Higgs decays in Standard Model Effective Field Theory at one-loop

YTF8

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Outline

1. Introduction
   - Introduction to SMEFT
   - Motivation
   - Effect of Dimension-6 operators
2. Higgs decay in SMEFT
   - Tree-level decays
   - Renormalisation Proceedure
3. Implications for Phenomenology
4. Conclusion

Standard Model

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &+ i\bar{\psi}\not{D}\psi \\ &+ \bar{\psi}_i Y_{ij} H \psi_j + \mathrm{h.c.} \\ &+ |D_\mu H|^2 - V(H)\end{aligned}$$

Standard Model Effective Field Theory

Standard Model

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}\not{D}\psi + \bar{\psi}_i Y_{ij} H \psi_j + \text{h.c.} + |D_\mu H|^2 - V(H)$$

SMEFT

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm D5} + \mathcal{L}_{\rm D6} + \mathcal{L}_{\rm D7} + \ldots$$

where,

$$\mathcal{L}_{{\rm D}k} = \sum_i C_{ki} Q_{ki}$$

- Q_{ki} are simply operators built from SM d.o.f of dimension k, while i runs over all operators available at that dimension which satisfy Lorentz and $SU(3) \times SU(2) \times U(1)$ gauge symmetry

Standard Model Effective Field Theory

For this talk we'll restrict ourselves to

$$\begin{aligned}\mathcal{L}_{\mathrm{SMEFT}} &= \mathcal{L}_{\mathrm{SM}} \; + \; \mathcal{L}_{\mathrm{D6}} \\ &= \mathcal{L}_{\mathrm{SM}} \; + \; \sum_{i=1}^{59} C_i Q_i\end{aligned}$$

- Baryon number conserving operators only
- 80 Generated from Buchmuller & Wyler, but over complete basis
- Minimal basis of 59 operators in 1008.4884
- Choice of basis
- The Wilson coefficients are dimensionful, $C_i = \frac{\tilde{C}_i}{\Lambda^2_{\mathsf{NP}}}$

Motivation for SMEFT

- The rationale behind extending the standard model in this manner stems from the idea that the SM is simply a low energy effective field theory
- If new physics exists *at high energy*, then the effects of integrated out new particles should manifest itself as non-renormalisable operators
- Same idea as four-quark operators, Fermi-theory, Higgs EFT (large m_t limit) ...
- In the abscence of direct hint of a particular model, this is a general way to proceed

Effect of Dimension-6 operators

- We get a new vacuum, v_T, since $(H^\dagger H)^3$ alters the potential.
- We also get contributions to the mass and yukawa matrices.

$$
\begin{aligned}
[M_f]_{rs} &= \frac{v_T}{\sqrt{2}}\left([Y_f]_{rs} - \frac{1}{2}v_T^2 C^*_{\substack{fH\\sr}}\right)\\
[\mathcal{Y}_f]_{rs} &= \frac{1}{\sqrt{2}}\left([Y_f]_{rs}\left[1 + C_{H,\mathrm{kin}}\right] - \frac{3}{2}v_T^2 C^*_{\substack{fH\\sr}}\right)\\
&= \frac{1}{v_T}[M_f]_{rs}\left[1 + C_{H,\mathrm{kin}}\right] - \frac{v_T^2}{\sqrt{2}}C^*_{\substack{fH\\sr}}
\end{aligned}
$$

- Can lead to flavour violating effects
- Impose MFV: essentially require that the mass and Yukawa matrices are simultaneously diagonalisable

Effect of Dimension-6 operators

- We also need to redefine the gauge fields and couplings..
- The actual expressions for the new terms are not relevant to the talk however
- We'll denote objects with a bar as those that appear in the covariant derivative in the broken phase of the theory

$$D_\mu = \partial_\mu + i\frac{\bar{g}_2}{\sqrt{2}}\left[W^+_\mu T^+ + W^-_\mu T^-\right] + i\bar{g}_Z\left[T_3 - \bar{s}^2 Q\right] Z_\mu + i\bar{e}QA_\mu$$

$$\bar{e} = \bar{g}_2 \sin\bar{\theta} - \frac{1}{2}\cos\bar{\theta}\bar{g}_2 v_T^2 C_{\mathsf{HWB}}$$

Input Parameters

- Before proceeding, it's necessary to specify the input parameters. How do we want the answer expressed?
- Choose to work with the following independent, physical parameters
  - $\bar{e}, m_H, M_W, M_Z, m_f, C_i$
- In practise, we'll choose to eliminate M_W in terms of the Fermi-constant G_F, though this will not be discussed here.

