FOUR/FIVE FLAVOUR SCHEMES AND THE BBH CASE

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In collaboration with:

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* [M. Lim (Cambridge), F. Maltoni (Louvain), G. Ridolfi (Genova)]

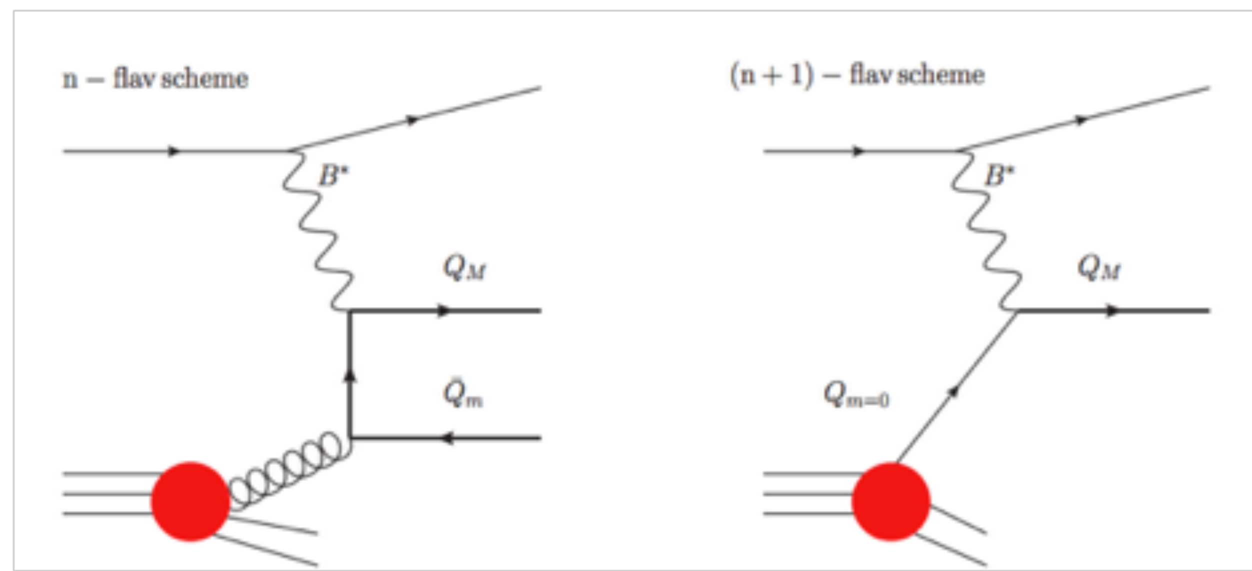
Outline of the talk

- 4F and 5F schemes: pros and cons
- An appraisal of current understanding
- A consistent matching procedure: FONLL
- Bottom-fusion initiated Higgs production
- Conclusion and outlook

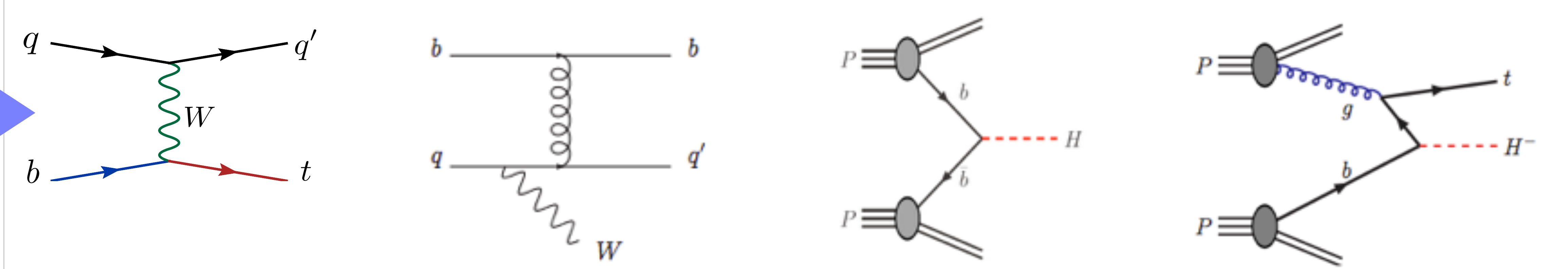
A rich phenomenology

$$\frac{d\sigma_H^{pp\to AB}}{dX} = \sum_{i,j=1}^{N_f} f_i(x_1,\mu_F)\, f_j(x_2,\mu_F)\, \frac{d\hat{\sigma}^{ij\to ab}}{dX}(x_1 x_2 S_{\rm had}, \alpha_S(\mu_R), \mu_R, \mu_F, m) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^{2n}}{S_{\rm had}^{n}}\right)$$

DIS



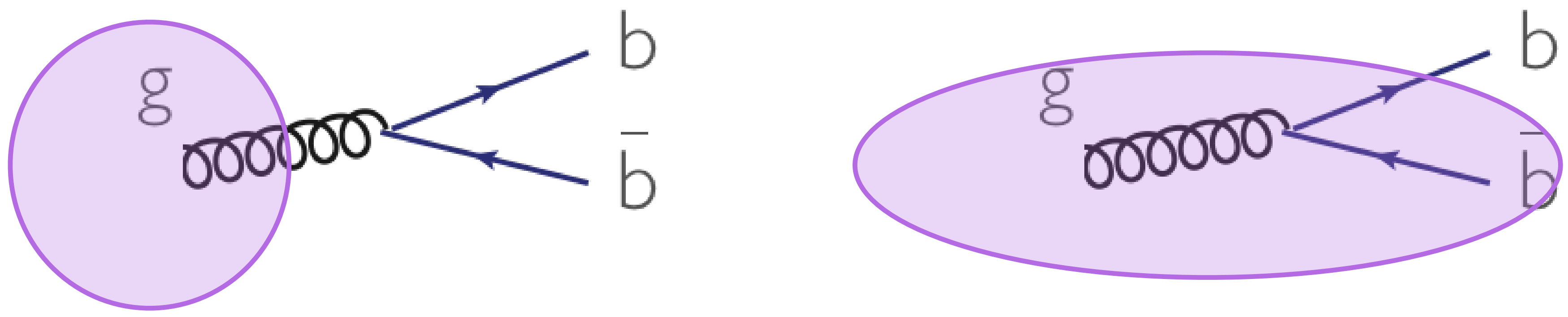
Hadron colliders



single top b+V production bb > H charged Higgs

4F and 5F schemes

- For all processes that feature bottom quarks at the hard-process level there are two ways of performing computations: **4F** and **5F** schemes



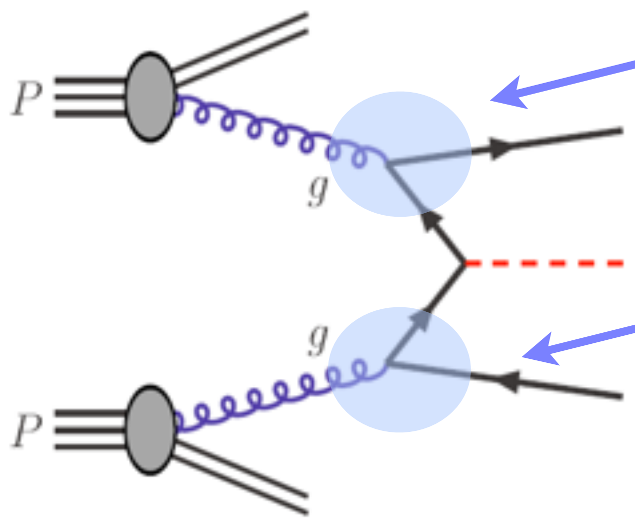
- Each supports the issues that arise in different kinematical regimes

If $m_b \sim Q$

4F scheme

- b quark treated as massive object at the level of short-distance xsec
- b quark never appears in the initial state
- In the short-distance xsec logarithms arise

4F and 5F schemes



$$\alpha_S^2 \log^2 \frac{\hat{s}}{m_b^2}$$

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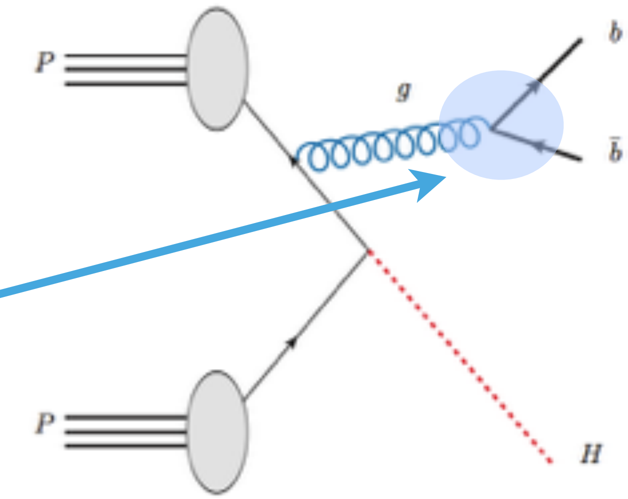


t-channel kinematics
Initial state

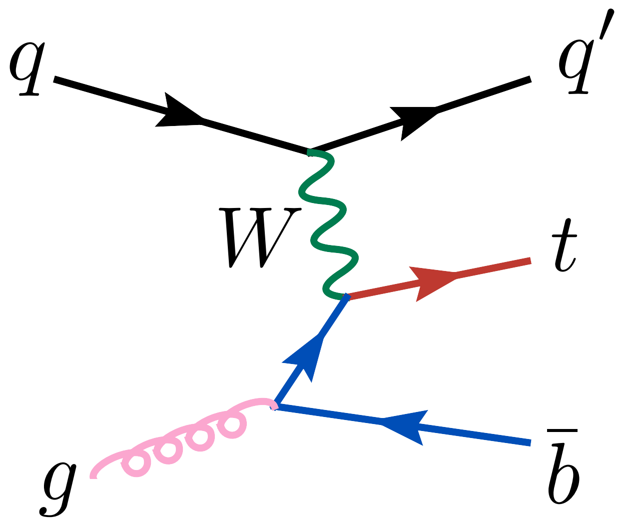
s-channel kinematics
Final state

These logs for $m_b << s$, might be large, possibly spoiling perturbation theory!!

