Heavy Flavour in MSTW/MMHT PDFs

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I will present details of heavy flavour treatments with the MSTW/MMHT framework.

This will involve the construction of a general mass variable flavour number scheme GM-VFNS, and difference in choices between these.

I will also present work on differences between PDFs in FFNS and GM-VFNS.

Will discuss Charm $(m_c^{\rm pole} \sim 1.4{\rm GeV})$, bottom $(m_b^{\rm pole} \sim 4.75{\rm GeV})$, as heavy flavours with variable schemes. Top always treated as a final state particle so far.

Choices for Heavy Flavours in DIS.

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state.

Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x, Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Does not sum $\alpha_S^n \ln^n Q^2/m_H^2$ terms in perturbative expansion. Usually achieved by definition of heavy flavour parton distributions and solution of evolution equations.

Additional problem FFNS known up to NLO (Laenen *et al.*), but are not fully known at NNLO – $\alpha_S^3 C_{2,Hi}^{FF,3}$ unknown.

Approximations based on some or all of threshold, low-x and high-Q^2 limits (last a continuing project by Blümlein *et al*) can be derived, see Kawamura, *et al.*,, and are sometimes used in fits, e.g. ABM and MSTW/MMHT (at low Q^2). Generally not large except at threshold and very low x.

Variable Flavour - at high scales $Q^2 \gg m_H^2$ heavy quarks behave like massless partons. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x,Q^2) = C_j^{ZM,n_f} \otimes f_j^{n_f}(Q^2).$$

Partons in different number regions related to each other perturbatively.

$$f_j^{n_f+1}(Q^2) = A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2),$$

Perturbative matrix elements $A_{jk}(Q^2/m_H^2)$ (Buza *et al.*, and Blümlein *et al* at $\mathcal{O}(\alpha_S^3)$) containing $\ln(Q^2/m_H^2)$ terms relate $f_i^{n_f}(Q^2)$ and $f_i^{n_f+1}(Q^2)$ $\rightarrow$ correct evolution for both.

Want a **General-Mass Variable Flavour Number Scheme** (GM-VFNS) taking one from the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

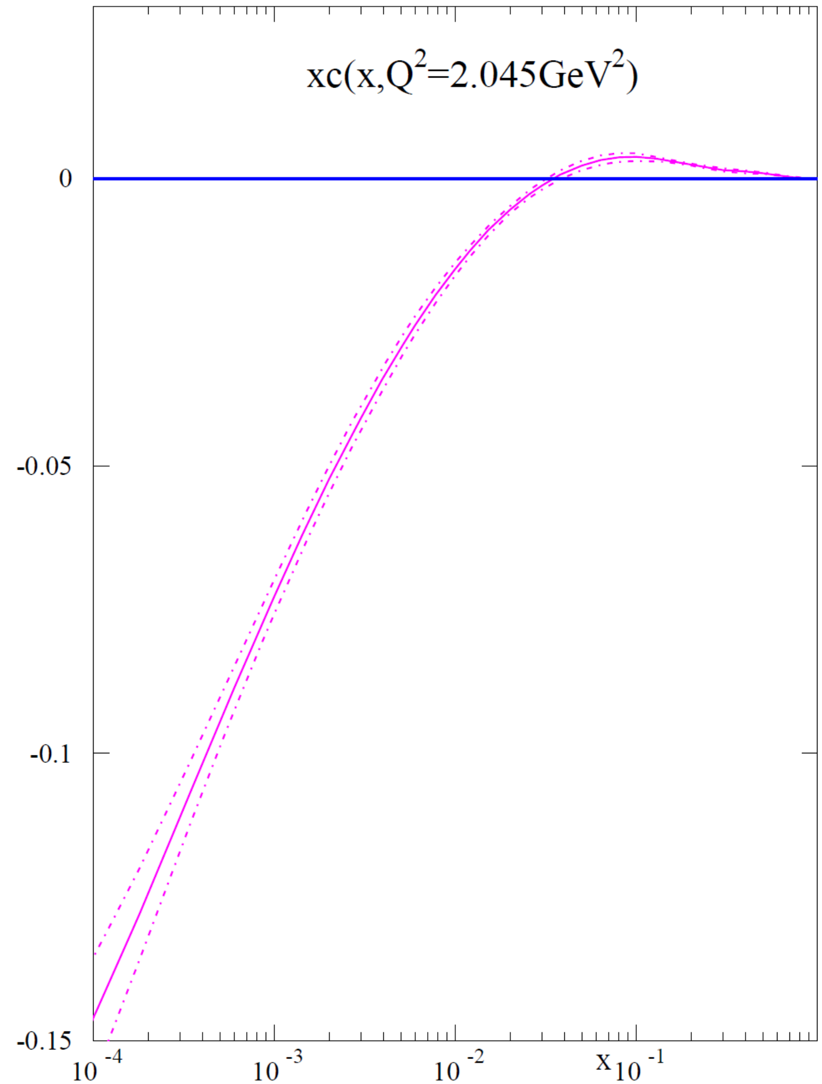
At NLO the partons remain continuous if transition point is taken as $Q^2 = m_H^2$. ZM-VFNS possible, if inaccurate.

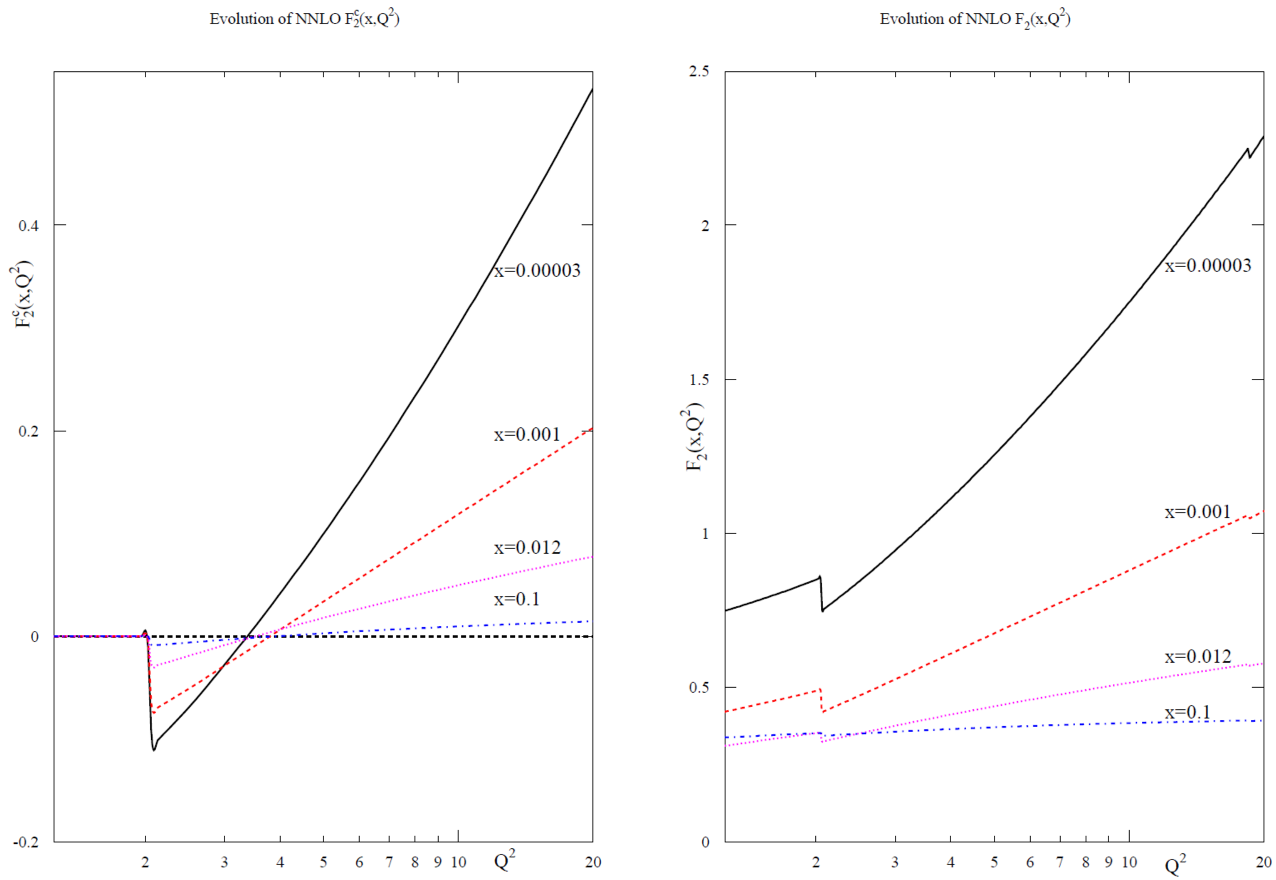
At NNLO lead to discontinuities in partons.

Heavy flavour no longer turns on from zero at $\mu^2 = m_c^2$

$$(c + \bar{c})(x, m_c^2) = A^2_{Hg}(m_c^2) \otimes g(m_c^2)$$

In practice turns on from negative value, (for general gluon).





Leads to huge discontinuity in $F_2^c(x,Q^2)$. Still significant in $F_2^{Tot}(x,Q^2)$.

ZM-VFNS not really feasible at NNLO. Want $\rightarrow$ Need.

The GM-VFNS can be defined by demanding equivalence of the n_f light flavour and n_f+1 light flavour descriptions at all orders – above transition point $n_f \to n_f+1$

$$F(x,Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2)$$
$$\equiv C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2).$$

Hence, the VFNS coefficient functions satisfy

$$C_k^{FF,n_f}(Q^2/m_H^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2),$$

which at $\mathcal{O}(\alpha_S)$ gives (in $\overline{MS}$ scheme)

$$C_{2,Hg}^{FF,n_f,(1)}(\frac{Q^2}{m_H^2}) = C_{2,HH}^{VF,n_f+1,(0)}(\frac{Q^2}{m_H^2}) \otimes P_{qg}^0 \ln(Q^2/m_H^2) + C_{2,Hg}^{VF,n_f+1,(1)}(\frac{Q^2}{m_H^2}),$$

The VFNS coefficient functions tend to the $m=0$ limits as $Q^2/m_H^2 \to \infty$.

However, $C_j^{VF}(Q^2/m_H^2)$ only uniquely defined in this limit.

Can swap $\mathcal{O}(m_H^2/Q^2)$ terms between $C_{2,HH}^{VF,0}(Q^2/m_H^2)$ and $C_{2,g}^{VF,1}(Q^2/m_H^2)$.

Various prescriptions (ACOT, TR, Chuvakin-Smith).

Some earlier versions violated threshold $W^2 > 4m_H^2$ in individual terms.

(TR-VFNS) highlighted freedom in choice and enforced kinematics in each term by making $(d\,F_2/d\,\ln Q^2)$ continuous at transition (in gluon sector). Complicated to extend.

(S)ACOT(χ) (Tung, *et al*) prescription says make simple choice

$$C_{2,HH}^{VF,0}(Q^2/m_H^2, z) = \delta(z - Q^2/(Q^2 + 4m_H^2)).$$

$$\rightarrow F_2^{H,0}(x, Q^2) = (h + \bar{h})(x/x_{max}, Q^2), \qquad x_{max} = Q^2/(Q^2 + 4m_H^2)$$

$\rightarrow C_{2,HH}^{ZM,0}(z) = \delta(1-z)$ for $Q^2/m_H^2 \rightarrow \infty$. Also $W^2 = Q^2(1-x)/x \geq 4m_H^2$.

Have adopted this and obvious extensions to higher orders (and now simple modifications). Though with different prefactor – chosen by analogy to $F_2^{\rm CC}$.

Still another difference.

ACOT type schemes have used e.g.

NLO $\frac{\alpha_S}{4\pi} C^{FF,n_f,(1)}_{2,Hg} \otimes g^{n_f} \rightarrow \frac{\alpha_S}{4\pi}(C^{VF,n_f+1,(1)}_{2,HH} \otimes (h+\bar{h}) + C^{VF,n_f+1,(1)}_{2,Hg} \otimes g^{n_f+1})$,

i.e., same order of α_S above and below.

But LO FFNS and evolution below and NLO definition and evolution above.

TR have used e.g.

LO $$\frac{\alpha_S(Q^2)}{4\pi} C^{FF,n_f,(1)}_{2,Hg}(Q^2/m_H^2) \otimes g^{n_f}(Q^2) \rightarrow \frac{\alpha_S(M^2)}{4\pi} C^{FF,n_f,(1)}_{2,Hg}(1) \otimes g^{n_f}(M^2)$$
$$+ C^{VF,n_f+1,(0)}_{2,HH}(Q^2/m_H^2) \otimes (h+\bar{h})(Q^2),$$

i.e. freeze higher order α_S term when going upwards through $Q^2 = m_H^2$.

This difference in choice can be phenomenologically important.

In order to define our VFNS at NNLO, need $\mathcal{O}(\alpha_S^3)$ heavy flavour coefficient functions for $Q^2 \leq m_H^2$ and to be frozen for $Q^2 > m_H^2$. However, not calculated. Needs modelling.

Different type of Definition

Both the BMSN (Buza *et al*) and FONLL (Forte *et al*) applied a similar type of reasoning. In general terms (for structure functions)

$$F^{\rm GMVFNS}(x,Q^2) = F_2^{\rm FFNS}(x,Q^2) - F_2^{\rm asymp}(x,Q^2) + F_2^{\rm ZMVFNS}(x,Q^2)$$

where the second (subtraction) term is the asymptotic version of the first, i.e., all terms $\mathcal{O}(m_H^2/Q^2)$ omitted.

Differences in exactly how the second and third terms are defined in detail (e.g. Blümlein *et al* do not resum $\ln Q^2/m_H^2$ terms from PDF evolution in $F_2^{\rm ZMVFNS}$).

In FONLL approach each term in the combination ($F_2^{\rm ZMVFNS} - F_2^{\rm asymp}$) can be modified by corrections which fall like m_H^2/Q^2.

In simplest application α_S order of $F^{\rm FFNS}(x,Q^2)$ at low Q^2 same as that of $F^{\rm ZMVFNS}(x,Q^2)$ as $Q^2 \to \infty$, like ACOT.

Modification in FONLL – can avoid this at NLO, but leads to extra (higher order) term as $Q^2 \to \infty$ – not exact cancellation in first two terms.

Ordering tricky problem. Would like any GMVFNS to reduce to exactly correct order FFNS at low Q^2 and exactly correct order ZMVFNS as $Q^2 \rightarrow \infty$.

Return to original TR version of the GMVFNS. Reason for violation of the above is frozen term $\alpha_S^n(m_H^2) \sum_i C_{2,i}^{\mathrm{FFNS}}(m_H^2) \otimes f_i(m_H^2)$ which still persists as $Q^2 \rightarrow \infty$ at order N^{n-1}LO.

Depends on PDFs at low scales, so rather small effect at large Q^2.

However, not strictly necessary. Frozen in original TR prescription from exact condition on derivative of $d\,F_2/d\,\ln Q^2$. Could have instead

$$\left(\frac{m_H^2}{Q^2}\right)^a \alpha_S^n(m_H^2) \sum_i C_{2,i}^{\mathrm{FF}}(m_H^2) \otimes f_i(m_H^2)$$

or

$$\left(\frac{m_H^2}{Q^2}\right)^a \alpha_S^n(Q^2) \sum_i C_{2,i}^{\mathrm{FF}}(Q^2) \otimes f_i(Q^2),$$

Any $a > 0$ provides both exactly correct asymptotic limits.

Also have the freedom to modify the heavy quark coefficient function, by default

$$C^{VF,0}_{2,HH}(Q^2/m_H^2, z) = \delta(z - x_{\max}).$$

Appears in convolutions for higher order subtraction terms, so do not want complicated x dependence. Simple choice.

$$C^{VF,0}_{2,HH}(Q^2/m_H^2, z) \to (1 + b(m_H^2/Q^2)^c)\delta(z - x_{\max})),$$

where c really encompasses (m_H^2/Q^2) with logarithmic corrections.

Can also modify argument of δ-function, as in Intermediate Mass (IM) scheme of Nadolsky, Tung. Let argument of heavy quark contribution change like

$$\xi = x/x_{\max} \to x\big(1 + (x(1 + 4m_H^2/Q^2))^d 4m_H^2/Q^2\big),$$

so kinematic limit stays the same, but if $d > 0$ small x less suppressed, or if $d < 0$ (must be > -1) small x more suppressed.

Default a, b, c, d all zero. Limit either by fit quality or *sensible* choices.

6 extreme variations tried.

GMVFNS1 – $b = -1, c = 1$.

GMVFNS2 – $b = -1, c = 0.5$.

GMVFNS1 – $a = 1$.