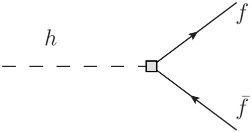
Tree-level Higgs decay: SMEFT style

With the SMEFT framework now in place, it is possible to study the decay of the Higgs in this context. The tree-level decay amplitude for the Higgs to fermions is straight forward. Simply the effective Yukawa coupling from earlier dressed with external spinors.

$$i\mathcal{M}^{(0)}(h \to f\bar{f}) = -i\bar{u}(p_f)\left(\mathcal{M}^{(0)}_{f,L}P_L + \mathcal{M}^{(0)*}_{f,L}P_R\right)v(p_{\bar{f}})$$

where

$$\mathcal{M}^{(0)}_{f,L} = \frac{m_f}{v_T}\left[1 + C_{H,\text{kin}}\right] - \frac{v_T^2}{\sqrt{2}}C^*_{fH}$$



Renormalisation Proceedure

- One-loop calculation proceeds in two parts:
  - Bare one-loop matrix elements
  - UV counter-terms
- Renormalise masses and electric charge in the on-shell scheme
- Renormalise Wilson coefficients in the $\overline{\text{MS}}$ scheme
  - Standard for EFT calculations

Renormalisation

- Wavefunction, mass, and electric charge renormalisation
- Defining the renormalised fields in terms of bare ones, indicated with the superscript (0)

$$
\begin{aligned}
h^{(0)} &= \sqrt{Z_h} h = \left(1 + \frac{1}{2}\delta Z_h\right) h \\
f_L^{(0)} &= \sqrt{Z_f^L} f_L = \left(1 + \frac{1}{2}\delta Z_f^L\right) f_L \\
f_R^{(0)} &= \sqrt{Z_f^R} f_R = \left(1 + \frac{1}{2}\delta Z_f^R\right) f_R
\end{aligned}
\tag{1}
$$

$$
M^{(0)} = M + \delta M \qquad \bar{e}_0 = \bar{e} + \delta\bar{e}
\tag{2}
$$

Renormalisation

- The on-shell scheme gives us our renormalisation conditions

$$\begin{aligned}
\delta Z_f^L =& - \widetilde{\mathrm{Re}}\, \Sigma_f^L(m_f^2) + \Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2) \\
&- m_f^2 \frac{\partial}{\partial p^2} \widetilde{\mathrm{Re}} \left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2) \right] \Big|_{p^2 = m_f^2} \\
\delta Z_f^R =& - \widetilde{\mathrm{Re}}\, \Sigma^{f,R}(m_f^2) \\
&- m_f^2 \frac{\partial}{\partial p^2} \widetilde{\mathrm{Re}} \left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2) \right] \Big|_{p^2 = m_f^2} \\
\delta Z_h =& - \mathrm{Re} \frac{\partial \Sigma^H(k^2)}{\partial k^2} \Big|_{k^2 = m_H^2}
\end{aligned}$$

Renormalisation

- The mass counterterms are computed as

$$\delta m_f = \frac{m_f}{2}\widetilde{\mathrm{Re}}\left(\Sigma_f^L(m_f^2) + \Sigma_f^R(m_f^2) + \Sigma_f^S(m_f^2) + \Sigma_f^{S*}(m_f^2)\right)$$

$$\frac{\delta M_W}{M_W} = \widetilde{\mathrm{Re}}\,\frac{\Sigma_T^W(M_W^2)}{2M_W^2}$$

- The electric charge renormalisation can also be computed from two-point functions

$$\frac{\delta\bar{e}}{\bar{e}} = \frac{1}{2}\frac{\partial\Sigma_T^{AA}(k^2)}{\partial k^2}\bigg|_{k^2=0} + \frac{(v_f - a_f)}{Q_f}\frac{\Sigma_T^{AZ}(0)}{M_Z^2}$$

Wilson Coefficient renormalisation

We use the $\overline{MS}$ scheme for the renormalisation of the Wilson coefficients. To one-loop order, we can write

$$C_i^{(0)} = C_i(\mu) + \frac{\delta C_i(\mu)}{16\pi^2} = C_i(\mu) + \frac{1}{2\hat{\epsilon}}\frac{1}{16\pi^2}\dot{C}_i(\mu)$$

$$\dot{C}_i(\mu) \equiv 16\pi^2 \left(\mu \frac{d}{d\mu} C_i(\mu)\right)$$

But the anomalous dimension mixes the operators $\mu \frac{d}{d\mu} C_i(\mu) = \Gamma_{ij} C_j(\mu)$.