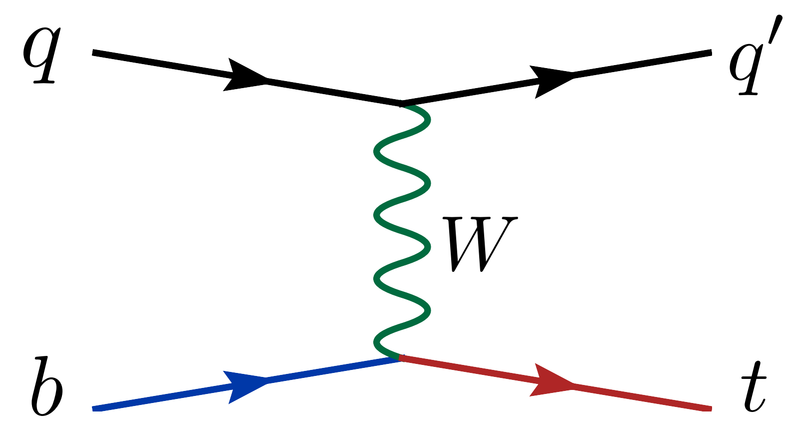
If logs dominate **5F scheme**

- b quark treated as a light parton generated at threshold $\mu_b \sim m_b$ from DGLAP evolution
- Set $m_b = 0$ in the short-distance xsec
- Resummation of the collinear logs achieved through DGLAP evolution equations for bottom PDFs

4F and 5F schemes



NLO correction in the 5FS



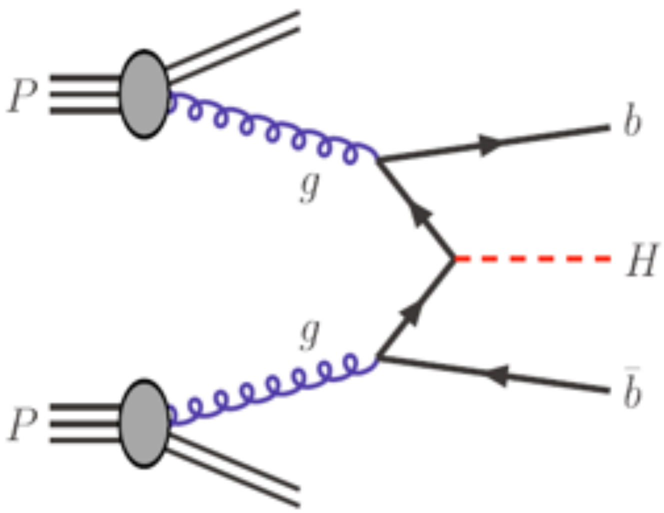
4F scheme

✘ It does not resum possibly large logs, yet it has them explicitly
✘ Computing higher orders is more difficult
✔ Mass effects are there at any order
✔ Straightforward implementation in MC event generators at LO and NLO

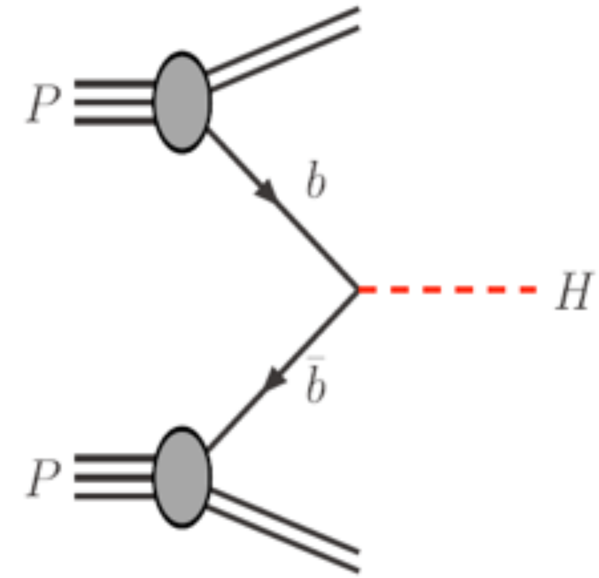
5F scheme

✔ It resums initial state large logs into b-PDFs leading to more stable predictions
✔ Computing higher orders is easier
✘ p_T of bottom enters at higher orders
✘ Implementation in MC depends on the gluon splitting model in the PS

4F and 5F schemes



NNLO correction in the 5FS



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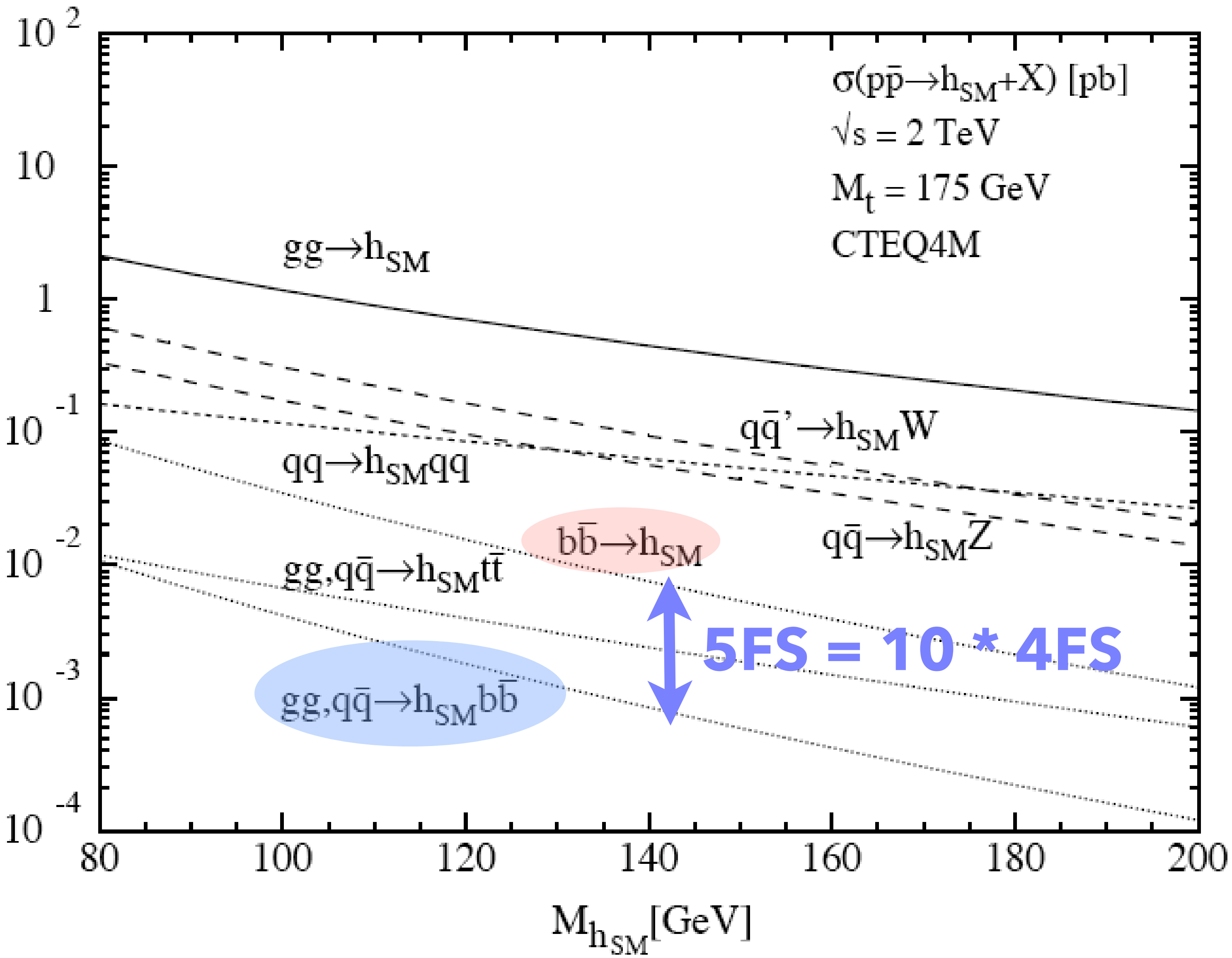
A lot of (open) questions



Higgs Tevatron Workshop 1998

- ➡ Why do the two schemes often lead to very different results?
- ➡ Why differences become smaller is a softer scale is used?
- ➡ For exclusive/differential observables: how to proceed?

  See Fabio's talk
- ➡ For inclusive observables: how to combine/match the two schemes to maximise the pros?

  This talk

Combining the 4F and 5F schemes

- There are cases when both **mass** terms and **resummation** of collinear logs must be included, as they both play a role in getting accurate predictions (e.g. DIS)
- What about predictions for partonic cross sections at the LHC?