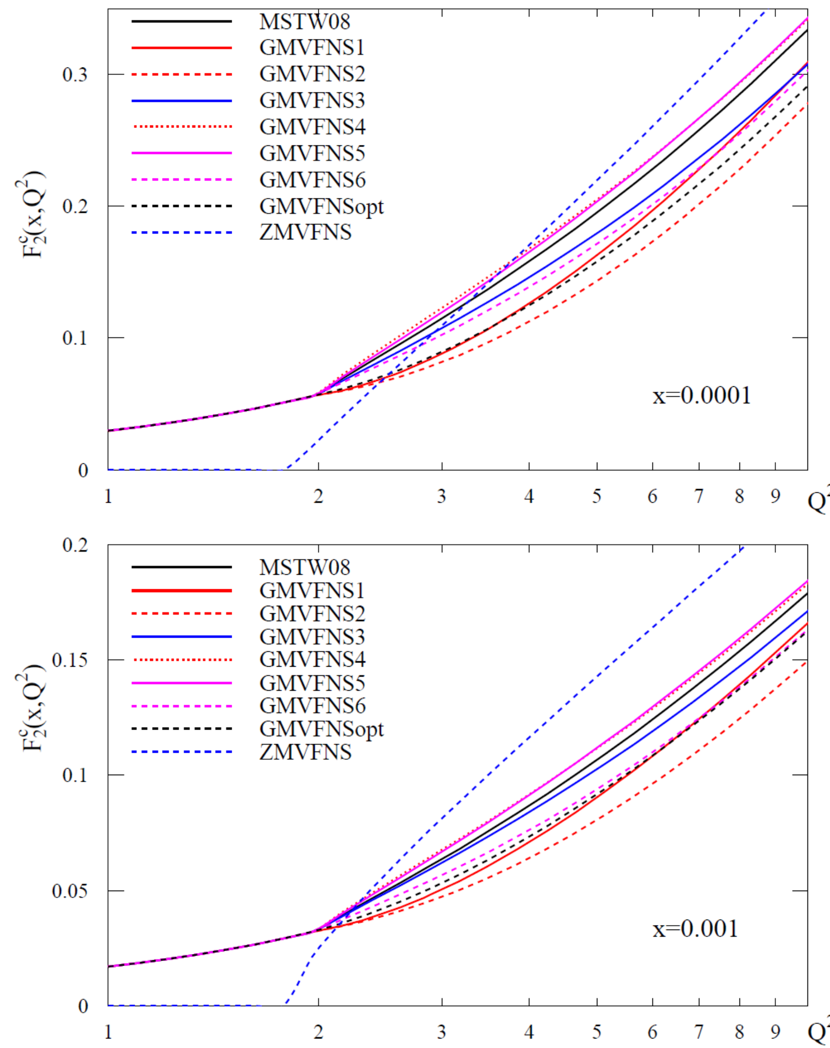
GMVFNS1 – $b = +0.3, c = 1$ – fit.

GMVFNS1 – $d = 0.1$ – fit.

GMVFNS1 – $d = -0.2$ – fit.

Variations in $F_2^c(x, Q^2)$ near the transition point at NLO due to different choices of GM-VFNS.

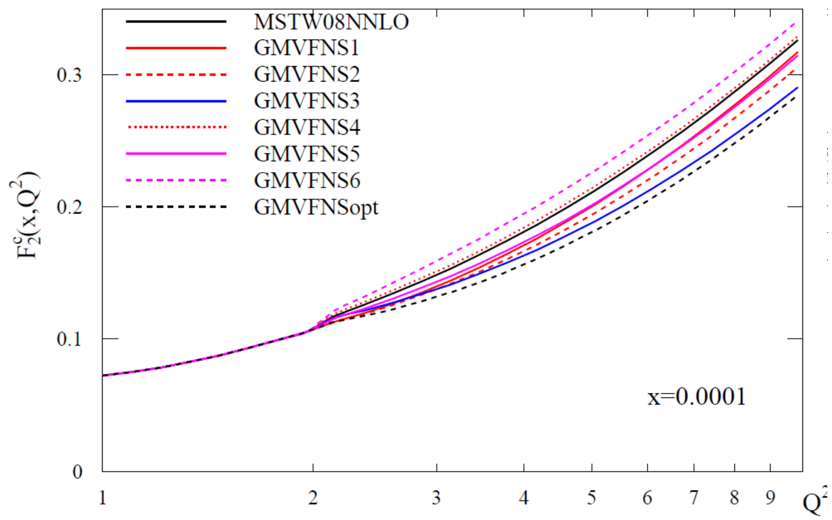
Optimal, $a = 1, b = -2/3, c = 1$, smooth behaviour.

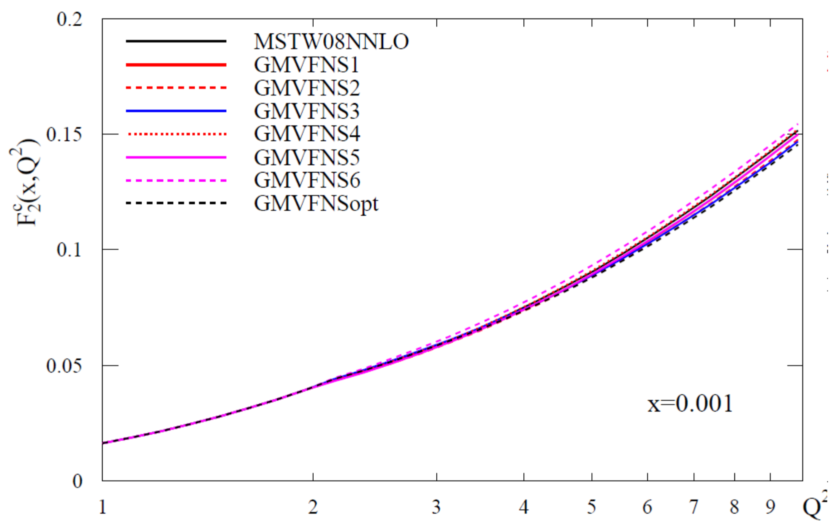


Variations in $F_2^c(x,Q^2)$ near the transition point due to different choices of GM-VFNS at NNLO.

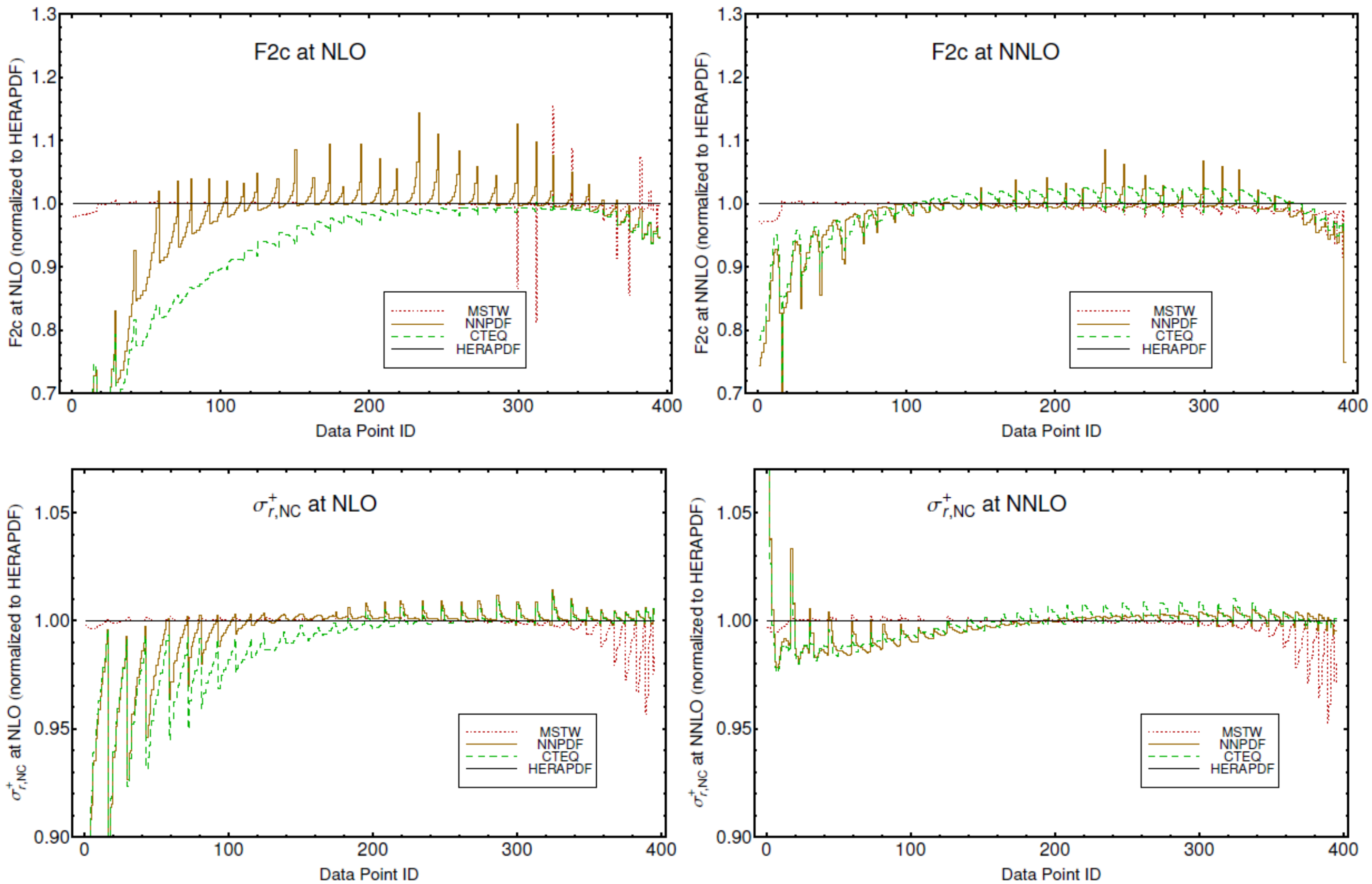
Very much reduced, almost zero variation until very small x.

Shows that NNLO evolution effects most important in this regime.





Also see convergence between groups in Les Houches benchmark study.



NNLO TR scheme larger at lowest Q^2 due to use of $\mathcal{O}(\alpha_S^3)$ coefficient function.

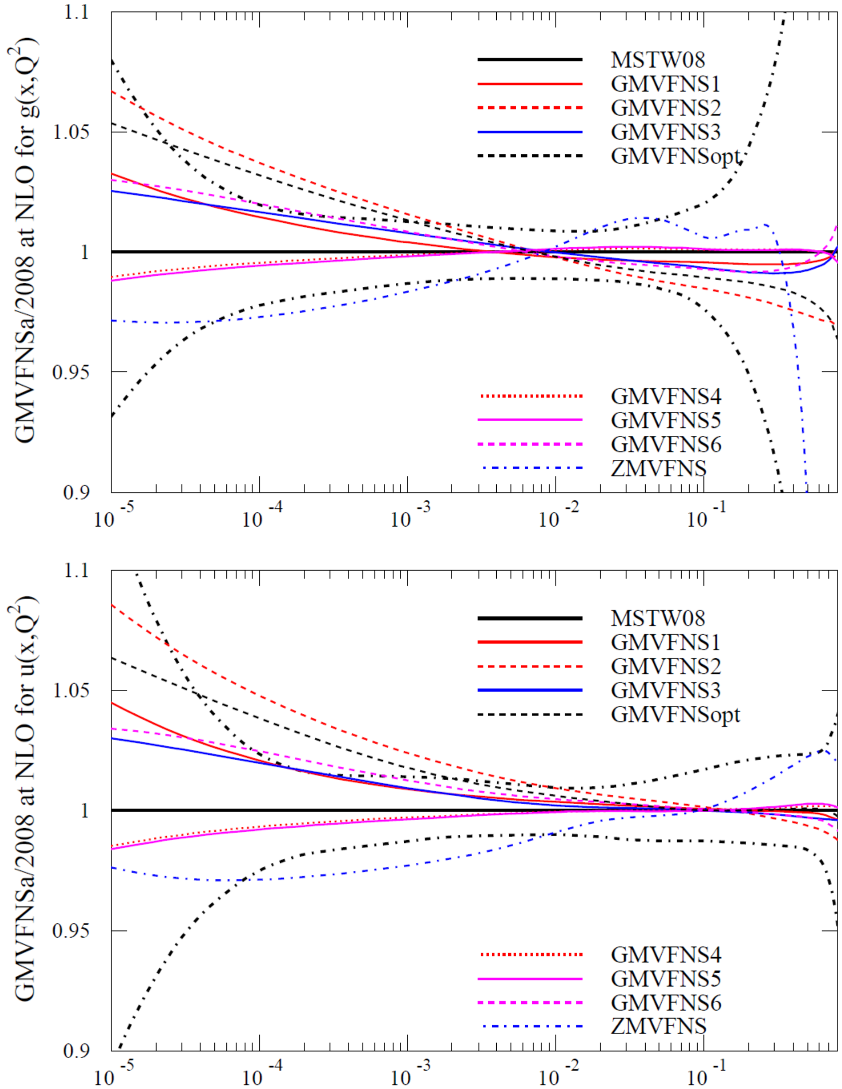
Variations in partons extracted from global fit due to different choices of GM-VFNS at NLO.

Initial χ^2 can change by 250.

Converges to at most about 15 of original.

Better fit for GMVFNS1, GMVFNS3 and GMVFNS6.

Some changes in PDFs large compared to one-sigma $uncertainty$.



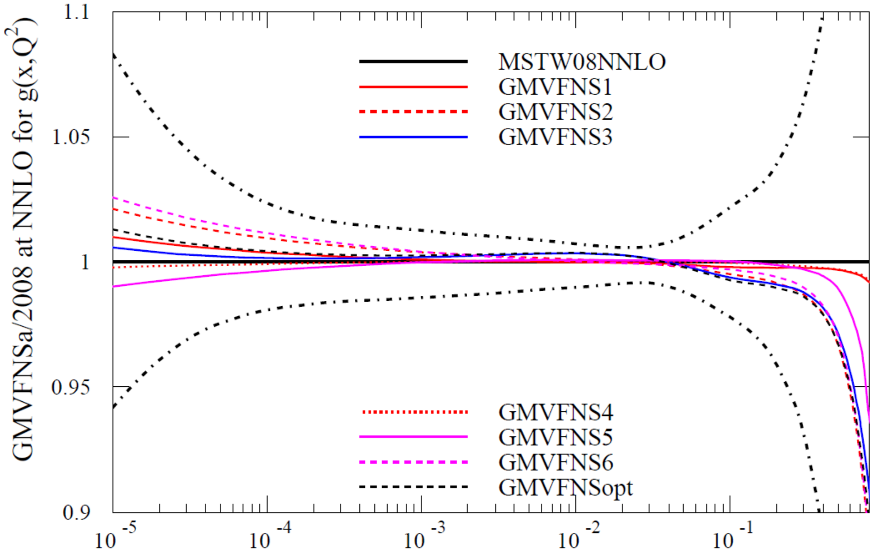
Variations in partons extracted from global fit due to different choices of GM-VFNS at NNLO.

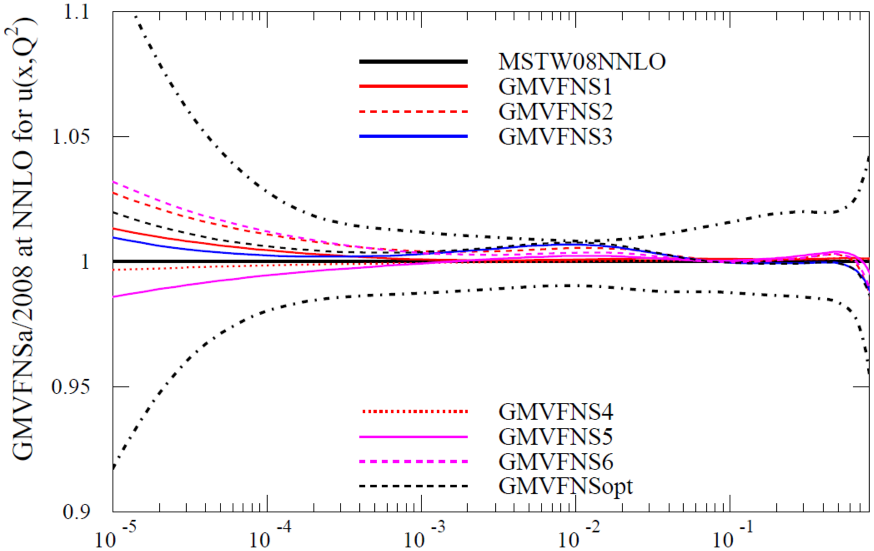
Initial changes in $\chi^2 < 20$.

Converge to about 10. None a marked improvement.

At worst changes approach *uncertainty*.

Biggest variation in high-x gluon, which has large uncertainty.



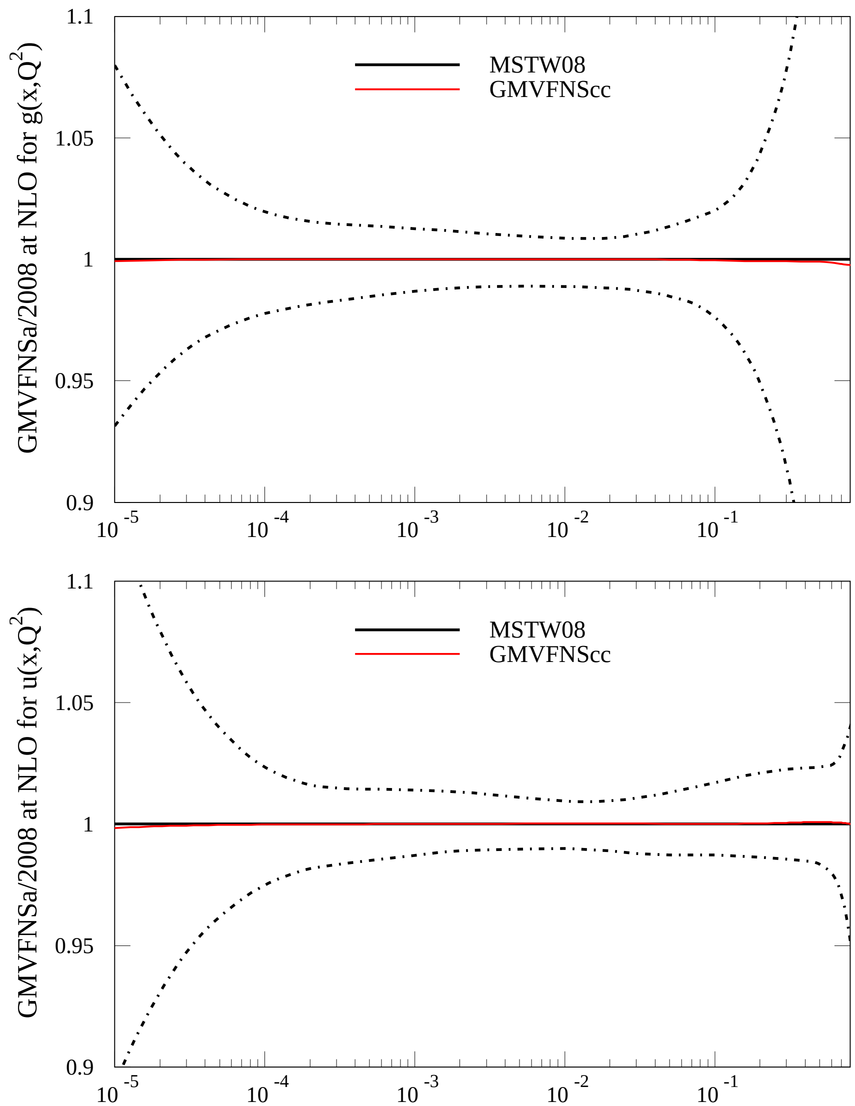


Also implement similar variations in GM-VFNS for charged current.

