

ANATOMY OF B-INITIATED PROCESSES: IMPLICATIONS FOR LHC SIMULATIONS

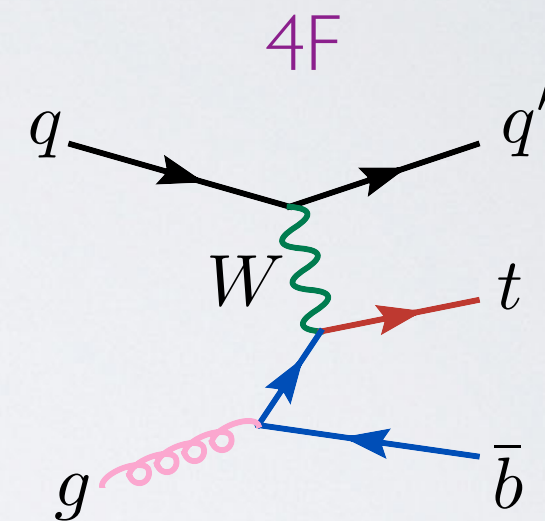
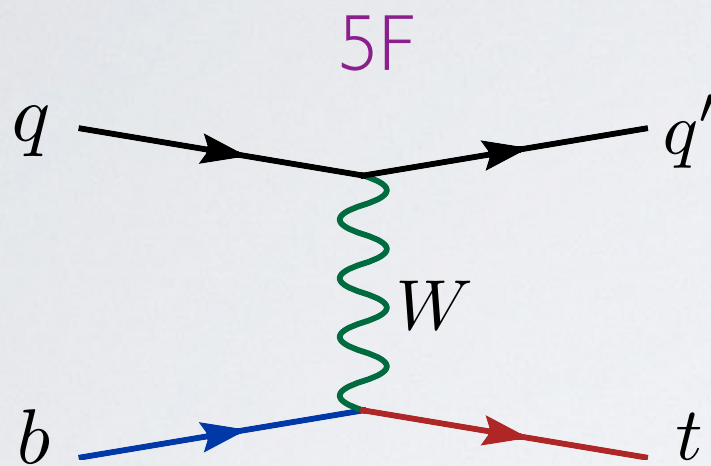
FABIO MALTONI

CENTRE FOR COSMOLOGY, PARTICLE PHYSICS AND PHENOMENOLOGY (CP3), LOUVAIN

WORK IN COLLABORATION WITH
GIOVANNI RIDOLFI AND MARIA UBIALI

AND RESULTS FROM
WIESEMANN, FREDERIX, FRIXIONE, HIRSCHI, FM, TORRIELLI. 1409.5301

5F OR 4F? THIS IS THE DILEMMA



1. It resums initial state large logs in the b pdf
2. Going NLO easy and NNLO possible.
3. Mass effects enter at higher orders.
4. Implementation in MC depends on the gluon splitting model in the PS.

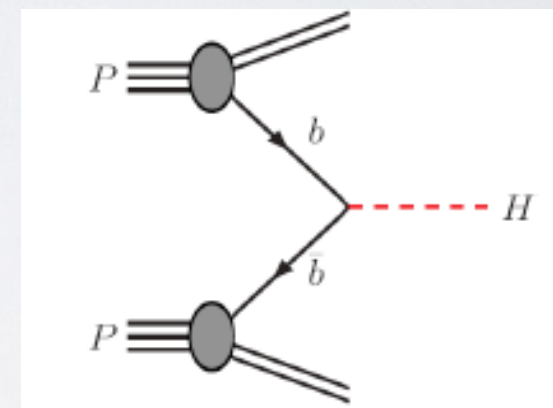
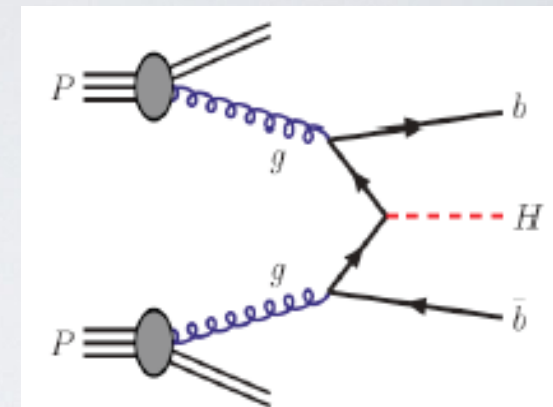
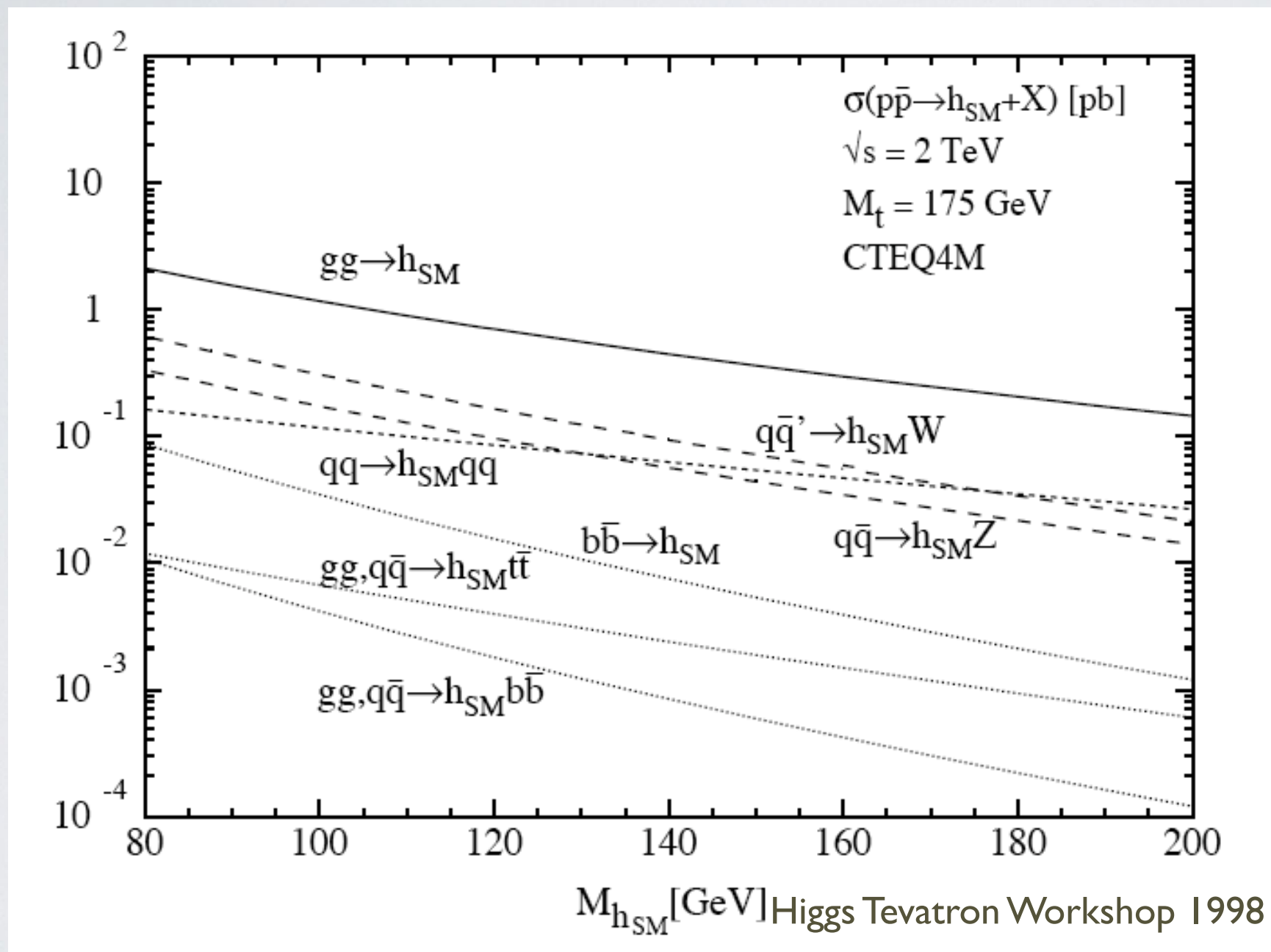
1. It does not resum (possibly) large logs, yet it has them explicitly at fixed order.
2. Going NLO WAS difficult.
3. Mass effects are there at any order in PT.
4. MC at LO and NLO no problem.

see Maria's talk

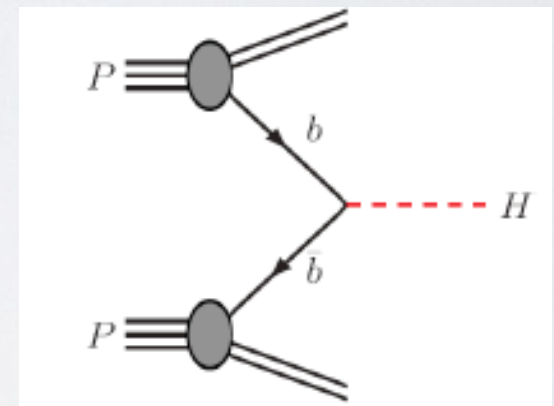
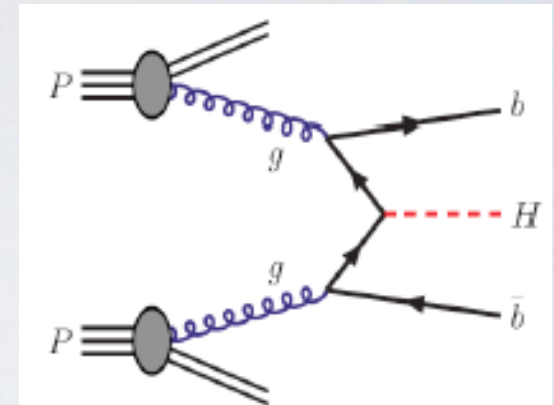
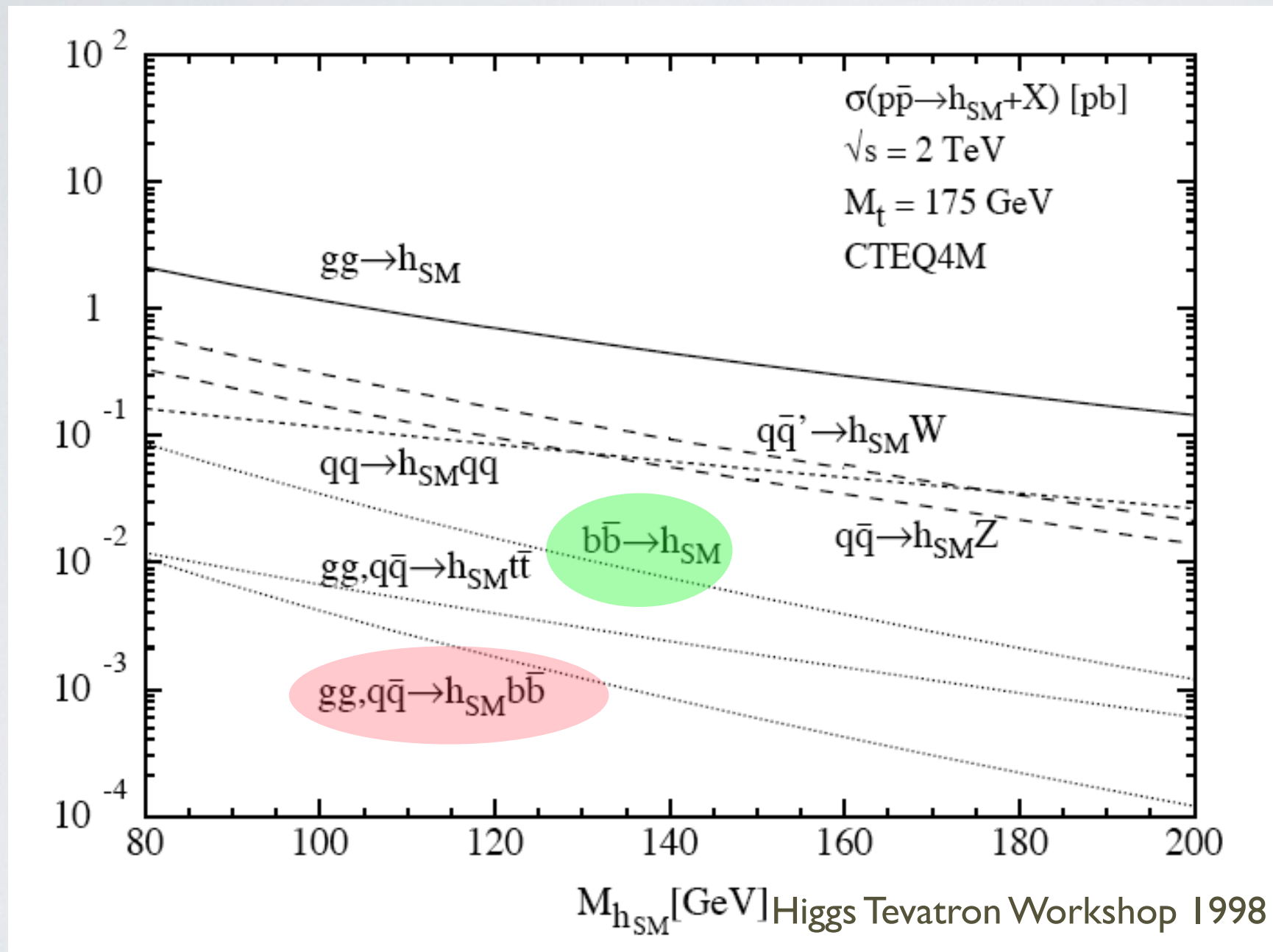
CONCLUSIONS

- Consistency between 4F and 5F calculations for both single and double heavy-quark initiated processes at the LHC has been found. 4F/5F calculations can be used in different contexts.
- 5F calculations at NNLO clearly give the most stable and reliable predictions for total cross sections and should be certainly used for normalisation. In particular, 5F calculations should be used for processes involving very high Bjorken- x .
- 4F calculations are (currently) easily and more reliably obtained at NLO in the form of an event generator and can provide a wider spectrum of observables at NLO accuracy. Behaviour of the NLO+PS predictions consistent with the analysis of the logs in the FO calculations.

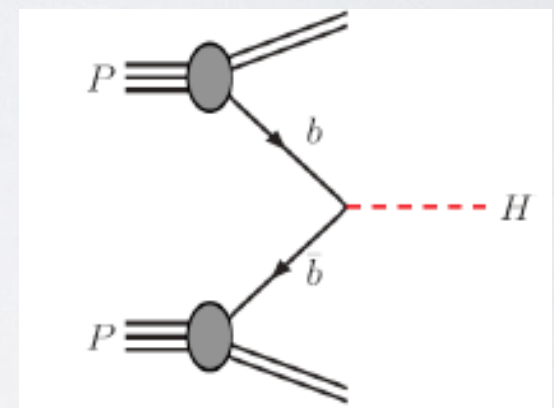
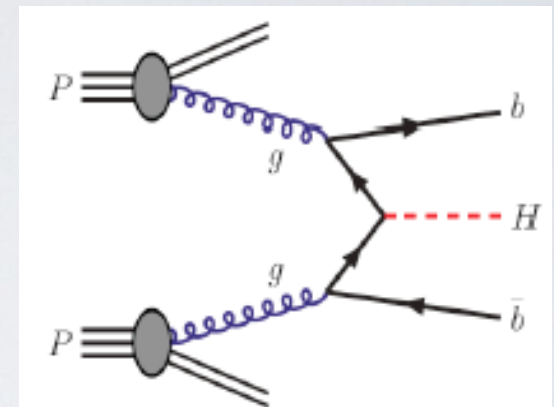
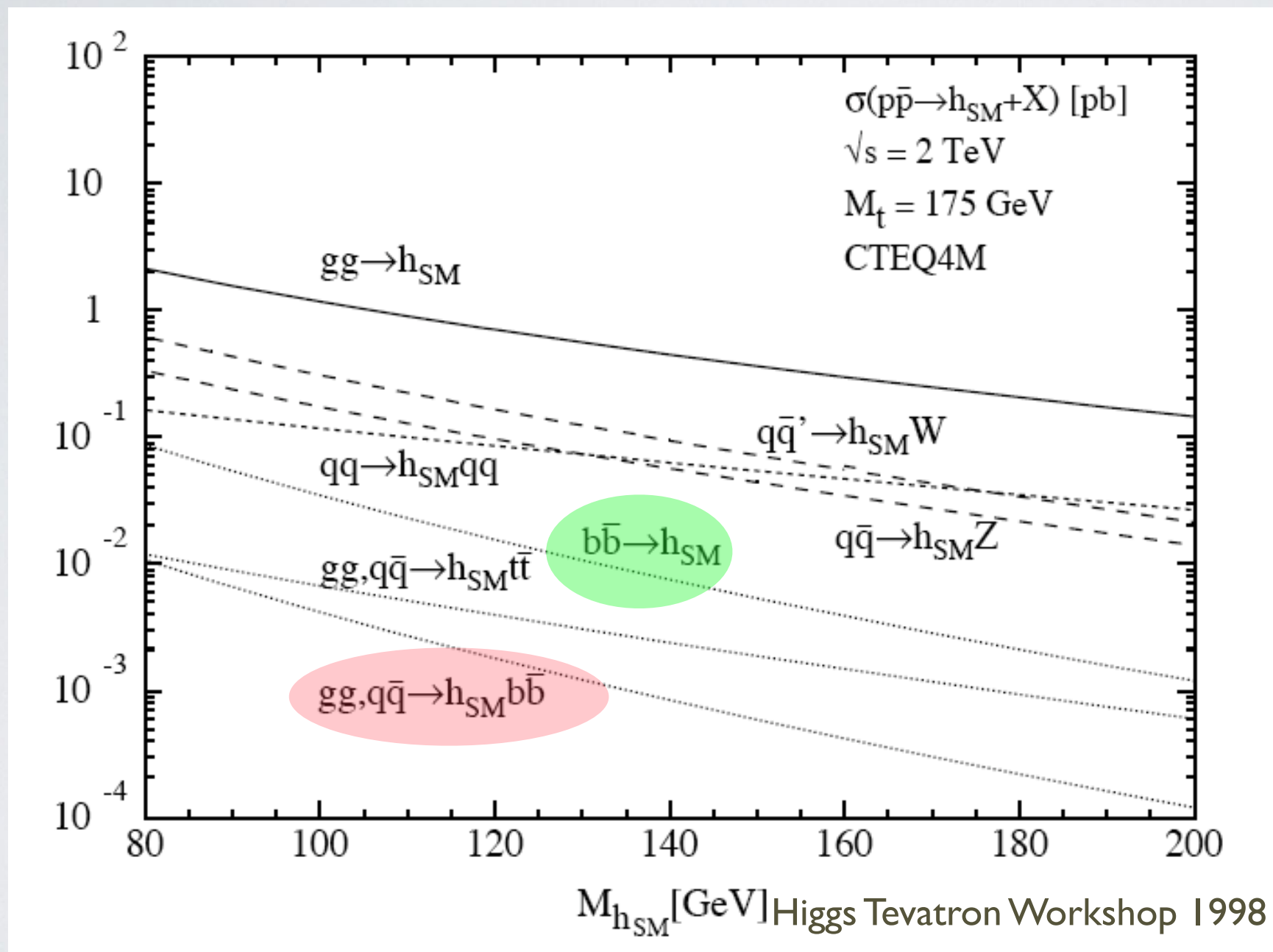
QUESTIONS AND PUZZLES: LEVEL 1



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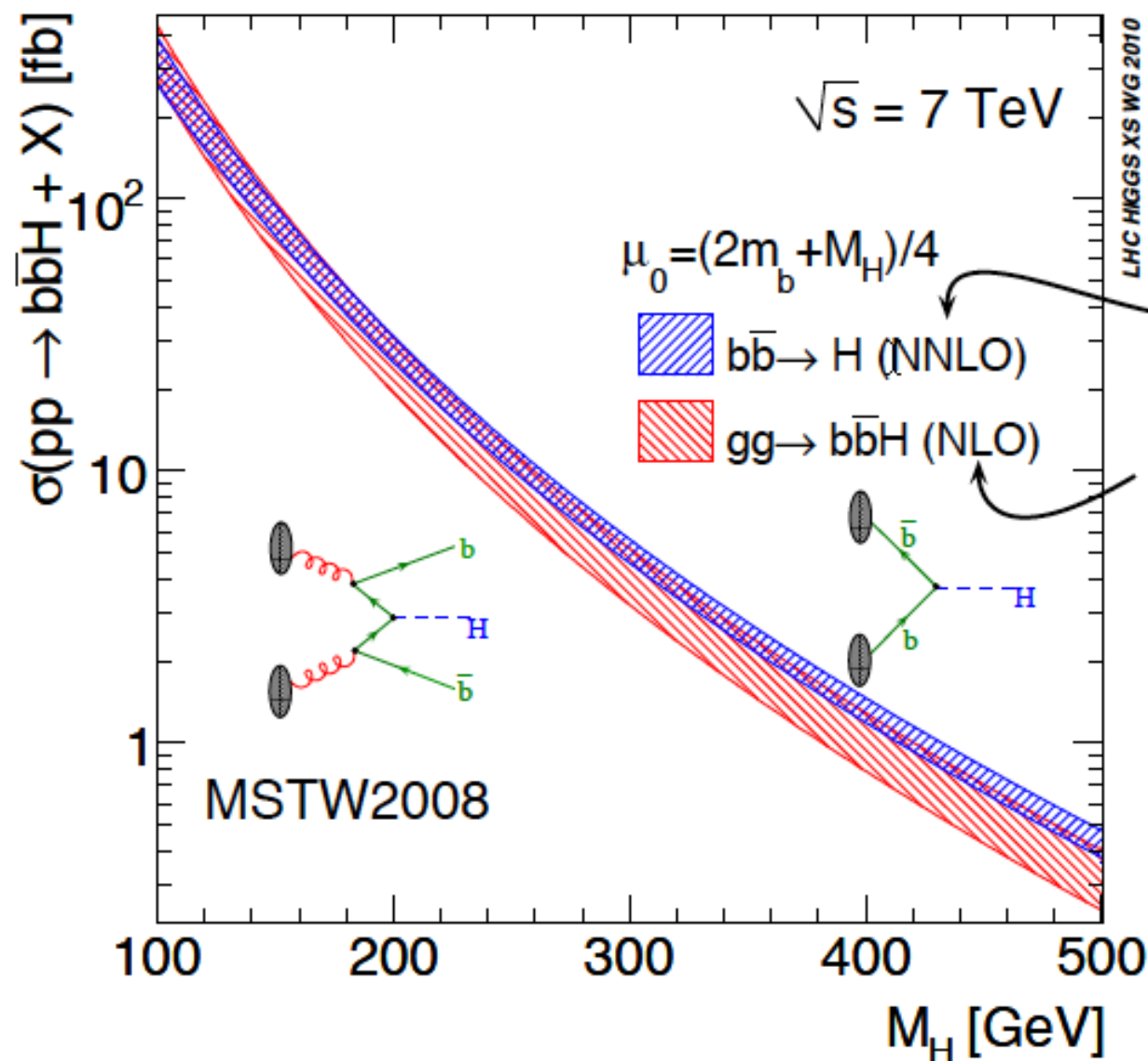


QUESTIONS AND PUZZLES: LEVEL 1



Factor of ten difference??? Is this the effects of the logs? How can that be?