4F scheme	5F scheme
pp → bb Nason et al (1989), Mangano et al (1992)	
pp → bbbb Greiner et al (2011)	
pp → ttbb Bevilacqua et al (2009), Bredenstein et al (2010)	pp → tW Campbell et al (2005),Frixione et al (2008)
pp → tbj Campbell et al (2009)	pp → tj Harris et al (2002), Campbell et al (2005)
pp → tbH± Dittmaier et al (2009), Degrande et al (2015)	pp → tH± Plehn et al (2003), Weydert et al (2010)
pp → Φbb Dawson et al (2005), Dittmaier et al (2004)	pp → Φ(bb),Φb(b) Campbell et al (2003), Harlander et al (2003)
pp → Vbb Ellis et al (1999,2000), Reina et al (2008,2009), Badger et al (2011), Frederix et al (2011)	pp → Z(bb),Vbj,Vb Campbell et al (2004,2006,2007,2009), Maltoni et al (2005)

Combining the 4F and 5F schemes

- Independently of the size of the mass effects and of collinear resummation effects, a prediction that combines the best available 4F and 5F scheme predictions based on standard QCD factorisation is the best one could get
- For inclusive cross sections a "phenomenological approach" is often adopted (HXSWG). Not too harmful is predictions do not differ much, but not theoretically sound!

Santander matching:

Weighted average between the 4F and the 5F scheme predictions

$$\sigma = \frac{\sigma^{(4F)} + w\,\sigma^{(5F)}}{1+w}$$

$$w = \log\left(\frac{M}{m_b}\right) - 2$$

Harlander, Kramer, Schumacher, 1112.3478

Combining the 4F and 5F schemes

- Independently of the size of the mass effects and of collinear resummation effects, a prediction that combines the best available 4F and 5F scheme predictions based on standard QCD factorisation is the best one could get
- Can we do better than that?

  - DIS [ACOT (1993), TR(2002), FONLL(2010)]
  - b hadro-production [Cacciari et al (1998)]
  - single top t -channel [MCFM, Campbell et al (2002,2009)]
  - W+Q, Z+Q [MCFM, Campbell et al (2004)]
  - ttH' [Han et al (2015)]
  - bbH [Forte et al (2015), Bonvini et al (2015)]

The FONLL approach

- Based on standard QCD collinear factorisation
- Match a fixed order calculation N^{p}LO with DGLAP-resummed N^{q}LL calculation
  - First applied to b-quark hadro-production [Cacciari, Greco, Nason (1998)]
  - Then to Deep-Inelastic-Scattering [Forte, Laenen, Nason, Rojo (2010)] [Ball et al (2016)]
  - Recently to b-fusion-initiated Higgs production [Forte, Napoletano, MU (2015)]

$$
\begin{aligned}
\sigma^{(FONLL)} &= \sigma^{(4)} + \sigma^{(5)} - \text{double counting} \\
&= \mathcal{L}_{ij}(x_1, x_2, \mu^2) \otimes \sum_p^N \left(\alpha_S^{(5)}(\mu^2)\right)^p \\
&\times \left\{ \mathcal{B}_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right) + \sum_{k=0}^{\infty} \mathcal{A}_{ij}^{(p),(k)}(x_1, x_2) \left(\alpha_S^{(5)}(\mu^2) L\right)^k \right\} \\
&- \text{double counting}
\end{aligned}
$$

$$L = \log \frac{\mu_F^2}{m_b^2}$$

The FONLL approach

- For a consistent subtraction of double counting we need to re-express **both** the 5FS cross section and the 4FS one in terms of the same α_S and PDFs
- Take the **5FS cross section**

$$\sigma^{(5)} = \int dx_1\, dx_2 \sum_{i,j=g,q,b} f_i^{(5)}(x_1,\mu^2) f_j^{(5)}(x_2,\mu^2)\, \hat{\sigma}_{ij}^{(5)}(x_1,x_2,\alpha_s^{(5)}(\mu^2))$$

- If no intrinsic bottom component, then b-PDF is determined in terms of gluon and light quarks by DGLAP evolution:

$$f_b^{(5)}(x,\mu^2) = \sum_{i=q,g} \int_x^1 \frac{dy}{y} f_i^{(5)}(y,\mu^2) C_{bi}\left(\frac{x}{y}, \alpha_S^5(\mu^2), L\right)$$

$$\sigma^{(5)} = \iint dx_1 dx_2 \sum_{ij=q,g} f_i^{(5)}(x_1,\mu^2) f_j^{(5)}(x_2,\mu^2) A_{ij}^{(5)}\left(x_1,x_2,L,\alpha_s^{(5)}(\mu^2)\right)$$

with $A_{ij}^{(5)} = C_{bk} \otimes \hat{\sigma}_{ij}^{(5)}$

The FONLL approach

- For a consistent subtraction of double counting we need to re-express **both** the 5FS cross section and the 4FS one in terms of the same α_S and PDFs
- Take the **4FS cross section**

$$\sigma^{(4)} = \int dx_1 dx_2 \sum_{i,j=g,q} f_i^{(4)}(x_1,\mu^2) f_j^{(4)}(x_2,\mu^2)\,\hat{\sigma}_{ij}^{(4)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}, \alpha_s^{(4)}(\mu^2)\right)$$

  - Both α_S and PDFs can be re-expressed in terms of their 5F counterparts
  - K_{ij} polynomial in L
  - Invert Eqns and obtain

$$\alpha_s^{(5)}(\mu^2) = \alpha_s^{(4)}(\mu^2) + \sum_{i=2}^{\infty} c_i(L) \times \left(\alpha_s^{(4)}(m_b^2)\right)^i,$$

$$f_i^{(5)}(x,\mu^2) = \int_x^1 \frac{dy}{y} \sum_j K_{ij}\left(y, L, \alpha_s^{(4)}(\mu^2)\right) f_j^{(4)}\left(\frac{x}{y}, \mu^2\right)$$

$$\sigma^{(4)} = \iint dx_1 dx_2 \sum_{ij=q,g} f_i^{(5)}(x_1,\mu^2) f_j^{(5)}(x_2,\mu^2) B_{ij}^{(4)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}, \alpha_s^{(5)}(\mu^2)\right)$$

The FONLL approach

- For a consistent subtraction of double counting we need to re-express **both** the 5FS cross section and the 4FS one in terms of the same α_S and PDFs

$$\sigma^{(5)} = \iint dx_1 dx_2 \sum_{ij=q,g} f_i^{(5)}(x_1,\mu^2) f_j^{(5)}(x_2,\mu^2) A_{ij}^{(5)}\left(x_1, x_2, L, \alpha_s^{(5)}(\mu^2)\right)$$

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- To identify the double-counting, expand the cross sections at the order N

$$A_{ij}^{(5)}\left(x_1, x_2, L, \alpha_s^{(5)}(\mu^2)\right) = \sum_{p=0}^{N} \left(\alpha_s^{(5)}(\mu^2)\right)^p \sum_{k=0}^{\infty} A_{ij}^{(p),(k)}(x_1, x_2) \left(\alpha_s^{(5)}(\mu^2) L\right)^k$$

$$B_{ij}^{(4)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}, \alpha_s^{(5)}(\mu^2)\right) = \sum_{p=0}^{N} \left(\alpha_s^{(5)}(\mu^2)\right)^p B_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right)$$

The FONLL approach

- Subtraction term: take the logarithmic (massless) terms in the B expansion that appear both in the 5FS and in the 4FS expansions

$$\sigma^{(4),(0)} = \iint dx_1 dx_2 \sum_{ij=q,g} f_i^{(5)}(x_1,\mu^2) f_j^{(5)}(x_2,\mu^2) B_{ij}^{(0)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}, \alpha_s^{(5)}(\mu^2)\right)$$

$$B_{ij}^{(0),(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right) = \sum_{k=0}^{p} A_{ij}^{(p-k),(k)}\left(x_1, x_2\right) L^k$$