HERA data completely insensitive due to large Q^2.

Some effect on fixed target (anti)neutrino data in fit. χ^2 changes by at most 4 units and almost no change in this, or PDFs, with refit.

Also make changes in cross-sections for di-muon data. In practice χ^2 changes by at most 1 unit. Essentially no change in PDFs.



Lower-Q^2 nuclear data most sensitive to (until recently) unknown NNLO corrections.

Result by Berger *et al*, `arXiv:1601.05430`.

Negative at smaller-x.

Hopefully available in usable form soon.

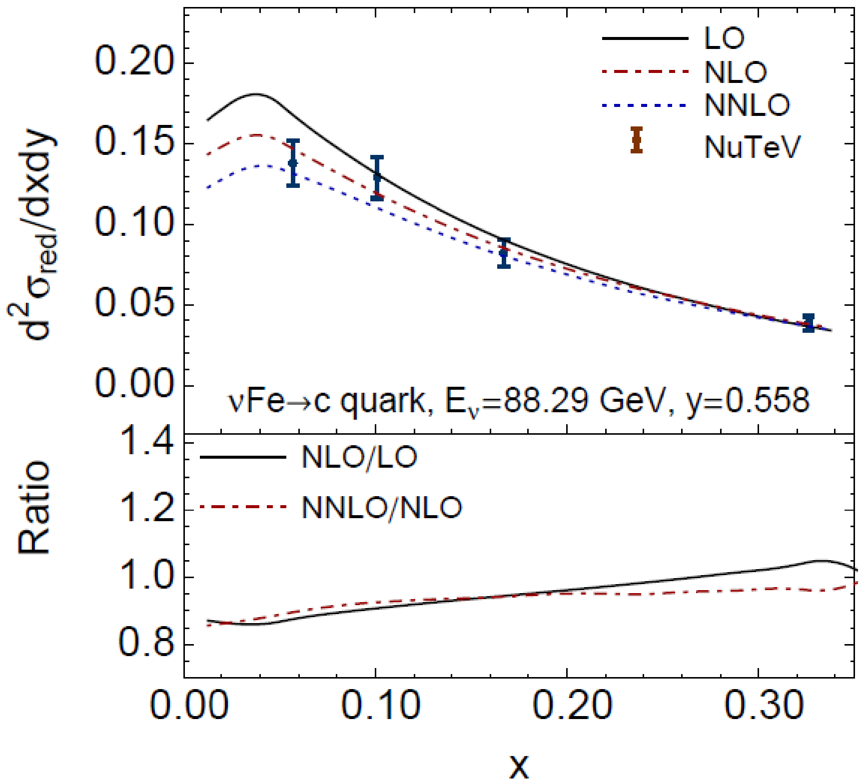
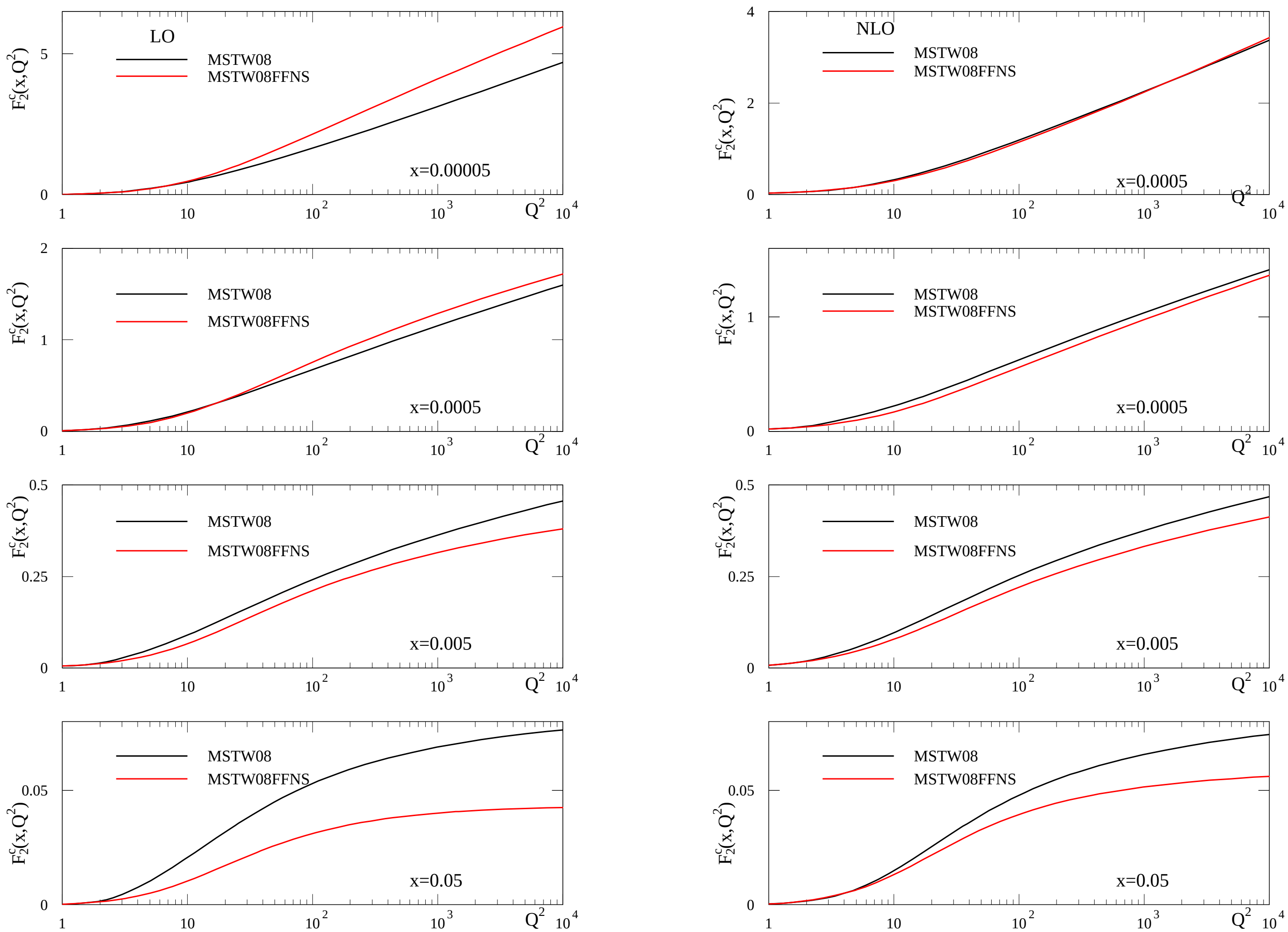
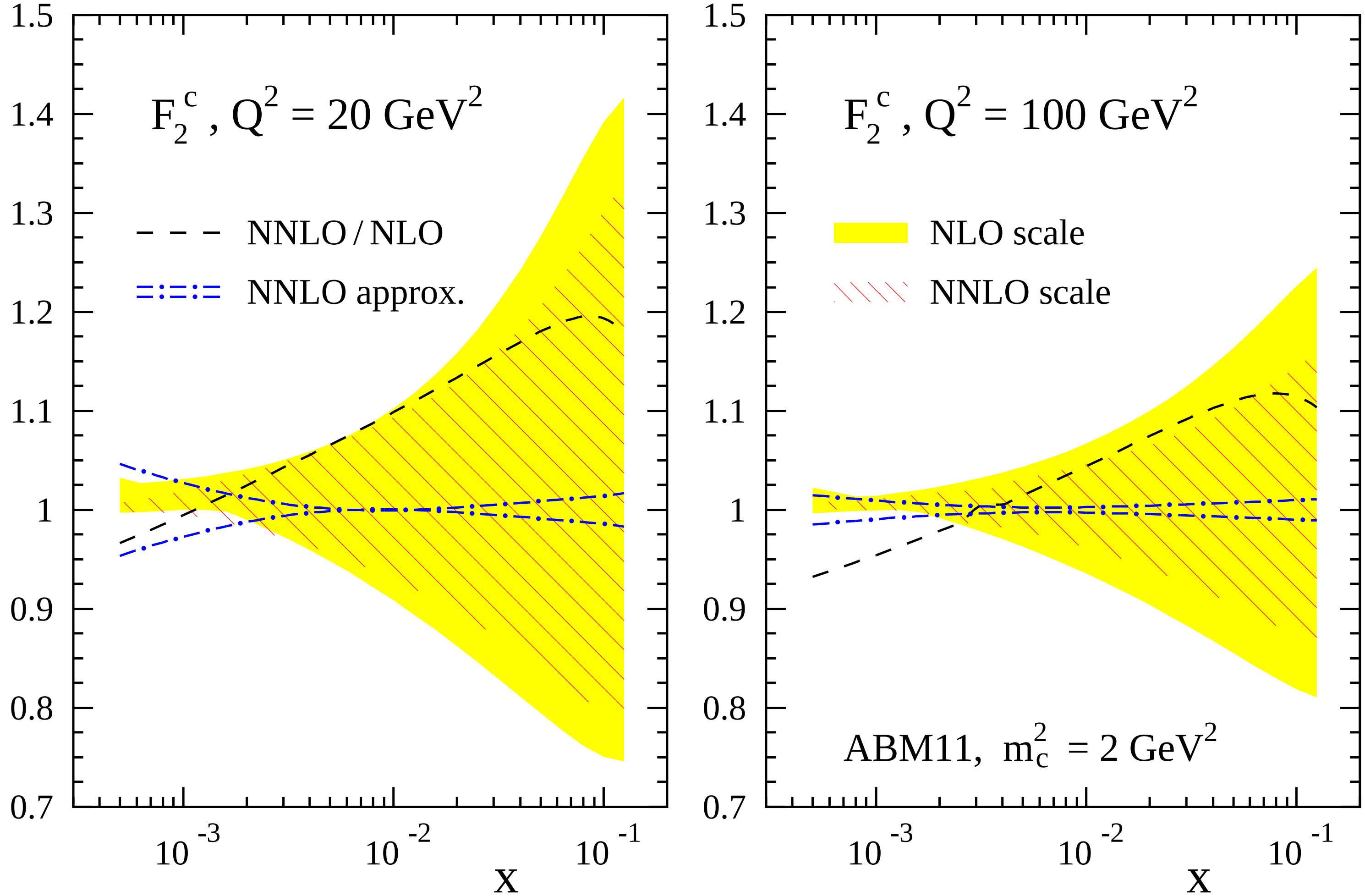


FIG. 3. Comparison of theoretical predictions to the doubly-differential cross sections measured by NuTeV for charm-quark production through neutrino DIS from iron.

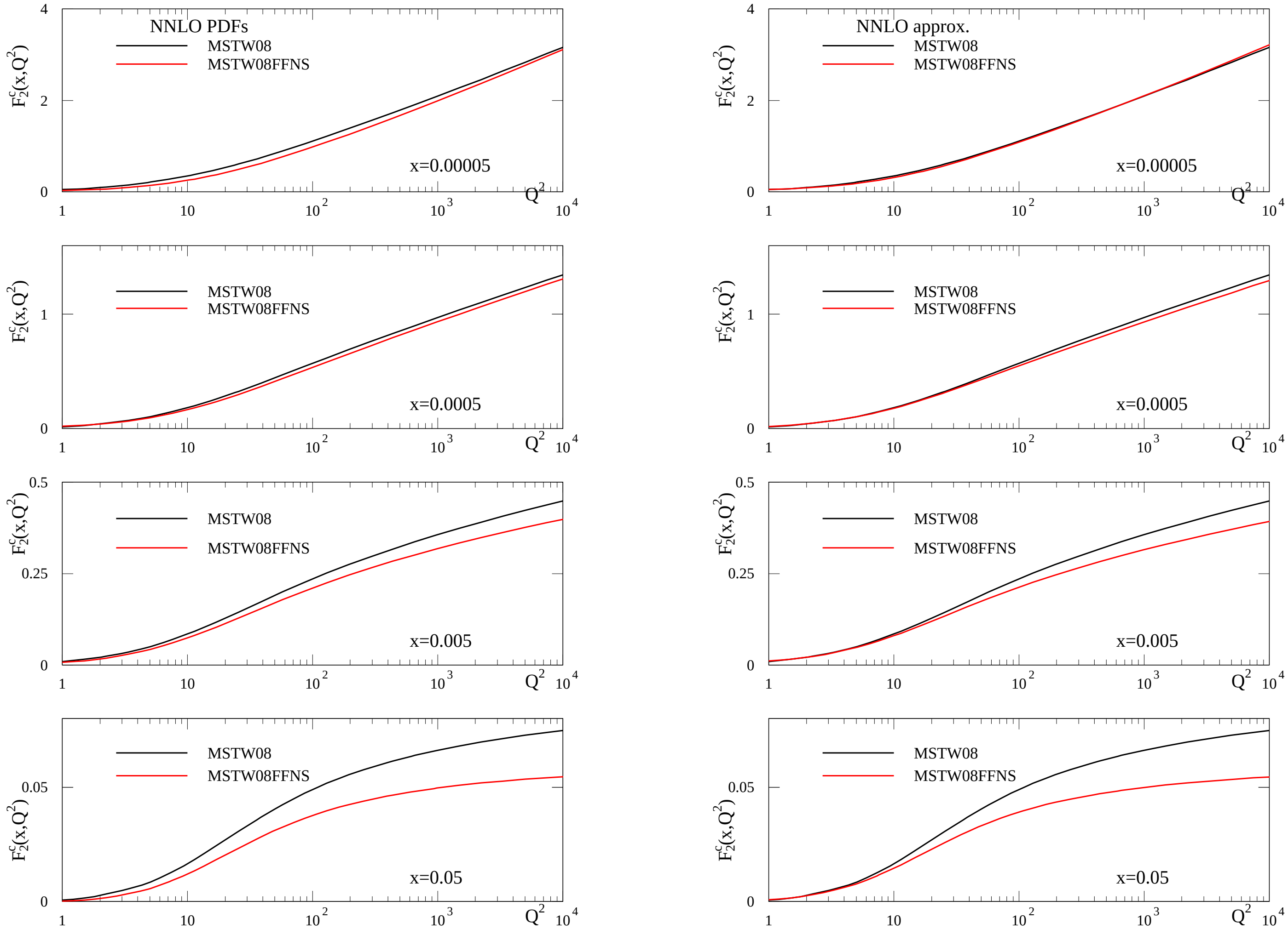
Difference between FFNS and GM-VFNS



Big difference at LO. At higher Q^2 charm structure function for FFNS nearly always lower than any GM-VFNS at NLO, but mainly at higher x.



Approximate $\mathcal{O}(\alpha_S^3)$ corrections to $F_2^c(x, Q^2)$ by Kawamura *et al.* in Nucl.Phys. B864 (2012) 399-468.



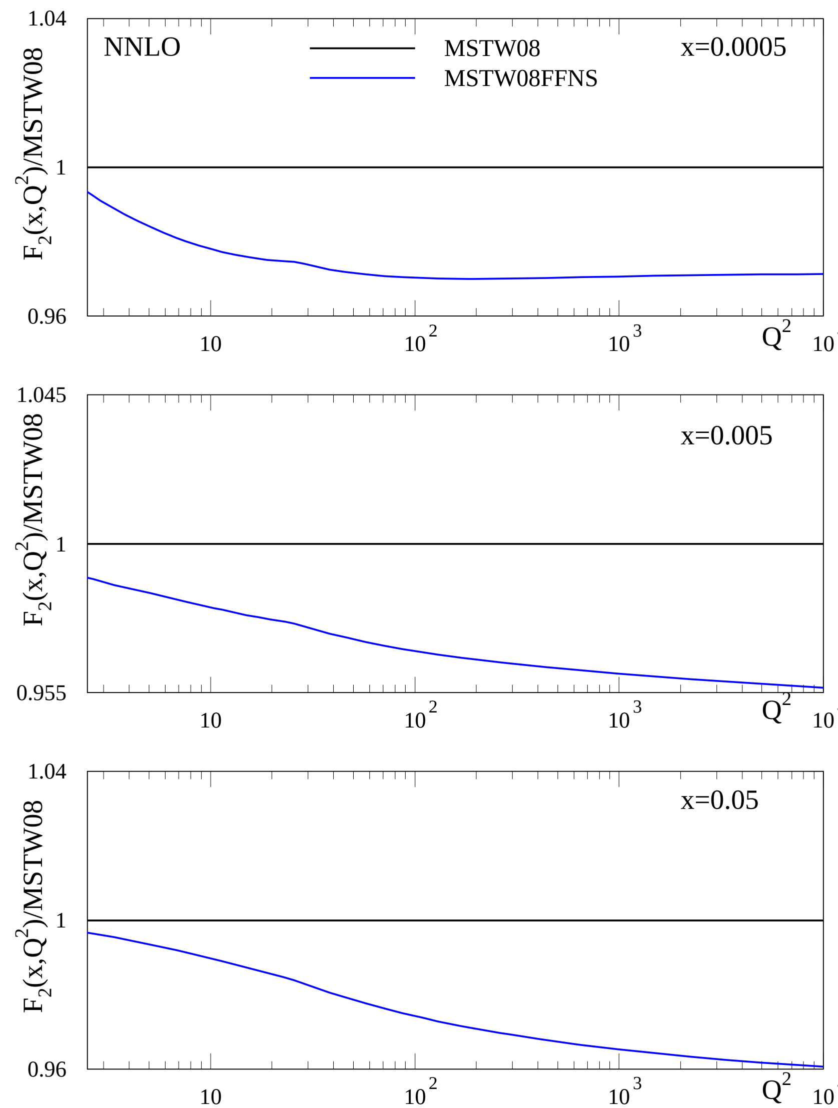
No dramatic change or improvement at NNLO. Left only NNLO PDFs, right uses $\mathcal{O}(\alpha_S^2)$ coefficient functions for $F_2^c(x,Q^2)$. Little difference at high Q^2.