These were recently fully worked out in a set of three papers by Alonso, Manohar, Jenkins & Trott.

Counter-term Construction

The counterterm for the $h \to f\bar{f}$ decay amplitude can now be written as

$$i\mathcal{M}^{\text{C.T.}}(h \to f\bar{f}) = -i\bar{u}(p_f)\left(\delta\mathcal{M}_L P_L + \delta\mathcal{M}_L^* P_R\right)v(p_{\bar{f}})$$

where we distinguish SM and dimension-6 contributions through the notation

$$\delta\mathcal{M}_L = \frac{1}{16\pi^2}\left(\delta\mathcal{M}_L^{(4)} + \delta\mathcal{M}_L^{(6)}\right) + \dots$$

Counter-term Construction

From considering the tree-level amplitude:

$$\delta\mathcal{M}_L^{(4)} = \frac{m_f}{v_T}\left(\frac{\delta m_f^{(4)}}{m_f} - \frac{\delta v_T^{(4)}}{v_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_f^{(4),L} + \frac{1}{2}\delta Z_f^{(4),R*}\right)$$

$$\begin{aligned}\delta\mathcal{M}_L^{(6)} = &\left(\frac{m_f}{v_T}C_{H,\mathrm{kin}}\right)\left(\frac{\delta m_f^{(4)}}{m_f} - \frac{\delta v_T^{(4)}}{v_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_f^{(4),L} + \frac{1}{2}\delta Z_f^{(4),R*}\right)\\ &- \frac{v_T^2}{\sqrt{2}}C_{bH}^*\left(2\frac{\delta v_T^{(4)}}{v_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_f^{(4),L} + \frac{1}{2}\delta Z_f^{(4),R*}\right)\\ &+ \frac{m_f}{v_T}\left(\frac{\delta m_f^{(6)}}{m_f} - \frac{\delta v_T^{(6)}}{v_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_f^{(6),L} + \frac{1}{2}\delta Z_f^{(6),R*}\right)\\ &+ \frac{m_f}{v_T}\delta C_{H,\mathrm{kin}} - \frac{v_T^2}{\sqrt{2}}\delta C_{fH}^*\end{aligned}$$

Recap

Right, where do we stand...

- Chosen input parameters
  - $\bar{e}, m_H, M_W, M_Z, m_f, C_i$
- Calculated tree-level decay $i\mathcal{M}^{(0)}(h \to f\bar{f})$
- Chosen renormalisation proceedure
  - Masses & electric charge in on-shell scheme
  - Wilson coefficients in $\overline{MS}$ scheme
  - These gave prescriptions for how to construct the counter-terms
- We have expressions for the counter-terms $i\mathcal{M}^{\text{C.T.}}(h \to f\bar{f})$

We now have the ingredients to calculate the one-loop corrections...

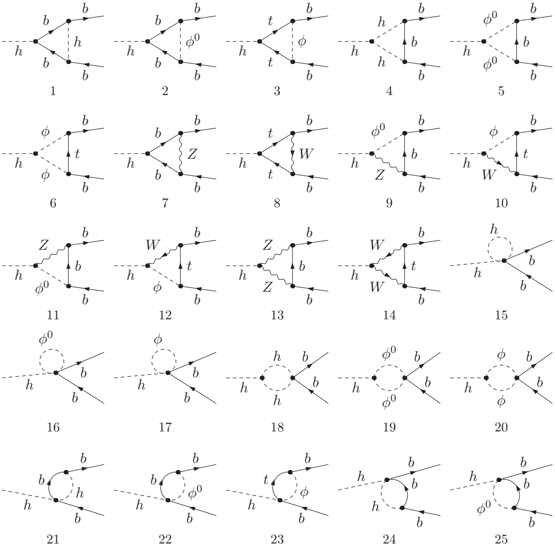
$$\mathcal{M}^{(1)}(h \to f\bar{f}) = \mathcal{M}^{(1),\text{bare}} + \mathcal{M}^{\text{C.T.}}$$

We will do this calculation in the limit of vanishing gauge couplings and further, only keep the log dependence or pieces proportional to m_t in the finite part.