- To identify the double-counting, expand the cross sections at the order N

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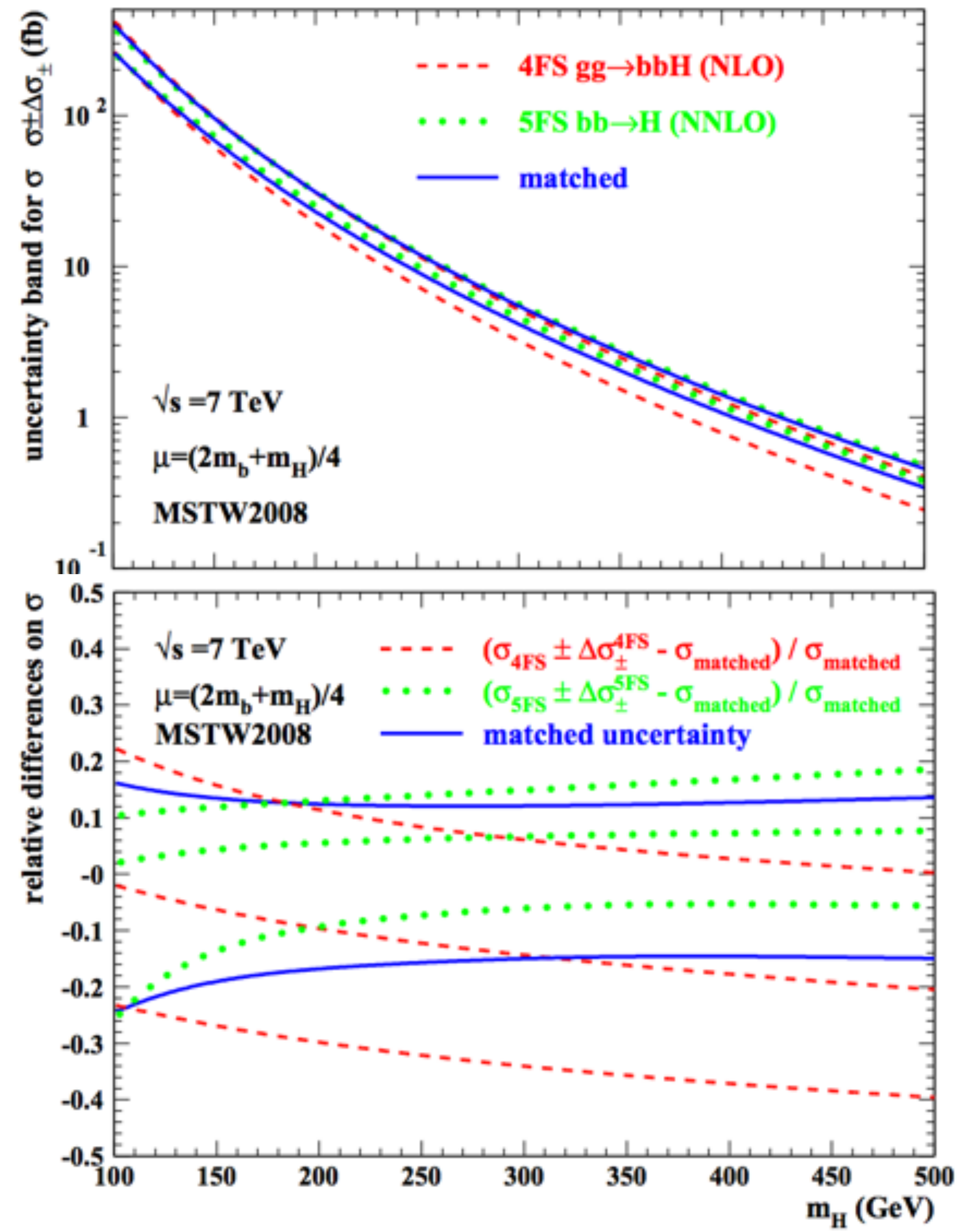
$$B_{ij}^{(4)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}, \alpha_s^{(5)}(\mu^2)\right) = \sum_{p=0}^{N} \left(\alpha_s^{(5)}(\mu^2)\right)^p B_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right)$$

The bbH case

- Bottom-fusion initiated H production relevant in models in which bH coupling is enhanced (e.g. 2HDM with large tanB)
- 5F known up to NNLO (diff.)
  - Dicus et al (1999)
  - Ballasz et al (1999)
  - Harlander et al (2003)
  - Busheler et al (2012)
- 4FS known up to NLO (+PS)
  - Dittmaier et al (2004)
  - Dawson et al (2004)
  - Wiesemann et al (2015)



MSTW08 PDFs

Scale + PDF + a_s uncertainties,
m_b^{pole} = 4.75 GeV, y_b evolved at μ_R at n+1 loops

HXSWG, YR3, 1201.3084

The bbH case

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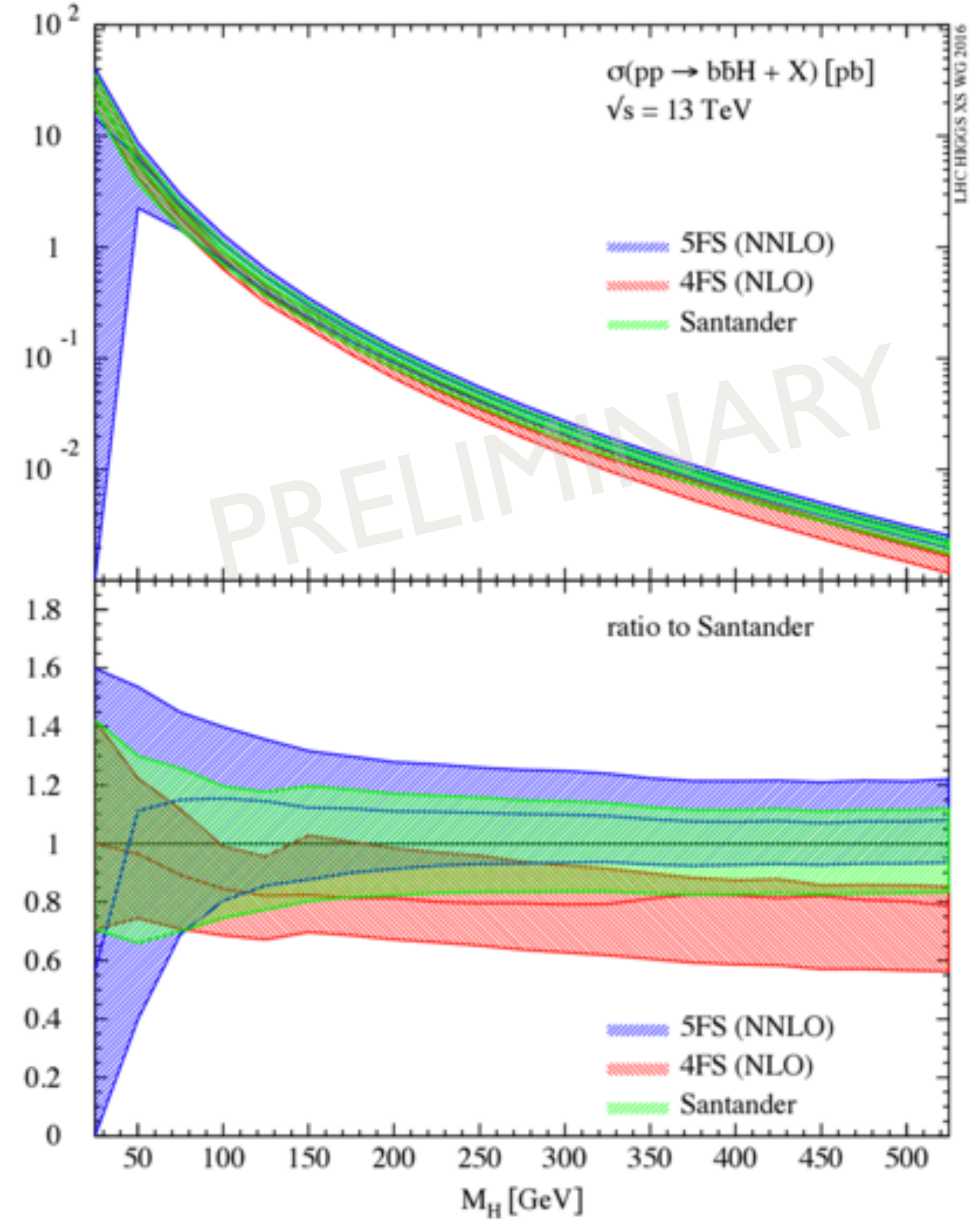


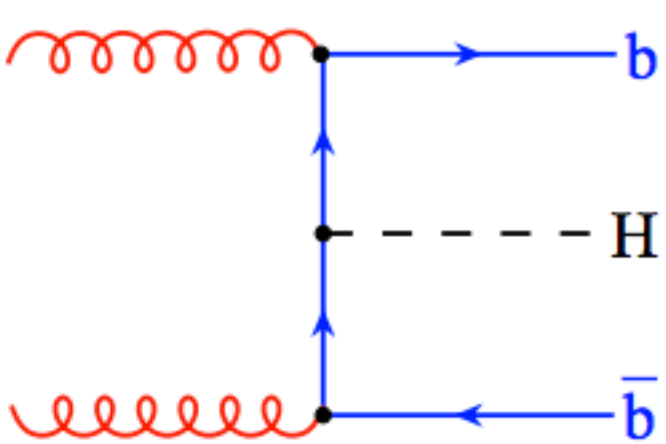
PDF4LHC15_mb4.58 PDFs, μb = 4.58 GeV
Scale + PDF + a_s + mb uncertainties,
4.44 GeV< m_b^{pole} < 4.72 GeV
y_b evolved at M at 4 loops