Scheme can lead to over 4% changes in the total $F_2(x,Q^2)$ if the same input PDFs are used.

At higher x mainly due to $F_2^c(x,Q^2)$.

At lower x there is a large contribution from light quarks evolving slightly more slowly in FFNS.

At much higher x difference dies away. Charm component becomes very small and light quark evolution not much different. (Light quarks slightly bigger at the highest x.)

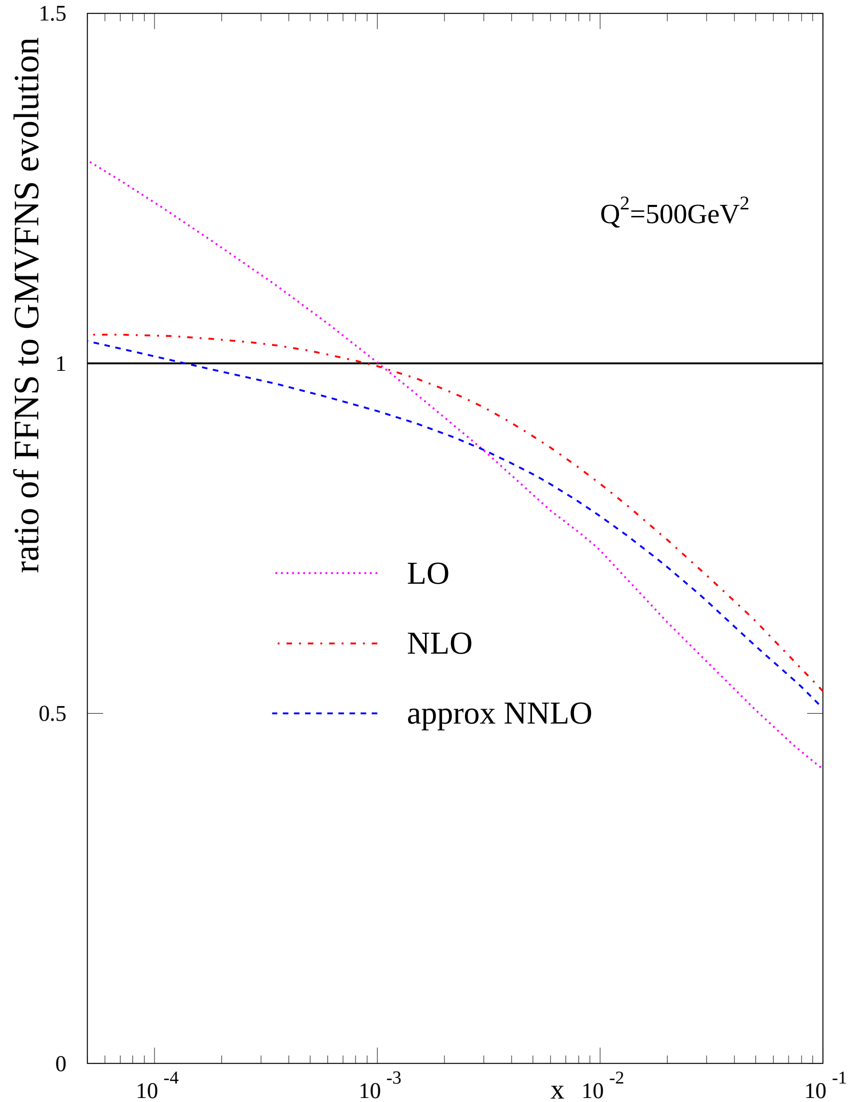


Understanding the differences between FFNS and GM-VFNS

Consider comparison of evolution, i.e. $dF_2^c/d\ln Q^2$ at high Q^2 where $\mathcal{O}(m_c^2/Q^2)$ contributions negligible.

General form of difference in evolution of F_2^c at $Q^2 = 500\mathrm{GeV}^2$.

Can we understand this?



Start at LO where (setting all scales as Q^2)

$$F_2^{c,1,FF} = \alpha_S \ln(\frac{Q^2}{m_c^2}) p^0_{qg} \otimes g + \mathcal{O}(\alpha_S \cdot g) \equiv \alpha_S A^{1,1}_{Hg} \otimes g + \mathcal{O}(\alpha_S \cdot g).$$

Calculating rate of change of evolution

$$\frac{d\,F_2^{c,1,FF}}{d\ln Q^2} = \alpha_S p^0_{qg} \otimes g + \ln(\frac{Q^2}{m_c^2}) \frac{d\,(\alpha_S p^0_{qg} \otimes g)}{d\ln Q^2}.$$

At leading-log in GM-VFNS where $F_2^{c,1,VF} = (c+\bar{c}) = c^+$

$$\frac{d\,c^+}{d\ln Q^2} = \alpha_S\, p^0_{qg} \otimes g + \alpha_S\, p^0_{qq} \otimes c^+$$

where

$$c^+ \equiv \alpha_S \ln(\frac{Q^2}{m_c^2}) p^0_{qg} \otimes g + \cdots \equiv \alpha_S A^{1,1}_{Hg} \otimes g + \cdots$$

so the second term is formally $\mathcal{O}(\alpha_S^2 \ln(\frac{Q^2}{m_c^2}))$.

The first two terms are of order α_S and are equivalent, but the difference between the two evolutions at LO is

$$\frac{d\,(F_2^{c,1,VF} - F_2^{c,1,FF})}{d\ln Q^2} = \alpha_S^2 \ln(\frac{Q^2}{m_c^2})\left(p^0_{qg}\otimes p^0_{qq}\otimes g - \frac{d\,(\alpha_S p^0_{qg}\otimes g)}{d\ln Q^2}\right) + \cdots$$

$$\equiv \alpha_S^2 \ln(\frac{Q^2}{m_c^2}) p^0_{qg}\otimes (p^0_{qq} + \beta_0 - p^0_{gg})\otimes g + \cdots$$

where $\beta_0 = \frac{9}{4\pi}$ and the effect of p^0_{gg} is negative at high x and positive at small x and that of p^0_{qq} is negative at high x, but smaller than of p^0_{gg}.

Hence the difference is positive and large at high x and large and negative at small x, exactly as observed.

Moreover, this difference can only be eliminated at NLO by defining the leading-log term in the NLO FFNS expression precisely to provide cancellation, i.e.

$$F_2^{c,2,FF} = \alpha_S^2 A^{2,2}_{Hg}\otimes g = \frac{1}{2}\alpha_S^2 \ln^2(\frac{Q^2}{m_c^2}) p^0_{qg}\otimes(p^0_{qq}+\beta_0-p^0_{gg})\otimes g + \mathcal{O}(\alpha_S^2 \ln(\frac{Q^2}{m_c^2})).$$

up to corrections involving quark mixing in evolution and possible sub-dominant scheme-dependent terms.

Looking at evolution at NLO all previous $\mathcal{O}(\alpha_S^2 \ln(\frac{Q^2}{m_c^2}))$ terms cancel between GM-VFNS and FFNS.

However, the derivative of $F_2^{c,2,FF}$ contains a contribution

$$\frac{1}{2}\ln^2(\frac{Q^2}{m_c^2})\frac{d\left(\alpha_S^2 p_{qg}^0 \otimes (p_{qq}^0 + \beta_0 - p_{gg}^0) \otimes g\right)}{d\,\ln Q^2}$$

which does not cancel. This leads to

$$\frac{1}{2}\alpha_S^3 \ln^2(\frac{Q^2}{m_c^2}) p_{qg}^0 \otimes (p_{qq}^0 + \beta_0 - p_{gg}^0) \otimes (p_{qq}^0 + 2\beta_0 - p_{gg}^0) \otimes g + \cdots .$$

The additional factor of $(p_{qq}^0 + 2\beta_0 - p_{gg}^0)$ is large, positive at high x and negative at small x, but not until smaller x than previously. Therefore, the term which convolutes the gluon is large and positive at high x, negative for a range of smaller x and positive for extremely small x. Explains behaviour correctly.

Moreover, to cancel this term at NNLO the dominant part of $F_2^{c,2,FF}$ at leading-log is (up to quark-mixing and scheme-dependent terms)

$$\alpha_S^3 A_{Hg}^{3,3} \otimes g = \frac{1}{6}\alpha_S^3 \ln^3(\frac{Q^2}{m_c^2}) p_{qg}^0 \otimes (p_{qq}^0 + \beta_0 - p_{gg}^0) \otimes (p_{qq}^0 + 2\beta_0 - p_{gg}^0) \otimes g.$$

Repeating the argument we find that at NNLO the dominant high-Q^2 uncancelled term between GM-VFNS and FFNS is

$$\frac{1}{6}\alpha_S^4 \ln^3(\frac{Q^2}{m_c^2}) p_{qg}^0 \otimes (p_{qq}^0 + \beta_0 - p_{gg}^0) \otimes (p_{qq}^0 + 2\beta_0 - p_{gg}^0) \otimes (p_{qq}^0 + 3\beta_0 - p_{gg}^0) \otimes g.$$

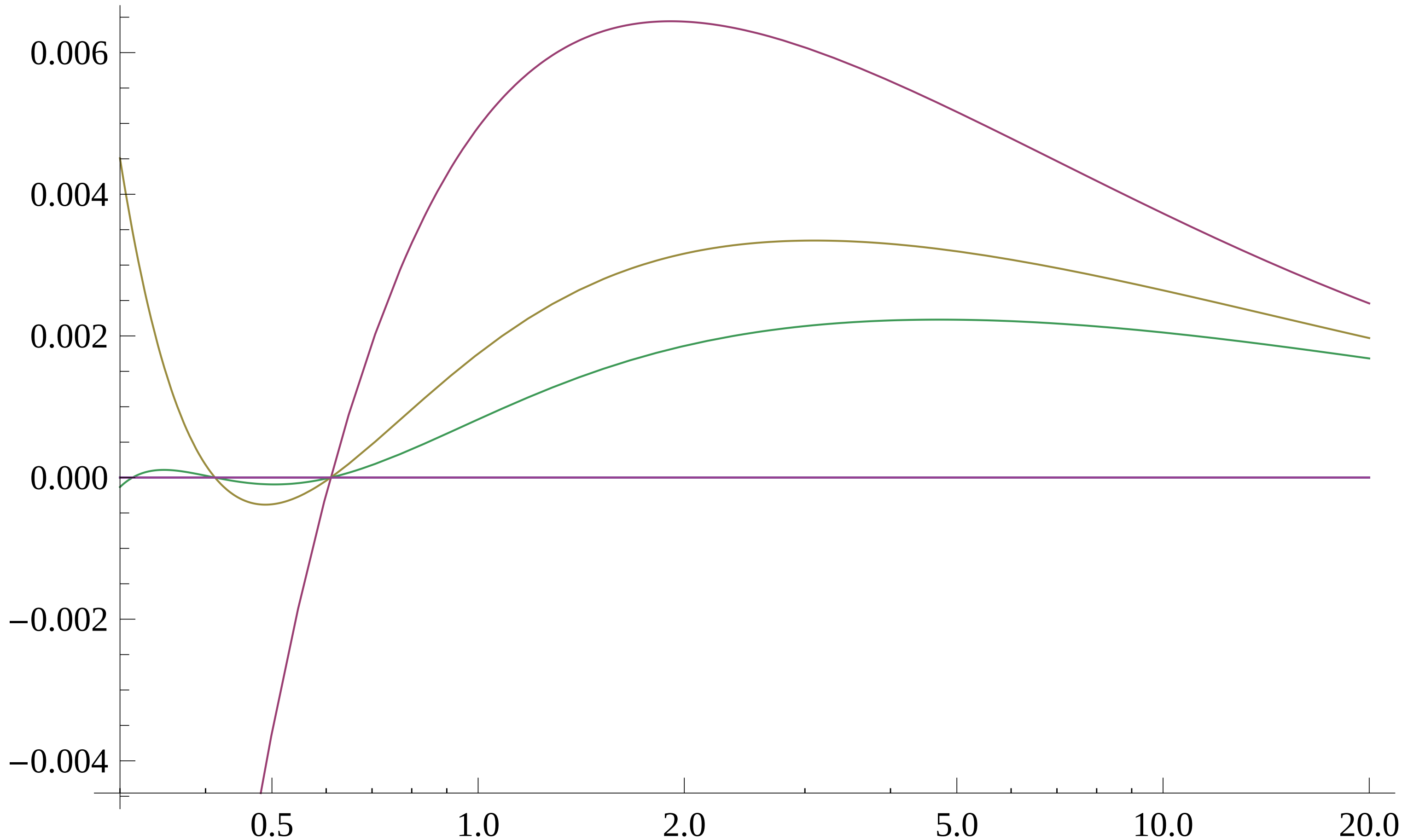
This remains large and positive at high x and changes sign twice but stays small at smaller x until becoming negative at tiny x.

Again explains behaviour correctly.

Can be generalised to higher orders. Similar in some sense to results from expression in Maltoni, Ridolfi and Ubiali, JHEP 1207 (2012) 022 for bottom quark, but this neglected evolution of gluon and hence p_{gg}^0 terms – actually the dominant effect at lowish orders.

Can look at the effect of this dominant high-Q^2 difference between GM-VFNS and FFNS in more detail.

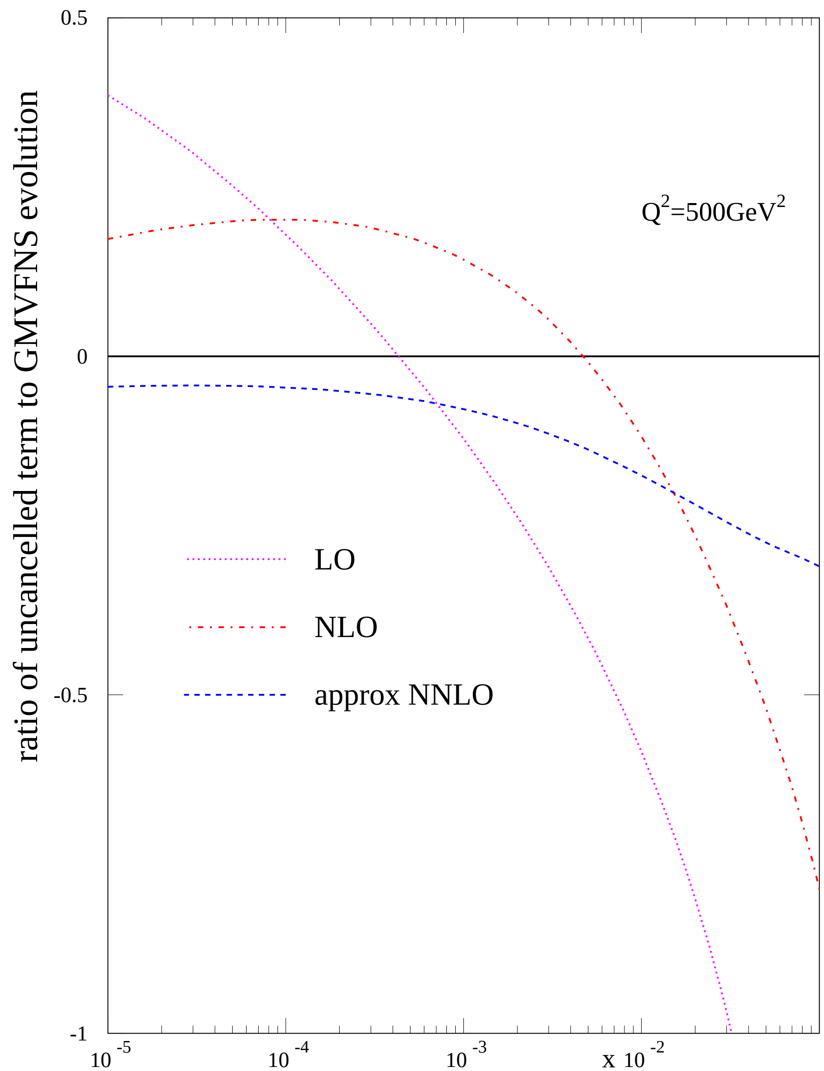
Moments of the dominant difference terms at LO, NLO and NNLO. LO in purple, NLO in brown and NNLO in green.



Fractional effect of dominant difference term between GM-VFNS and FFNS evolution at the various orders.

Precise form of the effect depends on form of gluon. Much steeper at LO than at NLO or NNLO.

Describes the general form of the difference in evolution between GM-VFNS and FFNS very well (though precise details depend on sub-dominant terms.