QUESTIONS AND PUZZLES: LEVEL 1



Two important ingredients helped in “solve” this puzzle:

1. Inclusion of higher order corrections

[Harlander and Kilgore, 2003]

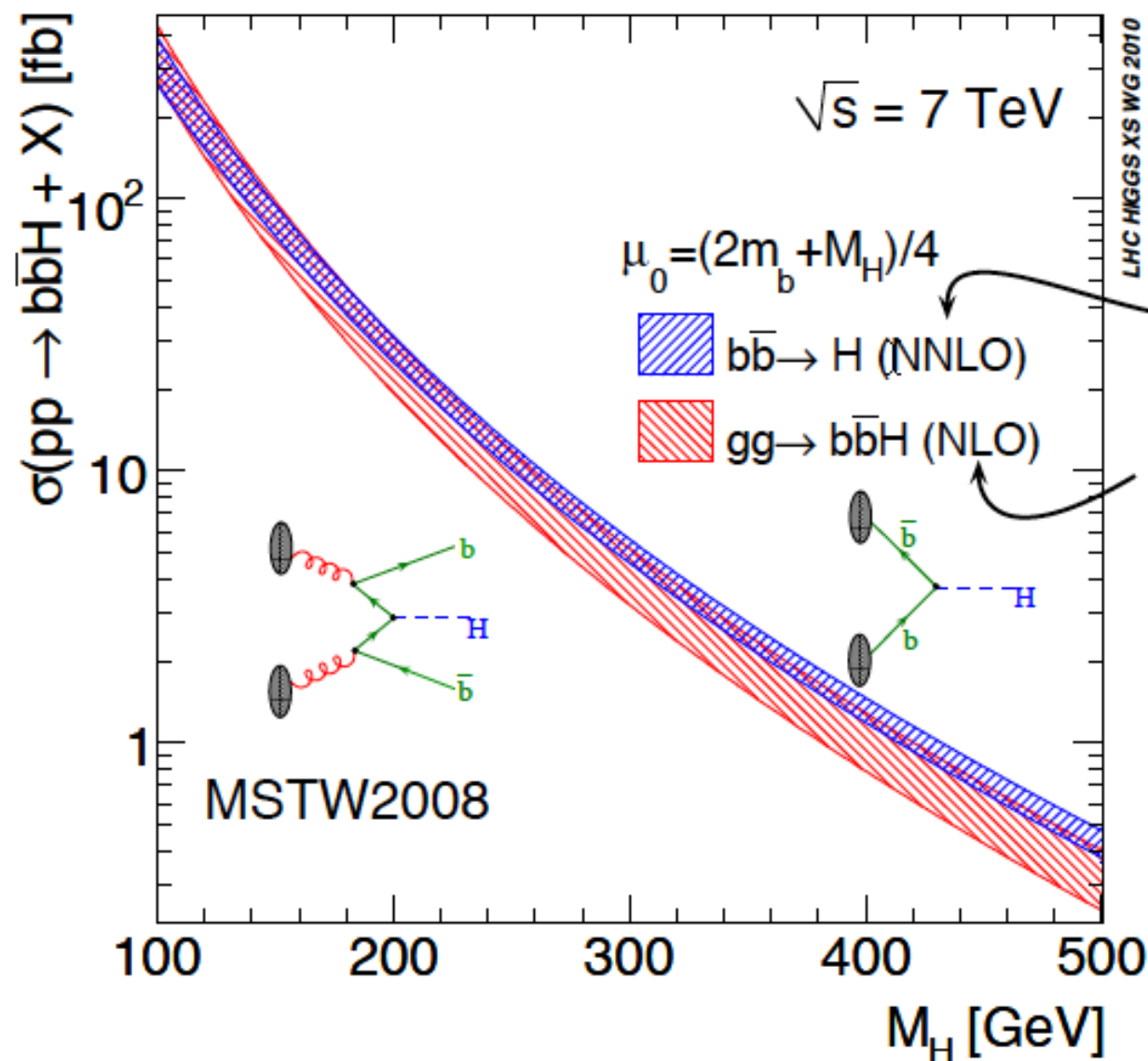
[Dittmaier, Krämer, Spira '04]

[Dawson, Jackson, Reina, Wackerroth '04]

[Hirschi et al. [1103.0621](#)]

2. Scale choices : better agreement when smaller than naive choices M_H .

QUESTIONS AND PUZZLES: LEVEL 2

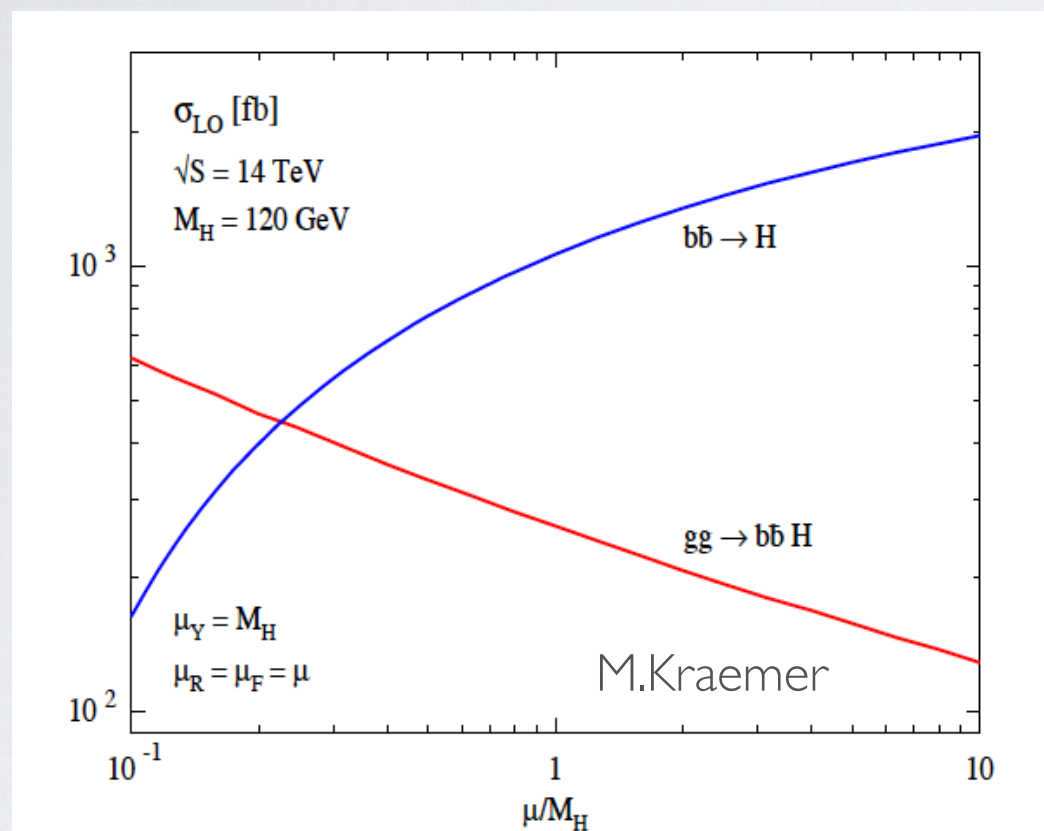


However, this plot now raises new burning questions:

1. Why the agreement is so good around $m_H = 100 \text{ GeV}$ and the uncertainty band comparable?
2. It looks like one needs a 500 GeV Higgs to really see the effects of the large logs? Is this the reason?
3. How is the smaller scale choice $m_H/4$ justified? Agreement seems ad hoc.
4. Is this behavior only proper of $b\bar{b} \rightarrow \text{Higgs}$ or it is general?

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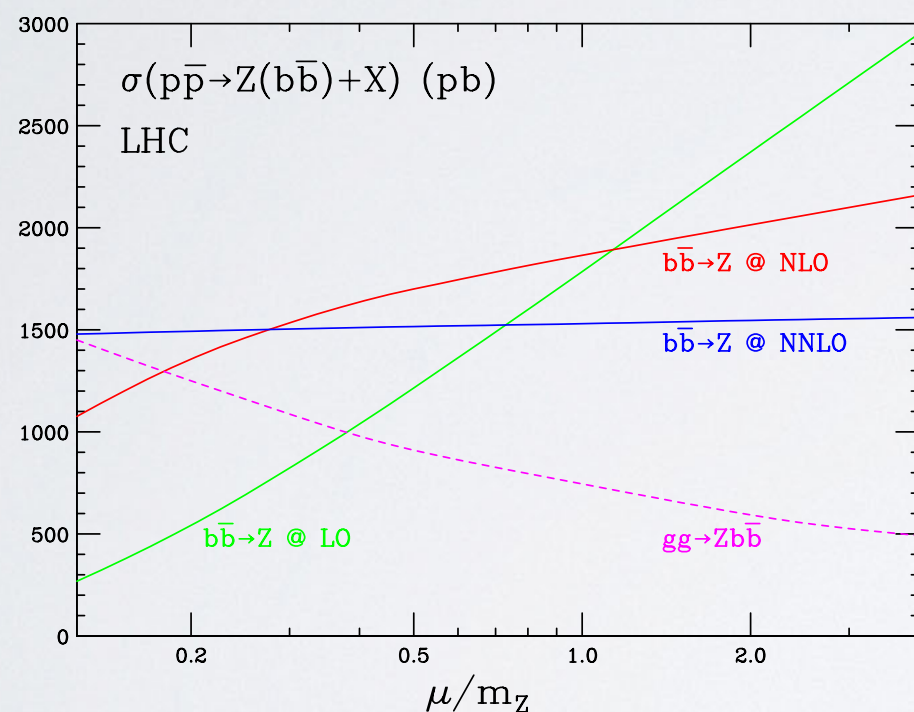
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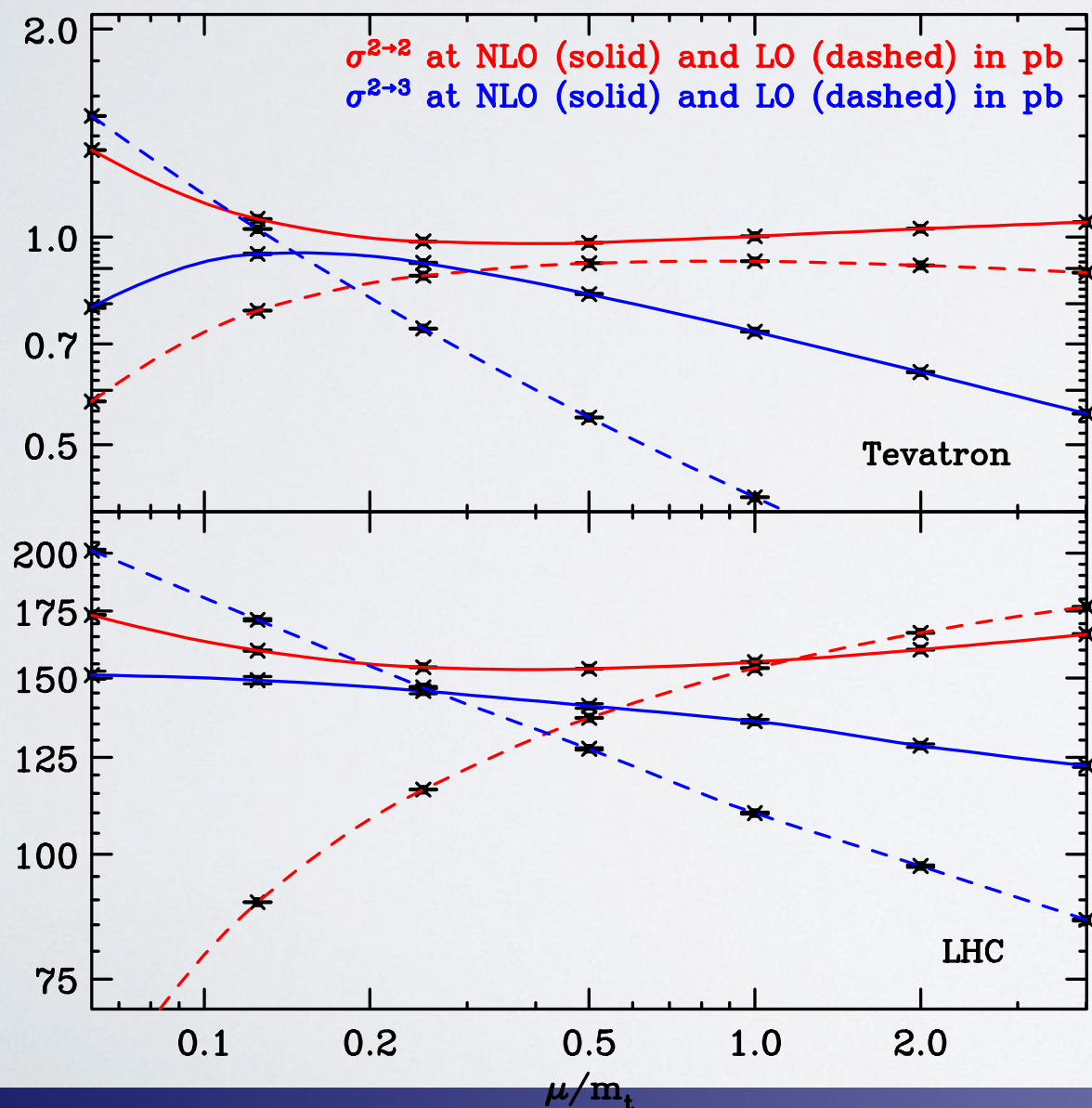
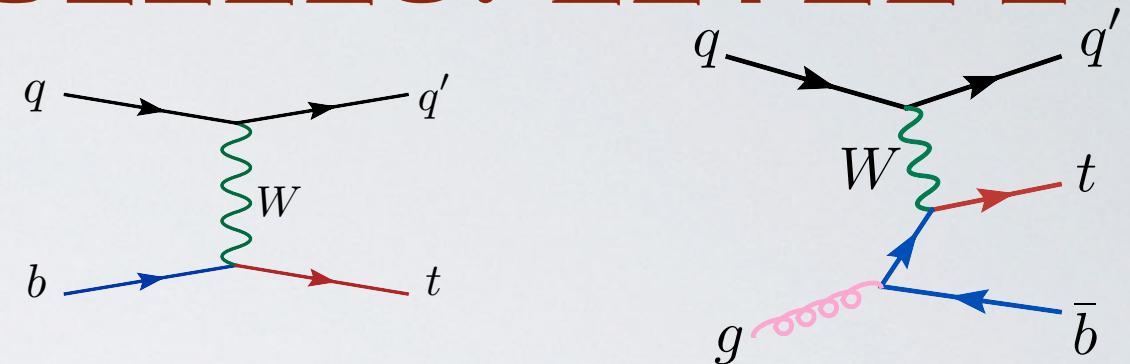
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QUESTIONS AND PUZZLES: LEVEL 2

t-channel single top:



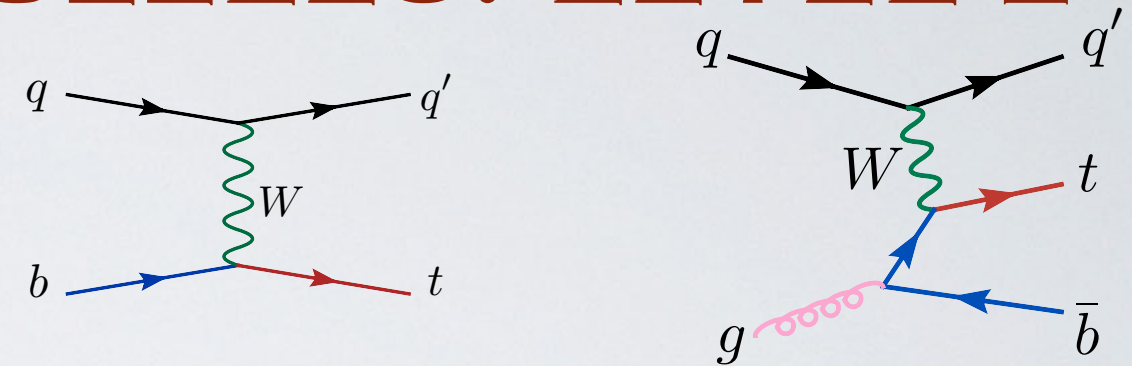
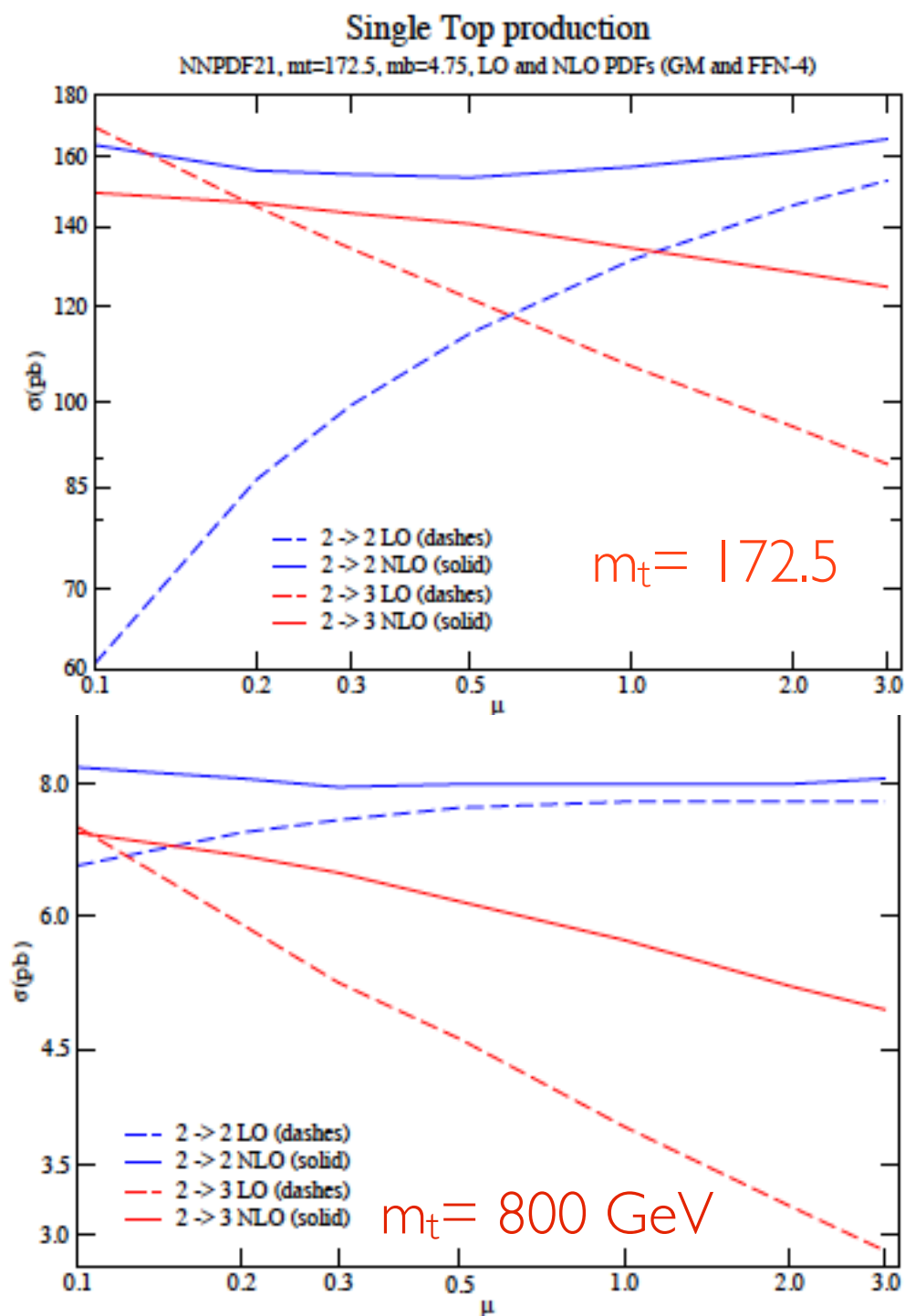
1. Differences at natural scales m_t become smaller at lower scales, $\mu \sim m_t/4$. Why?

2. At LHC both scale dependences are rather mild. 4F is as good as 5F. Where is the need for resummation?

3. Differences are smaller at the LHC than Tevatron. Why? The logs should have more space to develop at the LHC...

3. What happens for a heavier top?

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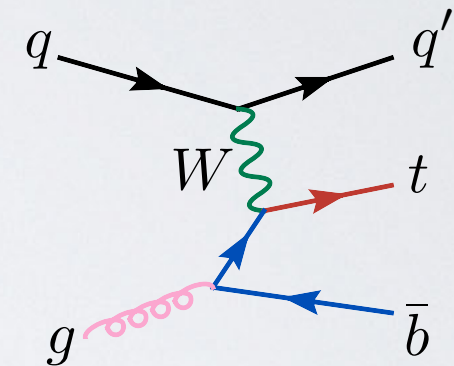
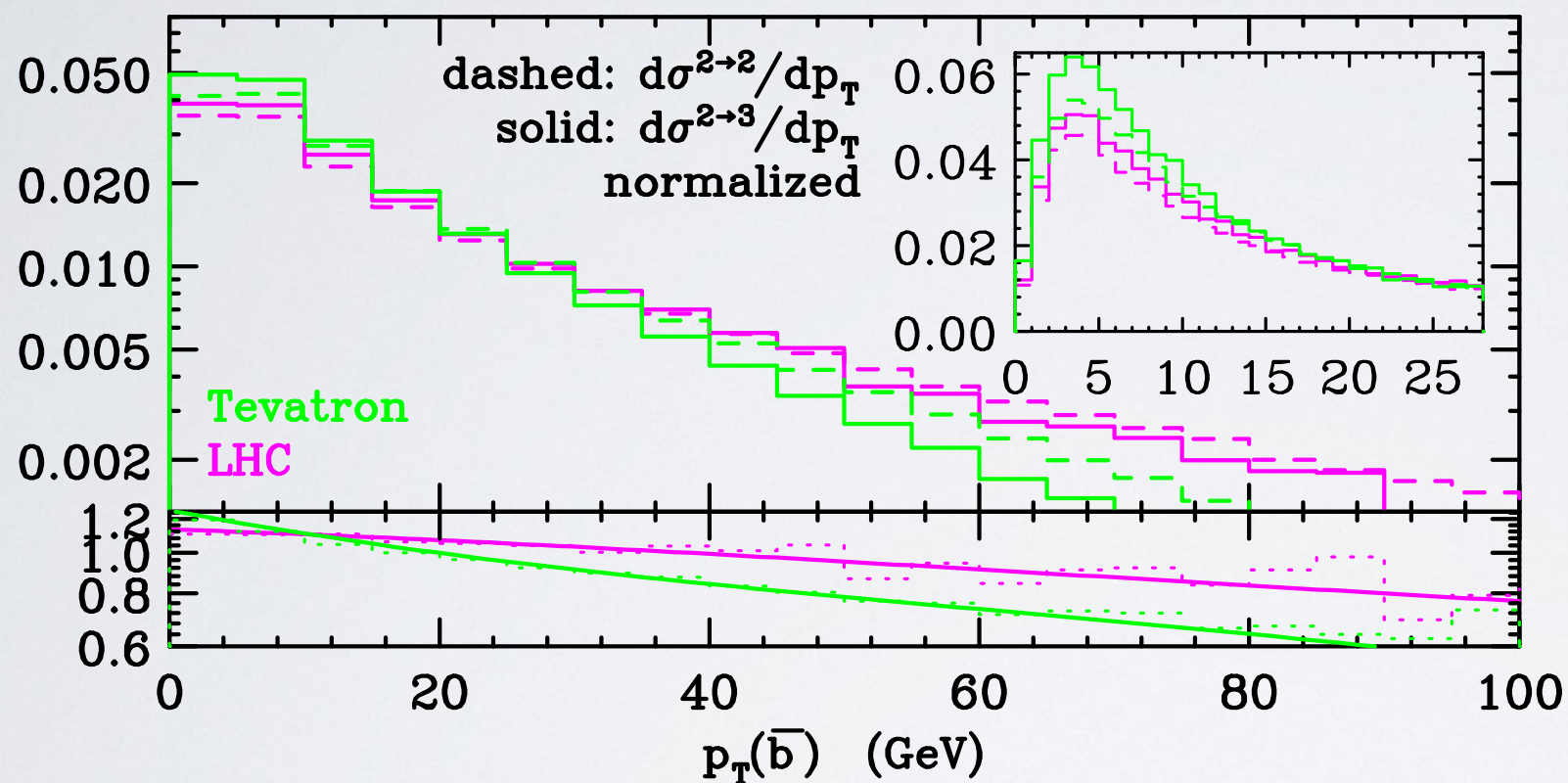
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QUESTIONS AND PUZZLES: LEVEL 3

- What about other more exclusive observables?

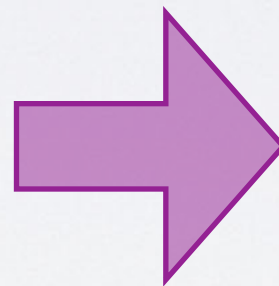
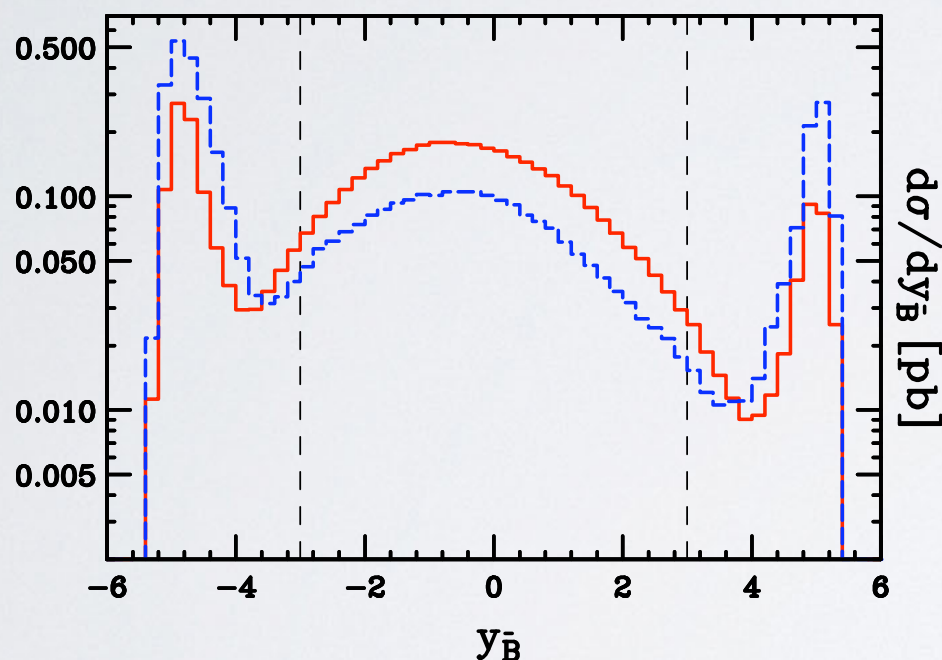


- This observable is NLO only in the 4F calculation.
- Slightly softer in 4F ($2 \rightarrow 3$), particularly at the Tevatron
- Deviations up to $\sim 20\%$: stable perturbative expansion, no large corrections
- A 4F calculation is much more EXP handy and useful in actual analyses.

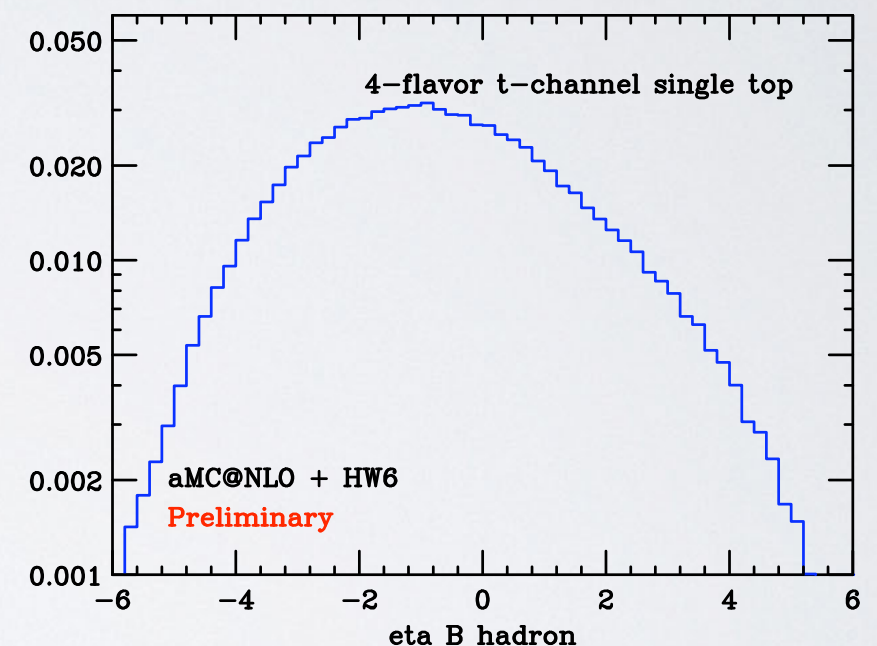
QUESTIONS AND PUZZLES: LEVEL 3

- What about other more exclusive observables?

5-flavor scheme with OLD Fortran Herwig



4-flavor scheme



- Not first example where leaving the shower to do gluon splitting is not exactly a good idea.
- 4F calculations are easily interfaced to MC@NLO or POWHEG.
- Many available automatically from aMC@NLO.

THE INVESTIGATION



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In arXiv:1203.6393 we argued that all the (apparent) puzzles and odd findings listed before can be understood in **processes with single heavy quark initiated state** by simply taking into account the following two main results:

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We have recently confirmed (to appear) that this picture holds unchanged for processes with two heavy-quarks in the initial state.

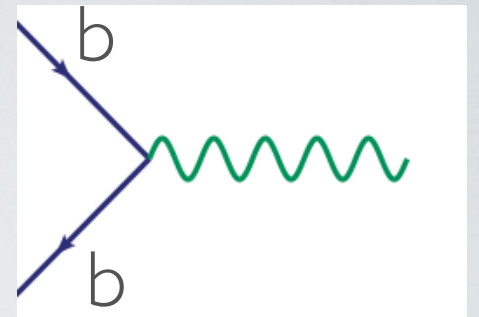
PLAN

I now dare to show you the
ANATOMY of the comparison
between 4F and 5F for pp \rightarrow bbH.