HXSWG, YR4, in preparation

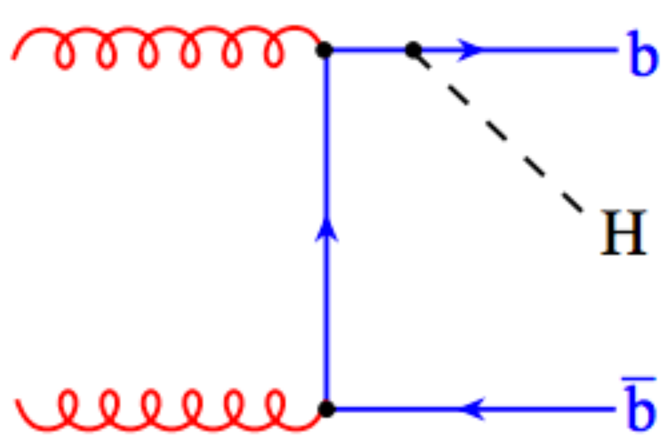
Courtesy of M. Spira

The bbH case

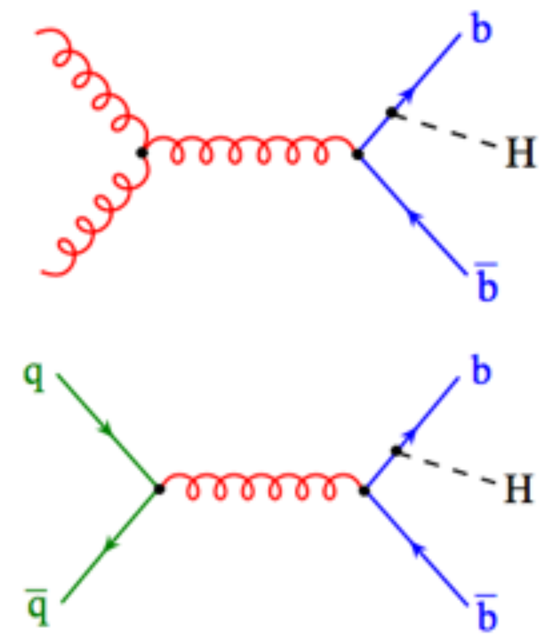
- Anatomy of bottom-fusion initiated Higgs production.
- For simplicity take 4FS **LO** diagrams (exclude cross diagrams and gluon emission from b)



O(α^2 L^2) + O(α^2 L^1) +O(α^2 L^0)



O(α^2 L^1) +O(α^2 L^0)

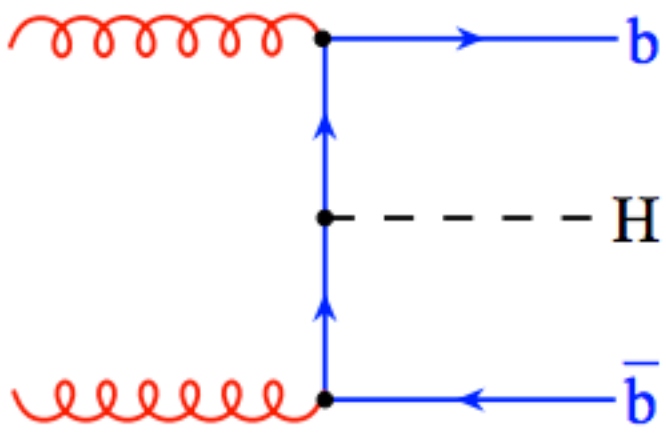


O(α^2 L^0)

$$L = \log \frac{Q^2}{m_b^2}$$

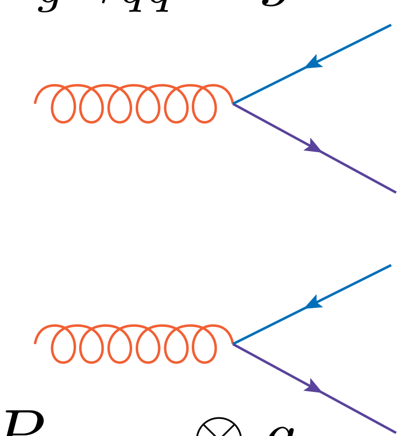
The bbH case

- In the massless/collinear limit, this diagram factorises into

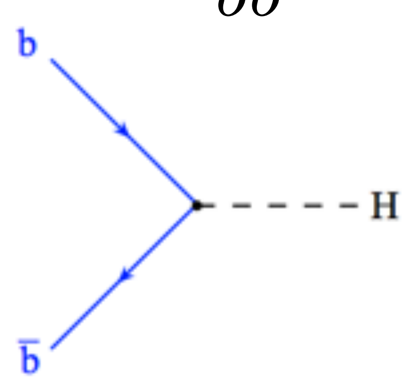


$$\tilde{f}_b^{(1)} = \frac{\alpha_s}{2\pi} L\, P_{g\to qq} \otimes g$$

$$\hat{\sigma}_{bb}^{(5F,0)}$$



$$\otimes$$



$$\tilde{f}_b^{(1)} = \frac{\alpha_s}{2\pi} L\, P_{g\to qq} \otimes g$$

These logs are double-counted in the 4FS and the 5FS. In the 4FS only the first one (two) log of the tower of logs resummed in the b PDFs are explicitly present and must be subtracted in the matching procedure

Truncated b PDFs

$$f_b(x,\mu^2) = \tilde{f}_b^{(1)}(x,\mu^2) + \tilde{f}_b^{(2)}(x,\mu^2) + \mathcal{O}(\alpha_S^3)$$

Truncated solution of DGLAP equations

with
$$\tilde{f}_b^{(1)}(x,\mu^2) = \left(\frac{\alpha_s(\mu)}{2\pi}\right) \log\left(\frac{\mu^2}{m_b^2}\right) P_{qg} \otimes g(x,\mu^2)$$
$$\tilde{f}_b^{(2)}(x,\mu^2) = \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \left[\log^2\left(\frac{\mu^2}{m_b^2}\right)\phi^{(2)}(x,\mu^2) + \log\left(\frac{\mu^2}{m_b^2}\right)\phi^{(1)}(x,\mu^2)\right]$$

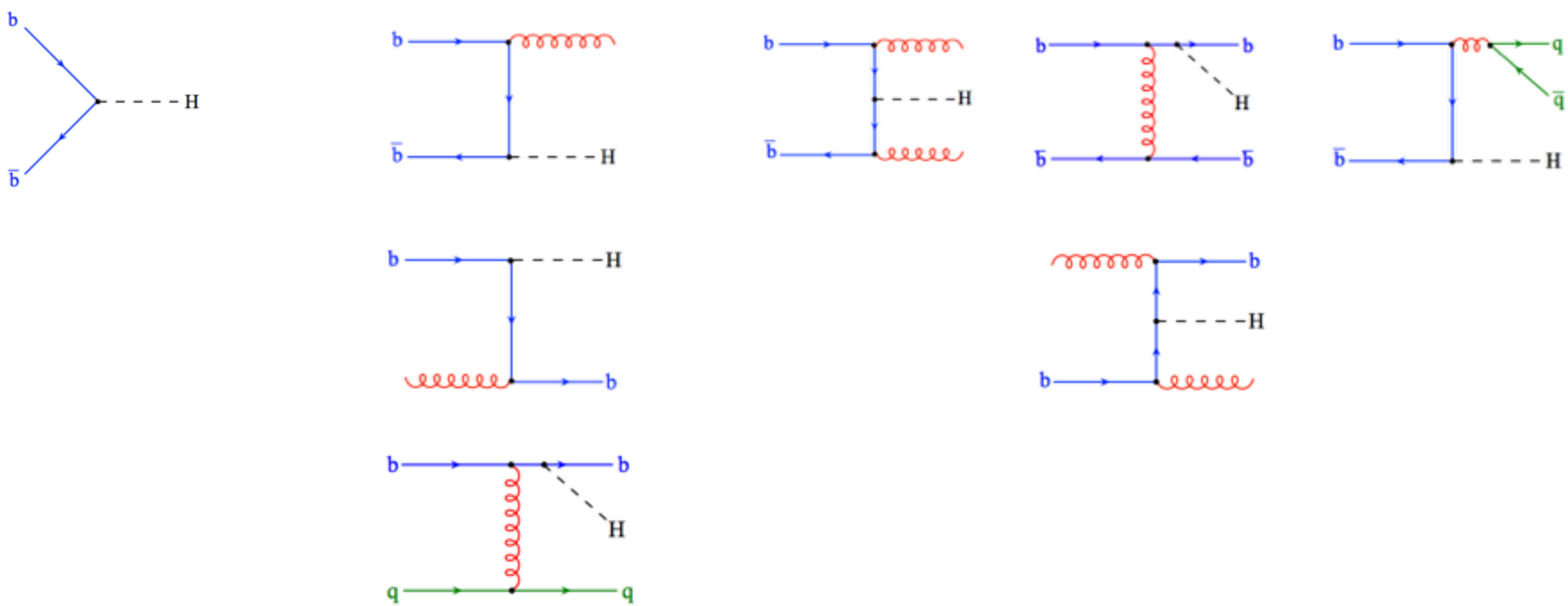
Truncated luminosities

$$\mathcal{L}_{bb}^{(2)}(x_1,x_2) = 2\tilde{f}_b^{(1)}(x_1)\tilde{f}_b^{(1)}(x_2)$$
$$\mathcal{L}_{bb}^{(3)}(x_1,x_2) = 2(\tilde{f}_b^{(2)}(x_1)\tilde{f}_b^{(1)}(x_2) + \tilde{f}_b^{(1)}(x_1)\tilde{f}_b^{(2)}(x_2))$$
$$\mathcal{L}_{bg}^{(1,2)}(x_1,x_2) = 4(f_g(x_1)\tilde{f}_b^{(1,2)}(x_2) + \tilde{f}_b^{(1,2)}(x_1)f_g(x_2))$$
$$\mathcal{L}_{b\Sigma}^{(1)}(x_1,x_2) = 4(f_\Sigma(x_1)\tilde{f}_b^{(1)}(x_2) + \tilde{f}_b^{(1)}(x_1)f_\Sigma(x_2))$$