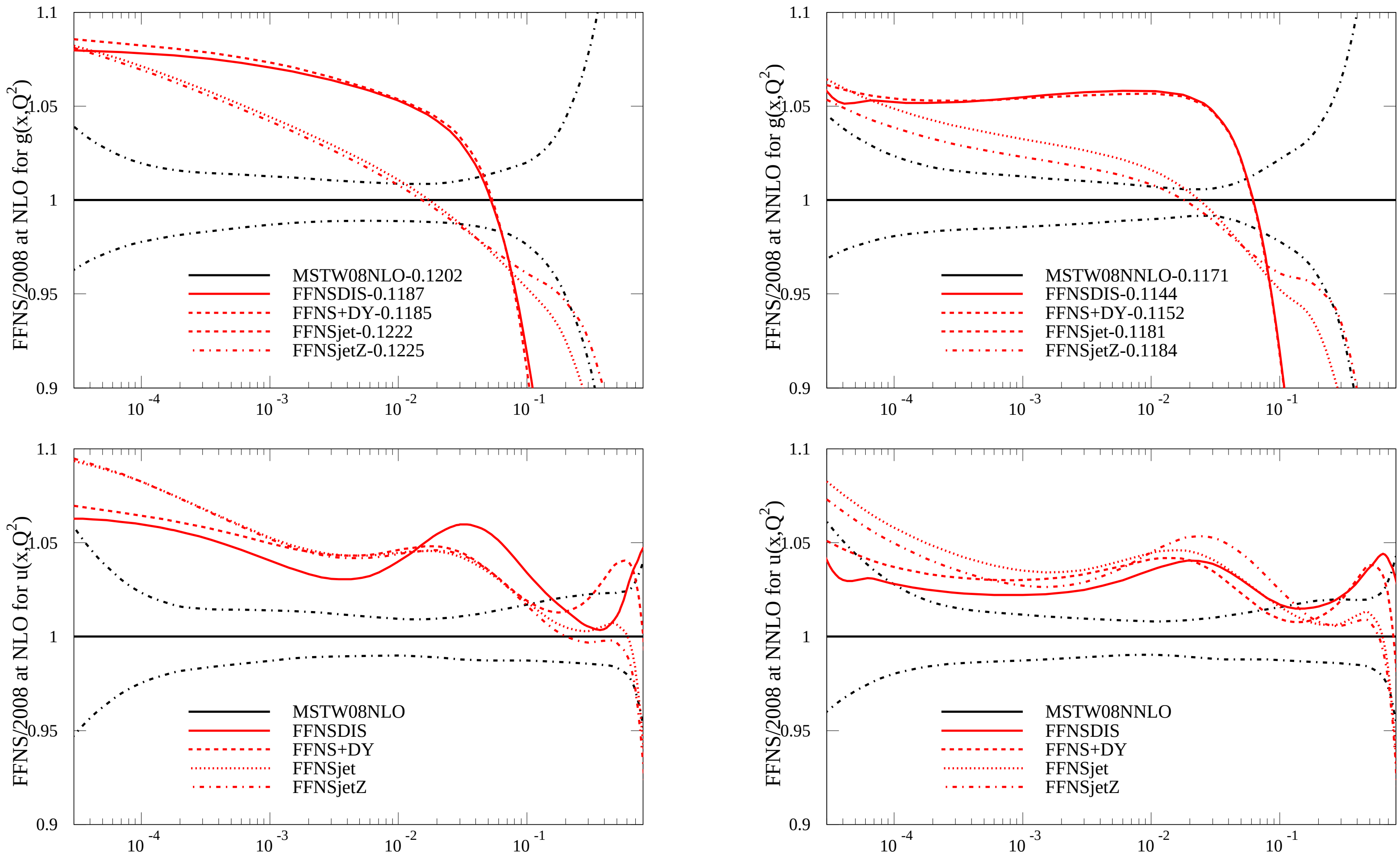
Results/consequences

Performed a series of NLO fits using the FFNS scheme and NNLO with up to $\mathcal{O}(\alpha_S^2)$ heavy flavour coefficient functions. (Approximations to the $\mathcal{O}(\alpha_S^3)$ expressions change results very little).

Fits to DIS and Drell-Yan data usually at least a few tens of units worse than MSTW08 to same data (even without refitting MSTW08 to restricted data sets). FFNS can be slightly better for published $F_2^c(x,Q^2)$ than GM-VFNS, but is flatter in Q^2 for $x \sim 0.01$ for inclusive structure function.

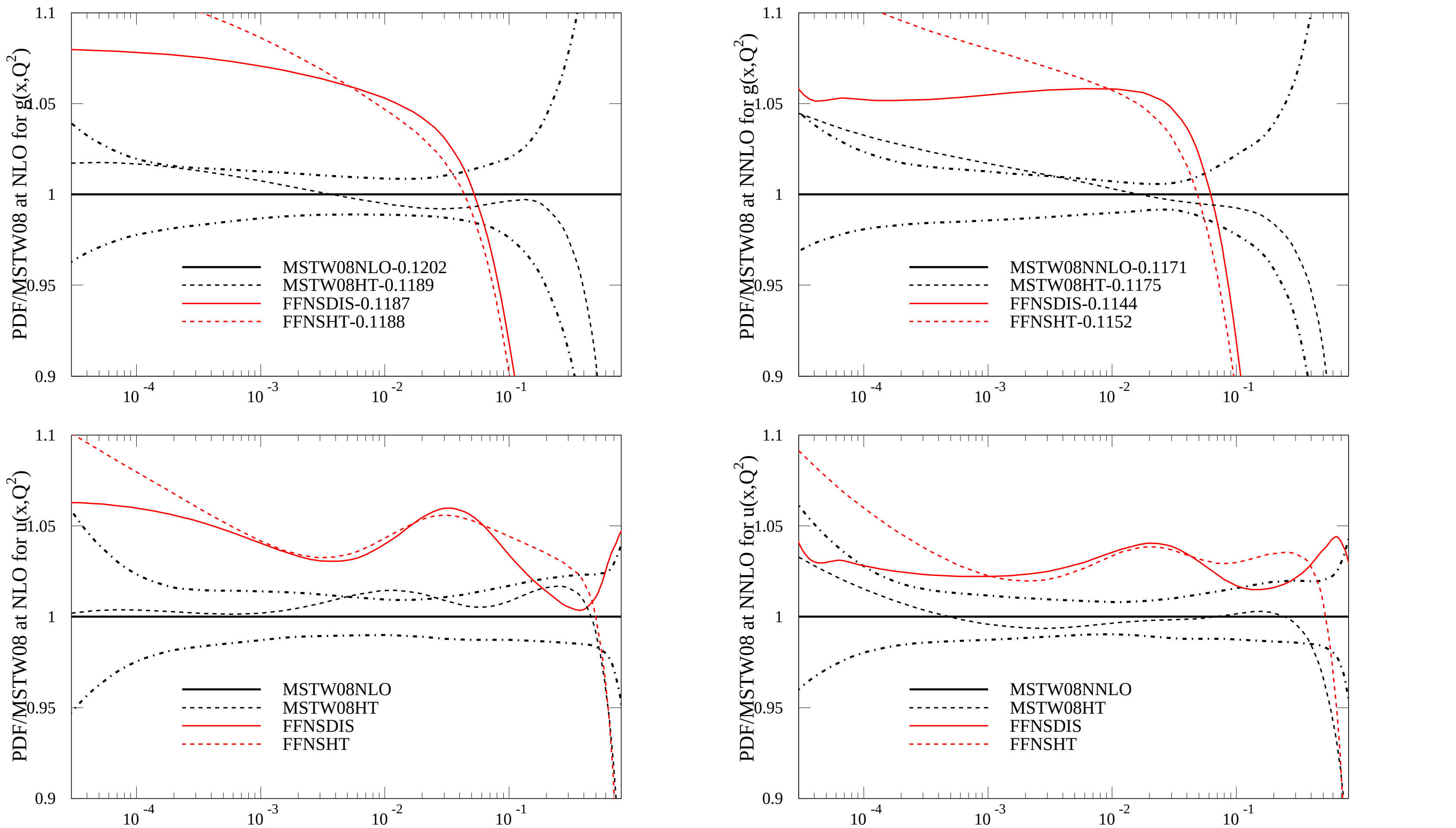
As well as (usually) a worse fit to DIS and Drell-Yan data only, in FFNS the fit quality for the DIS and low-energy Drell Yan data deteriorates by in general ~ 50 units when all jet data is included as opposed to < 10 units when using a GM-VFNS.

PDFs evolved up to $Q^2 = 10,000\mathrm{GeV}^2$ (using variable flavour evolution for consistent comparison) different in form to MSTW08. Similar differences found by NNPDF and older ZEUS fits.

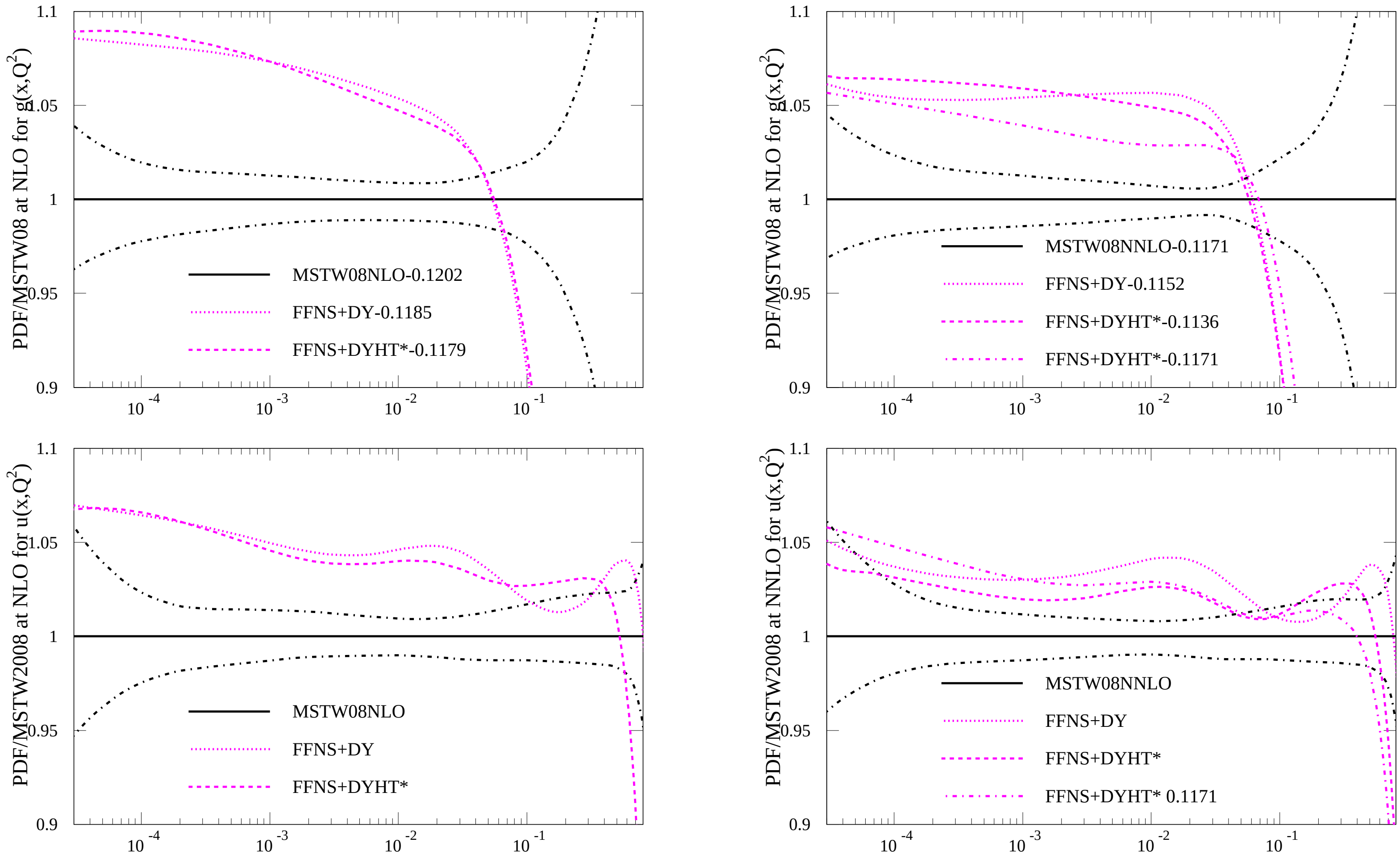


Using FFNS leads to much larger changes than any choice of GM-VFNS mainly due to fitting high-Q^2 DIS data.

Low Q^2 – Higher Twist.



Not a big effect. Largely washes out quickly with Q^2. Similar effect using FFNS as for GM-VFNS.



Restricting higher twist from lowest x and omitting nuclear target data (except dimuon for strangeness) $\rightarrow \alpha_S$ for FFNS lower by ~ 0.02. Fixing α_S reduces effect on gluon (see also NNPDF).

Total fit quality better using GM-VFNS

NNLO				
	χ^2 DIS 2198pts	χ^2 ftDY 199pts	χ^2 jets 186pts	$\alpha_S^{n_f=5}(M_Z^2)$
MSTW2008 HT	2039	241	175	0.1175
MSTW2008 HT* (DIS+ftDY)	2014	233	(193)	0.1175
MSTW$n_f = 3$ HT (DIS only)	2088		(>300)	0.1152
MSTW$n_f = 3$ HT* (DIS only)	2130		(>300)	0.1132
MSTW$n_f = 3$ HT* (DIS + ftDY)	2145	229	(>300)	0.1136
MSTW$n_f = 3$ HT* (jets)	2174	246	183	0.1152
MSTW$n_f = 3$ HT* (jets+Z)	2179	253	173	0.1174
MSTW$n_f = 3$ HT* (DIS+fyDY)	2150	232	(>300)	0.1171

Table 5: The χ^2 values for DIS data, fixed target Drell Yan (ftDY) data and Tevatron jet data for various NNLO fits performed using the GM-VFNS used in the MSTW 2008 global fit and using the $n_f = 3$ FFNS for structure functions with reduced cuts and higher twist terms added.

Indication from HERAI+II final inclusive data?

MMHT (without higher twist)

(χ^2 definitions may differ slightly.)

$$\chi^2/N_{pts} = 1443/1168 = 1.235$$

$$\chi^2/N_{pts} = 1310/1092 = 1.20$$

$$\chi^2/N_{pts} = 1197/1007 = 1.19$$

ABM (Alekhin – DIS2016)

Q^2(HERA)	χ^2/NDP(HERA)
>2.5 GeV^2	1505/1168=1.29
>5 GeV^2	1350/1092=1.24
>10 GeV^2	1225/1007=1.22

Can be many tens of units discrepancy in χ^2 for inclusive data. (NNPDF numbers slightly lower and CT slightly higher than MMHT.)

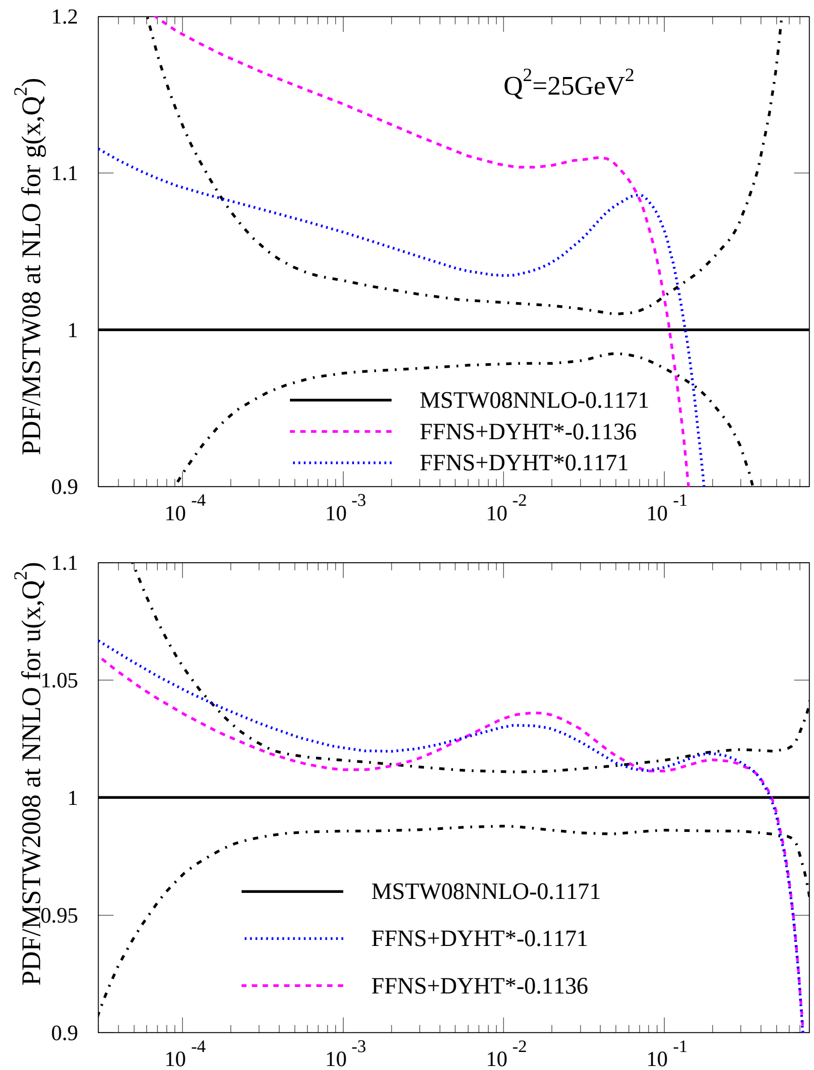
Why is α_S lower in FFNS?

FFNS fit 8 units worse if $\alpha_S(M_Z^2) = 0.1171$. HERA data better, fixed target worse.