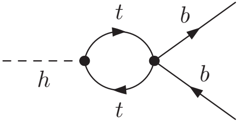
Sample Diagrams

The dimension-6 contributions are inserted onto the relevant vertices, and contributions to $\mathcal{O}(1/\Lambda_{NP}^2)$ are kept.

Note the presence of Diagrams 15-17 which are generated solely by Class 5 operators.



Bare one-loop Matrix Elements



We'll discuss the contribution from four-fermion operators.

Four-Fermion operators

We denote the non-vanishing contribution for the sum of all four-fermion diagrams to the bare matrix element by

$$i\mathcal{M}_8^{(1),\text{bare}}(h \to f\bar{f}) = -i\frac{1}{16\pi^2}\bar{u}(p_f)\left(C_{8,f}^{L,(1),\text{bare}} P_L + C_{8,f}^{R,(1),\text{bare}} P_R\right) v(p_{\bar{f}})$$

It is found that

$$\begin{aligned} C_{8,b}^{L,(1),\text{bare}} &= \frac{1}{v_T}\frac{1}{\epsilon}\Bigg[4m_b\left(3m_b^2 - \frac{m_H^2}{2}\right)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) + 2m_\tau\left(3m_\tau^2 - \frac{m_H^2}{2}\right)C_{l\tau bq}^* \\ &\quad - m_t\left(3m_t^2 - \frac{m_H^2}{2}\right)\left((1+2N_c)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right)\Bigg] + C_{8,b}^{L,(1),\text{fin}} \end{aligned}$$

$$\begin{aligned} C_{8,b}^{L,(1),\text{fin}} &= \frac{1}{v_T}\Bigg[m_b\left(4\hat{I}_8^b - 6m_b^2 + m_H^2\right)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) + 2m_\tau\hat{I}_8^\tau C_{l\tau bq}^* \\ &\quad - m_t\hat{I}_8^t\left((2N_c+1)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right)\Bigg] \end{aligned}$$

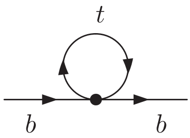
Four-Fermion operators

To renormalise we need to find all the four-fermion contributions from the expression for the counter term. Mass renormalisation as an example.

Recall: $\delta\mathcal{M}_L^{(6)} = \left(\frac{m_f}{v_T}C_{H,\text{kin}}\right)\left(\frac{\delta m_f^{(4)}}{m_f} - \frac{\delta v_T^{(4)}}{v_T} + \ldots\right) + \frac{m_f}{v_T}\delta C_{H,\text{kin}} - \frac{v_T^2}{\sqrt{2}}\delta C_{fH}^*$

$$\delta m_b^{(6)} = \frac{1}{\epsilon}\left[\frac{m_t^3}{2}\left((2N_c+1)\left(C_{qtqb}^{(1)} + C_{qtqb}^{(1)*}\right) + c_{F,3}\left(C_{qtqb}^{(8)} + C_{qtqb}^{(8)*}\right)\right)\right.$$
$$\left. - 4m_b^3\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) + m_\tau^3\left(C_{l\tau bq} + C_{l\tau bq}^*\right)\right] + \delta m_b^{\text{fin}}(\mu)\,,$$

$$\delta m_b^{\text{fin}}(\mu) = \frac{m_t}{2}\hat{A}_0(m_t^2)\left((2N_c+1)\left(C_{qtqb}^{(1)} + C_{qtqb}^{(1)*}\right) + c_{F,3}\left(C_{qtqb}^{(8)} + C_{qtqb}^{(8)*}\right)\right)$$