A consistent subtraction

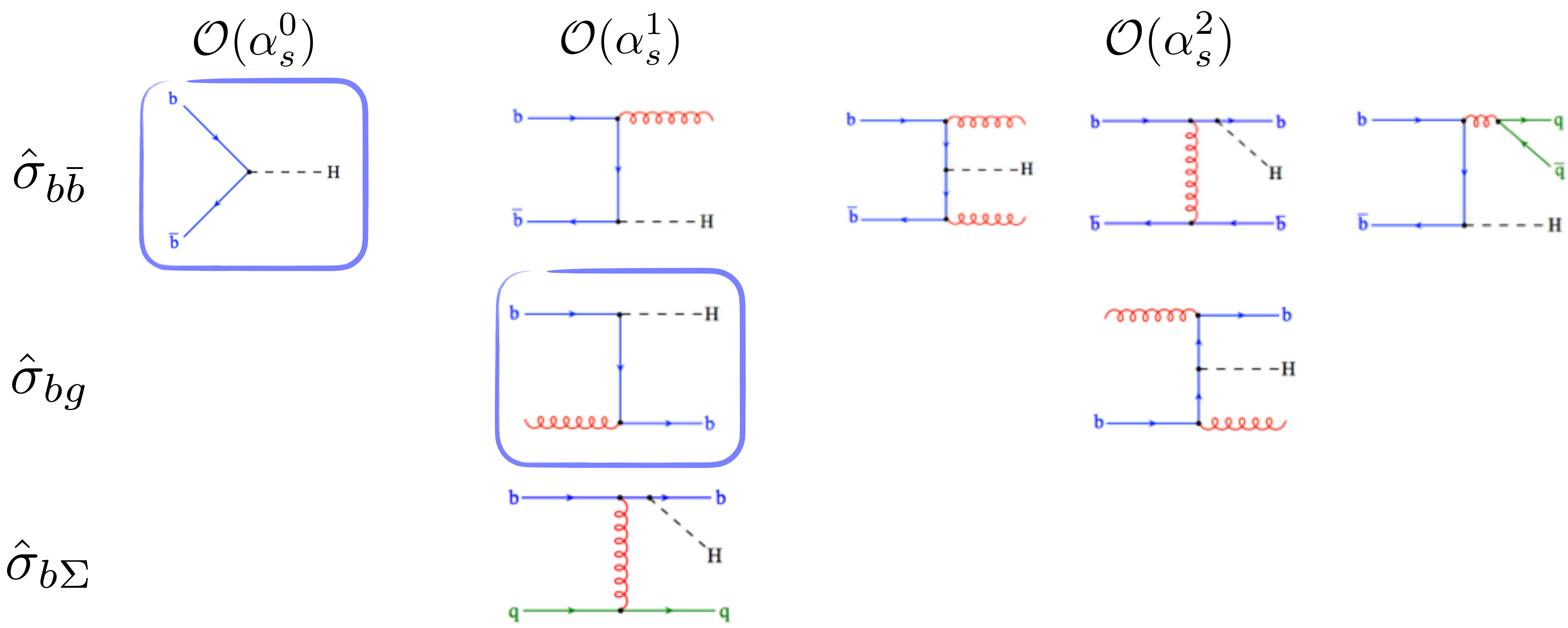
- Start from 5FS and take all diagrams that have bottom quark in the initial state

	$\mathcal{O}(\alpha_s^0)$	$\mathcal{O}(\alpha_s^1)$	$\mathcal{O}(\alpha_s^2)$
$\hat{\sigma}_{b\bar{b}}$			
$\hat{\sigma}_{bg}$			
$\hat{\sigma}_{b\Sigma}$			



A consistent subtraction

- Start from 5FS and take all diagrams that have bottom quark in the initial state
- Include convolution of truncated luminosity and matrix elements up to $O(\alpha_S^2)$

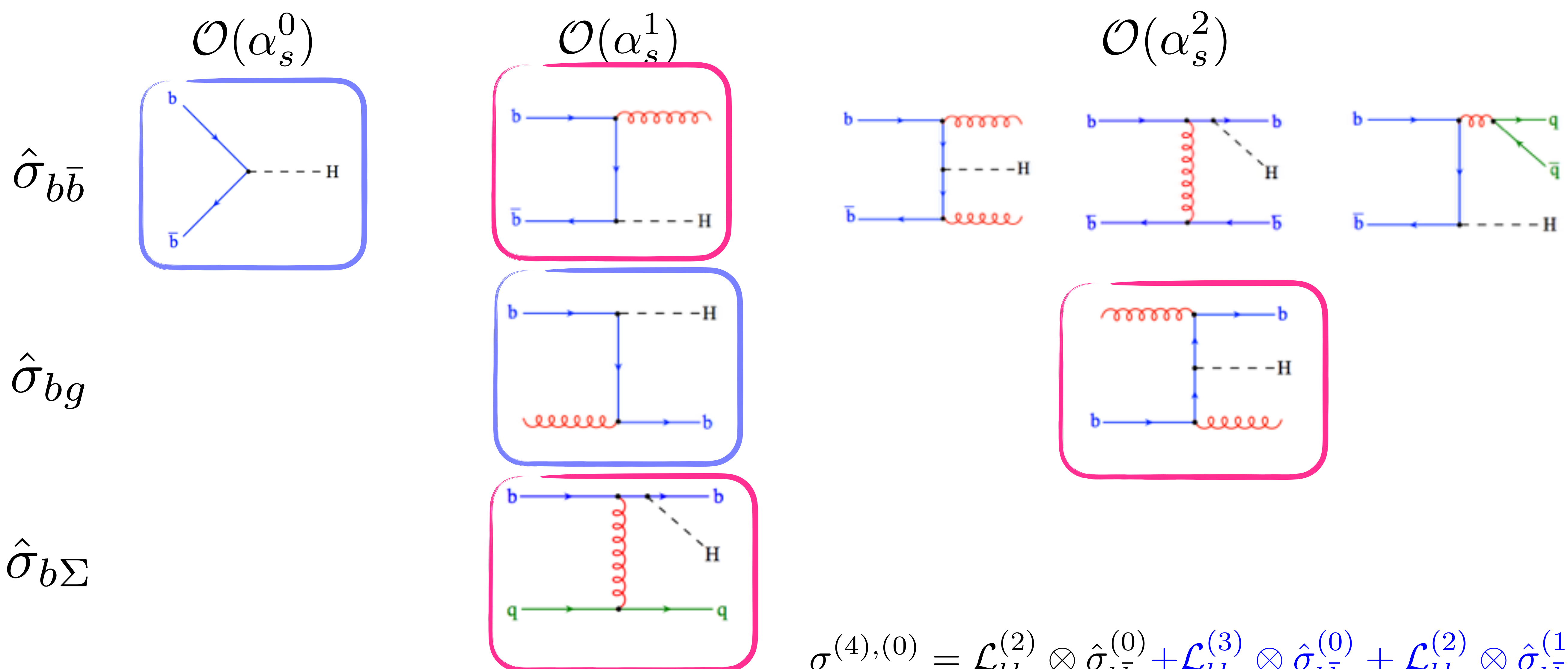


$$\sigma^{(4),(0)} = \mathcal{L}_{bb}^{(2)} \otimes \hat{\sigma}_{b\bar{b}}^{(0)} + \mathcal{L}_{gb}^{(1)} \otimes \hat{\sigma}_{bg}^{(1)}$$

FONLL-A:
NNLO 5FS + LO 4FS

A consistent subtraction

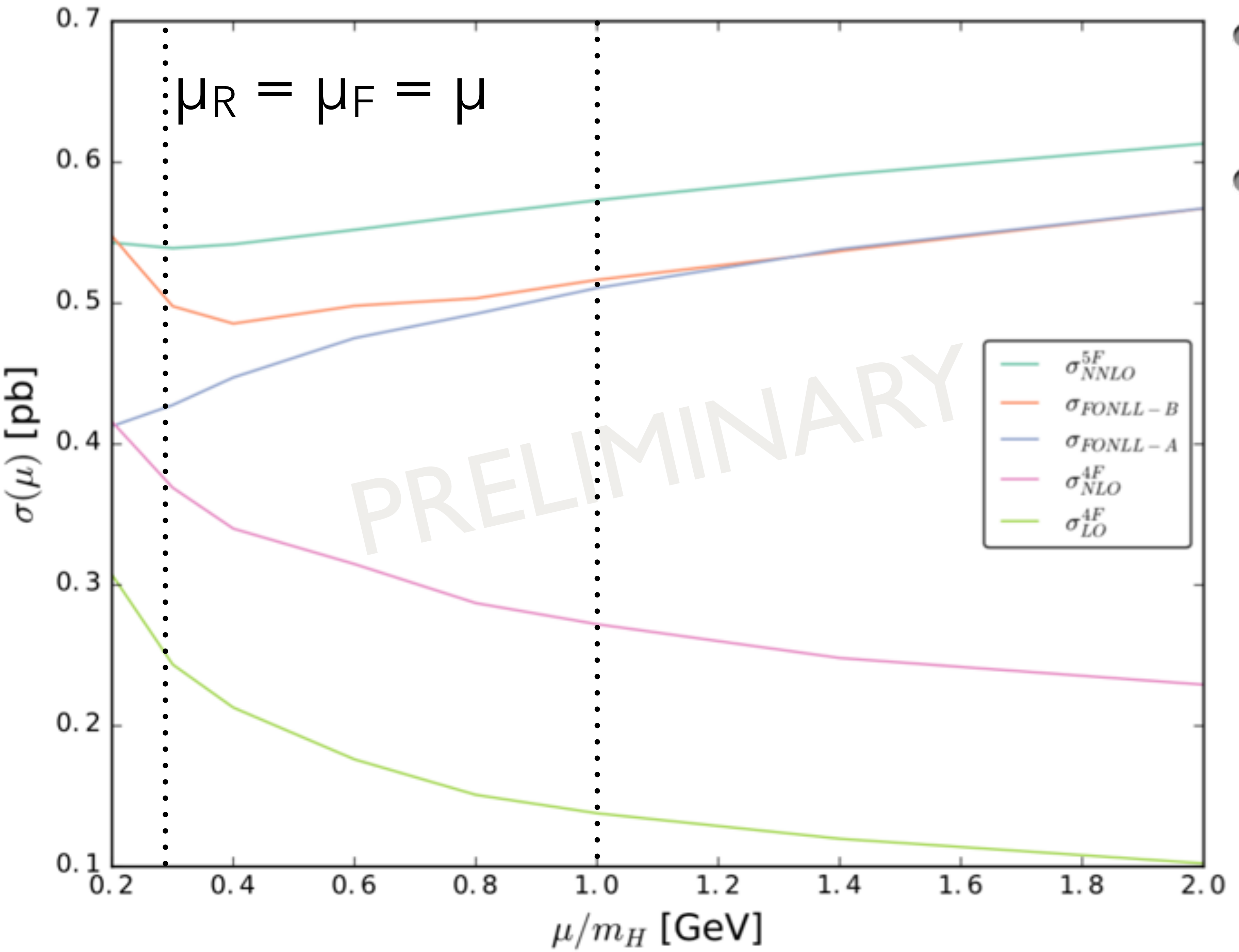
- Start from 5FS and take all diagrams that have bottom quark in the initial state
- Include convolution of truncated luminosity and matrix elements up to O(α_S^3)