Comparing schemes, look at parton ratios at lower Q^2 where evolution must match data, and respective $\alpha_S(M_Z^2)$ values are 0.1171 and 0.1136.

Gluon needs to be bigger at $x \sim 0.01$-0.1 – smaller at high x – to fit data. Feeds to lower x at higher Q^2.

Inverse correlation between high-x gluon and α_S. Without high-x gluon quark evolution too quick. Need lower α_S.



PDFs and Heavy Quarks for MMHT

As before we made the standard PDFs sets (i.e. exactly the same input at $Q_0^2 = 1\ \mathrm{GeV}^2$) available for three flavour and four flavour fixed-flavour number schemes (FFNS).

As default fix the number of flavours in α_S, but we also provide analogous sets with variable flavour α_S for $n_f = 4$ as there were some requests for this for MSTW2008.

We have also made available sets with fits done for m_c and m_b (defined in pole scheme) varying from default values of $m_c = 1.40\ \mathrm{GeV}$ and $m_b = 4.75\ \mathrm{GeV}$ in steps of $0.05\ \mathrm{GeV}$ and $0.25\ \mathrm{GeV}$ respectively.

Might expect $m_c^{\mathrm{pole}} = 1.5 \pm 0.2\ \mathrm{GeV} \quad \mathrm{and} \quad m_b^{\mathrm{pole}} = 4.9 \pm 0.2\ \mathrm{GeV}$ from conversion of m_b from $\overline{MS}$ definition and $m_b^{\mathrm{pole}} - m_c^{\mathrm{pole}} = 3.4\ \mathrm{GeV}$ with a very small uncertainty `(hep-ph/0509195, hep-ph/0408002)`, where renormalon ambiguity cancels. Fit preference for $m_c \sim 1.25\mathrm{GeV}$ - low but not inconsistent. Pole or $\overline{MS}$ definition most desired?

m_b constrained to fairly close to $m_b = 4.75\ \mathrm{GeV}$ from direct $F_2^{\bar{b}b}(x, Q^2)$ data from HERA.

Variation of Cross Sections with quark masses Use $\Delta m_c = \pm 0.15\ \mathrm{GeV}$ and $\Delta m_b = \pm 0.5\ \mathrm{GeV}$.

	σ	PDF unc.	m_c var.	m_b var.
W Tevatron (1.96 TeV)	2.78	$^{+0.0017}_{-0.056}\ \left(^{+2.0\%}_{-2.0\%}\right)$	$^{+0.0017}_{-0.0086}\ \left(^{+0.061\%}_{-0.31\%}\right)$	$^{-0.00092}_{-0.0015}\ \left(^{-0.033\%}_{-0.052\%}\right)$
Z Tevatron (1.96 TeV)	0.256	$^{+0.0052}_{-0.0046}\ \left(^{+2.0\%}_{-1.8\%}\right)$	$^{+0.00042}_{-0.0011}\ \left(^{+0.16\%}_{-0.43\%}\right)$	$^{-0.00029}_{-0.000016}\ \left(^{-0.11\%}_{-0.0059\%}\right)$
W^+ LHC (7 TeV)	6.20	$^{+0.103}_{-0.092}\ \left(^{+1.7\%}_{-1.5\%}\right)$	$^{+0.029}_{-0.040}\ \left(^{+0.48\%}_{-0.64\%}\right)$	$^{+0.0043}_{-0.014}\ \left(^{+0.070\%}_{-0.22\%}\right)$
W^- LHC (7 TeV)	4.31	$^{+0.067}_{-0.076}\ \left(^{+1.6\%}_{-1.8\%}\right)$	$^{+0.019}_{-0.022}\ \left(^{+0.44\%}_{-0.51\%}\right)$	$^{+0.0059}_{-0.0091}\ \left(^{+0.14\%}_{-0.21\%}\right)$
Z LHC (7 TeV)	0.964	$^{+0.014}_{-0.013}\ \left(^{+1.5\%}_{-1.3\%}\right)$	$^{+0.0074}_{-0.0088}\ \left(^{+0.77\%}_{-0.92\%}\right)$	$^{-0.00096}_{-0.00038}\ \left(^{-0.10\%}_{-0.039\%}\right)$
W^+ LHC (14 TeV)	12.5	$^{+0.22}_{-0.18}\ \left(^{+1.8\%}_{-1.4\%}\right)$	$^{+0.091}_{-0.12}\ \left(^{+0.73\%}_{-0.93\%}\right)$	$^{+0.0087}_{-0.037}\ \left(^{+0.069\%}_{-0.30\%}\right)$
W^- LHC (14 TeV)	9.3	$^{+0.15}_{-0.14}\ \left(^{+1.6\%}_{-1.5\%}\right)$	$^{+0.064}_{-0.075}\ \left(^{+0.69\%}_{-0.81\%}\right)$	$^{+0.012}_{-0.029}\ \left(^{+0.13\%}_{-0.31\%}\right)$
Z LHC (14 TeV)	2.06	$^{+0.035}_{-0.030}\ \left(^{+1.7\%}_{-1.5\%}\right)$	$^{+0.021}_{-0.025}\ \left(^{+1.03\%}_{-1.2\%}\right)$	$^{-0.0035}_{-0.0013}\ \left(^{-0.17\%}_{-0.062\%}\right)$

	σ	PDF unc.	m_c var.	m_b var.
$t\bar{t}$ Tevatron (1.96 TeV)	7.5	$^{+0.21}_{-0.20}\ \left(^{+2.8\%}_{-2.7\%}\right)$	$^{-0.059}_{+0.077}\ \left(^{-0.78\%}_{+1.0\%}\right)$	$^{+0.0088}_{+0.0015}\ \left(^{+0.12\%}_{+0.20\%}\right)$
$t\bar{t}$ LHC (7 TeV)	176	$^{+3.9}_{-5.5}\ \left(^{+2.2\%}_{-3.1\%}\right)$	$^{-1.1}_{+1.4}\ \left(^{-0.60\%}_{+0.77\%}\right)$	$^{+0.77}_{-0.009}\ \left(^{+0.44\%}_{-0.0051\%}\right)$
$t\bar{t}$ LHC (14 TeV)	970	$^{+16}_{-20}\ \left(^{+1.6\%}_{-2.1\%}\right)$	$^{-3.0}_{+3.1}\ \left(^{-0.31\%}_{+0.32\%}\right)$	$^{+3.1}_{-1.7}\ \left(^{-0.32\%}_{+0.17\%}\right)$

	σ	PDF unc.	m_c var.	m_b var.
Higgs Tevatron (1.96 TeV)	0.87	$^{+0.024}_{-0.030}\ \left(^{+2.7\%}_{-3.4\%}\right)$	$^{-0.0060}_{+0.0070}\ \left(^{-0.68\%}_{+0.79\%}\right)$	$^{+0.0042}_{-0.0011}\ \left(^{+0.48\%}_{-0.13\%}\right)$
Higgs LHC (7 TeV)	14.6	$^{+0.21}_{-0.29}\ \left(^{+1.4\%}_{-2.0\%}\right)$	$^{+0.025}_{-0.019}\ \left(^{+0.17\%}_{-0.13\%}\right)$	$^{+0.049}_{-0.044}\ \left(^{+0.34\%}_{-0.30\%}\right)$
Higgs LHC (14 TeV)	47.7	$^{+0.63}_{-0.88}\ \left(^{+1.3\%}_{-1.8\%}\right)$	$^{+0.27}_{-0.22}\ \left(^{+0.57\%}_{-0.48\%}\right)$	$^{+0.16}_{-0.16}\ \left(^{+0.34\%}_{-0.33\%}\right)$

Variations small but not insignificant. Easily understood from PDF behaviour. Suggest adding in quadrature.

Ratios of PDFs obtained with different active flavour numbers.

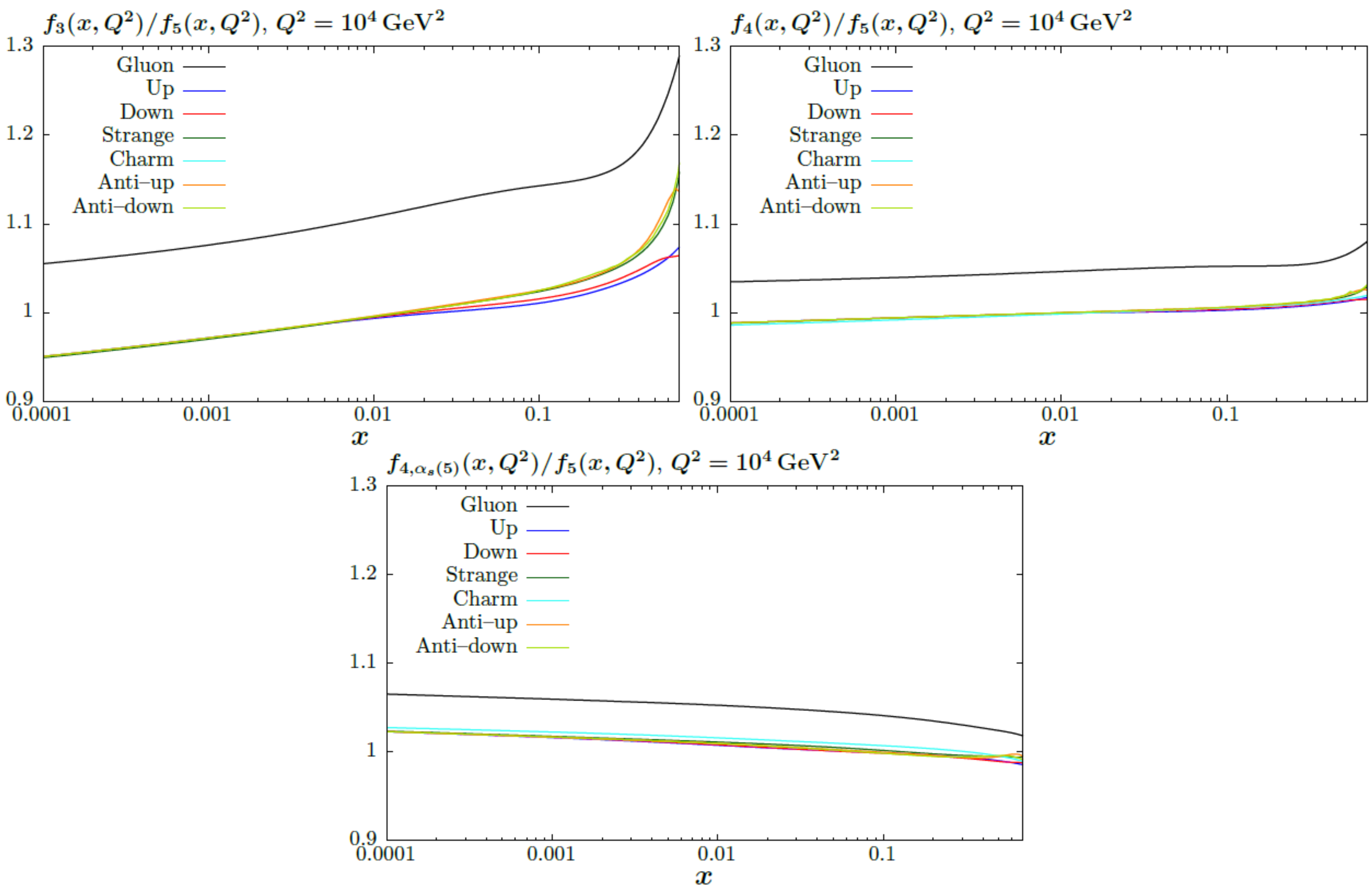


Figure 12: The ratio of the different fixed flavour PDFs to the standard 5 flavour PDFs at NNLO and at $Q^2 = 10^4$ GeV^2. The 3 and 4 flavour schemes are show in the top left and right plots, while the 4 flavour scheme with 5 flavours in the running of α_S is shown in the bottom plot.

Intrinsic charm

Formerly of higher twist, i.e. $\mathcal{O}(\Lambda^2/m_c^2)$.

Possible enhancement at high-x, like large higher twist expected at low W^2.

Therefore no expected constraint from HERA data.

Tried fitting EMC data. Overshoot lower x data even at NLO with dynamical charm.

High-x intrinsic charm with modified coefficient functions, $m_c^2 \to m_c^2 + \Lambda^2$, at threshold works ok.

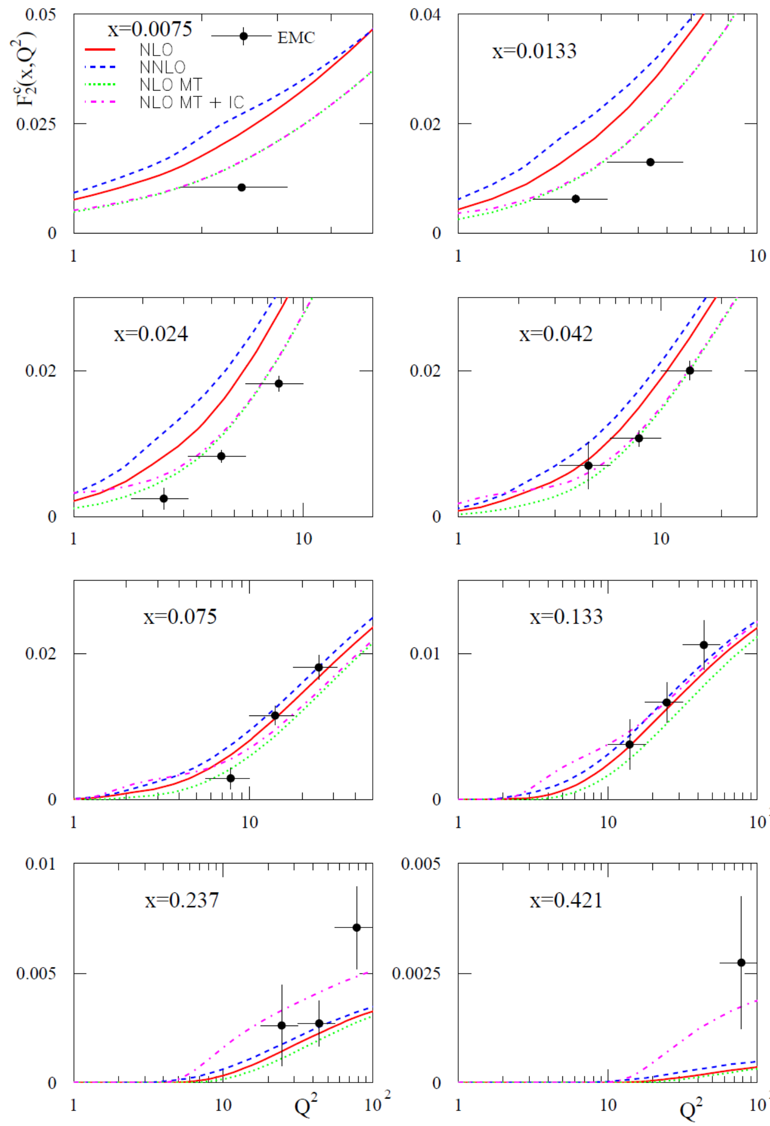


Figure 39: The comparison of the EMC charm data [165] to our predictions at NLO and NNLO. MT stands for the modified threshold approach and IC stands for inclusion of intrinsic charm.

Strictly speaking ambiguity in individual coefficient functions in GM-VFNS which vanishes at all orders for only dynamical heavy flavour is $\mathcal{O}(m_c^2/Q^2)$.

Coupled with magnitude of intrinsic charm, i.e. $\mathcal{O}(\Lambda^2/m_c^2)$ leads to an uncertainty/error of cross sections from intrinsic charm of $\mathcal{O}(\Lambda^2/Q^2)$, i.e. of standard higher twist corrections.

May be significant if intrinsic charm enhanced in some region, e.g. high-x, i.e. region of large higher twist effects to inclusive cross section.

Unsure about inclusion of significant component of higher twist away fro high x.

Conclusions

MMHT/MSTW/MRST have been using a GM-VFNS since 1998. Versions have evolved, but all based on the same basic principles.

Massless evolution for heavy PDFs $\rightarrow$ $\overline{MS}$ PDFs and cross sections as $Q^2/m^2 \rightarrow \infty$. All mass effects in coefficient functions. So far for structure functions – generalisation to other processes will be required.

Believe GM-VFNS preferable to FFNS since it leads to better fit quality (though prefers lowish $m_c^{\rm pole}$) with $\alpha_S(M_Z^2)$ happily consistent with world average. Use of PDFs normally at high scales.

Can be translated to $\overline{MS}$ mass definition. Is this what is wanted in practice?

Little investigation of intrinsic charm so far. Not a strong belief in significant low-x (< 0.1) contribution (some loss of predictive power?).

LHC predictions fairly insensitive to m_c, m_b values and choice of GM-VFNS scheme (particularly at NNLO). For masses probably settle on a common value/renormalisation scheme in future – like $\alpha_S(M_z^2)$, but far less urgent/important.