B B \rightarrow H (5F)

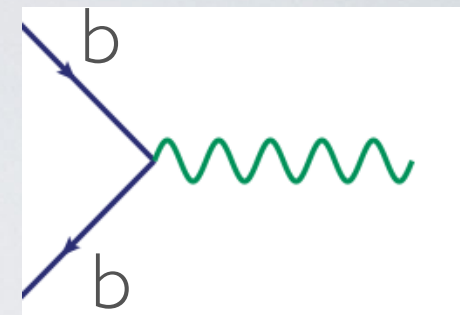
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$$\hat{\sigma}^{5F}(\hat{\tau}) = \frac{G_F \pi}{3\sqrt{2}} \frac{m_b^2}{M_H^2} \delta(1 - \hat{\tau}), \quad \hat{\tau} = \frac{M_H^2}{\hat{s}}$$

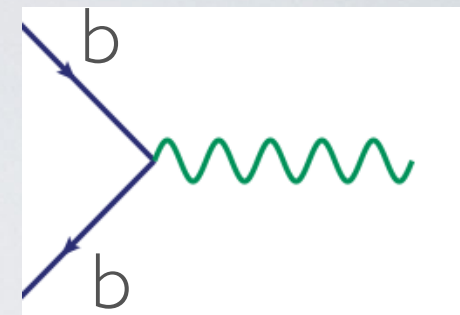


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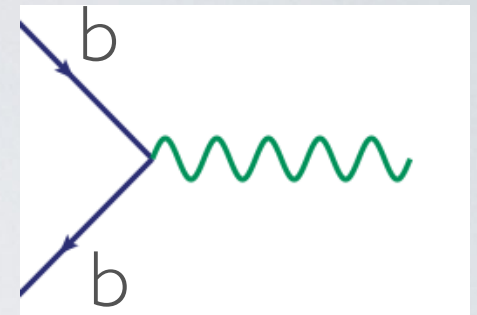
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I can now define b_{tilde} as the first log of b :

$$b(x, \mu_F^2) = \frac{\alpha_s}{2\pi} L_b \int_x^1 \frac{dy}{y} P_{qg}(y) g\left(\frac{x}{y}, \mu_F^2\right) + O(\alpha_s^2) = \tilde{b}^{(1)}(x, \mu_F^2) + O(\alpha_s^2) \quad L_b = \log \frac{\mu_F^2}{m_b^2}$$

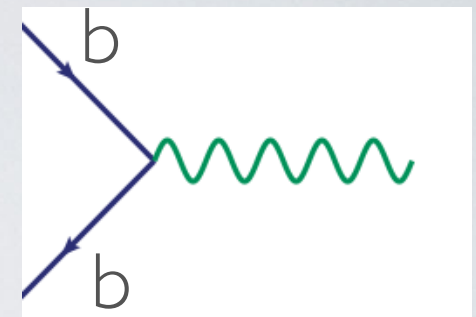


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and write the unresummed 5F cross section:

$$\tilde{\sigma}^{5F}(\tau) = 2 \int_{\tau}^1 dx_1 \int_{\frac{\tau}{x_1}}^1 dx_2 \hat{\sigma}^{5F}\left(\frac{\tau}{x_1 x_2}\right) \int_{x_1}^1 \frac{dy}{y} \left[\frac{\alpha_s}{2\pi} P_{qg}(y) L_b \right] g\left(\frac{x_1}{y}, \mu_F^2\right) \int_{x_2}^1 \frac{dz}{z} \left[\frac{\alpha_s}{2\pi} P_{qg}(z) L_b \right] g\left(\frac{x_2}{z}, \mu_F^2\right)$$

G G \rightarrow B B H (4F)

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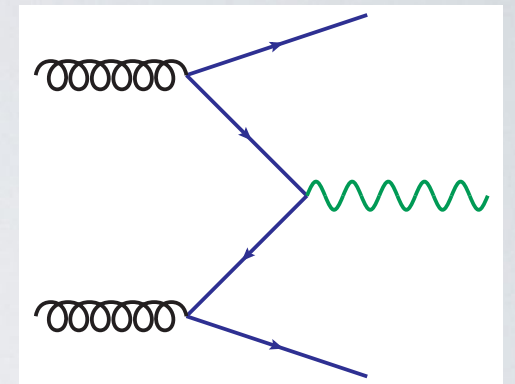
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I consider the double collinear limit where t_1 and t_2 tend to zero:

$$k_1 = (1 - z_1)p_1; \quad k_2 = (1 - z_2)p_2; \quad 0 \leq z_i \leq 1$$

$$L(z, \hat{\tau}) = \log \frac{M_H^2}{m_b^2} \frac{(1 - z)^2}{\hat{\tau}}$$

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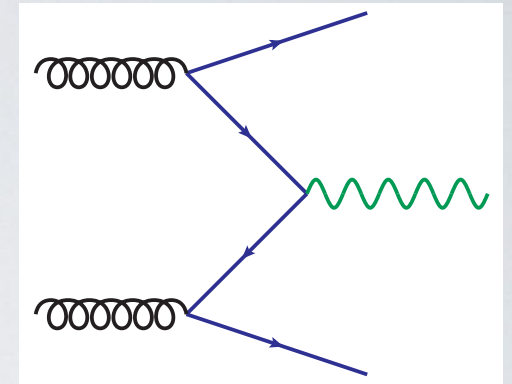
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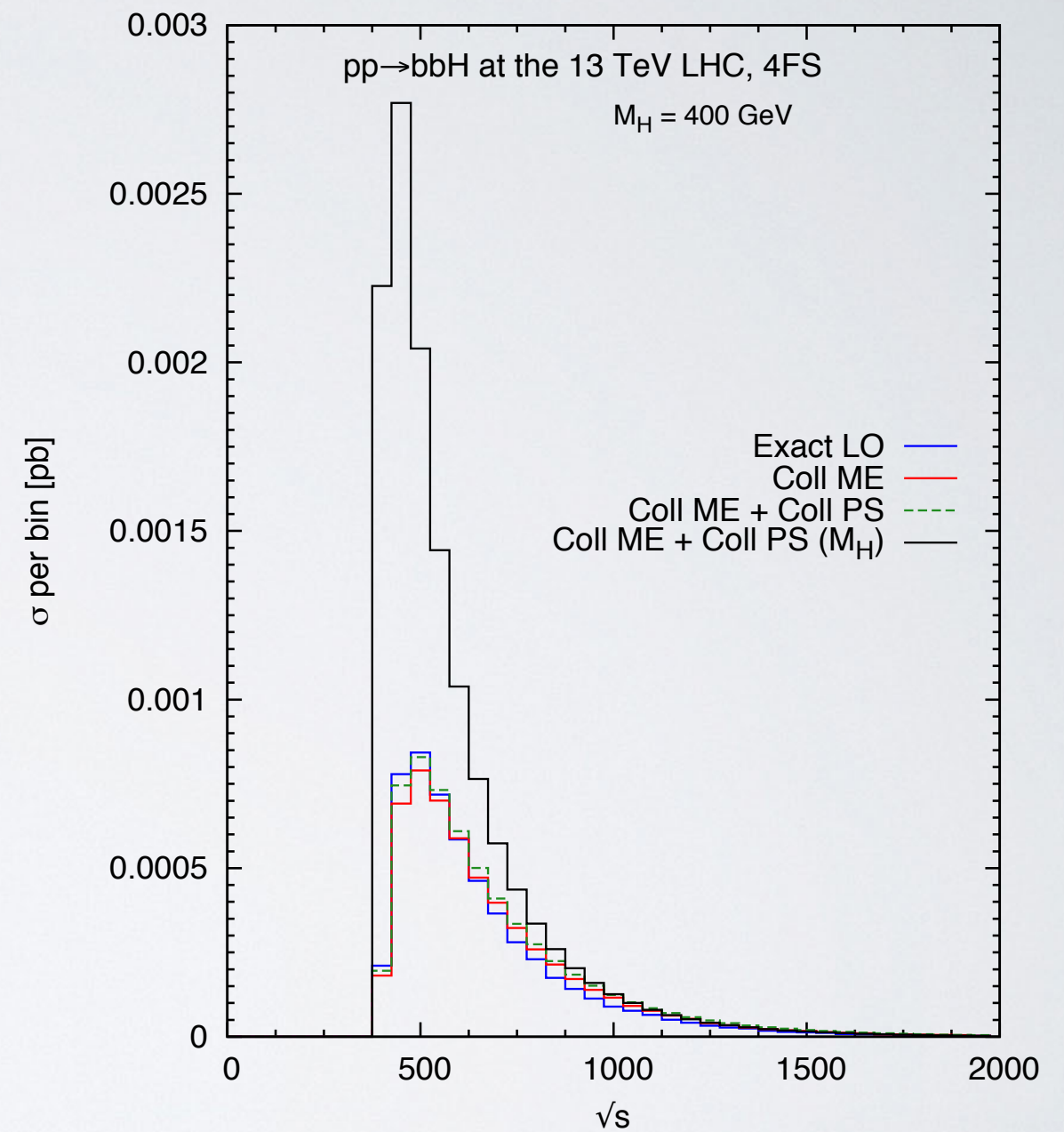
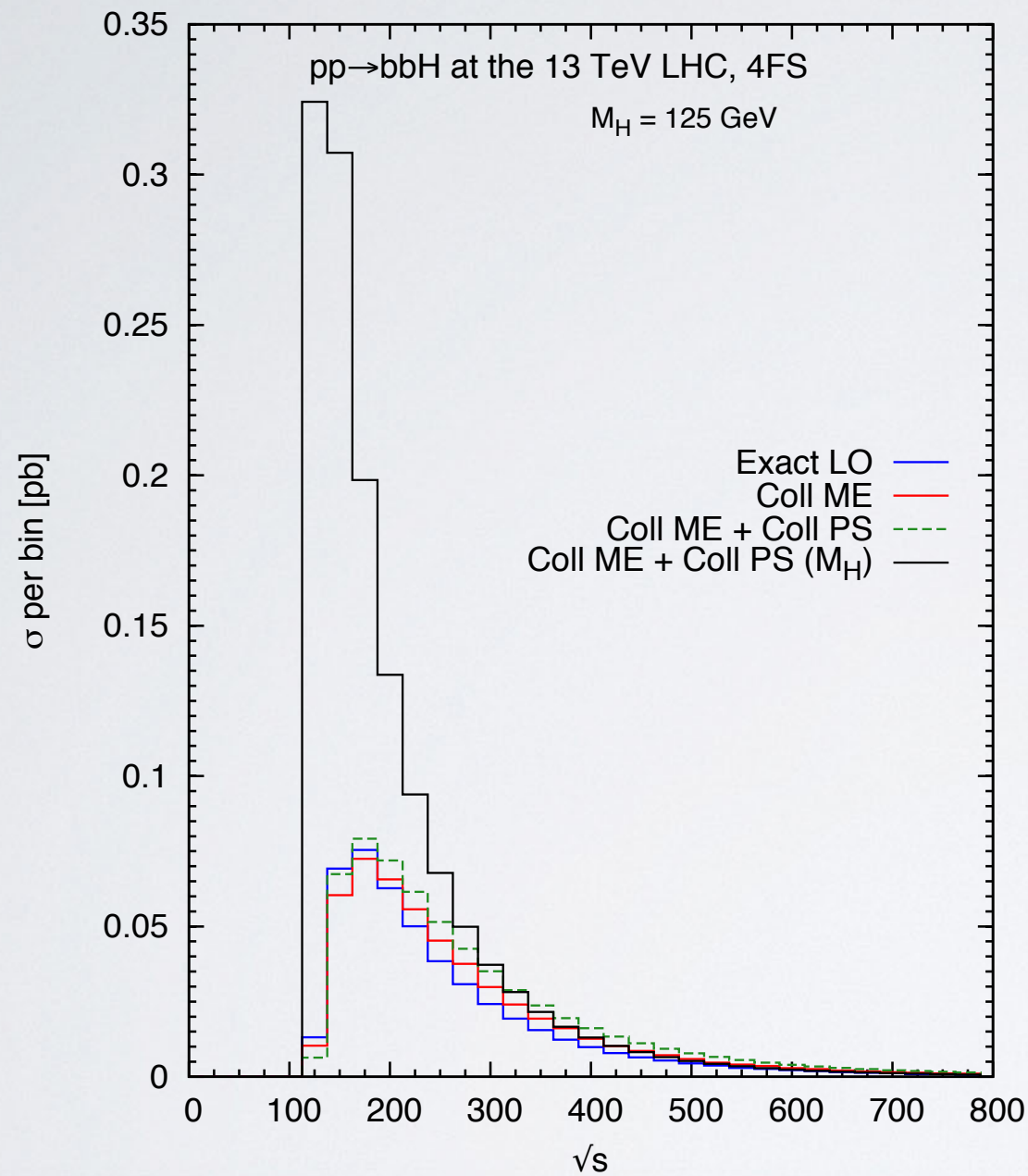
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A “straightforward” derivation based on taking the collinear limit of the full amplitude and completely factorising the phase space, leads to the following expression:

$$\begin{aligned} \hat{\sigma}^{4F, \text{coll}}(\hat{\tau}) &= \hat{\tau} \frac{\alpha_s^2}{4\pi^2} \frac{G_F \pi}{3\sqrt{2}} \frac{m_b^2}{M_H^2} 2 \int_0^1 dz_1 \int_0^1 dz_2 P_{qg}(z_1) P_{qg}(z_2) L(z_1, \hat{\tau}) L(z_2, \hat{\tau}) \delta(z_1 z_2 - \hat{\tau}) \\ &= 2 \int_{\hat{\tau}}^1 dz_1 \int_{\frac{\hat{\tau}}{z_1}}^1 dz_2 \left[\frac{\alpha_s}{2\pi} P_{qg}(z_1) L(z_1, \hat{\tau}) \right] \left[\frac{\alpha_s}{2\pi} P_{qg}(z_2) L(z_2, \hat{\tau}) \right] \hat{\sigma}^{5F} \left(\frac{\hat{\tau}}{z_1 z_2} \right), \end{aligned}$$

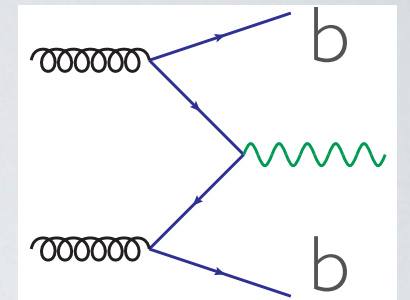
$G G \rightarrow B B H (4F)$



The “improved” collinear approximation compares well with the exact calculation.

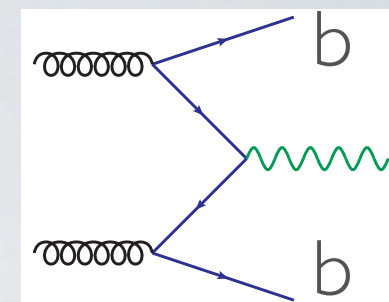
COMPARISON 4F/5F

$$\sigma^{4F, \text{coll}}(\tau) = 2 \int_{\tau}^1 dx_1 \int_{\frac{\tau}{x_1}}^1 dx_2 \hat{\sigma}^{5F} \left(\frac{\tau}{x_1 x_2} \right) \\ \int_{x_1}^1 \frac{dz_1}{z_1} \left[\frac{\alpha_s}{2\pi} P_{qg}(z_1) L(z_1, z_1 z_2) \right] g \left(\frac{x_1}{z_1}, \mu_F^2 \right) \int_{x_2}^1 \frac{dz_2}{z_2} \left[\frac{\alpha_s}{2\pi} P_{qg}(z_2) L(z_2, z_1 z_2) \right] g \left(\frac{x_2}{z_2}, \mu_F^2 \right)$$

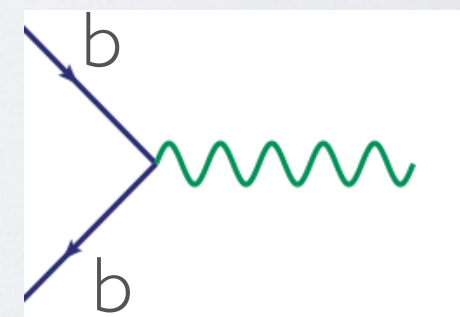


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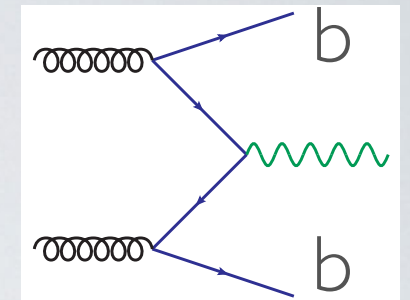


$$\tilde{\sigma}^{5F}(\tau) = 2 \int_{\tau}^1 dx_1 \int_{\frac{\tau}{x_1}}^1 dx_2 \hat{\sigma}^{5F} \left(\frac{\tau}{x_1 x_2} \right) \\ \int_{x_1}^1 \frac{dy}{y} \left[\frac{\alpha_s}{2\pi} P_{qg}(y) L_b \right] g \left(\frac{x_1}{y}, \mu_F^2 \right) \int_{x_2}^1 \frac{dz}{z} \left[\frac{\alpha_s}{2\pi} P_{qg}(z) L_b \right] g \left(\frac{x_2}{z}, \mu_F^2 \right)$$



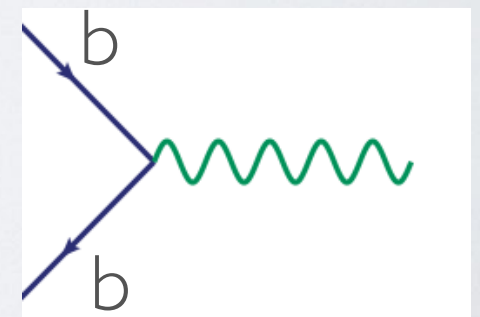
COMPARISON 4F/5F

$$\sigma^{4F, \text{coll}}(\tau) = 2 \int_{\tau}^1 dx_1 \int_{\frac{\tau}{x_1}}^1 dx_2 \hat{\sigma}^{5F} \left(\frac{\tau}{x_1 x_2} \right) \int_{x_1}^1 \frac{dz_1}{z_1} \left[\frac{\alpha_s}{2\pi} P_{qg}(z_1) L(z_1, z_1 z_2) \right] g \left(\frac{x_1}{z_1}, \mu_F^2 \right) \int_{x_2}^1 \frac{dz_2}{z_2} \left[\frac{\alpha_s}{2\pi} P_{qg}(z_2) L(z_2, z_1 z_2) \right] g \left(\frac{x_2}{z_2}, \mu_F^2 \right)$$



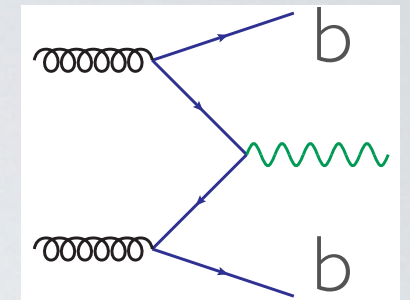
$$L(z, \hat{\tau}) = \log \frac{M_H^2}{m_b^2} \frac{(1-z)^2}{\hat{\tau}} \longleftrightarrow L_b = \log \frac{\mu_F^2}{m_b^2}$$

$$\tilde{\sigma}^{5F}(\tau) = 2 \int_{\tau}^1 dx_1 \int_{\frac{\tau}{x_1}}^1 dx_2 \hat{\sigma}^{5F} \left(\frac{\tau}{x_1 x_2} \right) \int_{x_1}^1 \frac{dy}{y} \left[\frac{\alpha_s}{2\pi} P_{qg}(y) L_b \right] g \left(\frac{x_1}{y}, \mu_F^2 \right) \int_{x_2}^1 \frac{dz}{z} \left[\frac{\alpha_s}{2\pi} P_{qg}(z) L_b \right] g \left(\frac{x_2}{z}, \mu_F^2 \right)$$



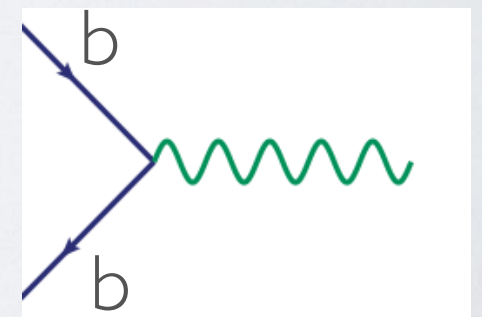
COMPARISON 4F/5F

$$\sigma^{4F, \text{coll}}(\tau) = 2 \int_{\tau}^1 dx_1 \int_{\frac{\tau}{x_1}}^1 dx_2 \hat{\sigma}^{5F} \left(\frac{\tau}{x_1 x_2} \right) \int_{x_1}^1 \frac{dz_1}{z_1} \left[\frac{\alpha_s}{2\pi} P_{qg}(z_1) L(z_1, z_1 z_2) \right] g \left(\frac{x_1}{z_1}, \mu_F^2 \right) \int_{x_2}^1 \frac{dz_2}{z_2} \left[\frac{\alpha_s}{2\pi} P_{qg}(z_2) L(z_2, z_1 z_2) \right] g \left(\frac{x_2}{z_2}, \mu_F^2 \right)$$

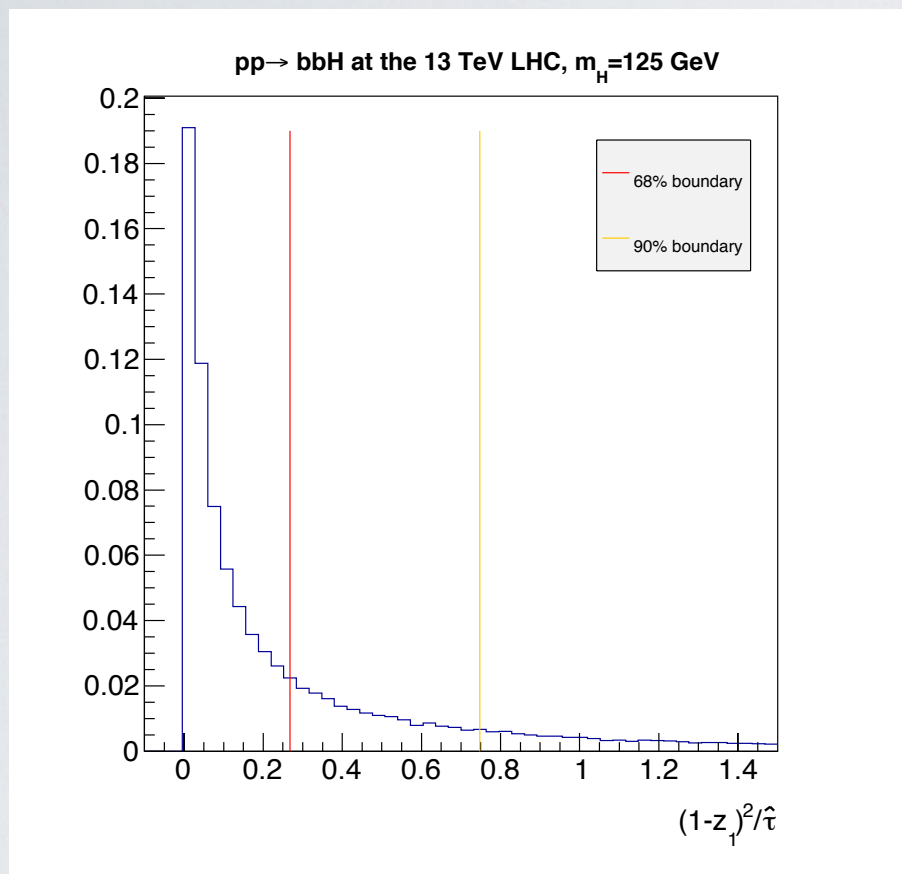


$$L(z, \hat{\tau}) = \log \frac{M_H^2}{m_b^2} \frac{(1-z)^2}{\hat{\tau}} \longleftrightarrow L_b = \log \frac{\mu_F^2}{m_b^2}$$

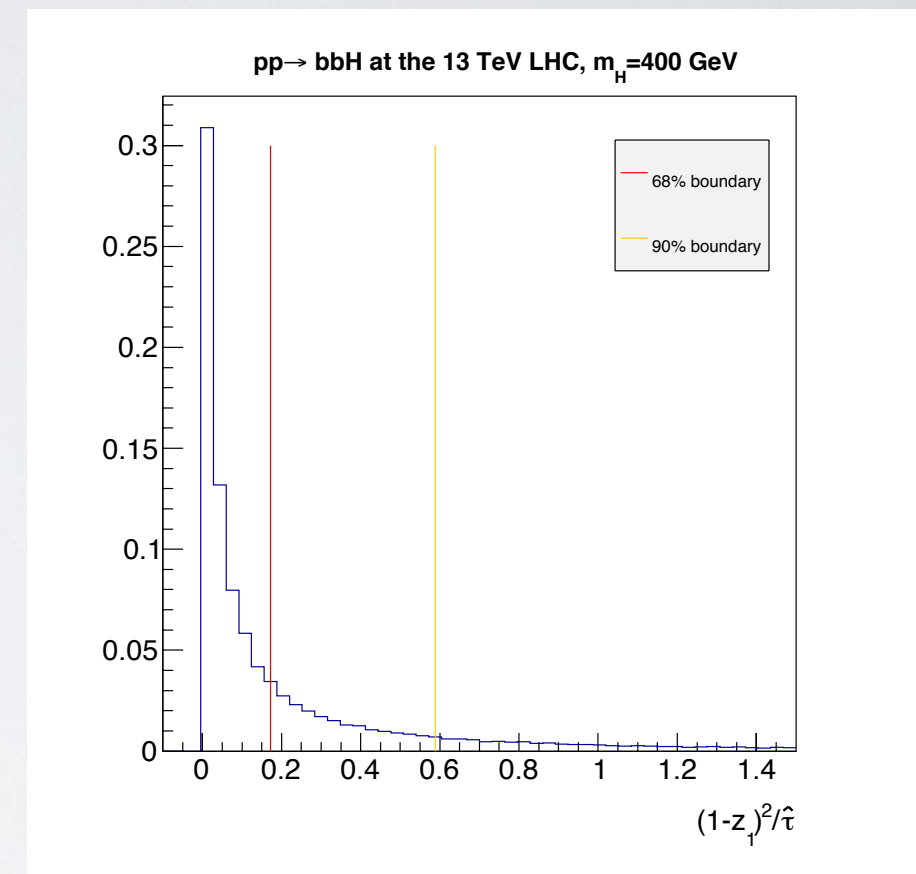
$$\tilde{\sigma}^{5F}(\tau) = 2 \int_{\tau}^1 dx_1 \int_{\frac{\tau}{x_1}}^1 dx_2 \hat{\sigma}^{5F} \left(\frac{\tau}{x_1 x_2} \right) \int_{x_1}^1 \frac{dy}{y} \left[\frac{\alpha_s}{2\pi} P_{qg}(y) L_b \right] g \left(\frac{x_1}{y}, \mu_F^2 \right) \int_{x_2}^1 \frac{dz}{z} \left[\frac{\alpha_s}{2\pi} P_{qg}(z) L_b \right] g \left(\frac{x_2}{z}, \mu_F^2 \right)$$