FONLL-B:
NNLO 5FS + NLO 4FS

$$\sigma^{(4),(0)} = \mathcal{L}^{(2)}_{bb} \otimes \hat{\sigma}^{(0)}_{b\bar{b}} + \mathcal{L}^{(3)}_{bb} \otimes \hat{\sigma}^{(0)}_{b\bar{b}} + \mathcal{L}^{(2)}_{bb} \otimes \hat{\sigma}^{(1)}_{b\bar{b}} + \mathcal{L}^{(1)}_{gb} \otimes \hat{\sigma}^{(1)}_{bg} + \mathcal{L}^{(2)}_{gb} \otimes \hat{\sigma}^{(1)}_{bg} + \mathcal{L}^{(1)}_{gb} \otimes \hat{\sigma}^{(2)}_{bg} + \mathcal{L}^{(1)}_{b\Sigma} \otimes \hat{\sigma}^{(2)}_{b\Sigma}$$

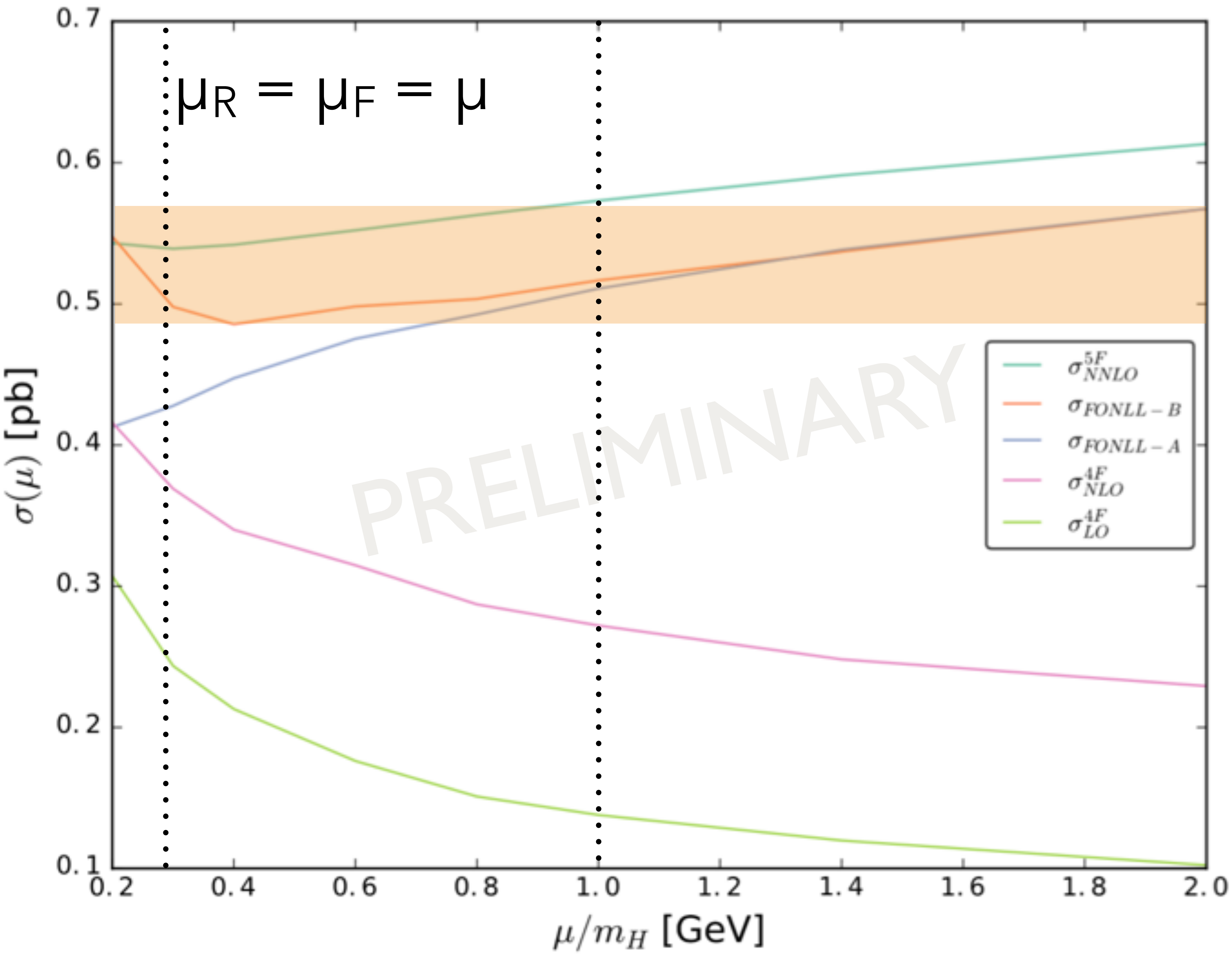
Results



- All cross sections flat wrt to μ_F variations
- 4FNLO xsec 40% lower than 5FNNLO at M_H but difference is reduced at lower μ