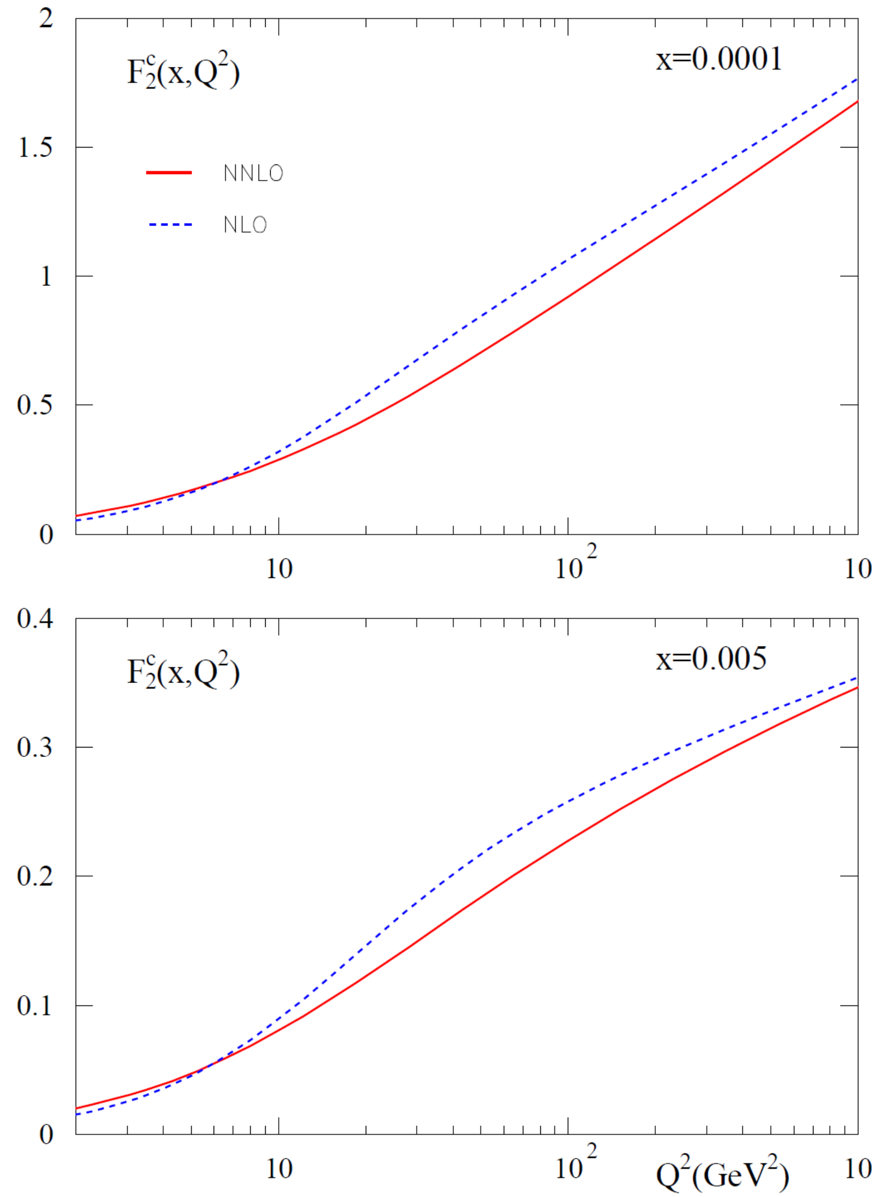
Back-up

NNLO consequences.

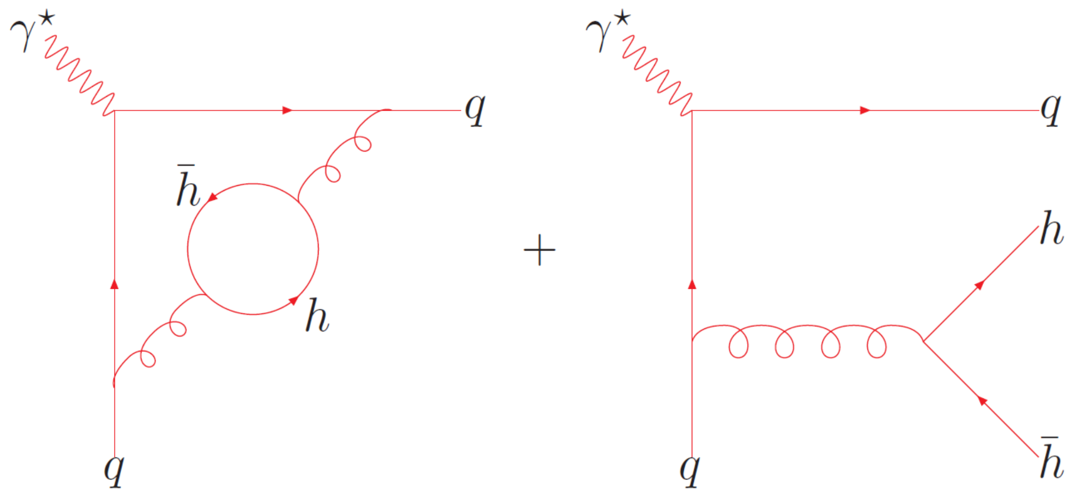
NNLO $F_2^c(x,Q^2)$ starts from higher value at low Q^2.

At high Q^2 dominated by $(c+\bar{c})(x,Q^2)$. This has started evolving from negative value at $Q^2=m_c^2$. Remains lower than at NLO for similar evolution.

General trend – $F_2^c(x,Q^2)$ flatter in Q^2 at NNLO than at NLO. Important effect on gluon distribution going from one to other.



Remember caveat at NNLO. At NNLO also get contribution due to heavy flavours away from photon vertex.

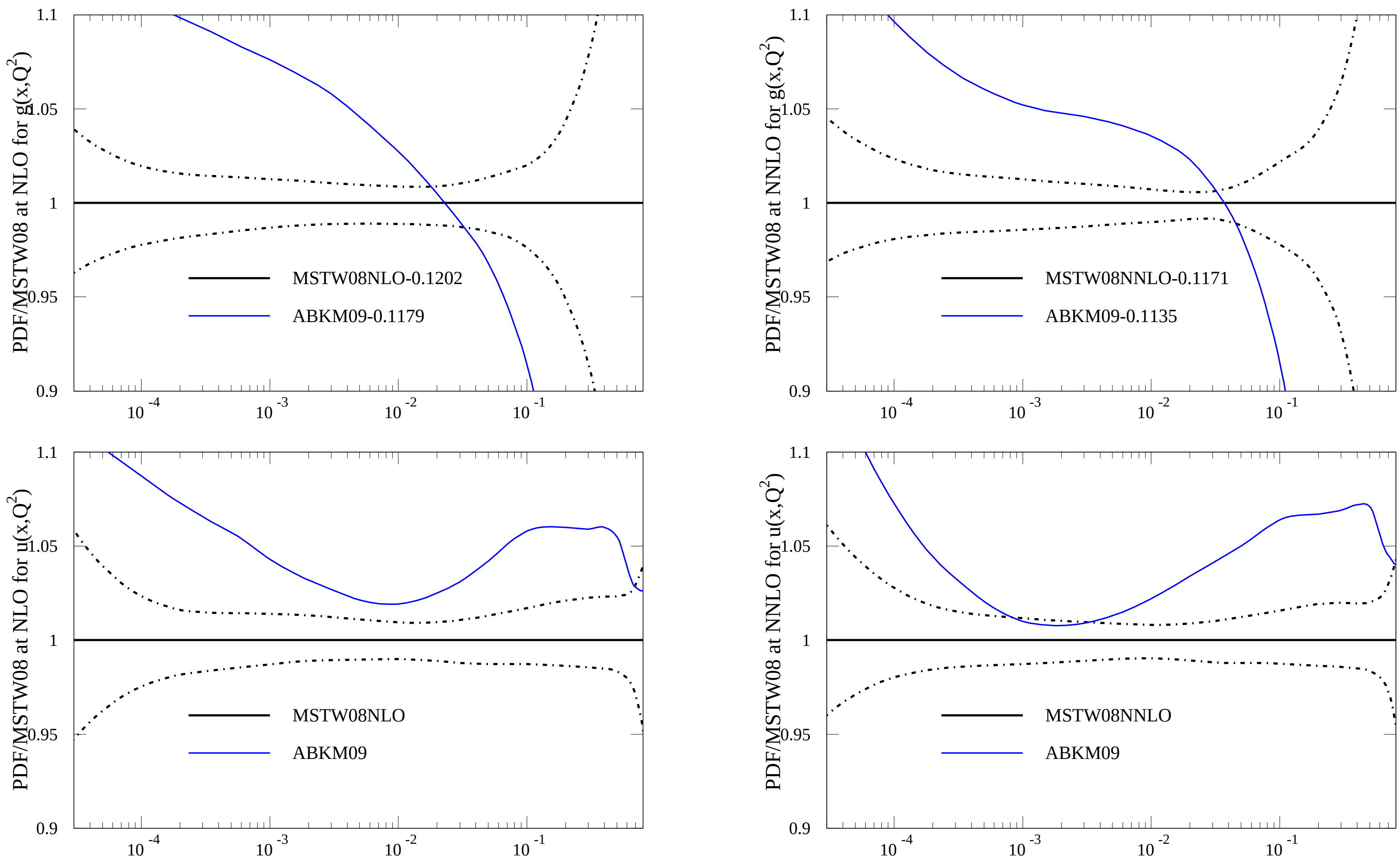


Strictly, left-hand type diagram and soft parts of right-hand type diagram should be light flavour structure function, and hard part of right-hand type diagram contributes to $F_2^H(x,Q^2)$ (Chuvakin, Smith, van Neerven).

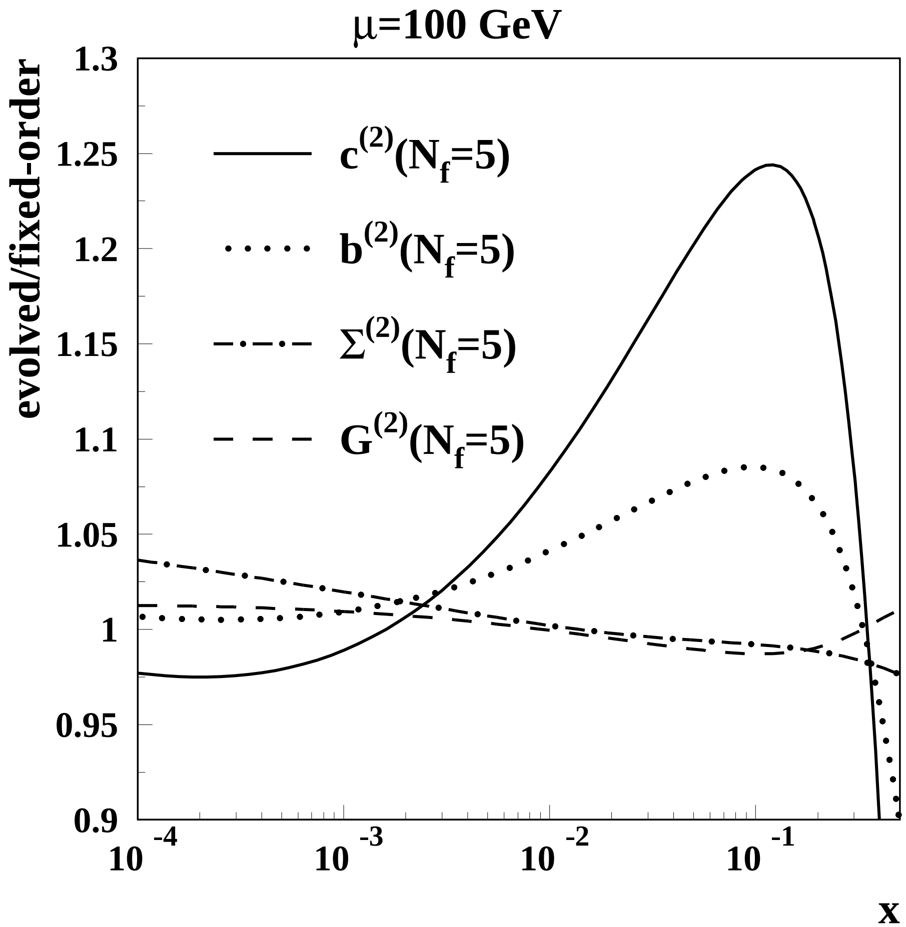
Soft part of right cancels $\ln^3(Q^2/m-H^2)$ divergences in virtual corrections (left).

Can be implemented (depends on separation parameter), but each contribution tiny. At moment all in light flavours. Not so small if $\ln^3(Q^2/m-H^2)$ terms not cancelled.

Explains some PDF differences? MSTW FFNS ratios and ABKM ratios.



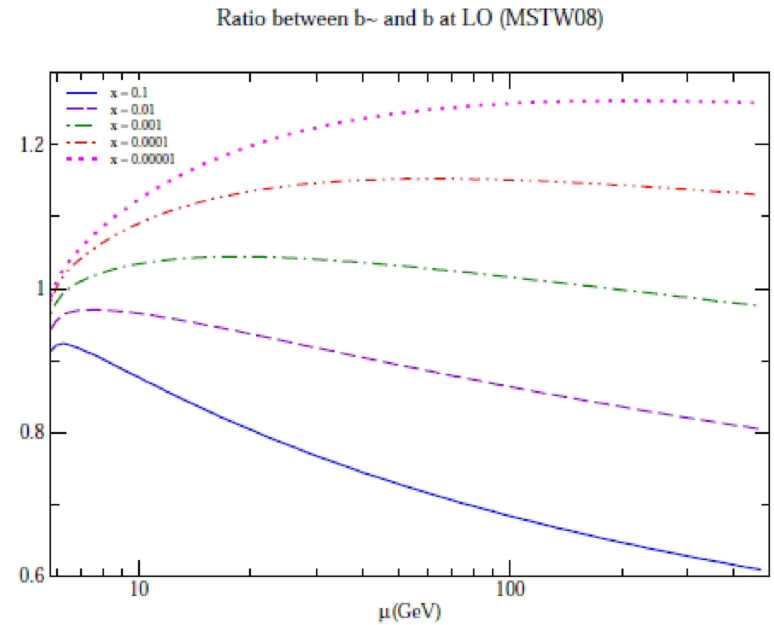
General trend is very similar to fits on previous page.

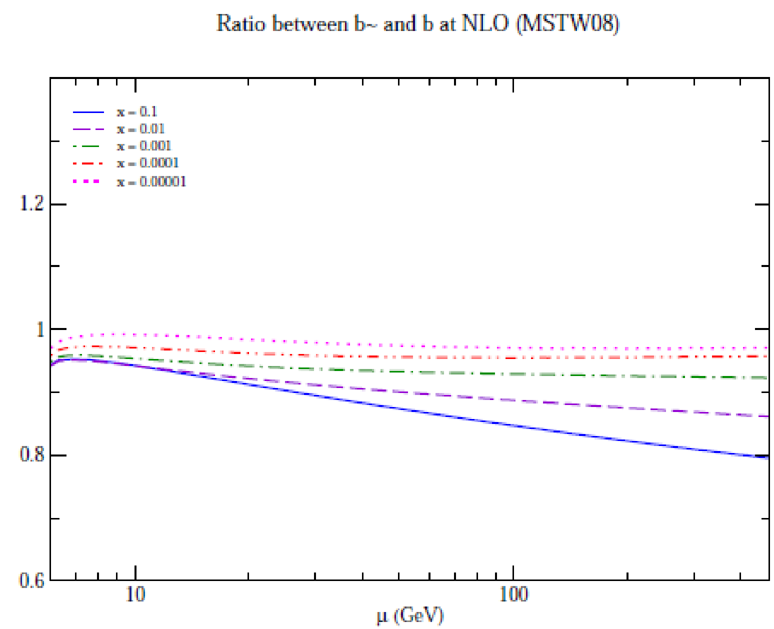


Results for $F_2^c(x, Q^2)$ in GM-VFNS compared to those for FFNS similar to results for PDFs by Alekhin *et al.* in Phys.Rev. D81 (2010) 014032 comparing NNLO evolution to the fixed order result up to $\mathcal{O}(\alpha_S^2)$. Details depend on PDF set and $\alpha_S(M_Z^2)$ value used.

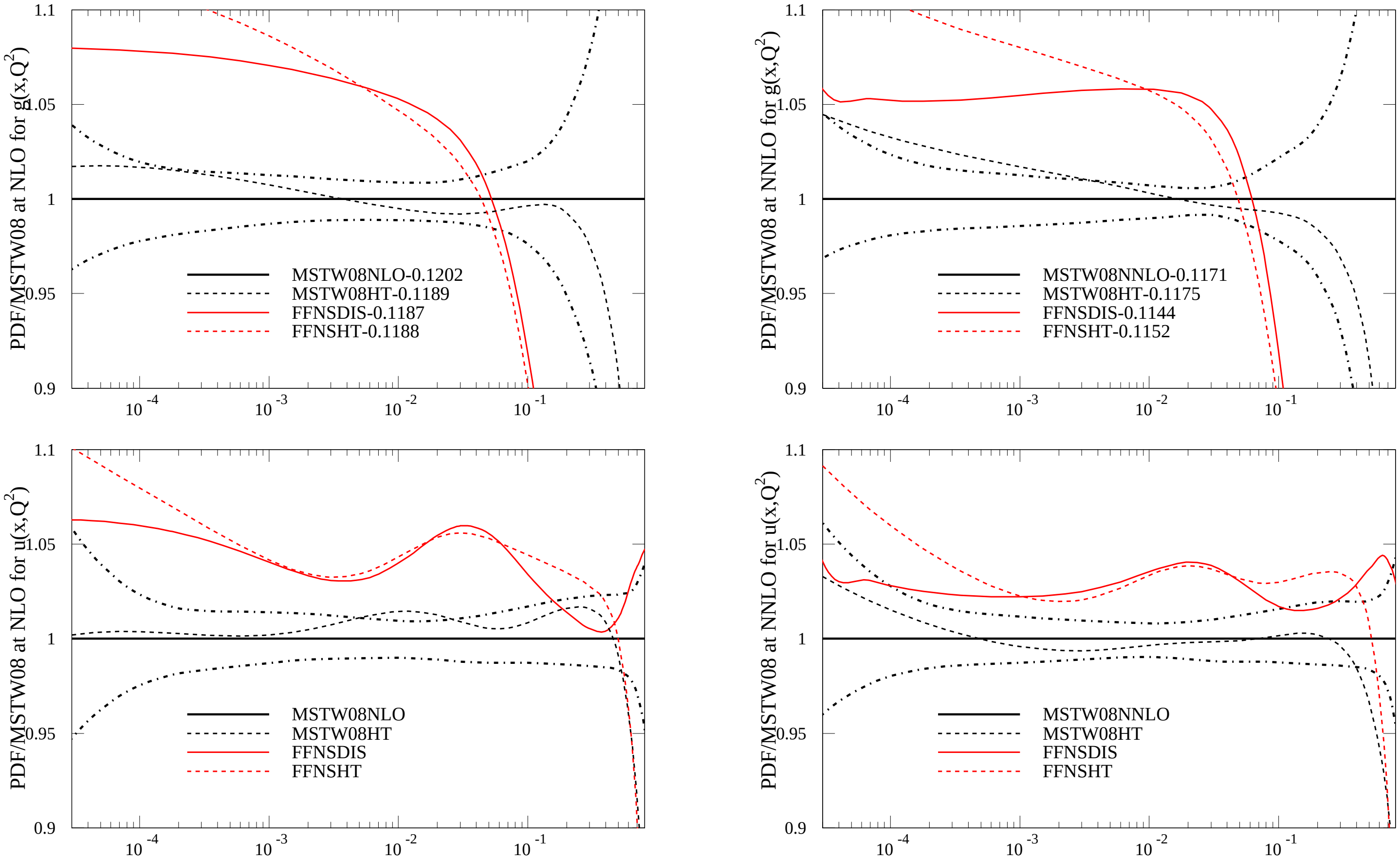
Also verified in evolution of bottom quark (Maltoni, *et al.*, JHEP 1207 (2012) 022).