Four-fermion Operators

After cancelling the divergences, we find

$$\begin{aligned} v_T C_{8,b}^{L,(1)} &= m_b(m_H^2 - 4m_b^2)\left(1 - 2\hat{b}_0(m_H^2, m_b^2, m_b^2)\right)\left(C_{qb}^{(1)} + c_{F,3} C_{qb}^{(8)}\right) \\ &+ m_\tau(m_H^2 - 4m_\tau^2)\hat{b}_0(m_H^2, m_\tau^2, m_\tau^2) C_{l\tau bq} \\ &+ \frac{m_t}{2}(m_H^2 - 4m_t^2)\hat{b}_0(m_H^2, m_t^2, m_t^2)\left((2N_c + 1) C_{qtqb}^{(1)*} + c_{F,3} C_{qtqb}^{(8)*}\right) \\ &- \frac{1}{2}\frac{v_T^2}{\sqrt{2}} \dot{C}_{bH}^{(4f)*} \ln\left(\frac{m_H^2}{\mu^2}\right) \end{aligned}$$

Implications for Phenomenology

It is possible to make some naïve estimates for the impact on SM phenomenology when also considering dimension-6 operators in fixed order.

The decay rate can be written as:

$$\Gamma(h \to f\bar{f}) = \underbrace{B_f}_{\text{Phase-space factor}} \Big[\overbrace{\underbrace{\Gamma_f^{(4,0)}}_{\mathcal{O}(1/\Lambda^0)} + \underbrace{\Gamma_f^{(6,0)}}_{\mathcal{O}(1/\Lambda^2)}}^{\text{Tree-level}} + \overbrace{\underbrace{\Gamma_f^{(4,1)}}_{\mathcal{O}(1/\Lambda^0)} + \underbrace{\Gamma_f^{(6,1)}}_{\mathcal{O}(1/\Lambda^2)}}^{\text{One-loop}} \Big]$$

$$\Gamma_f^{(4,0)} = \left[A_f^{(4,0)} \cdot A_f^{(4,0)}\right], \qquad \Gamma_f^{(4,1)} = \frac{1}{16\pi^2}\left[2A_f^{(4,0)} \cdot A_f^{(4,1)}\right],$$

$$\Gamma_f^{(6,0)} = \left[2A_f^{(4,0)} \cdot A_f^{(6,0)}\right], \quad \Gamma_f^{(6,1)} = \frac{1}{16\pi^2}\left[2\left(A_f^{(6,0)} \cdot A_f^{(4,1)} + A_f^{(4,0)} \cdot A_f^{(6,1)}\right)\right]$$

Implications for Phenomenology: Tree-level

Consider a tree-level comparison of dimension-6 and SM contributions. Numerically, at a scale of $\Lambda_{\rm NP} = 1$ TeV, for $h \to b\bar{b}$ this amounts to

$$\frac{\Gamma_b^{(6,0)}}{\Gamma_b^{(4,0)}} = -4.44\tilde{C}_{bH} + 0.03\left(4\tilde{C}_{H\Box} - \tilde{C}_{HD} - 2\left(\tilde{C}^{(3)}_{\substack{Hl\\ee}} + \tilde{C}^{(3)}_{\substack{Hl\\\mu\mu}}\right) + \left(\tilde{C}_{\substack{ll\\\mu ee\mu}} + \tilde{C}_{\substack{ll\\e\mu\mu e}}\right)\right)$$

Implications for Phenomenology: One-loop

$$\frac{\Gamma_b^{(4,1)}}{\Gamma_b^{(4,0)}} = \frac{G_F m_t^2}{8\pi^2}\left(\frac{-18+7N_c}{3\sqrt{2}}\right) = 0.003\,,$$

$$\frac{\Gamma_b^{(6,1)}}{\Gamma_{C_{bH}}^{(6,0)}} \simeq -0.12 + 0.03\frac{\tilde{C}_{Htb}}{\tilde{C}_{bH}} + 0.13\frac{\tilde{C}_{qtqb}^{(1)}}{\tilde{C}_{bH}} + 0.03\frac{\tilde{C}_{qtqb}^{(8)}}{\tilde{C}_{bH}} + \ldots\,.$$

Conclusion and Summary

- SMEFT is a model independent way to account for possible decoupled BSM effects
- Calculated Higgs decays to b quarks at one-loop:
  - Select renormalisation scheme
  - Calculate Feynman diagrams
  - Cancelation of divergences
  - Rough Pheno implications
- *Interesting to note that the electric charge required renormalisation even in the vanishing gauge coupling limit*
- Next step is to complete the calculation without vanishing gauge couplings...
- Can use renormalisation group running to resum higher order logs....
- *Work in progress..*