loop	tree	tree
NLL	2-loop	1-loop	tree
NNLL	3-loop	2-loop	1-loop
N3LL	4-loop	3-loop	2-loop

Conclusions and future work

- We have presented an automated algorithm to compute dijet soft functions for a wide class of observables in SCET.

- Our master formulas coupled with *SecDec* can quickly and easily produce predictions for a wide class of SCET_I soft functions at one and two-loops. We are currently working with the developers to implement a few additional features in *SecDec.*

- This is an important ingredient for NNLL resummations in SCET. We are currently utilizing the results to perform a first-ever theory comparison to data, with *Angularities*.

- Next steps: SCET_II observables, n-jet soft functions, jet functions (?)…

Thanks!