COMPARISON 4F/5F



$$L(z, \hat{\tau}) = \log \frac{M_H^2}{m_b^2} \frac{(1-z)^2}{\hat{\tau}}$$



$$\tilde{\sigma}^{5F}(\tau) = \sigma^{4F, \text{coll}}(\tau) \Rightarrow \tilde{\mu}_F$$

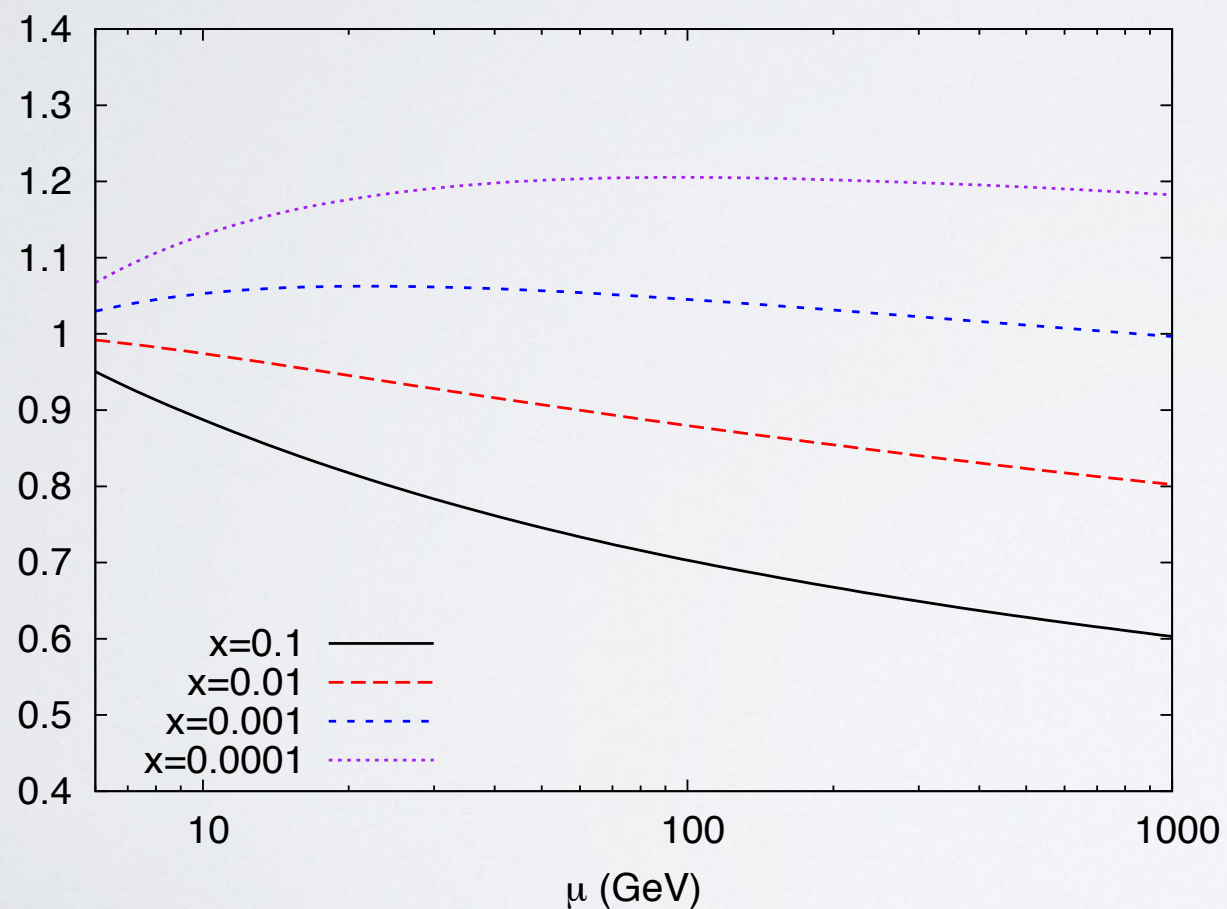
$$\begin{aligned} b\bar{b}H, M_H = 125 \text{ GeV} : \tilde{\mu}_F &\approx 0.36 M_H \\ b\bar{b}Z', M_{Z'} = 91.2 \text{ GeV} : \tilde{\mu}_F &\approx 0.38 M_{Z'} \\ b\bar{b}Z', M_{Z'} = 400 \text{ GeV} : \tilde{\mu}_F &\approx 0.29 M_{Z'}, \end{aligned}$$

COMPARISON: SUMMARY

- The $(1-z)^2/\tau$ kinematic factor leads to a suppression at the LHC. Analogous to the $(1-z)^2/z$ term found for the one-heavy-quark-initiated processes.
- The effective scale $\widetilde{\mu}_F$ is typically a fraction of the naive scale of the process M_h .

IMPACT OF RESUMMATION

$$\tilde{b}^{(1)}(x, \mu^2) = \frac{\alpha_S}{2\pi} \log \frac{\mu^2}{m_b^2} \int_x^1 \frac{dz}{z} P_{qg}(z) g\left(\frac{x}{z}, \mu^2\right) / b^{(1)}(x, \mu^2)$$



Comparison between the first log which the one included in the LO 4F calculation of single-top, and the full resummed result given by the AP equations.

The various curves correspond to different Bjorken x's.

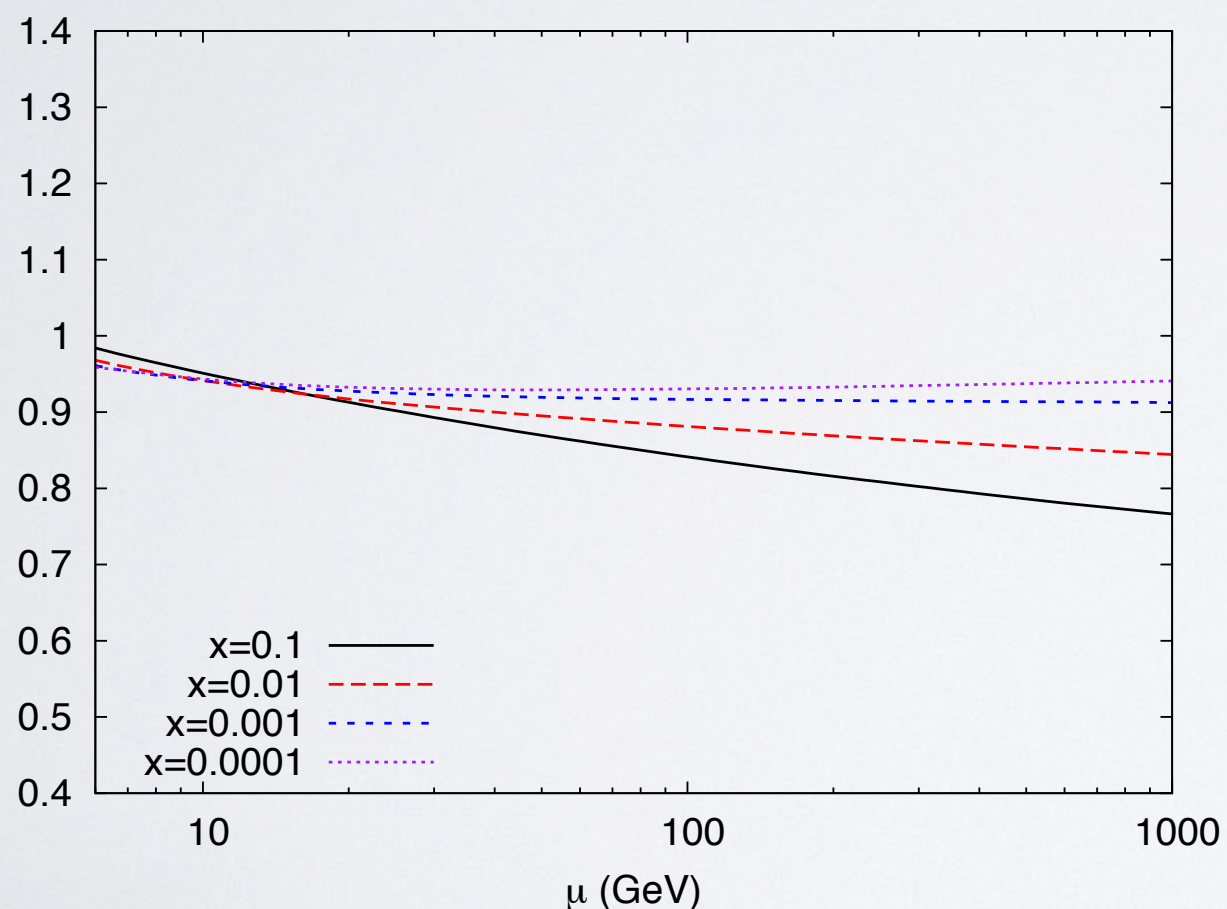
At small x the effect is positive, in other words \tilde{b} is a kind of bad overestimate.

At large x resummation effects are manifest.

LO approximation does not look good enough.

IMPACT OF RESUMMATION

$$\tilde{b}^{(2)}(x, \mu^2) = \int_x^1 \frac{dz}{z} \Sigma^{4F,(2)}\left(\frac{x}{z}, \mu^2\right) \left(\frac{\alpha_S}{4\pi}\right)^2 a_{\Sigma,b}^{(2)}(z, \mu^2/m_b^2) \\ + \int_x^1 \frac{dz}{z} g^{4F,(2)}\left(\frac{x}{z}, \mu^2\right) \left[\left(\frac{\alpha_S}{4\pi}\right) a_{g,b}^{(1)}(z, \mu^2/m_b^2) + \left(\frac{\alpha_S}{4\pi}\right)^2 a_{g,b}^{(2)}(z, \mu^2/m_b^2) \right] / b^{(2)}(x, \mu^2)$$



Comparison between the first $\log^2 + \log$ which are included in the NLO 4F calculation of single-top, and the full resummed result given by the AP equations at NLO.

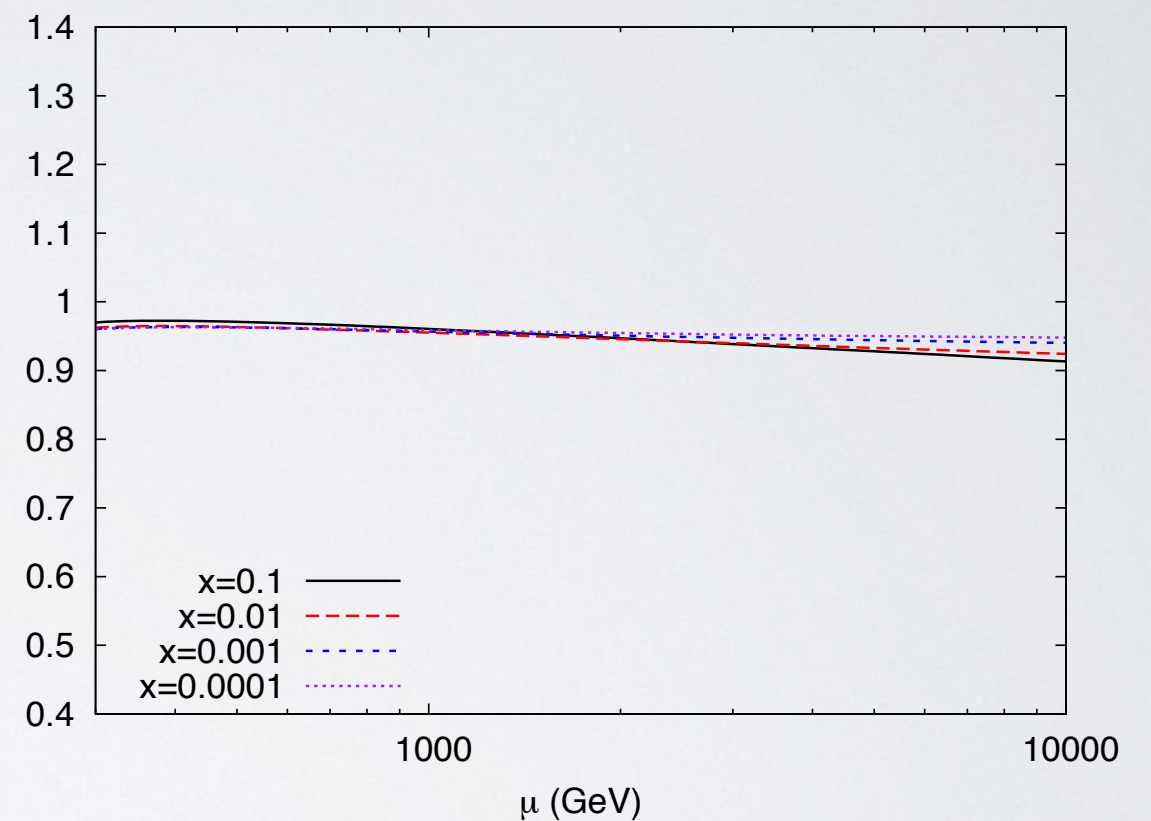
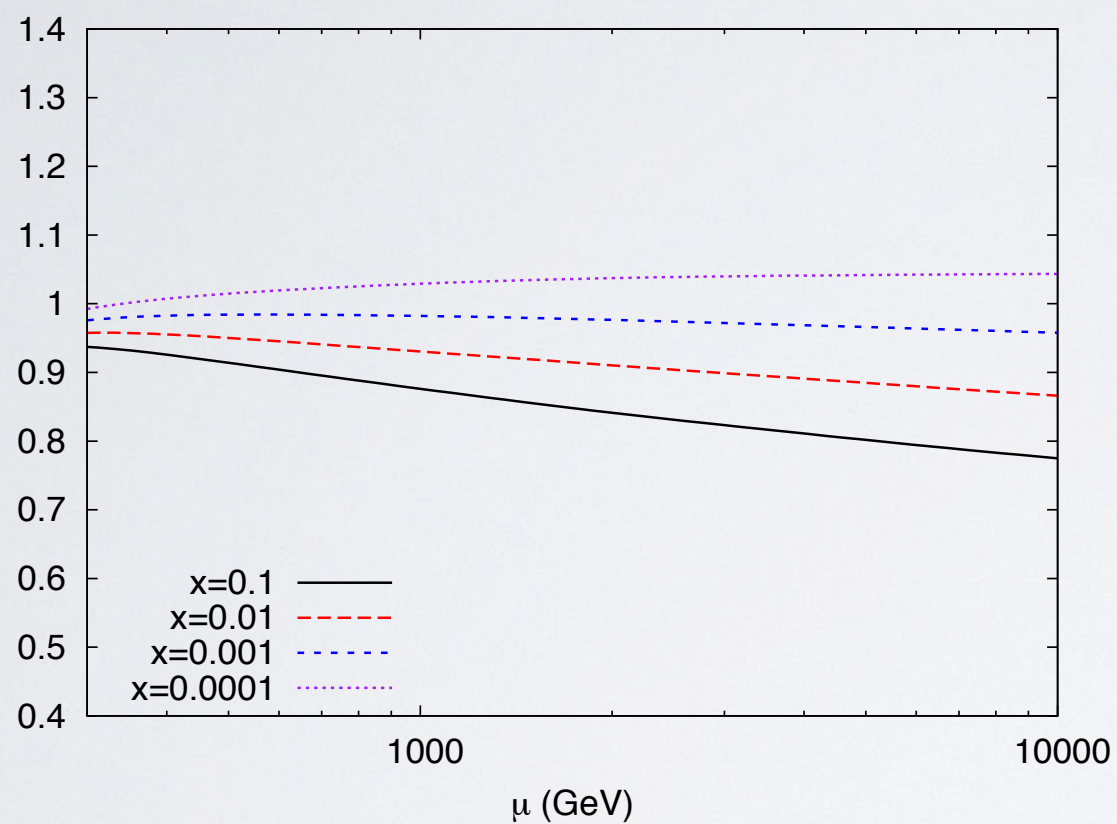
The various curves correspond to different Bjorken x 's.

At small x also now the resummation is visible yet is very small.

At large x resummation effects are manifest.

NLO approximation does look reasonably behaved.

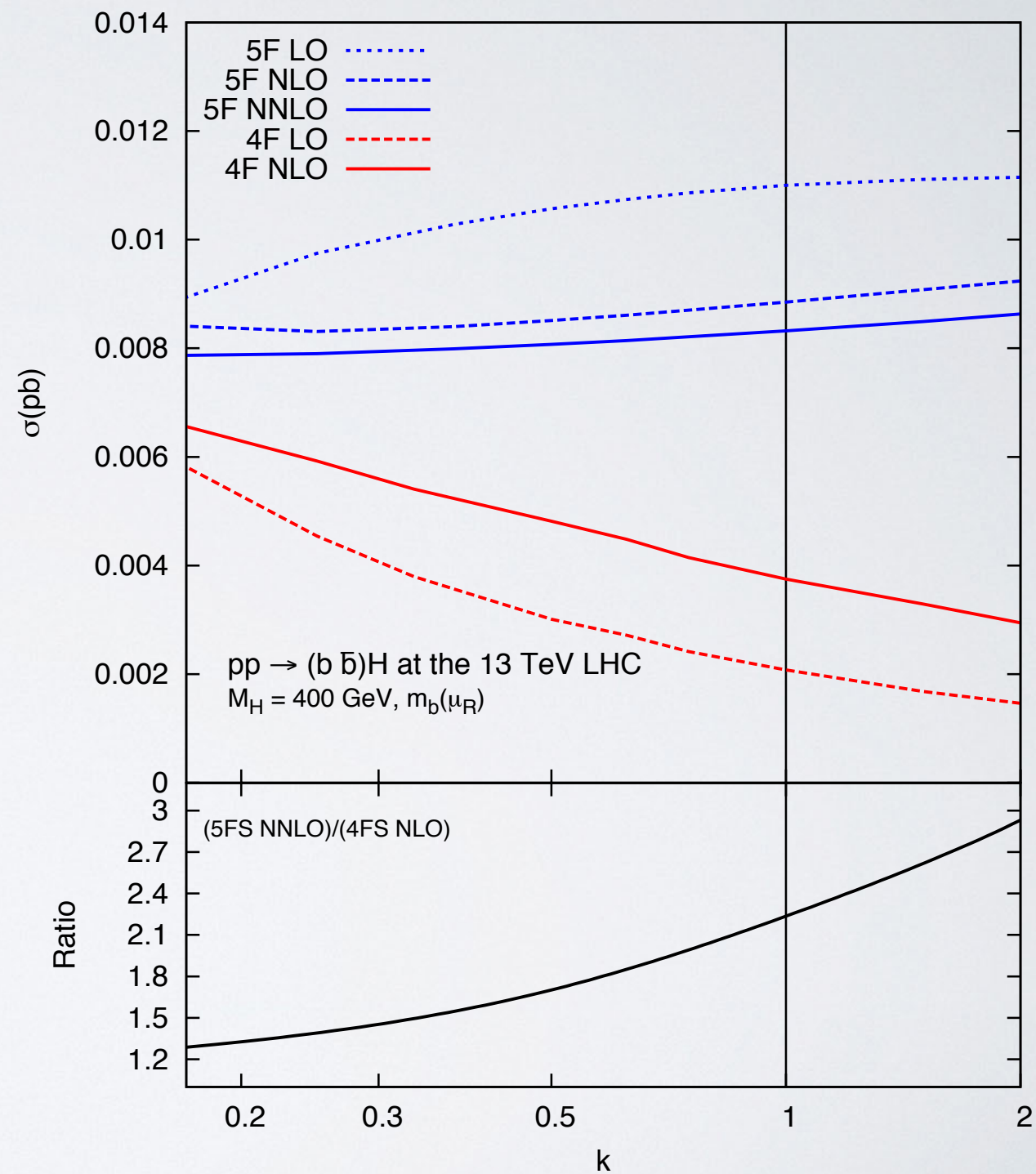
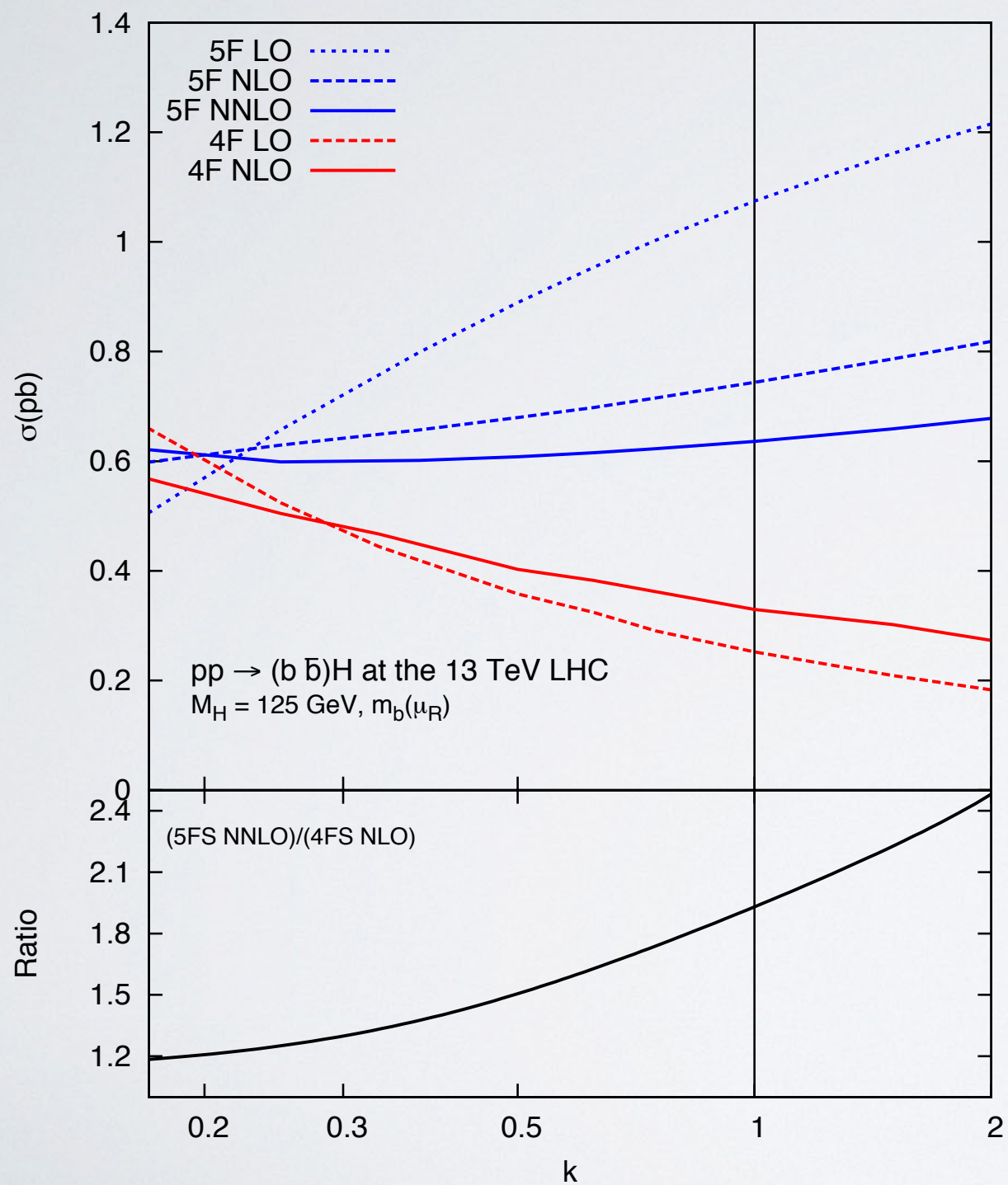
IMPACT OF RESUMMATION



