

First integrals of affine connections on surfaces

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Given a pseudo-Riemannian manifold, there is a natural notion of geodesics defined by the Levi-Civita connection. But the geodesic equations can be written just in terms of the Christoffel symbols Γ :

$$\begin{equation} \ddot{x}^a + \Gamma^a_{bc} \dot{x}^b \dot{x}^c = 0, \end{equation}$$

where x^a are local coordinates. Therefore, geodesics can be defined for any symmetric affine connection (not necessarily metric). The same is true for Killing forms K_a , which are defined to be solutions to $\nabla_{(a} K_{b)} := \frac{1}{2}(\nabla_a K_b - \nabla_b K_a) = 0$, and correspond to first integrals of the geodesic equations linear in the momenta: $K_a \dot{x}^a$.

I will present the method of prolongation and Frobenius theorem to determine necessary and sufficient conditions for an affine connection on a two-dimensional manifold (not necessarily endowed with a metric) to admit 0, 1, 2 or 3 Killing forms.

Reference: F. Contatto, M. Dunajski. (2015) First integrals of affine connections and Hamiltonian systems of hydrodynamic type. [arXiv:1510.01906]

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