In this case $\ln(Q^2/m_b^2)$ rather smaller.

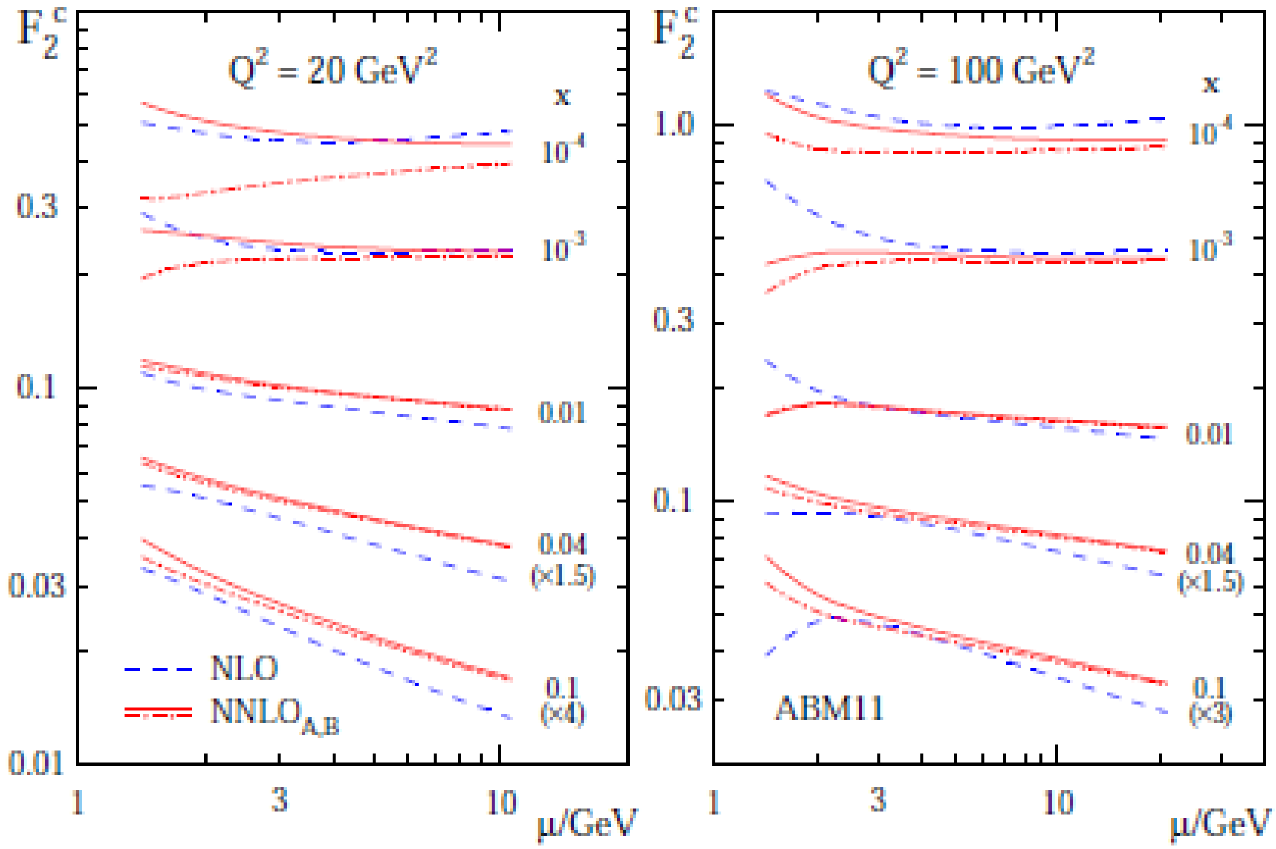




Low Q^2 – Higher Twist.



Now more evidence for positive contribution also at very low x. Leads to lower input quarks, more gluon for evolution. Largely washes out quickly with Q^2. Similar effect using FFNS as for GM-VFNS.

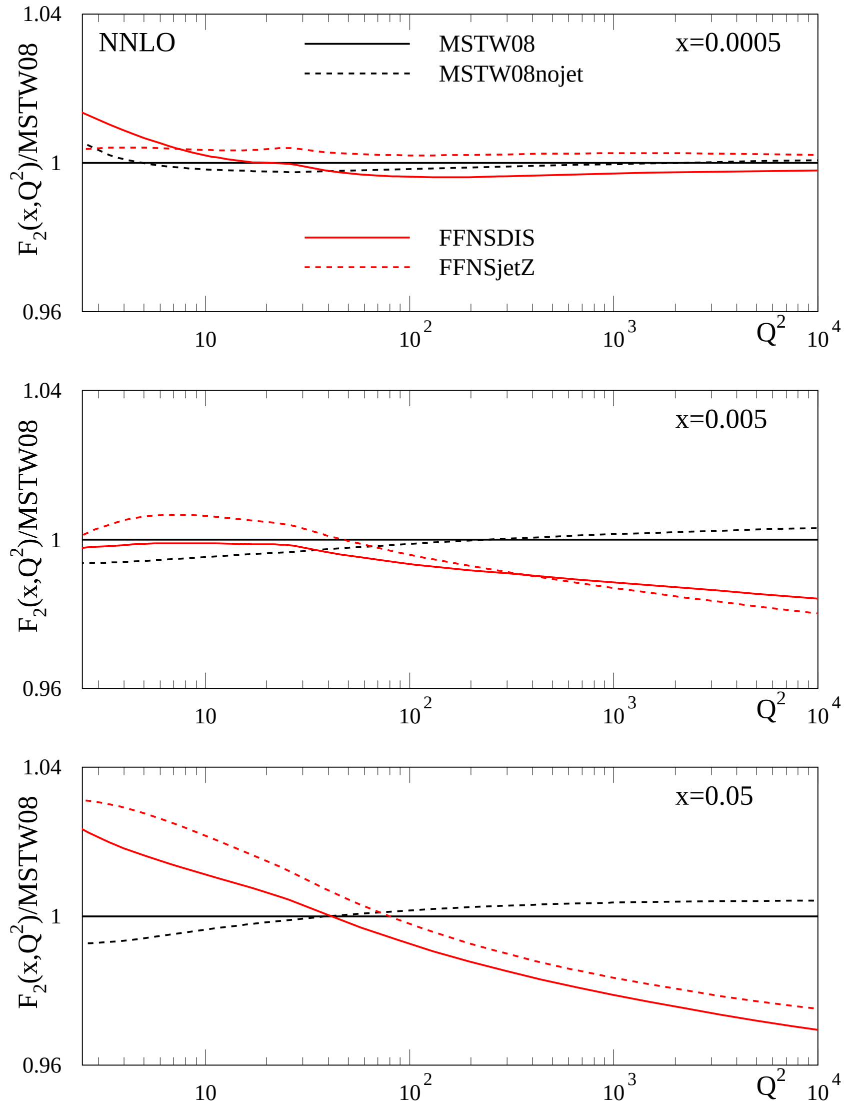


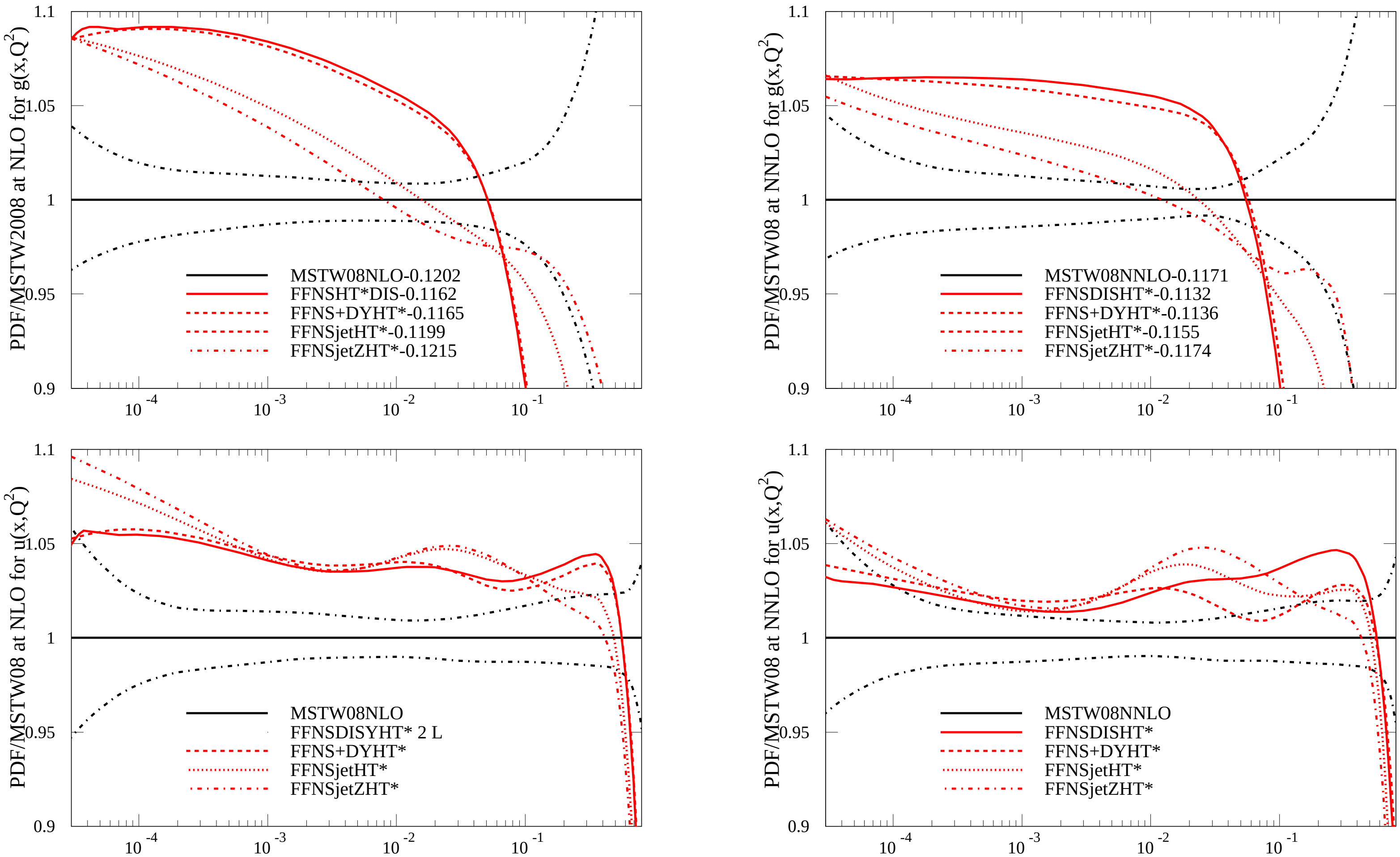
Scale dependence of $F_2^c(x, Q^2)$ using FFNS at NLO and approx. NNLO (Kawamura *et al.*).

The results for $F_2(x,Q^2)$ when refits are performed.

As seen very little change when using GM-VFNS with no jets.

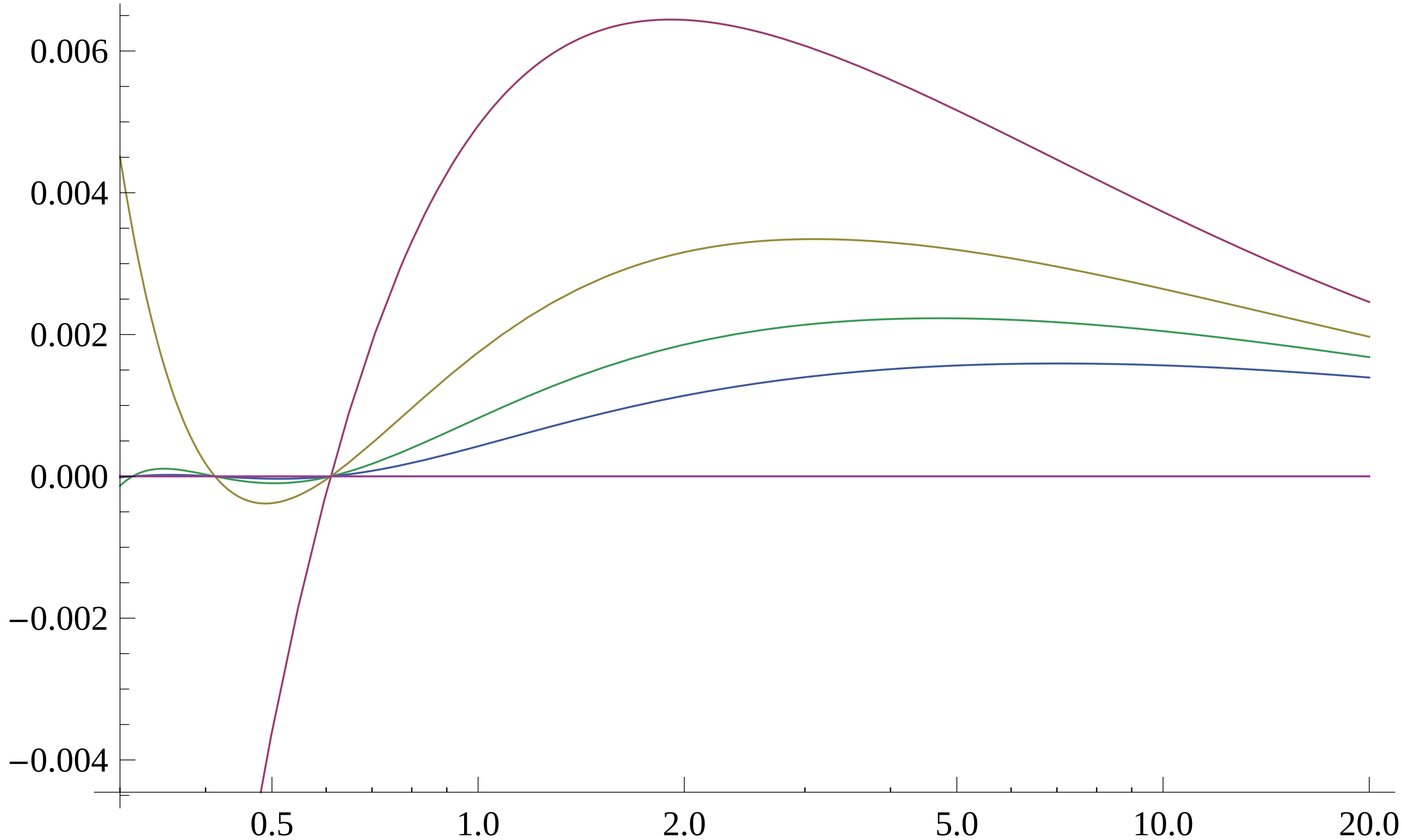
Much more tension and worse fits for FFNS.





Restricting higher twist from lowest x value and omitting nuclear target data (except dimuon for strangeness). Same trends as for standard fits but slightly lower α_S

Moments of the dominant difference terms at LO, NLO and NNLO, and also the term which would be dominant at NNNLO.



LO in purple, NLO in brown, NNLO in green and NNLO in blue.

MMHT2014 – Changes in theoretical treatment or procedures.

Continue to use extended parameterisation with Chebyshev polynomials, and freedom in deuteron nuclear corrections – change in $u_V - d_V$ distribution.

Now use "optimal" GM-VFNS choice which is smoother near to heavy flavour transition points (more so at NLO

Errors multiplicative not additive. Using χ^2 definition

$$\chi^2 = \sum_{i=1}^{N_{pts}} \left(\frac{D_i + \sum_{k=1}^{N_{corr}} r_k \sigma_{k,i}^{corr} - T_i}{\sigma_i^{uncorr}} \right)^2 + \sum_{k=1}^{N_{corr}} r_k^2,$$

where $\sigma_{k,i}^{corr} = \beta_{k,i}^{corr} T_i$ and $\beta_{k,i}^{corr}$ are the percentage error. Additive would use $\sigma_{k,i}^{corr} = \beta_{k,i}^{corr} D_i$.

Strange branching ratio. Now avoid those determined by fits to dimuon data relying on PDF input. Also apply error which feeds into PDFs. Use $B_\mu = 0.092 \pm 10\%$ from hep-ex/9708014.

Update in nuclear corrections (de Florian *et al*).

Intrinsic (Fitted) charm