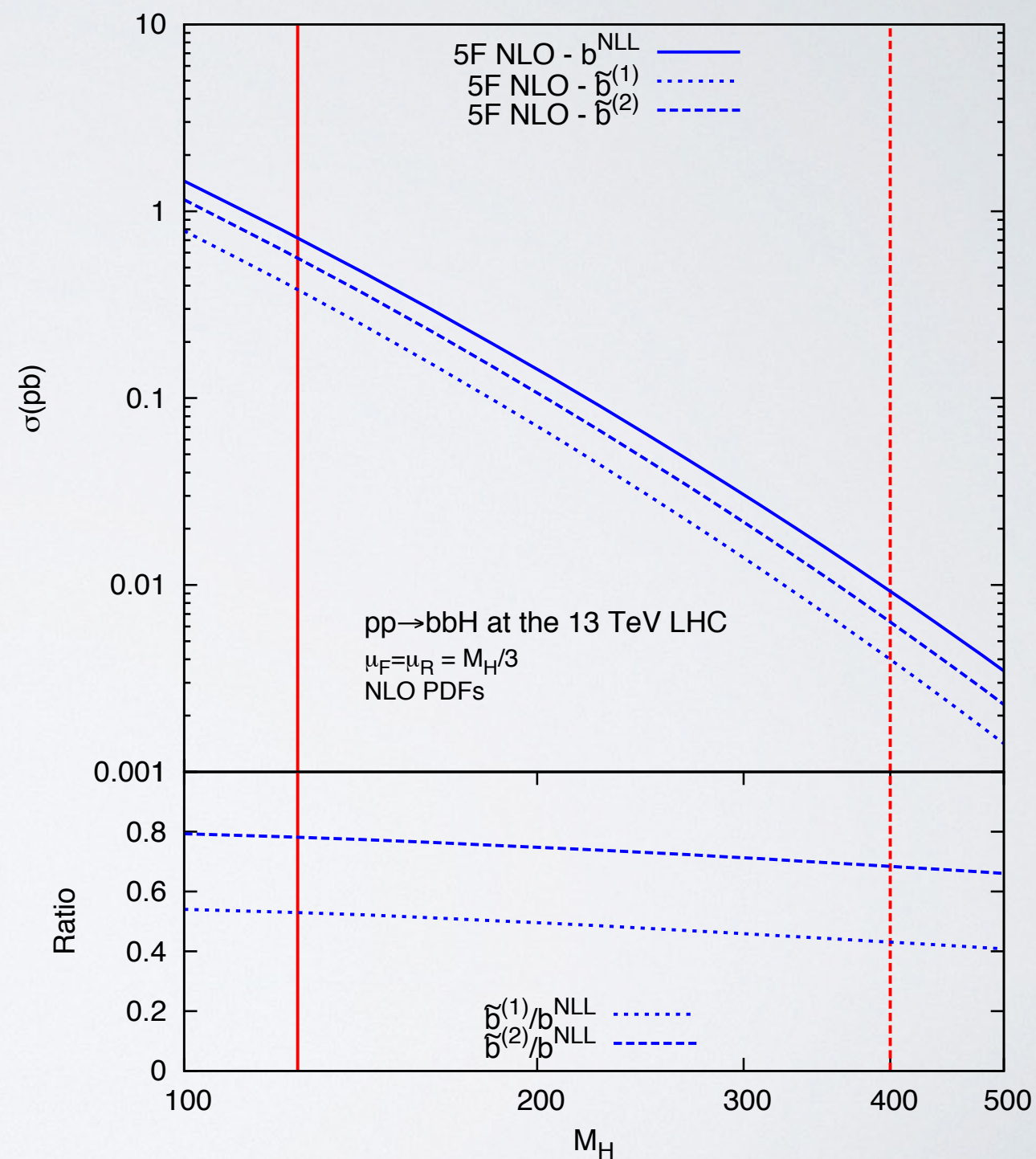
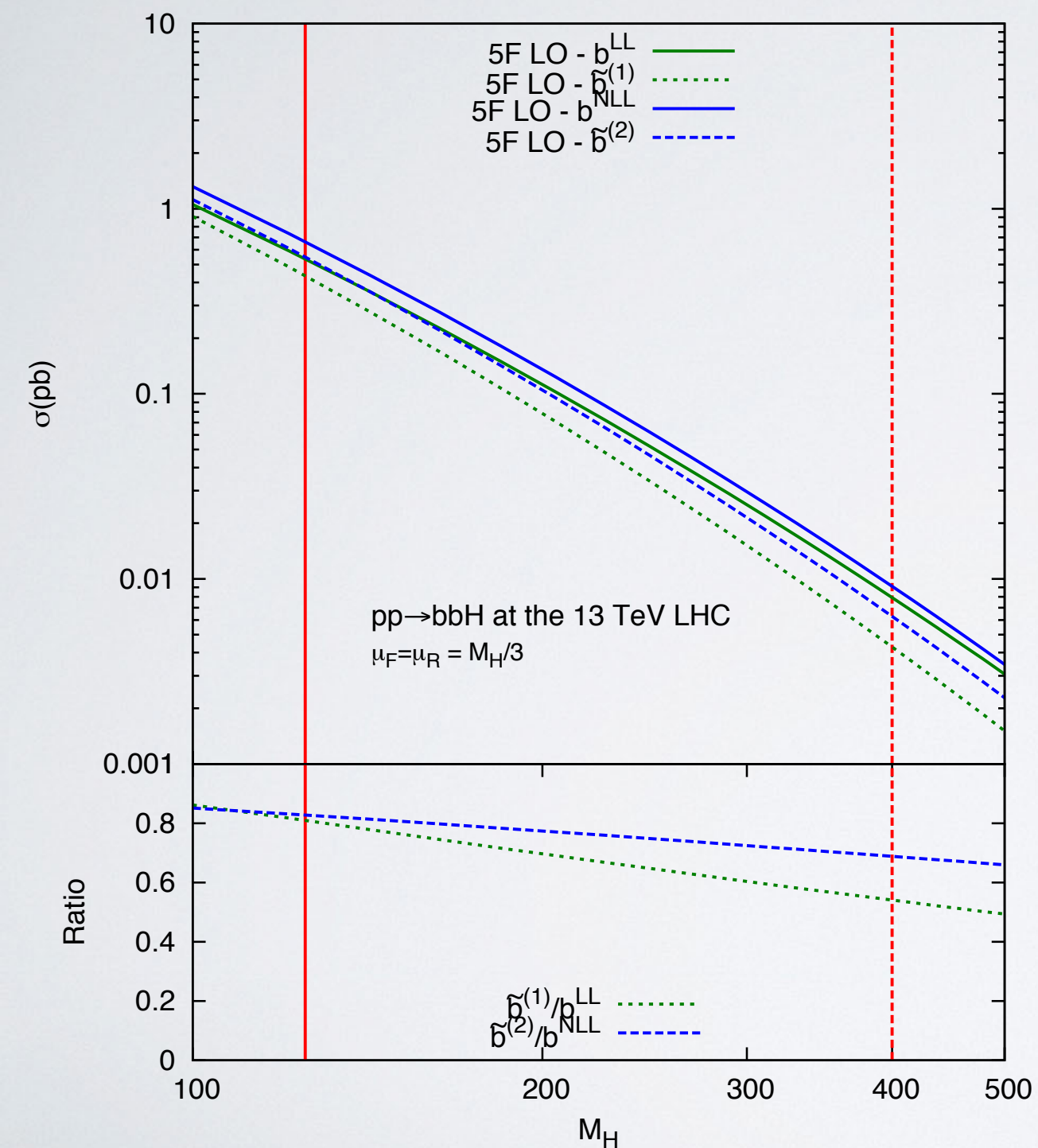
top quark pdf's. left : LO , right : NLO

APPLICATIONS AT THE LHC

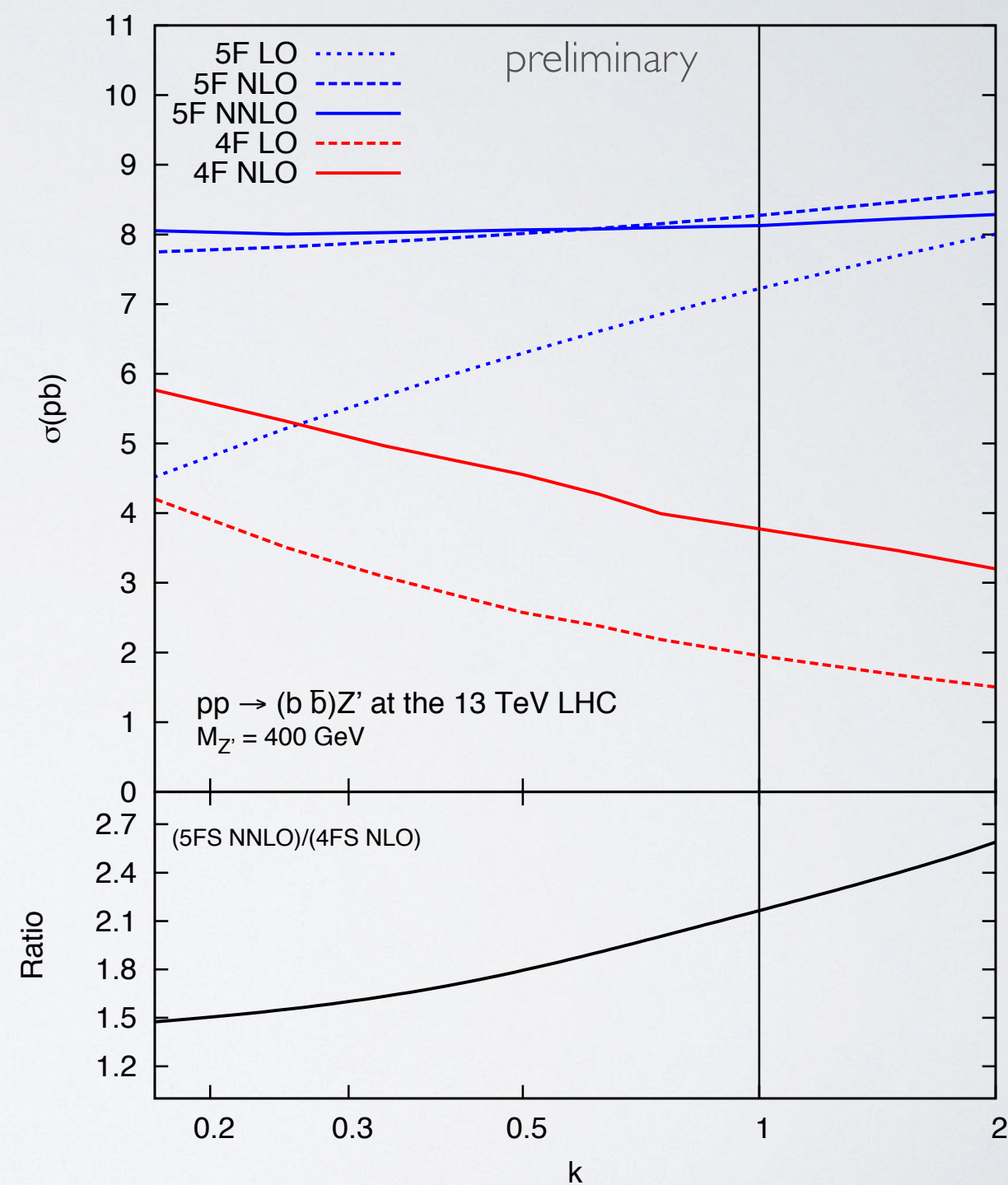
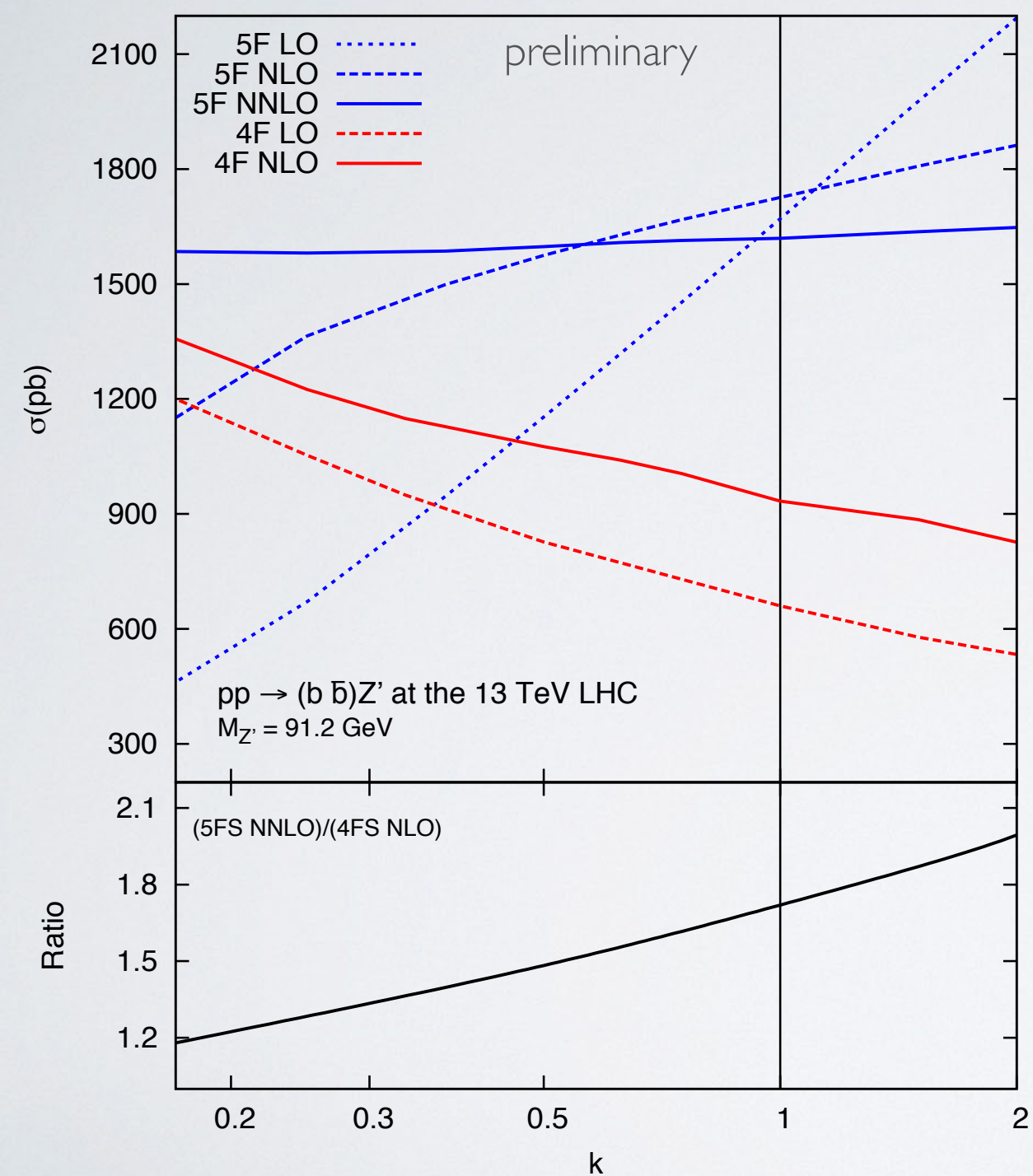
$PP \rightarrow HBB$



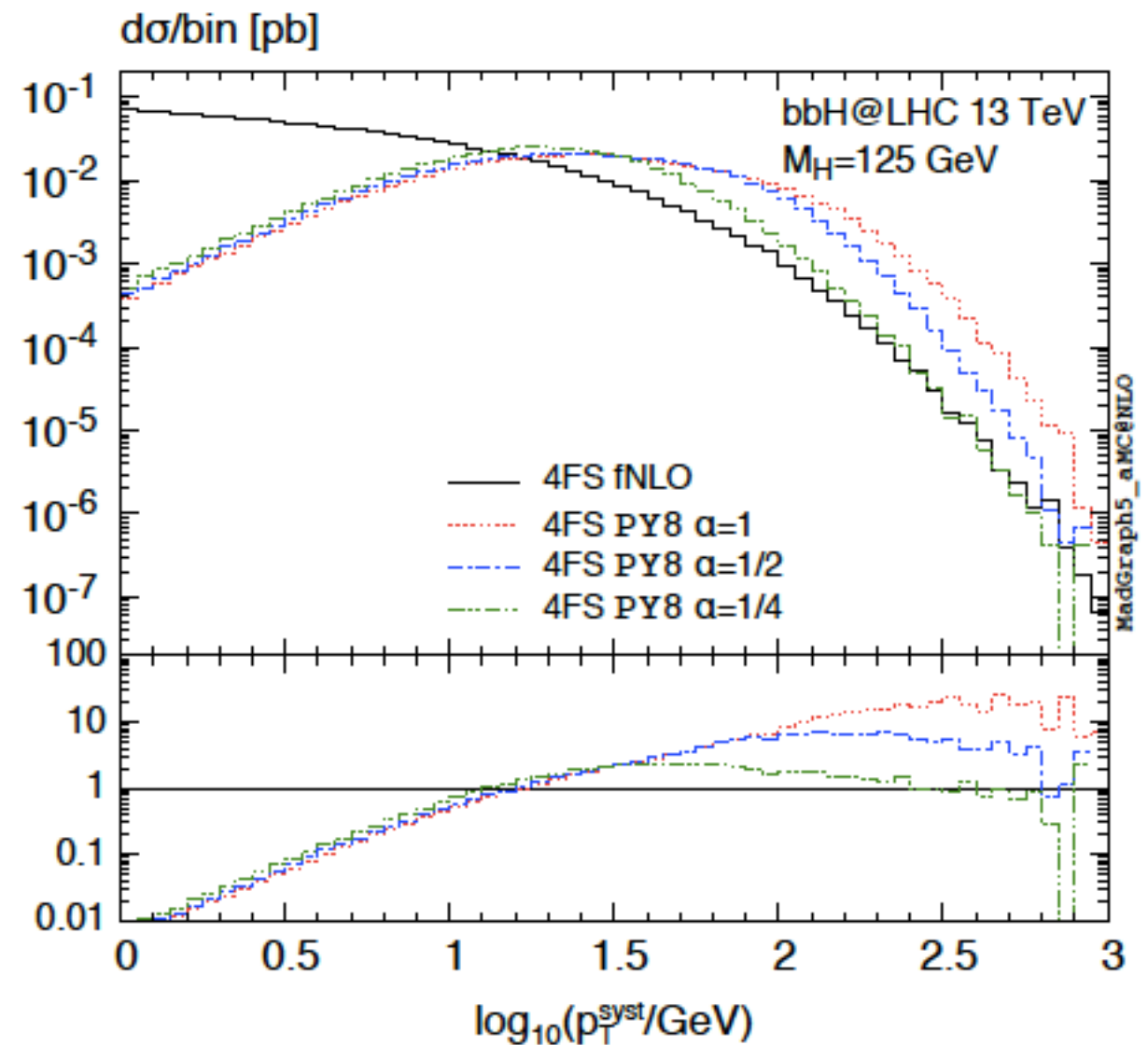
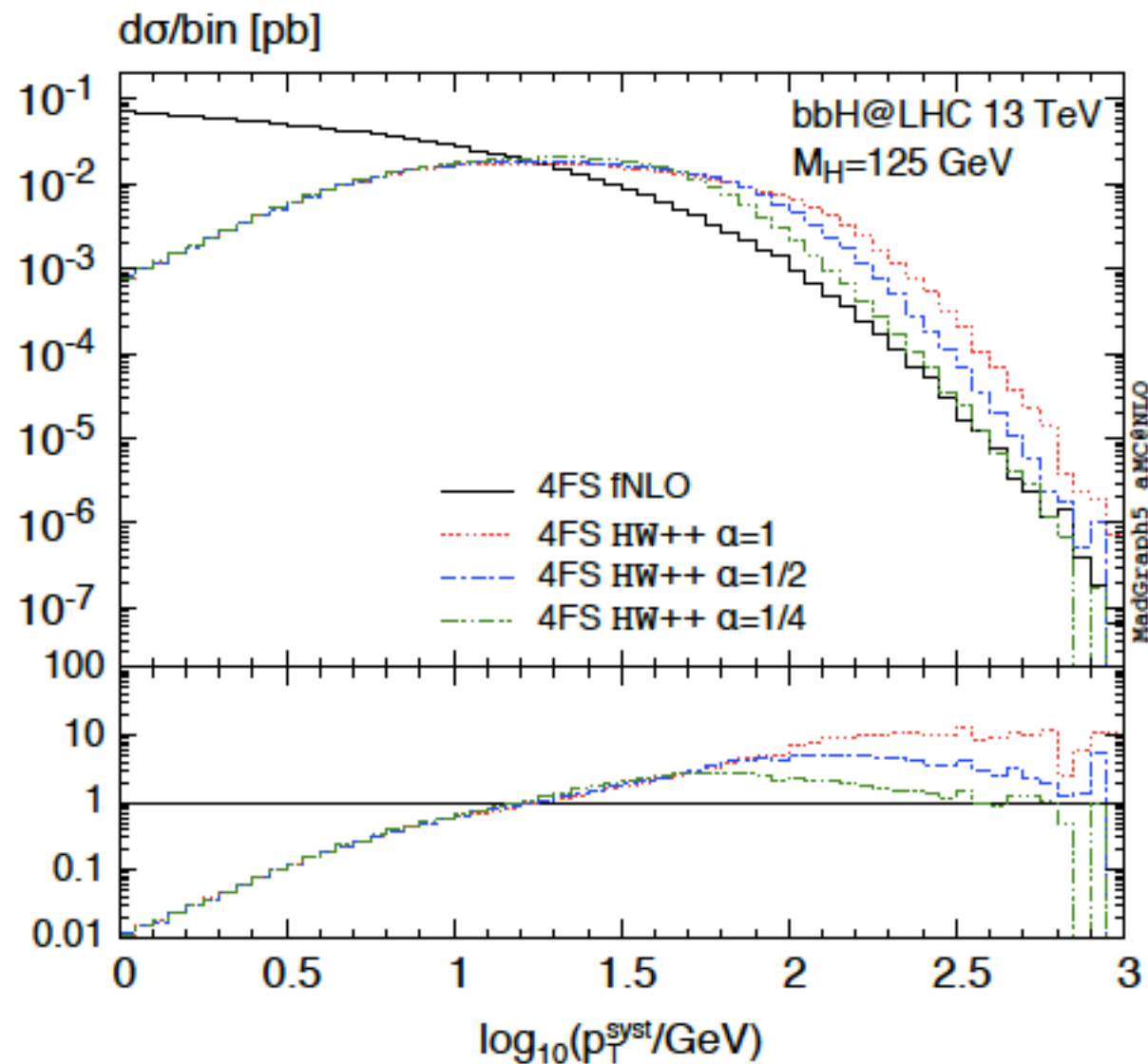
$pp \rightarrow HBB$



$PP \rightarrow ZBB$



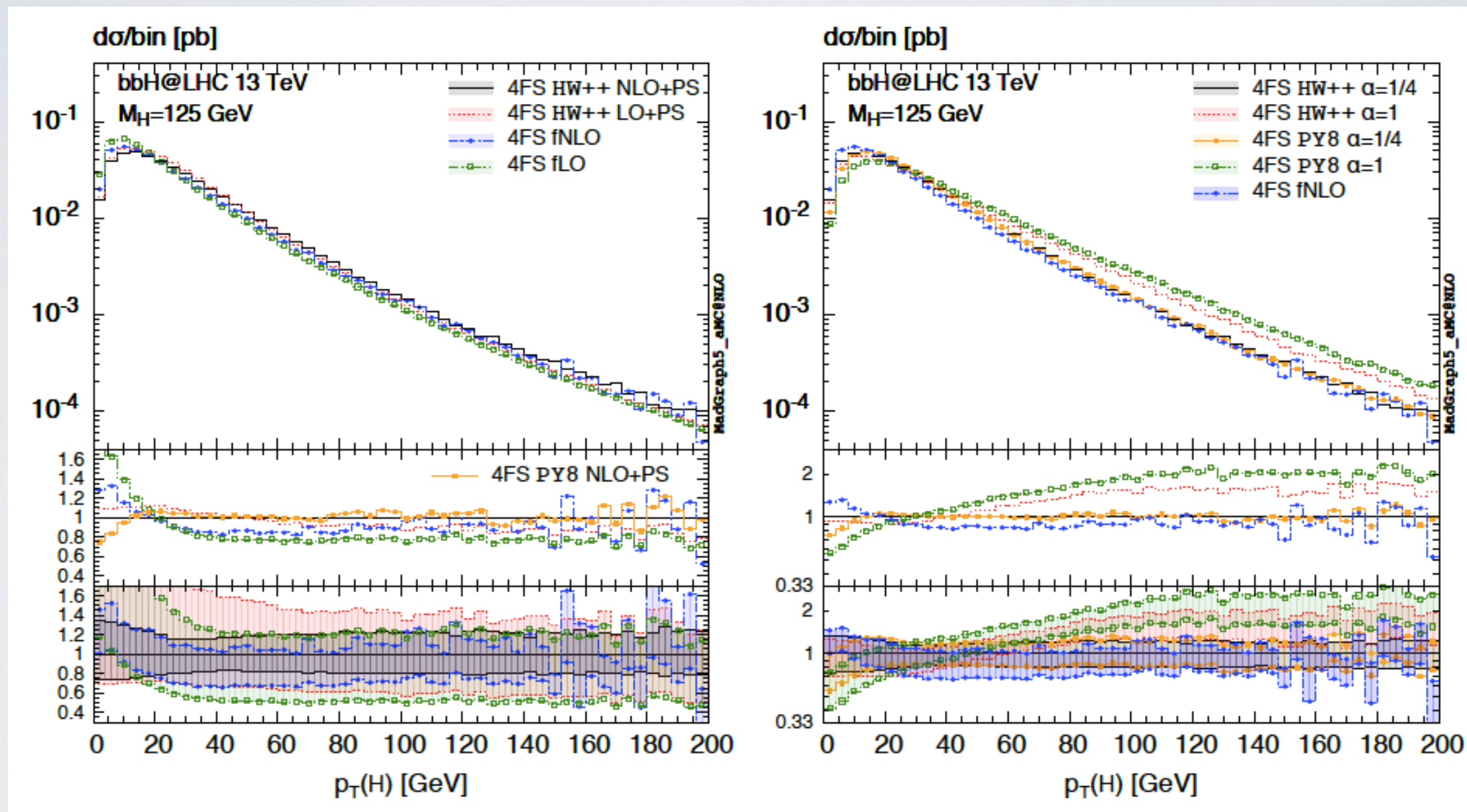
PP→BBH : MC SIMULATIONS



Choosing the range of the shower scale

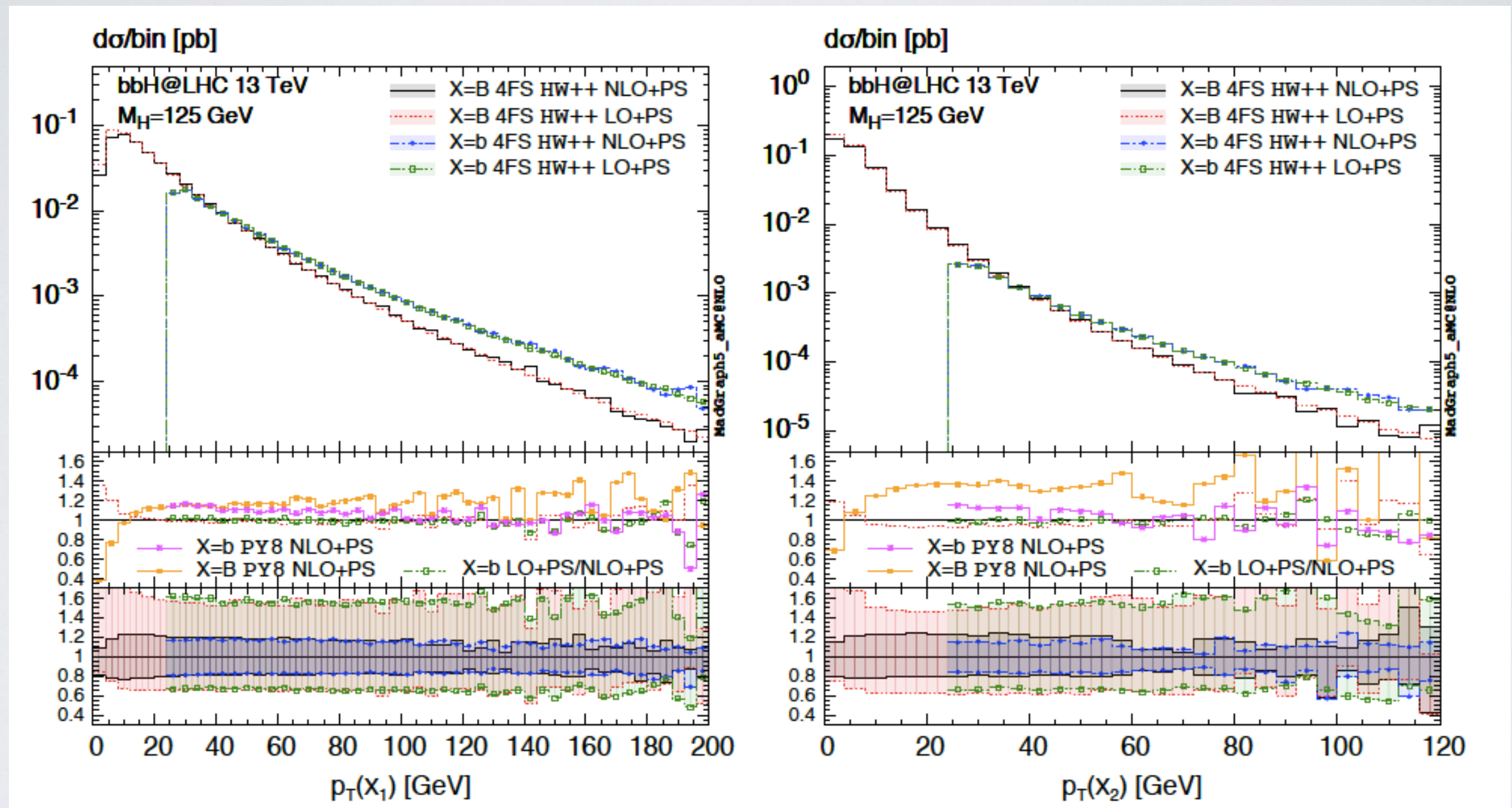
$$\alpha f_1 \sqrt{s_0} \leq Q_{\text{sh}} \leq \alpha f_2 \sqrt{s_0}$$

PP→BBH : MC SIMULATIONS



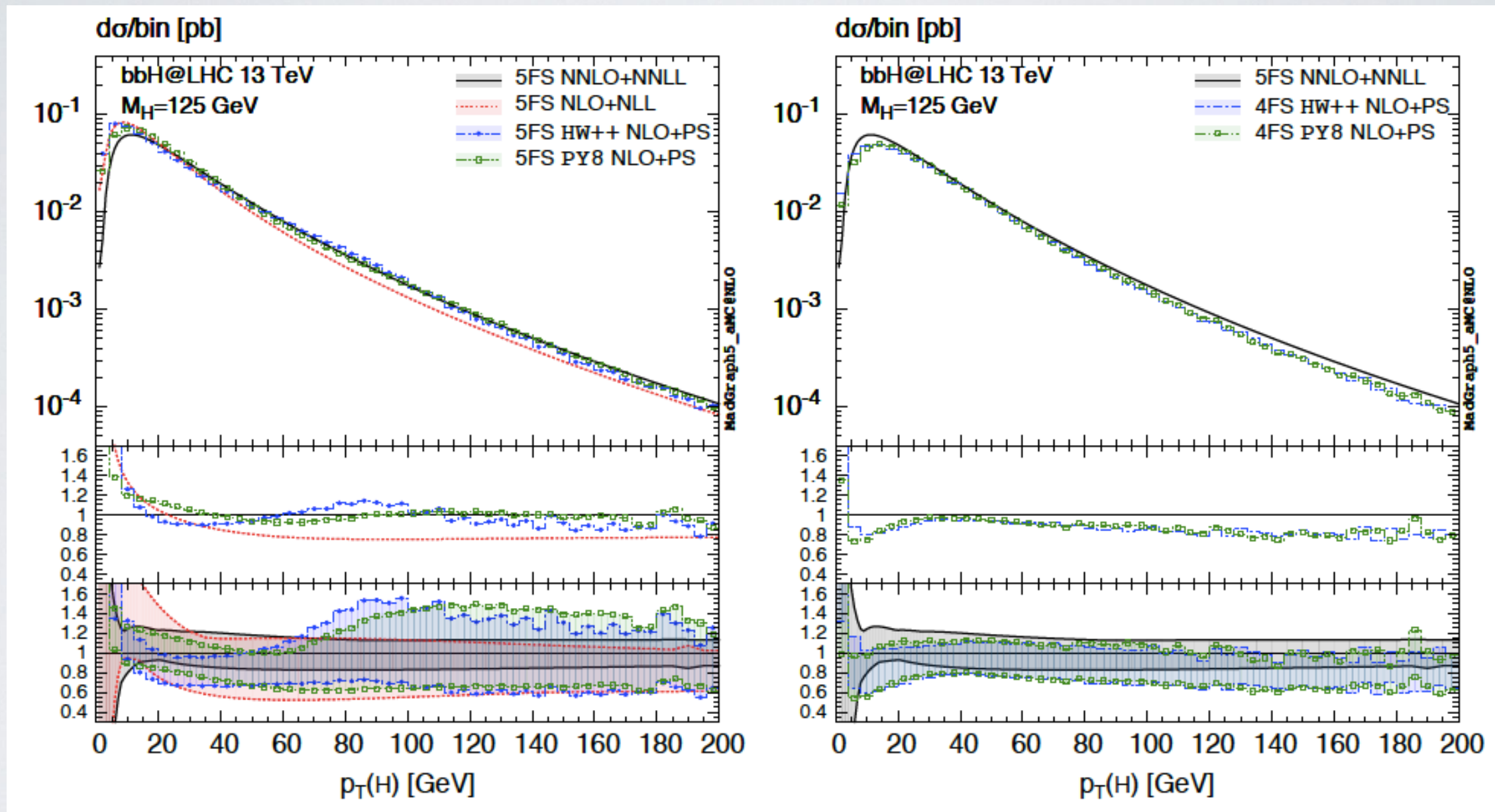
Transverse momentum of the Higgs : comparison with different shower scales

PP→BBH : MC SIMULATIONS



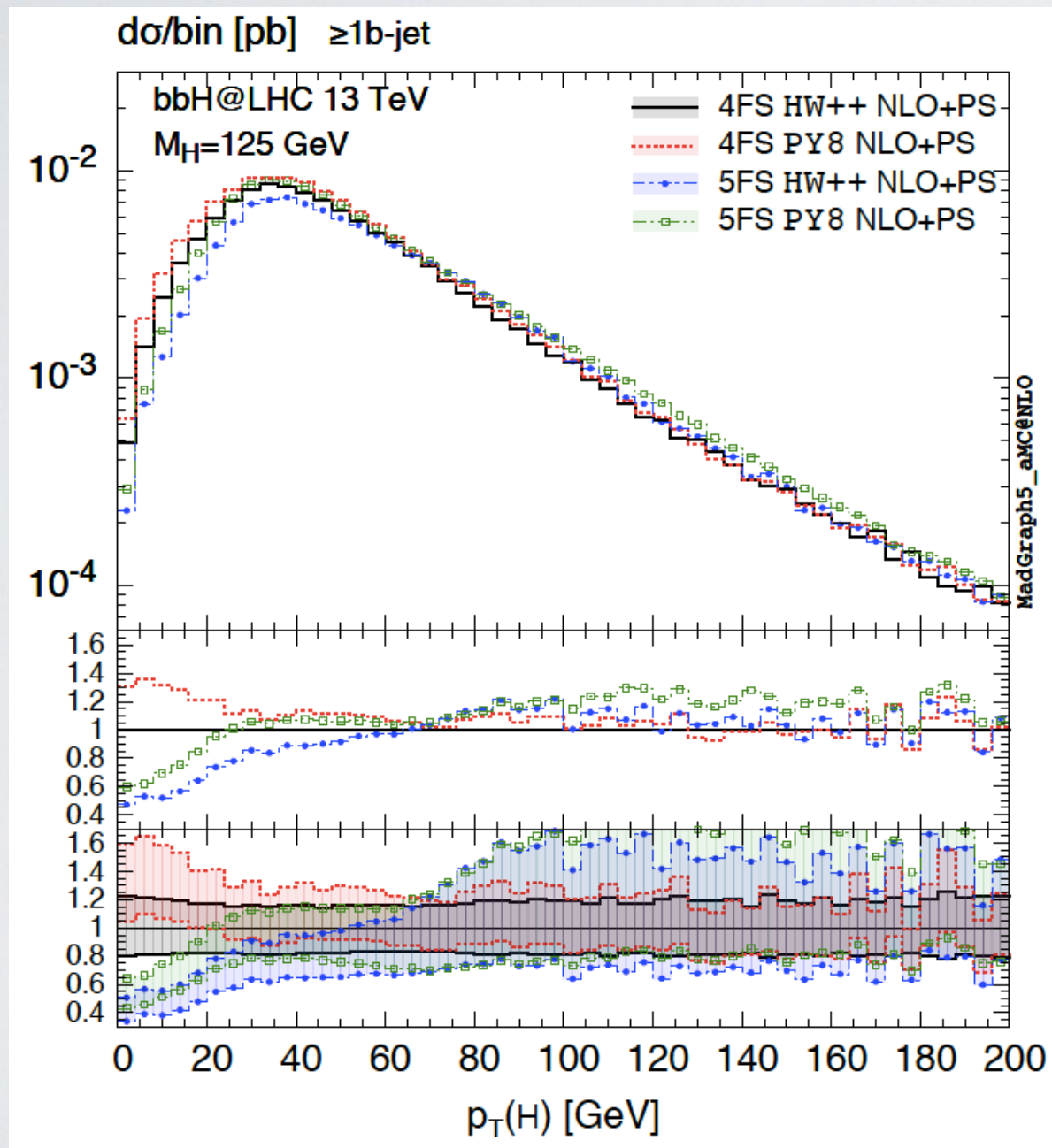
Transverse momenta of the b-mesons or b-jets with HW++.

PP→BBH : MC SIMULATIONS



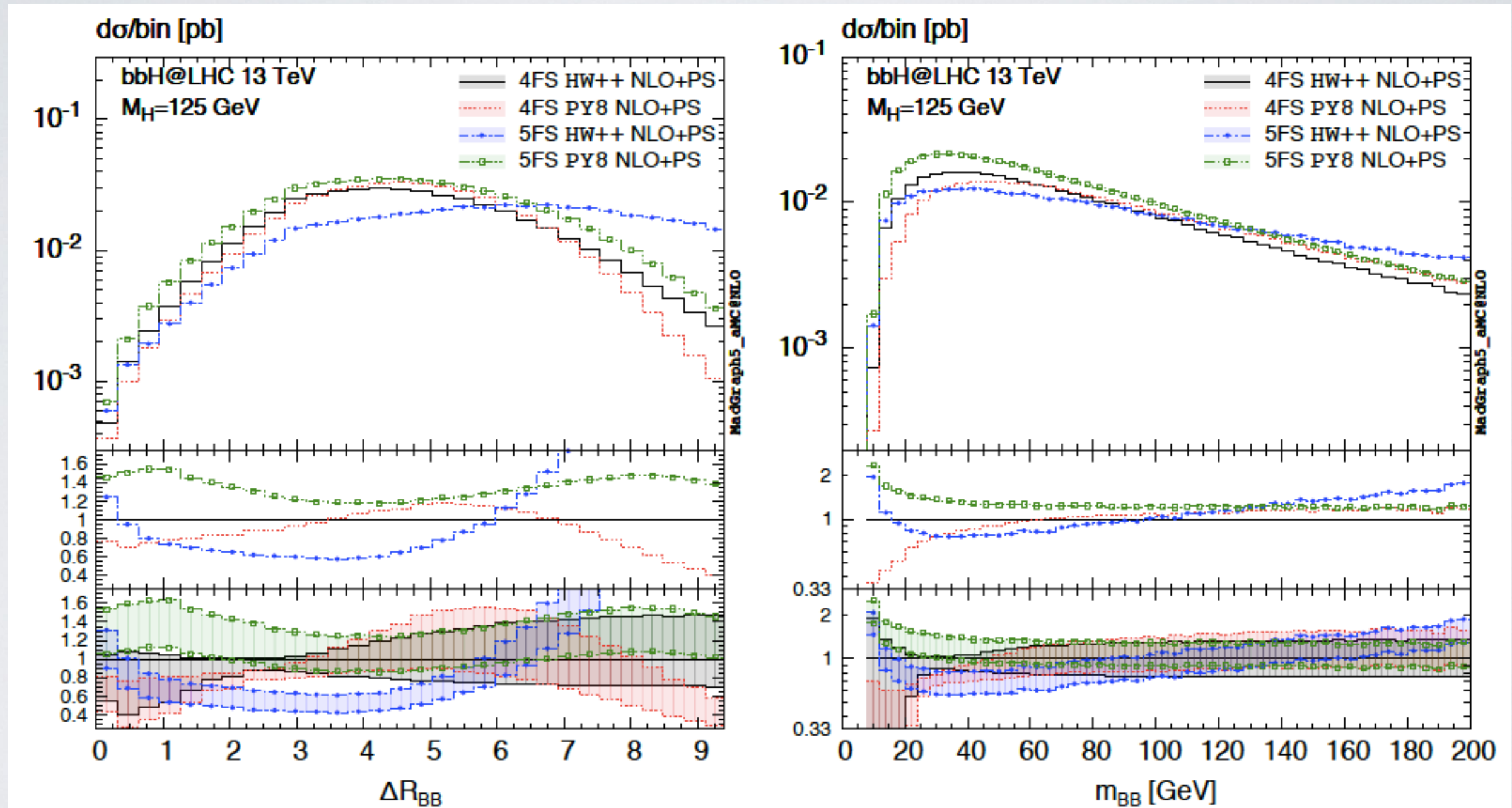
Transverse momentum of the Higgs : comparison with the analytic result

PP→BBH : MC SIMULATIONS



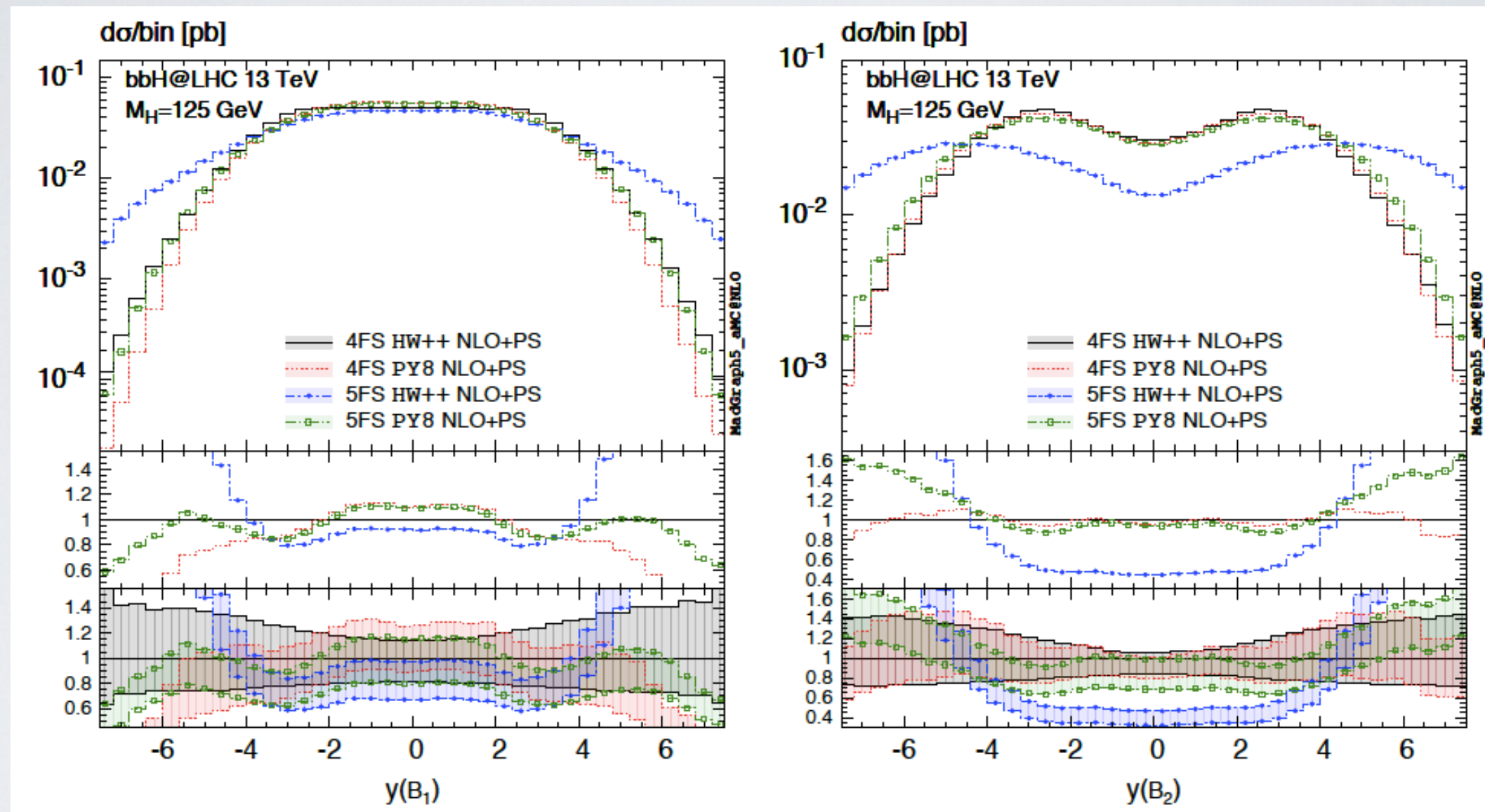
Transverse momentum of the Higgs
when a b-jet is required.

PP → BBH : MC SIMULATIONS



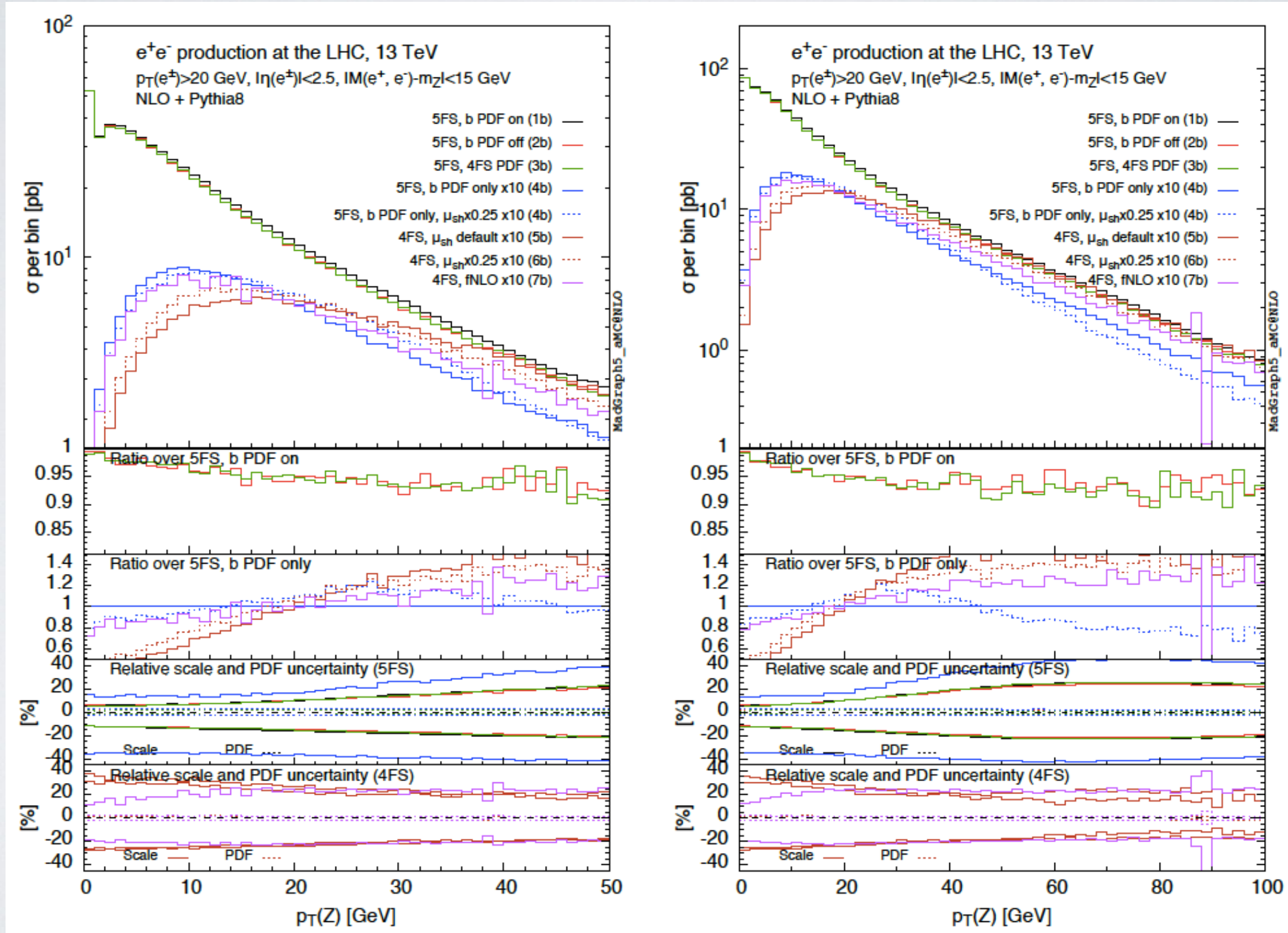
Characteristics of the BB pair : distance and invariant mass

PP→BBH : MC SIMULATIONS



Normalised distributions in rapidity of the first and second B meson.

PP → BBZ : MC SIMULATIONS



FINAL REMARKS

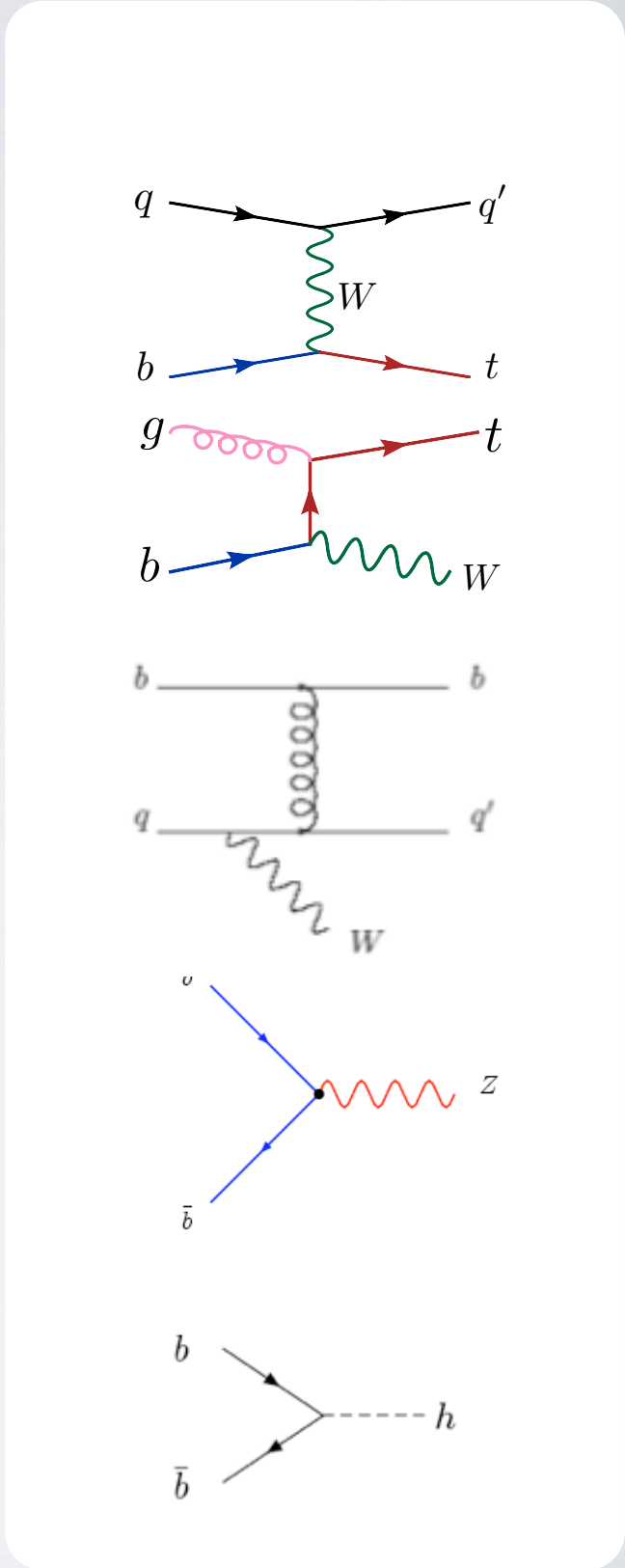
- We have conducted a detailed analytic and numerical study of double-heavy-quark initiated processes and confirmed the picture that raised in the first study on single-heavy-quark processes.
- 5F calculations at NNLO and mass improved ones, clearly give the most stable and reliable predictions for total cross sections and should be certainly used for normalisation. Differences between 4F and 5F are $O(\sqrt{s} * 10\%)$ for not too heavy states. In particular, 5F calculations should be used for processes involving very high Bjorken- x .
- Behaviour of the 4F NLO+PS predictions consistent with the analysis of the logs in the FO calculations. 4F NLO+PS simulations provide reliable predictions.
- More studies of final state gluon splitting at high p_T , and in particular 4F vs 5F approaches in the context of ME+PS merging and NLO+PS are urgently needed.



EXTRA SLIDES

B-INITIATED PROCESSES AT THE LHC

Class	Process	Interest
Top	$qb \rightarrow tq$ (t-channel)	SM, top EW couplings and polarization, V_{tb} . Anomalous couplings. H^+ : SUSY, 2HDM
	$gb \rightarrow t(W, H^+)$	
Vector Bosons	$pp \rightarrow Wb$ $pp \rightarrow Wbj$	SM, bkg to single top
	$bb \rightarrow Z$ $gb \rightarrow Zb$ $pp \rightarrow Zbj$	Standard candle: SM BSM bkg, b-pdf
	$gb \rightarrow \text{gamma} + b$	
Higgs	$bb \rightarrow (h, A)$ $gb \rightarrow (h, A) + b$	SUSY discovery/ measurements at large $\tan(\beta)$

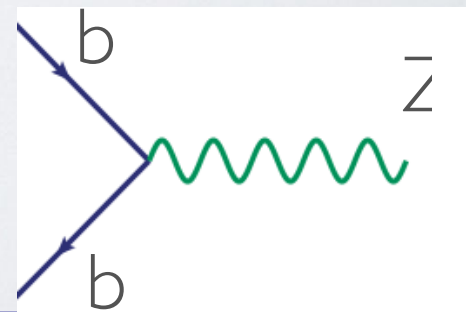


BOTTOM QUARKS AT THE LHC

- At face value these logs might be large, possibly spoiling perturbation theory

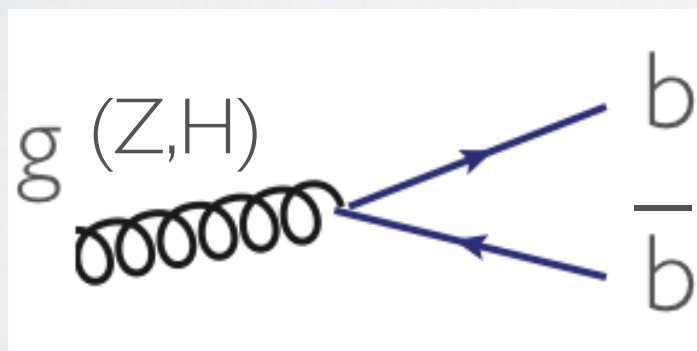
$$\alpha_S(\hat{s}) \log \frac{\hat{s}}{m_b^2} \sim \frac{1}{\log \frac{\hat{s}}{\Lambda_{QCD}^2}} \log \frac{\hat{s}}{m_b^2} \simeq 1$$

- Such worries can be lifted, by defining an 5 flavor QCD effective field theory where the effects of such logs are resummed using DGLAP equations into fragmentation function and b-pdf's, in the final and initial state respectively.
- This leads to the widely employed, yet sometimes confusing, concept of initial-state b's, i.e. b's considered as partons in the protons, on the same ground as u,d,s,(and c).
- So now $pp \rightarrow Z(bb)$ is seen as $2 \rightarrow 1$ process, much SIMPLER to calculate and also (apparently) more accurate than the $2 \rightarrow 3$ process in the 4F.

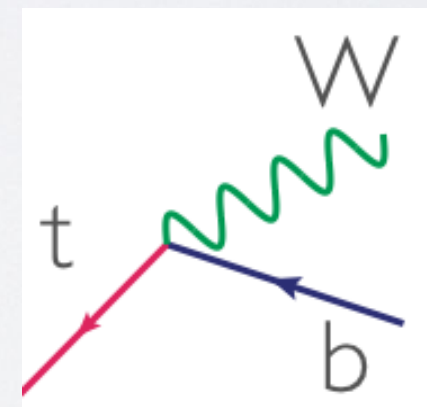


BOTTOM QUARKS AT THE LHC

- b quark phenomenology plays a key role at the LHC, from flavor (B mesons) to Higgs searches and measurements, and as a window to New Physics.
- The b is the only quark for which $\Lambda_{\text{QCD}} < m_Q \ll v$ ($=m_W, m_Z, m_h, m_t$).
- Understanding their production is a necessary ingredient to make accurate predictions for signals and backgrounds.
- Bottom quarks can enter in processes at the LHC in two main ways:



$$\Delta B=0$$

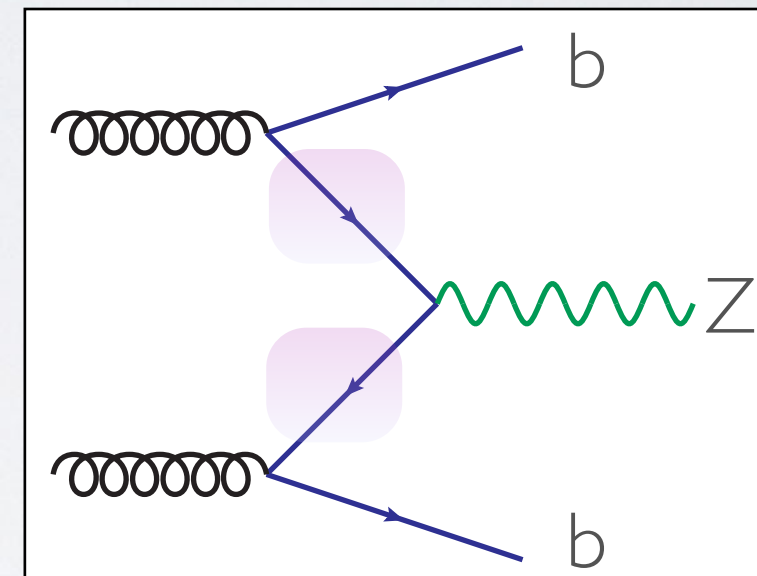
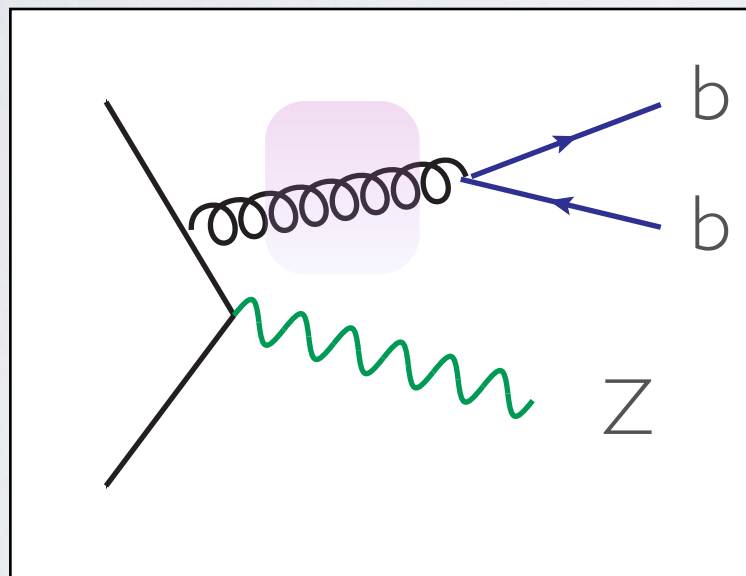


$$\Delta B=\pm 1$$

strong production (let us call it “gluon splitting”) being the dominant one.

BOTTOM QUARKS AT THE LHC

Now, gluon splitting can take place in a s-channel kinematics (in the final state) or in a t-channel kinematics (initial state). So take, for example, $pp \rightarrow Zbb$ associated production:

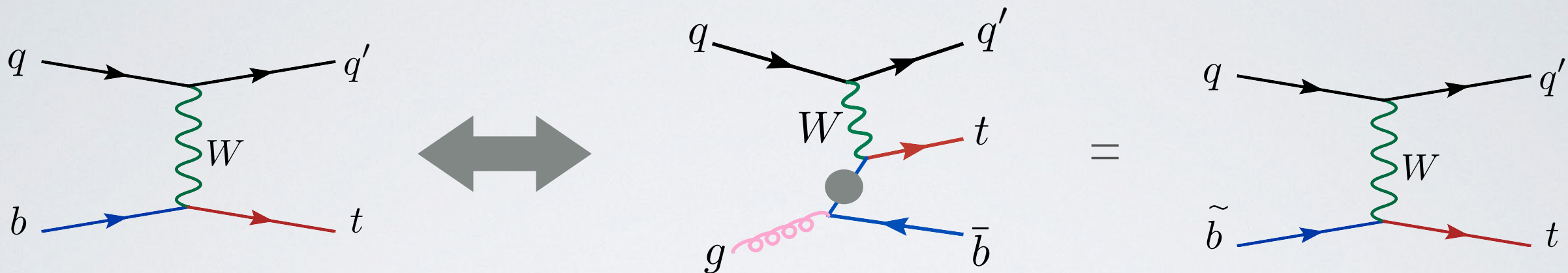


Both possibilities are affected by the same theoretical worries, which are related to the fact that $m_b \ll \sqrt{s}$ -partonic, and therefore one expects:

$$\sigma \sim \alpha_S^2 \log \frac{\hat{s}}{m_b^2}$$

$$\sigma \sim \alpha_S^2 \log^2 \frac{\hat{s}}{m_b^2}$$

IMPACT OF RESUMMATION



b-pdf has all the logs resummed

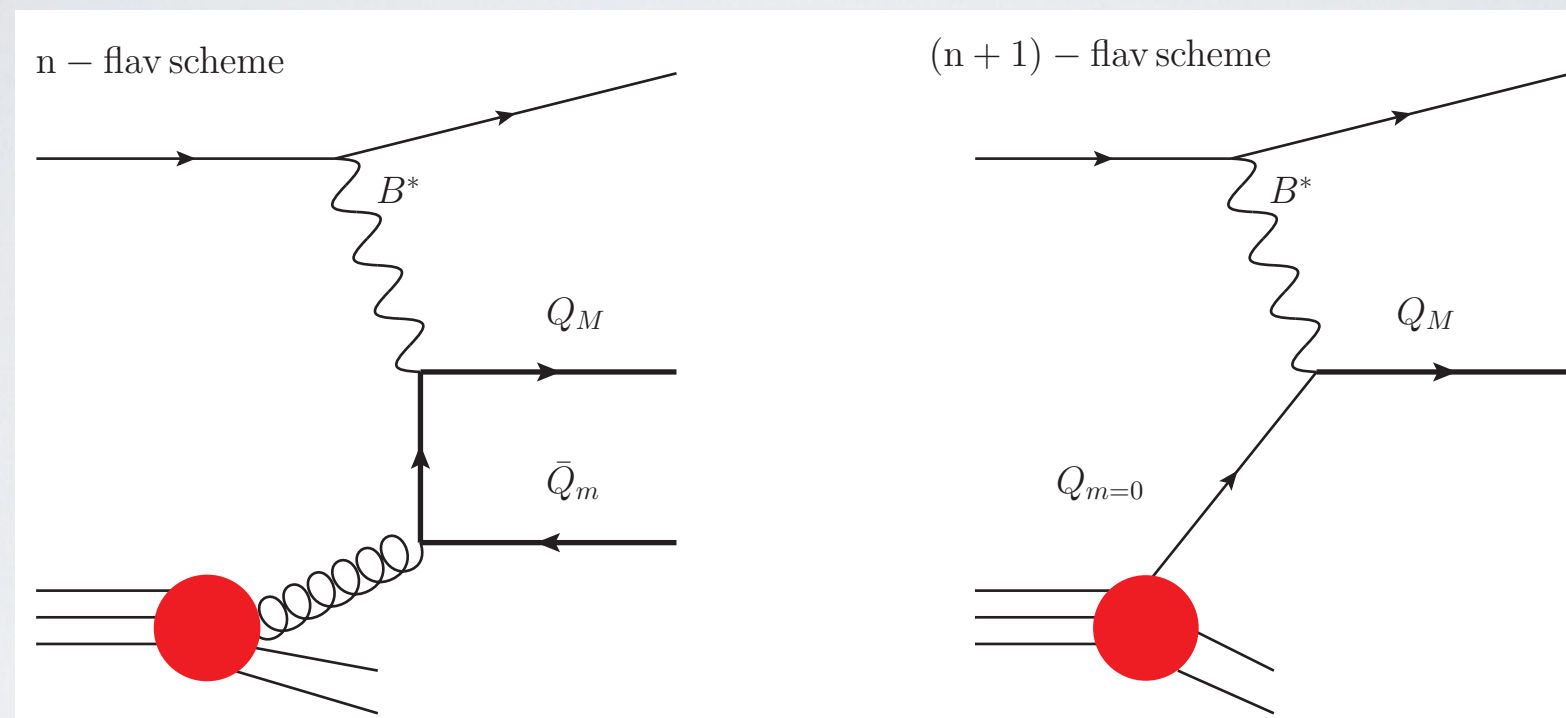
\tilde{b} -pdf has only the first log :

$$\int dx_1 dx_2 q(x_1, \mu_F^2) b(x_1, \mu_F^2) \hat{\sigma}(qb \rightarrow q't) \quad \int dx_1 dx_2 q(x_1, \mu_F^2) \tilde{b}(x_1, \mu_F^2) \hat{\sigma}(qb \rightarrow q't)$$

$$\tilde{b}(x, \mu_F) \sim \frac{\alpha_s}{2\pi} \log \frac{\mu^2}{m_b^2} \int_x^1 \frac{dy}{y} P_{qg} \left(\frac{x}{y} \right) g(y, \mu_f)$$

\tilde{b} is just the first log that one gets from a LO 4F calculation. The b-pdf resums the full tower of such logs that come from higher orders in the 4F calculation.

THE UNIVERSAL LOGS : DIS



$$\sigma_b(\mu^2) = \int_{y_{\min}}^{y_{\max}} dy \int_{Q_{\min}^2}^{Q_{\max}^2} dQ^2 \frac{2\pi\alpha_l\alpha_h}{y(M^2 + Q^2)^2} \left\{ [1 + (1 - y)^2] F_2^b(x, Q^2, m_b^2) \right. \\ \left. - y^2 F_L^b(x, Q^2, m_b^2) + [1 - (1 - y)^2] F_3^b(x, Q^2, m_b^2) \right\}$$

$$y = \frac{Q^2}{xS}$$

THE UNIVERSAL LOGS : DIS

Let's take the expression for the 4F process $\gamma^* + g \rightarrow b + b\bar{b}$ at small t :

$$\frac{d\hat{\sigma}_2}{dt} = \frac{\pi\alpha_e e_b^2 \alpha_S C_F}{16} \left[-\frac{4z}{Q^2(t - m_b^2)} \frac{z^2 + (1-z)^2}{2} \right] + \text{non-singular terms}$$

Integrating over t gives:

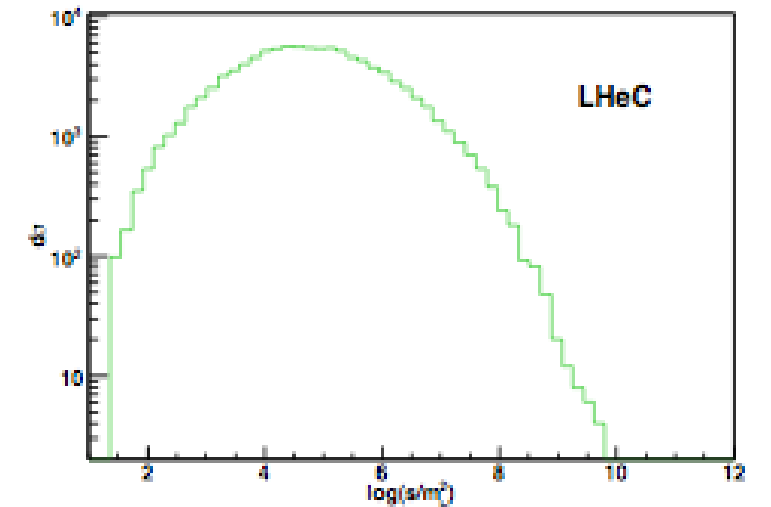
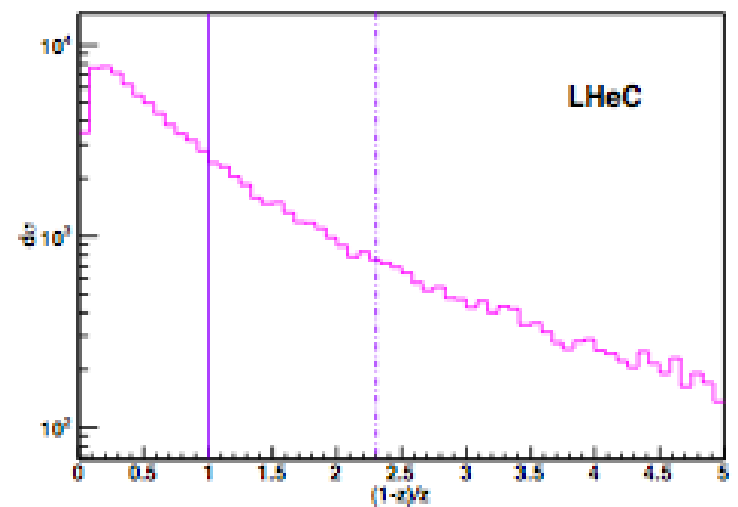
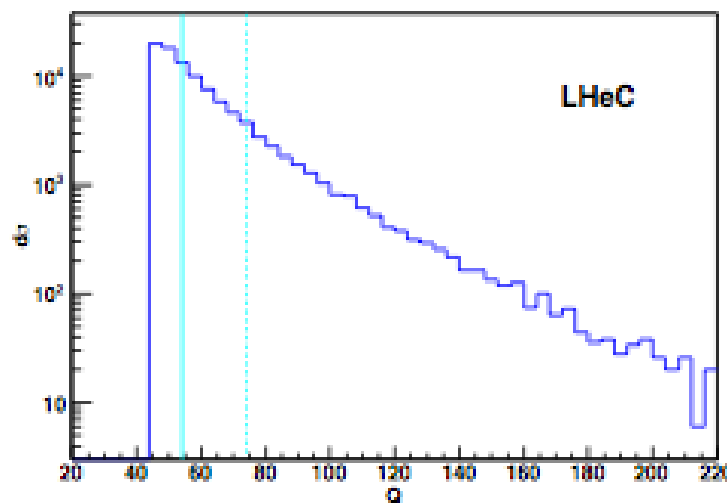
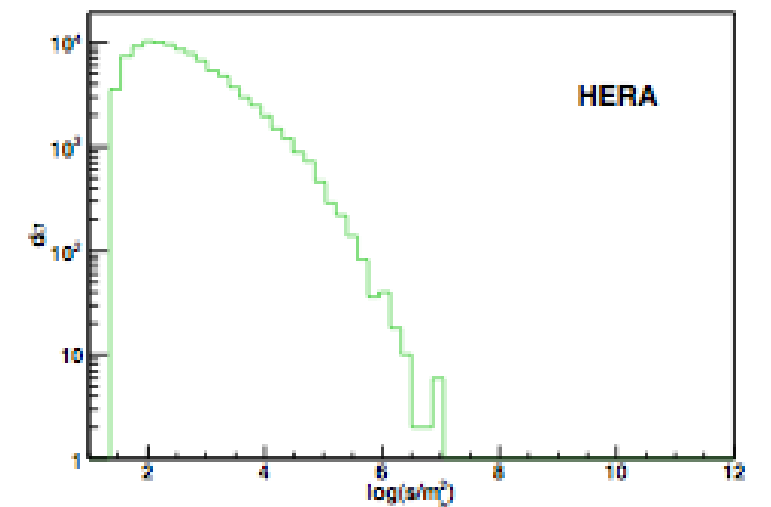
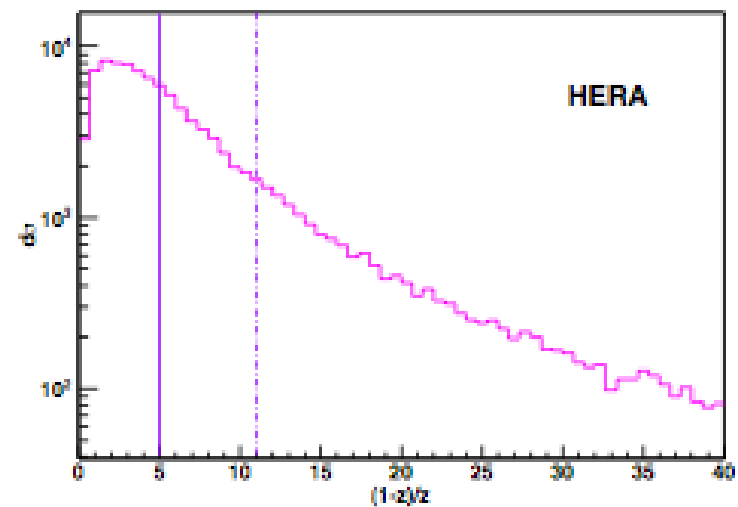
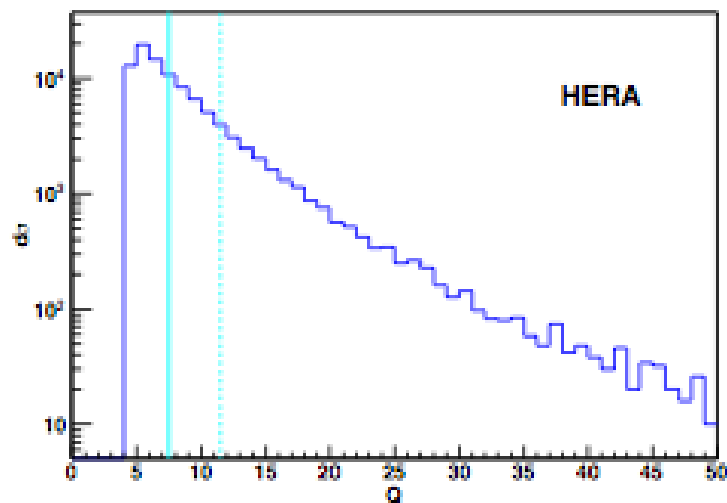
$$\begin{aligned} \int_{t_-}^{t_+} dt \frac{d\hat{\sigma}_2}{dt} &= \frac{\pi\alpha_e e_b^2 \alpha_S C_F}{4Q^2} z P_{qg}(z) \log \frac{1+\beta}{1-\beta} & t_{\pm} &= m_b^2 - \frac{s+Q^2}{2}(1 \pm \beta); & \beta &= \sqrt{1 - \frac{4m_b^2}{s}} \\ &= \left(\frac{\pi^2 \alpha_e e_b^2 C_F}{2Q^2} \right) \frac{\alpha_S}{2\pi} z P_{qg}(z) \left[\log \frac{m_b^2}{s} + O\left(\frac{m_b^2}{s}\right) \right], \end{aligned}$$

i.e., doing it properly, one sees that the naively expected $\log Q^2/m_b^2$ is actually:

$$L_{\text{DIS}} \equiv \log \left[\frac{Q^2}{m_b^2} \frac{1-z}{z} \right] = \log \frac{M_{b\bar{b}}^2}{m_b^2}$$

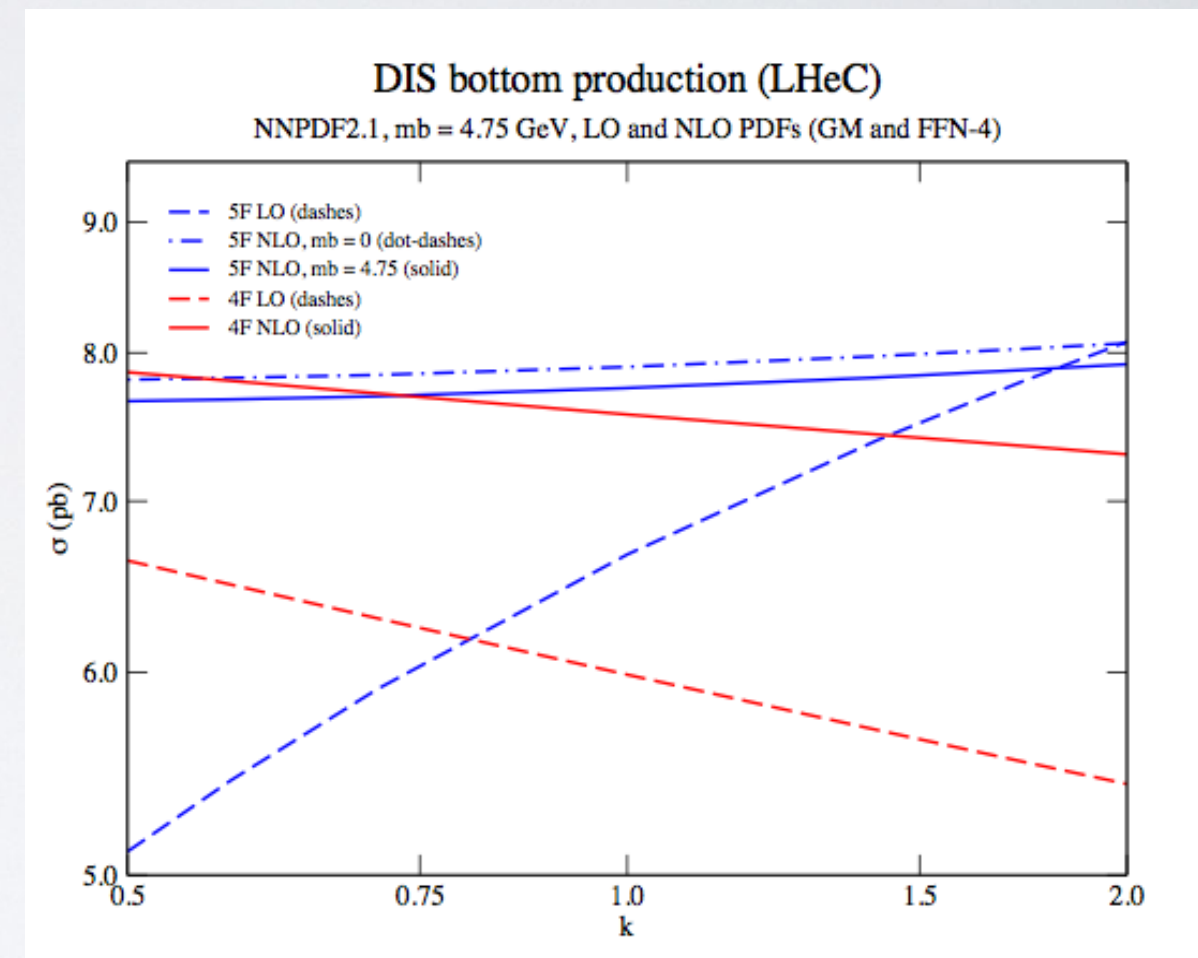
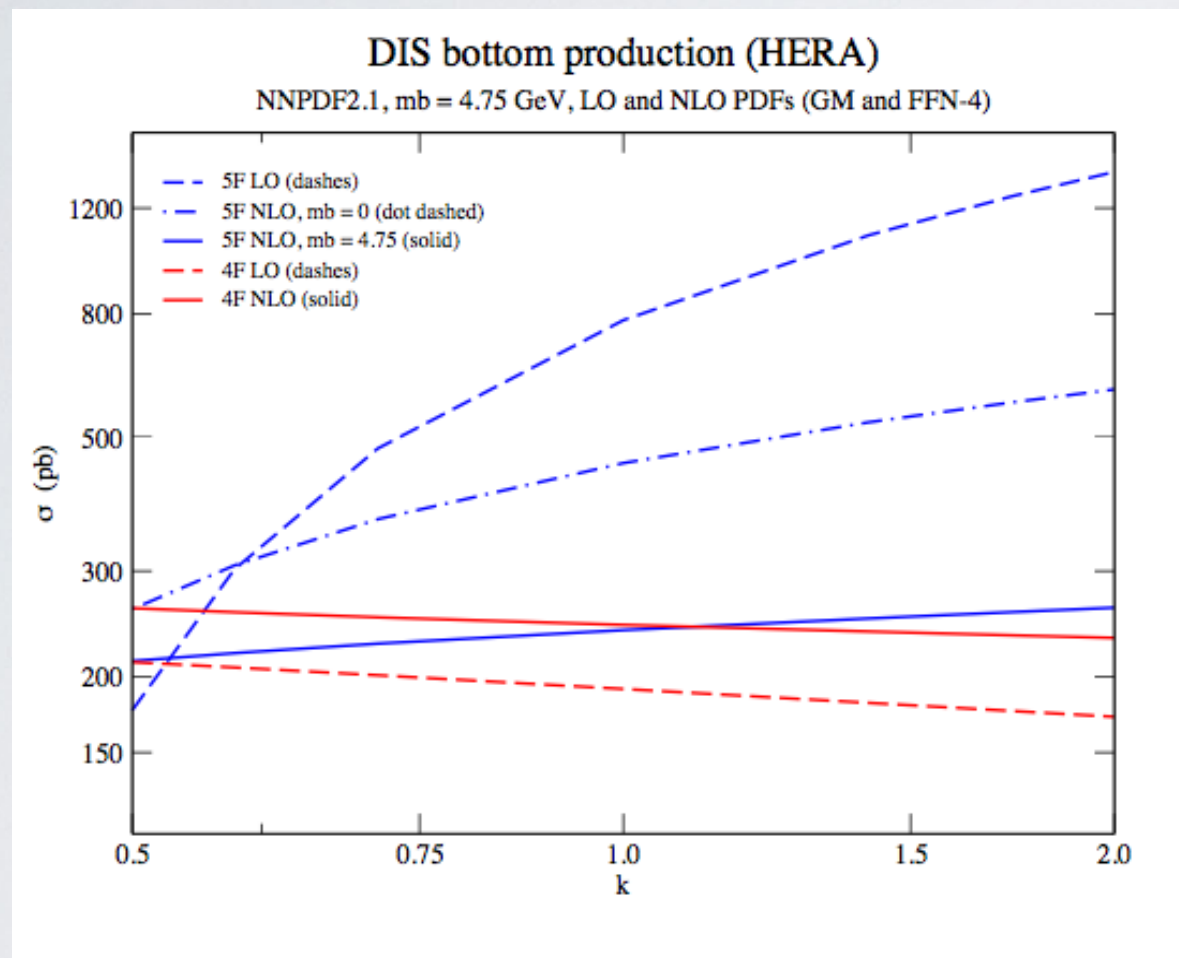
THE UNIVERSAL LOGS : DIS

$$L_{\text{DIS}} \equiv \log \left[\frac{Q^2}{m_b^2} \frac{1-z}{z} \right] = \log \frac{M_{b\bar{b}}^2}{m_b^2}$$

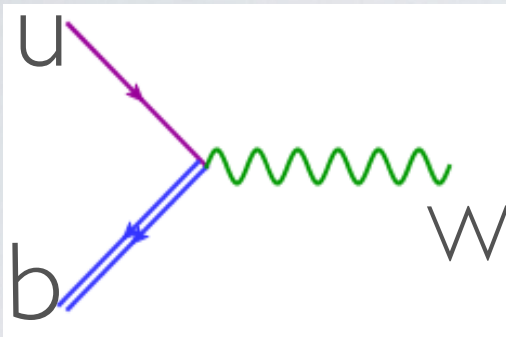


The typical values for $(1-z)/z$ lead to an enhancement of the log at HERA and ~ 1 at the LHeC

THE UNIVERSAL LOGS : DIS

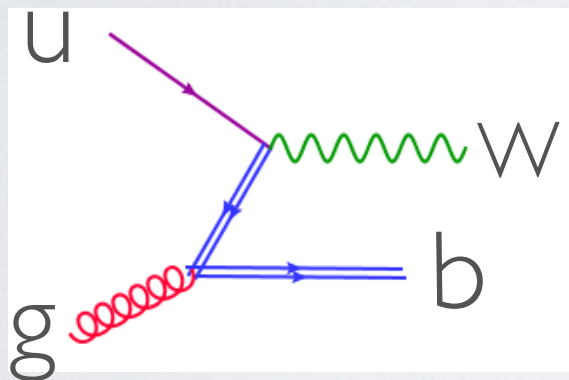


THE UNIVERSAL LOGS : DY



$$b(k_1) + u(k_2) \longrightarrow W(k)$$

$$\sigma^{5F}(\tau) = \left(\frac{\pi\sqrt{2}}{3} G_F \tau \right) \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ug} \left(\frac{\tau}{z}, \mu_F^2 \right) \frac{\alpha_S}{2\pi} P_{qg}(z) \log \frac{\mu_F^2}{m_b^2} + \dots$$



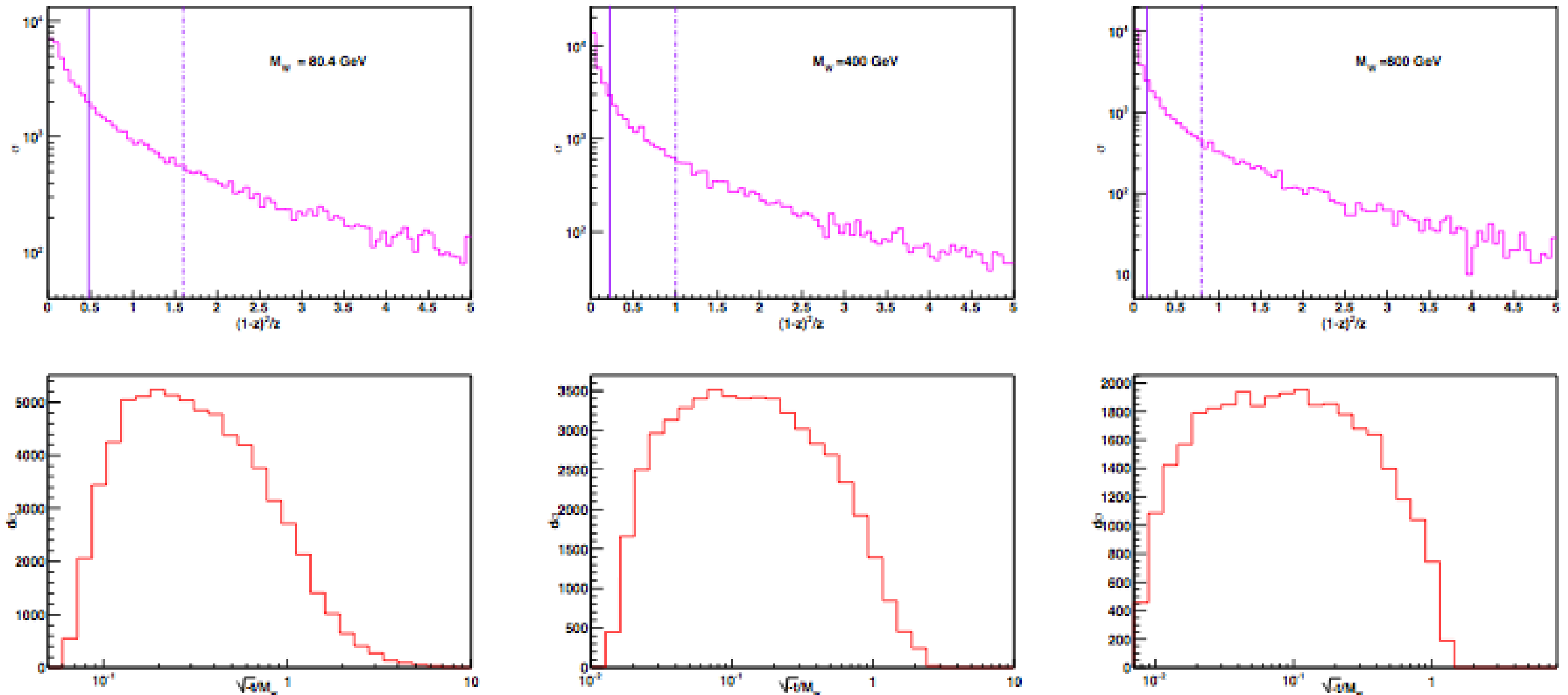
$$g(p_1) + u(p_2) \longrightarrow b(p_3) + W(p_4)$$

$$\hat{\sigma}^{4F}(z) = \int_{t_-}^{t_+} dt \frac{d\hat{\sigma}}{dt}(s, t, \alpha_S) = \frac{\alpha_s}{2\pi} \left(\pi \frac{\sqrt{2}}{3} G_F \right) z \frac{z^2 + (1-z)^2}{2} \log \left[\frac{M_W^2}{m_b^2} \frac{(1-z)^2}{z} \right] + \mathcal{O}(m_b^0)$$

$$\sigma^{4F}(\tau) = \left(\pi \frac{\sqrt{2}}{3} G_F \tau \right) \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ug} \left(\frac{\tau}{z} \right) \frac{\alpha_S}{2\pi} P_{qg}(z) L_{DY} + \mathcal{O}(m_b^0)$$

$$L_{DY} \equiv \log \left[\frac{M_W^2}{m_b^2} \frac{(1-z)^2}{z} \right].$$

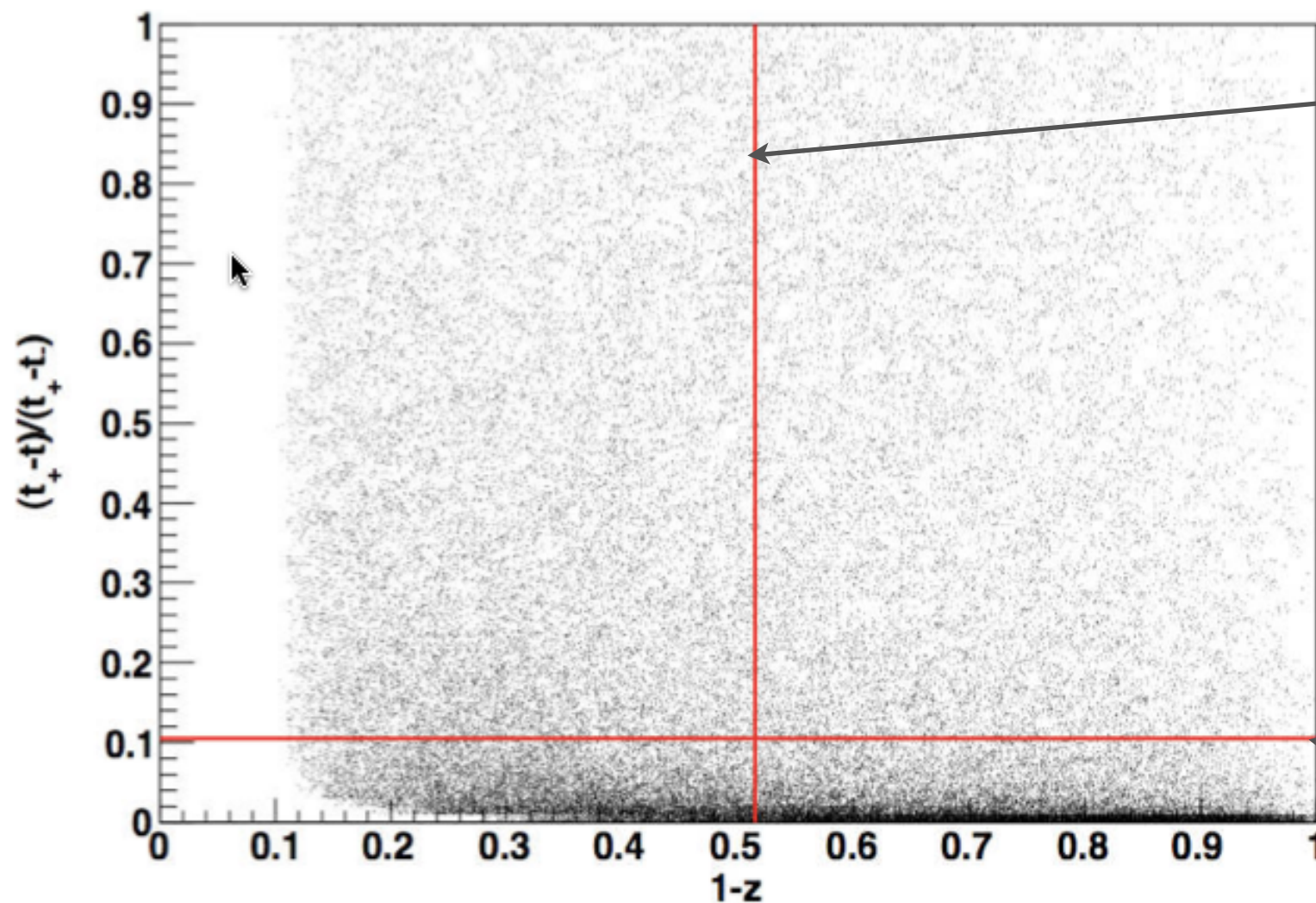
THE UNIVERSAL LOGS : DY



The typical values for $(1-z)^2/z$ and t lead to a suppressed L_{DY}

THE UNIVERSAL LOGS : DY

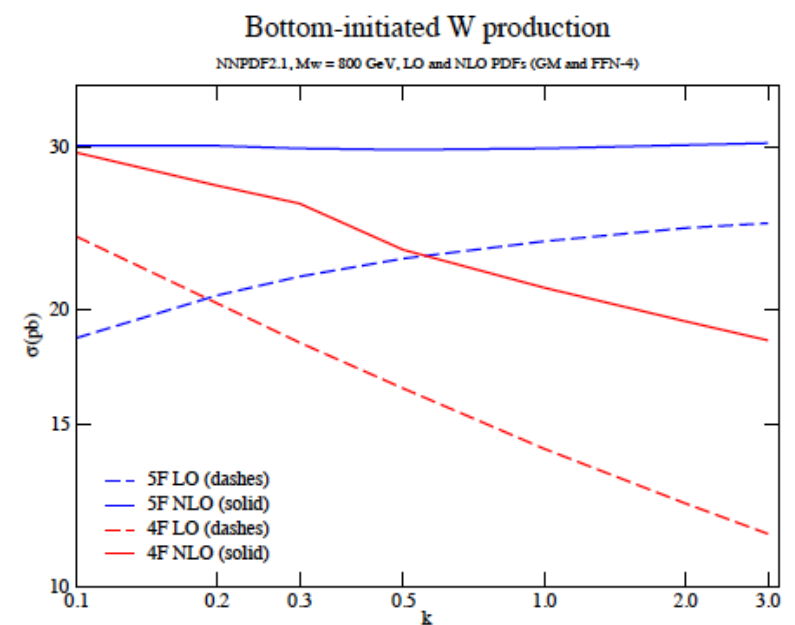
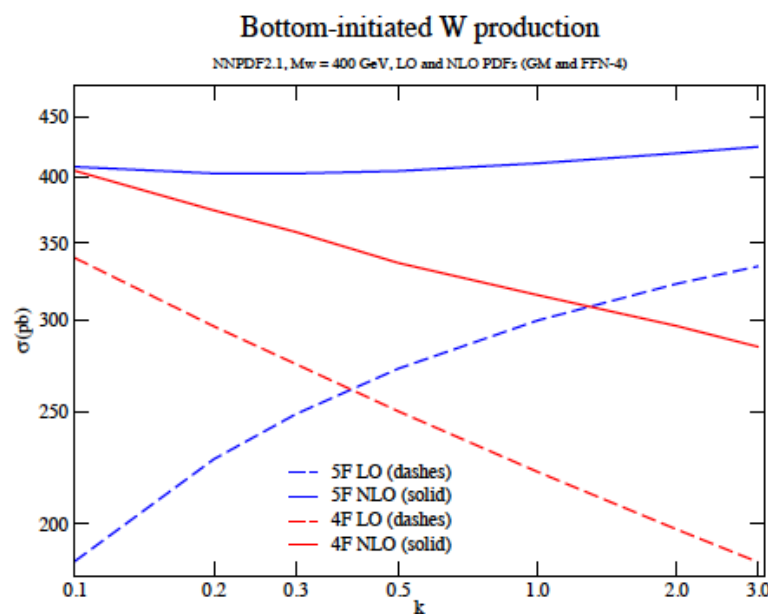
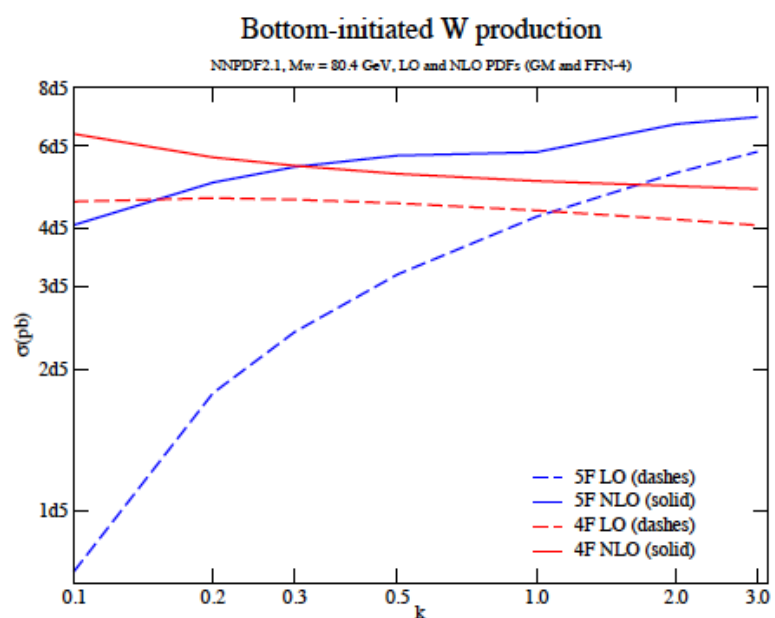
Are these logs “soft” or “collinear”?



Soft Median

Collinear Median

THE UNIVERSAL LOGS : DY



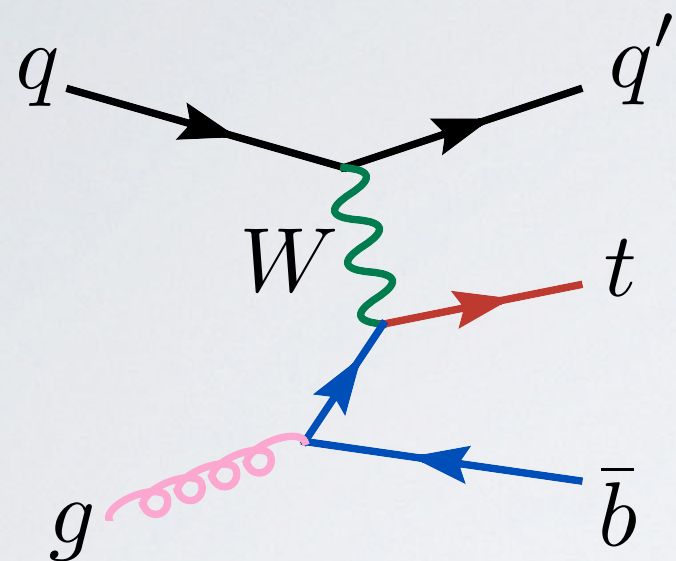
$$\log \frac{\tilde{\mu}_F^2}{m_b^2} = \frac{\int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ug} \left(\frac{\tau}{z} \right) P_{qg}(z) \log \left[\frac{M_W^2}{m_b^2} \frac{(1-z)^2}{z} \right]}{\int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ug} \left(\frac{\tau}{z} \right) P_{qg}(z)}$$

$$M_W = 80 \text{ GeV} \quad , \quad \tilde{\mu}_F \simeq [0.4, 0.5] M_W$$

$$M_W = 400 \text{ GeV} \quad , \quad \tilde{\mu}_F \simeq [0.3, 0.4] M_W$$

$$M_W = 800 \text{ GeV} \quad , \quad \tilde{\mu}_F \simeq [0.25, 0.35] M_W$$

THE UNIVERSAL LOGS : SINGLE-TOP



$$Q^2 \rightarrow 0 \Rightarrow \text{Drell-Yan}$$

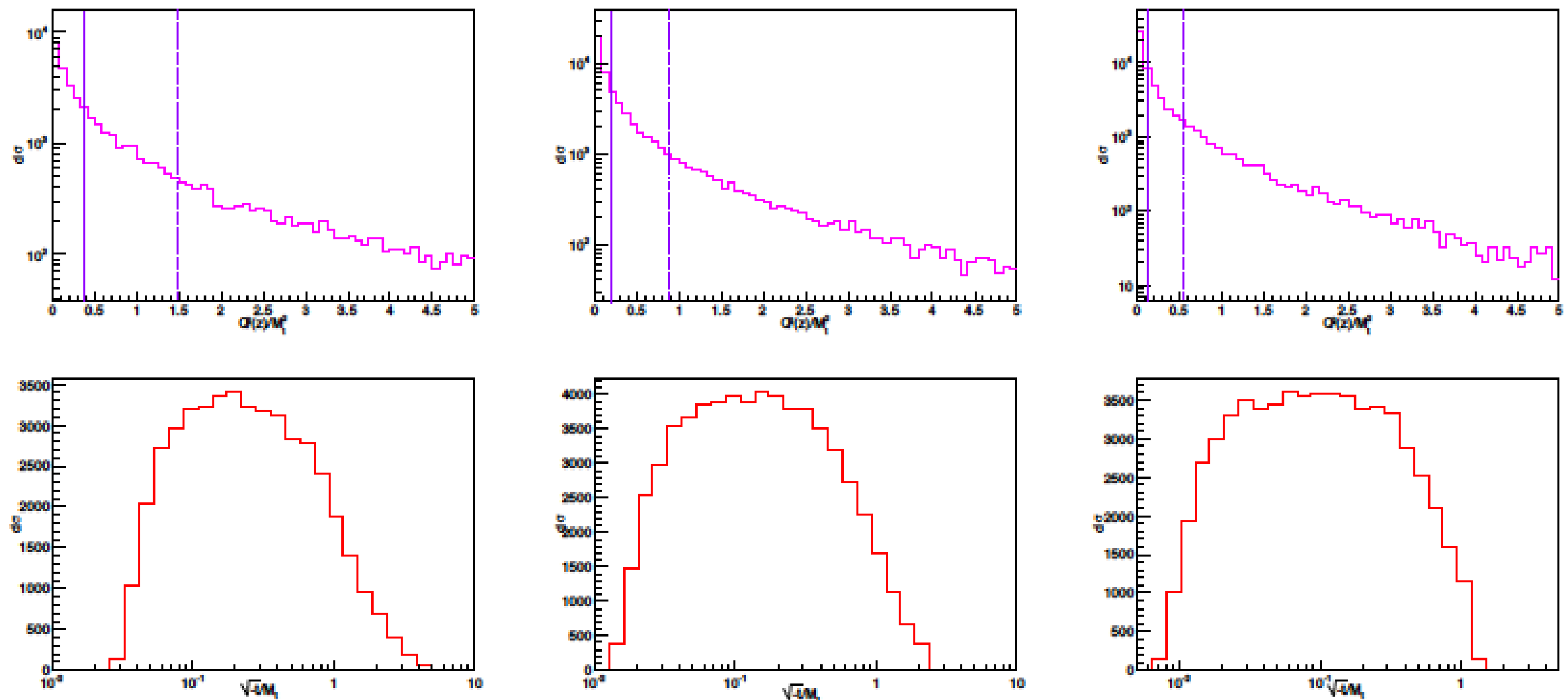
$$M_t \rightarrow m_b \Rightarrow \text{DIS}$$

The same procedure followed before leads to:

$$\int_{t_{\min}}^{t_{\max}} dt \frac{d\hat{\sigma}_2^{4F}}{dt} = \frac{3\alpha_S g_W^2 C_F}{64(s + Q^2)} \frac{z^2 + (1 - z)^2}{2} \log \frac{Q^2(z)}{m_b^2}, \quad z = \frac{M^2 + Q^2}{s + Q^2}$$

$$Q^2(z) = (M^2 + Q^2) \frac{(1 - z)^2}{z} \frac{1}{1 - \frac{zQ^2}{M^2 + Q^2}}$$

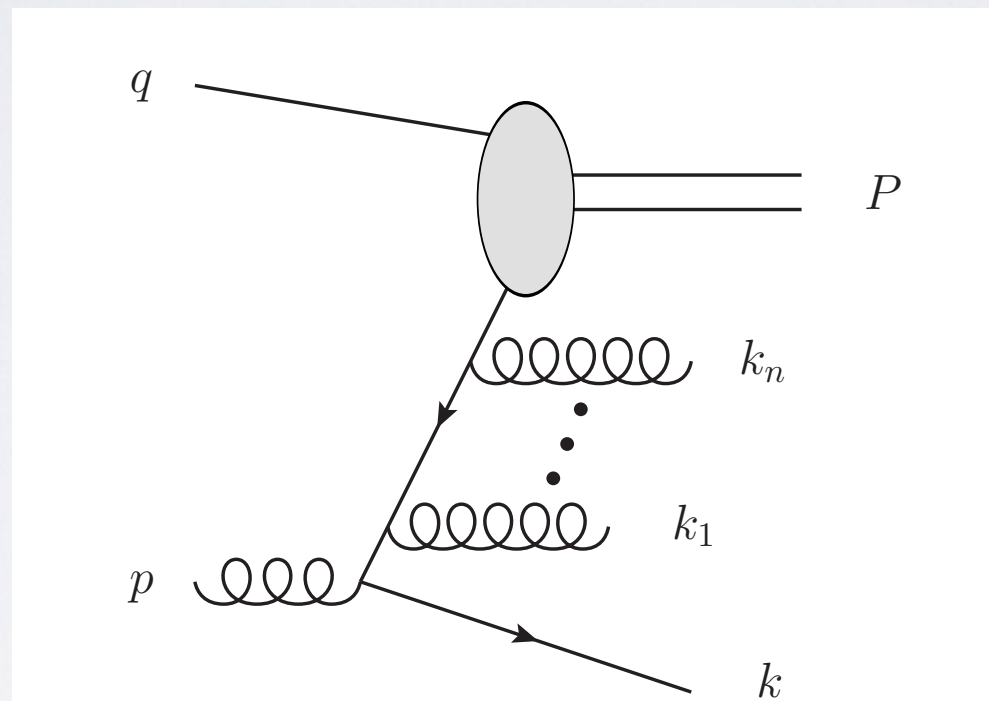
THE UNIVERSAL LOGS : SINGLE-TOP



The typical values for Q^2 and t lead to a suppressed Logarithm

THE UNIVERSAL LOGS

$$I(q) + g(p) \rightarrow b(k) + g(k_1) + \dots + g(k_n) + X(P)$$



$$L_{\text{UNIV}} = \log \frac{Q^2(z)}{m_b^2}$$

QUESTIONS AND PUZZLES: ANSWERS

- At the level of total cross section 5F predictions are in general better behaved than 4F.
 - However, a substantial and unexpected agreement between 4F and 5F is found when scales smaller than a naive choice is made.
 - Agreement is found even in regions where the logs should be large. Only exception seems to take place for very heavy object production.
 - Independently of 5F results: No sign of breakdown of the perturbative expansion for 4F in total cross sections as well as for more exclusive observables is found.
- ➔ 5F predictions formally always start at one power in α_s less, do resum logs (large or small) and therefore display a rather milder scale dependence. Finally for some procs we have NNLO calculations available.
 - ➔ The agreement is found for scales that are indicated by the 4F calculations themselves. No artificial tuning is necessary. This is due to the same kinematical mechanism that suppresses the possibly large logs. An average scale of the order of the p_T of the spectator b also falls in the same ball park.
 - ➔ The effect of resummation is in general small so a 4F calculation at NLO catches already the main logs. The logs are anyway smaller than what one would guess. Very heavy objects compared to the total energy available production demands large Bjorken- x and here the resummation effects are the largest.
 - ➔ There is no clear call for resummation from the calculation itself.