Forte, Napoletano, MU, in preparation

Results



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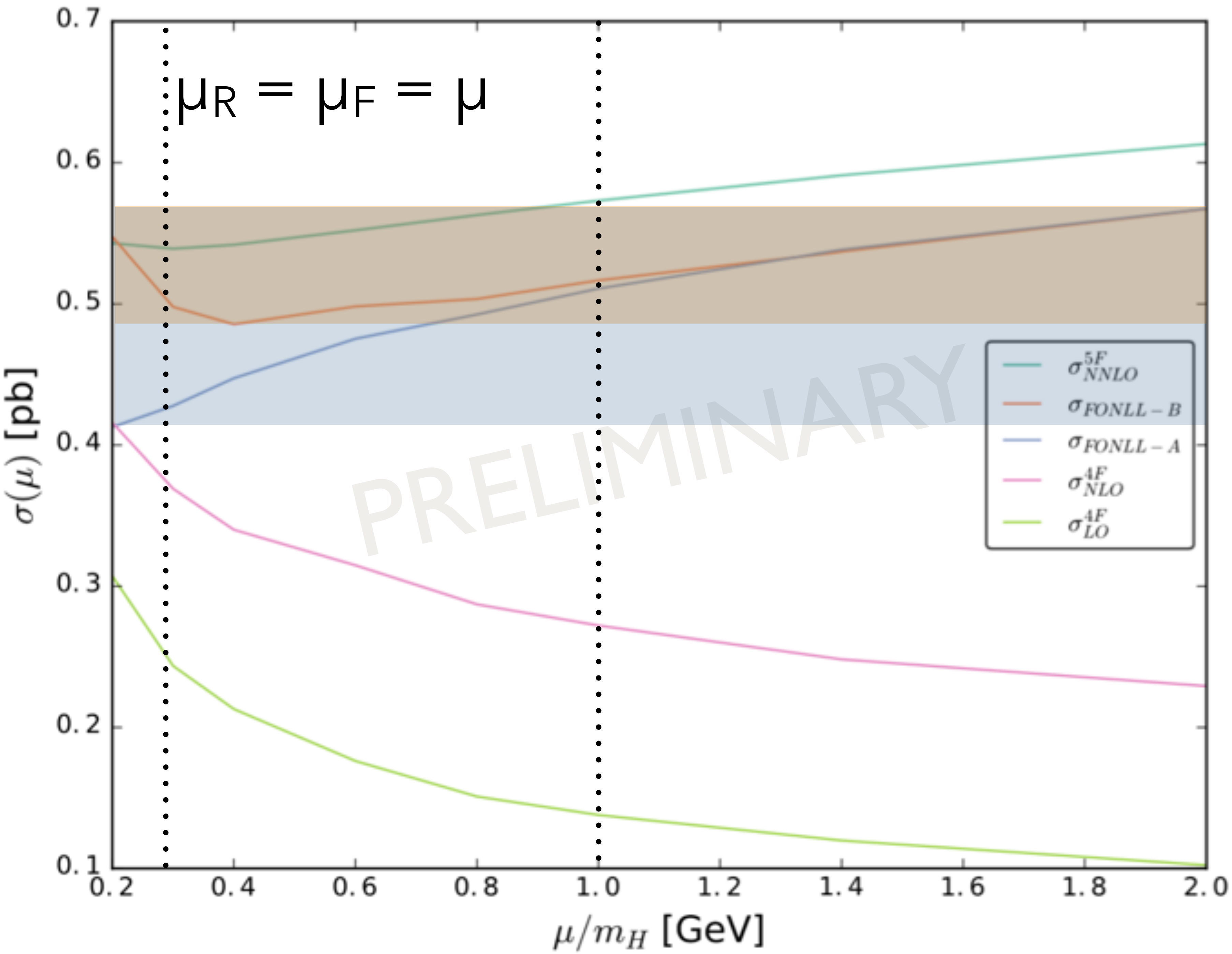
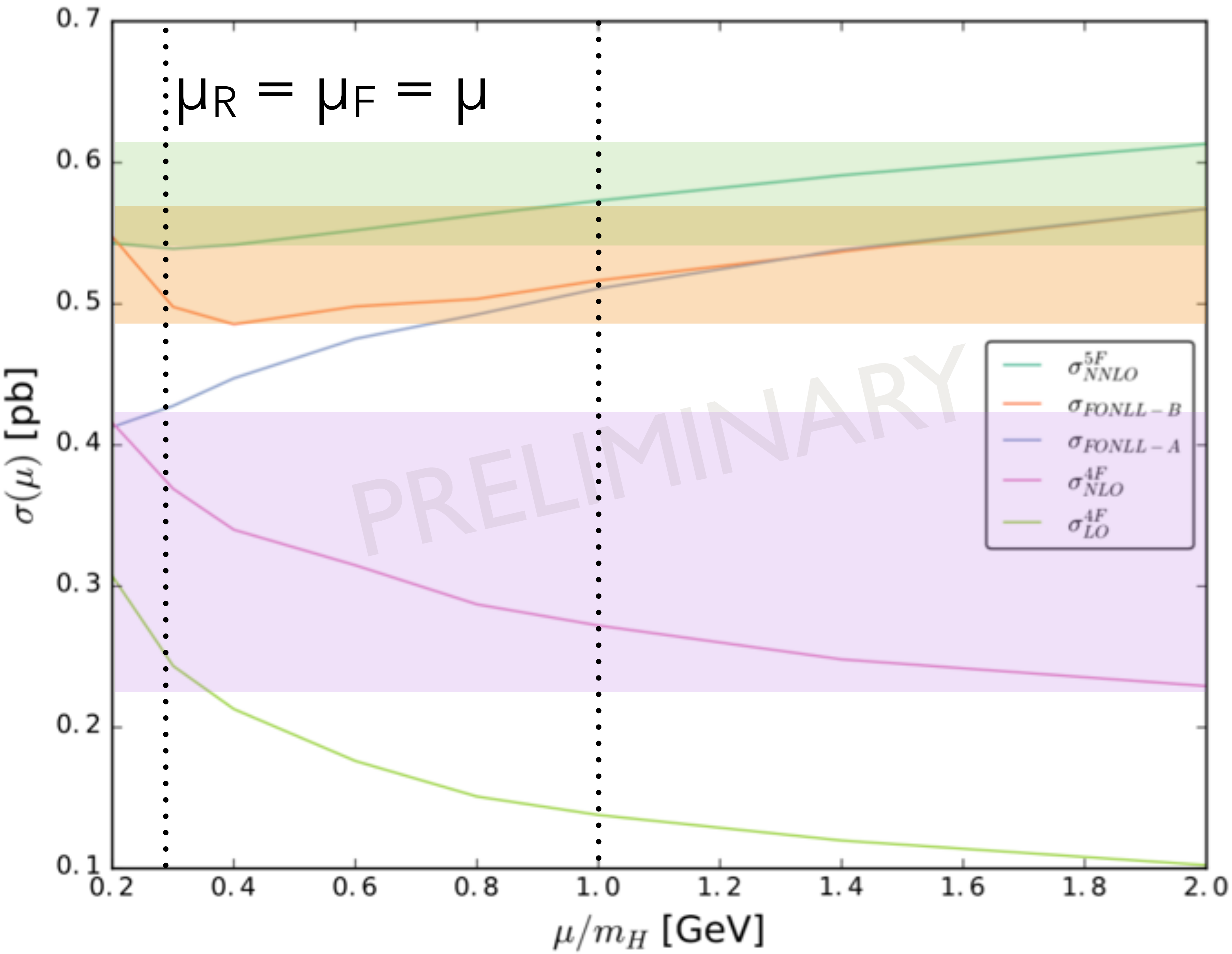


- All cross sections flat wrt to μ_F variations
- 4FNLO xsec 40% lower than 5FNNLO at M_H but difference is reduced at lower μ
- FONLL-B scale uncertainty is half the size of FONLL-A

Forte, Napoletano, MU, in preparation

Results



- All cross sections flat wrt to μ_F variations
- 4FNLO xsec 40% lower than 5FNNLO at M_H but difference is reduced at lower μ
- FONLL-B scale uncertainty is half the size of FONLL-A
- FONLL-B scale uncertainty less than half than 4FS one (resummation)

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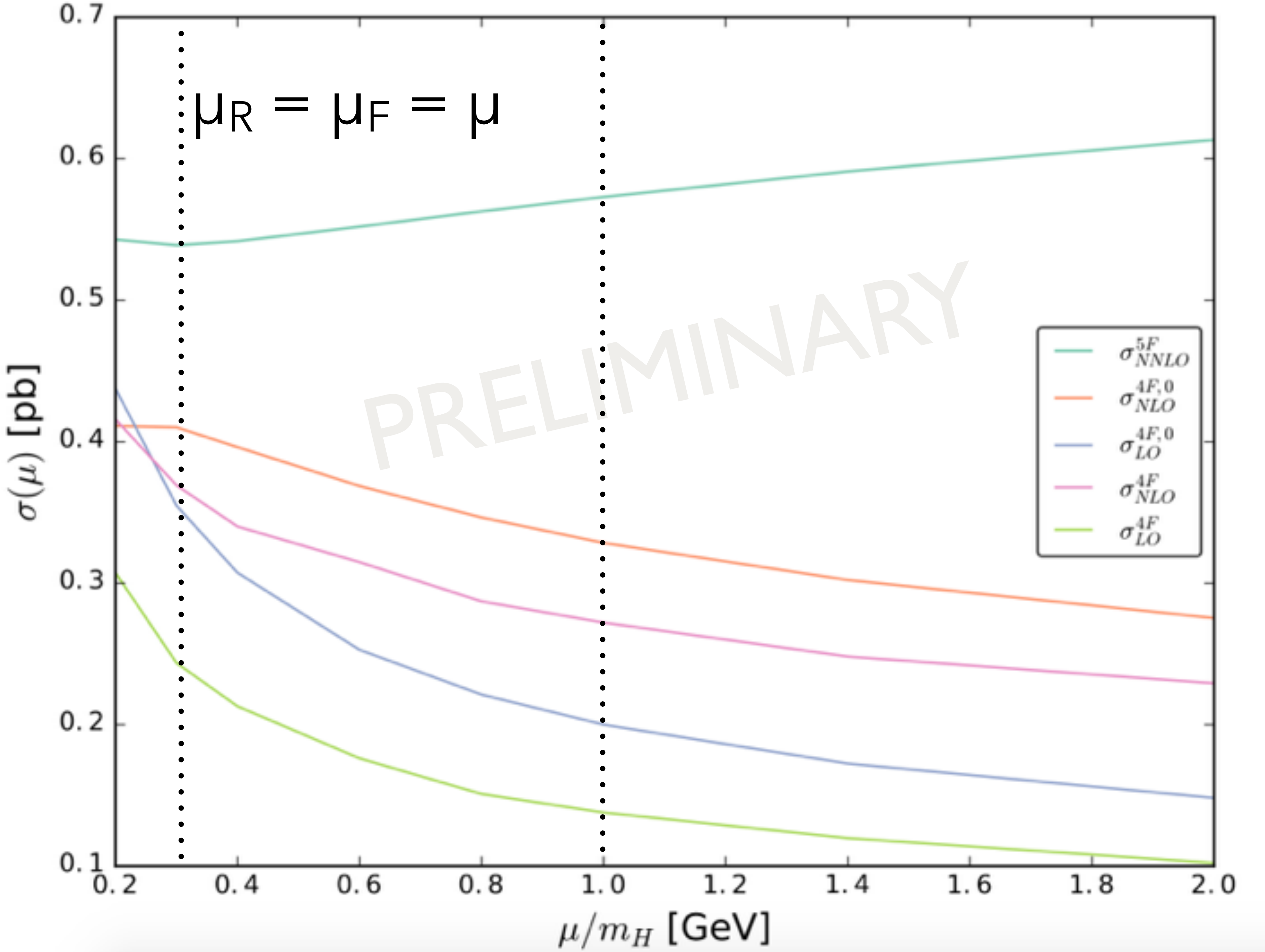
Interpretation(s)



- Difference between 5F NNLO and FONLL-B consistent with difference between "massless" 4FS and massive 4FS

➔ Effect of mass terms about 10% and constant with μ

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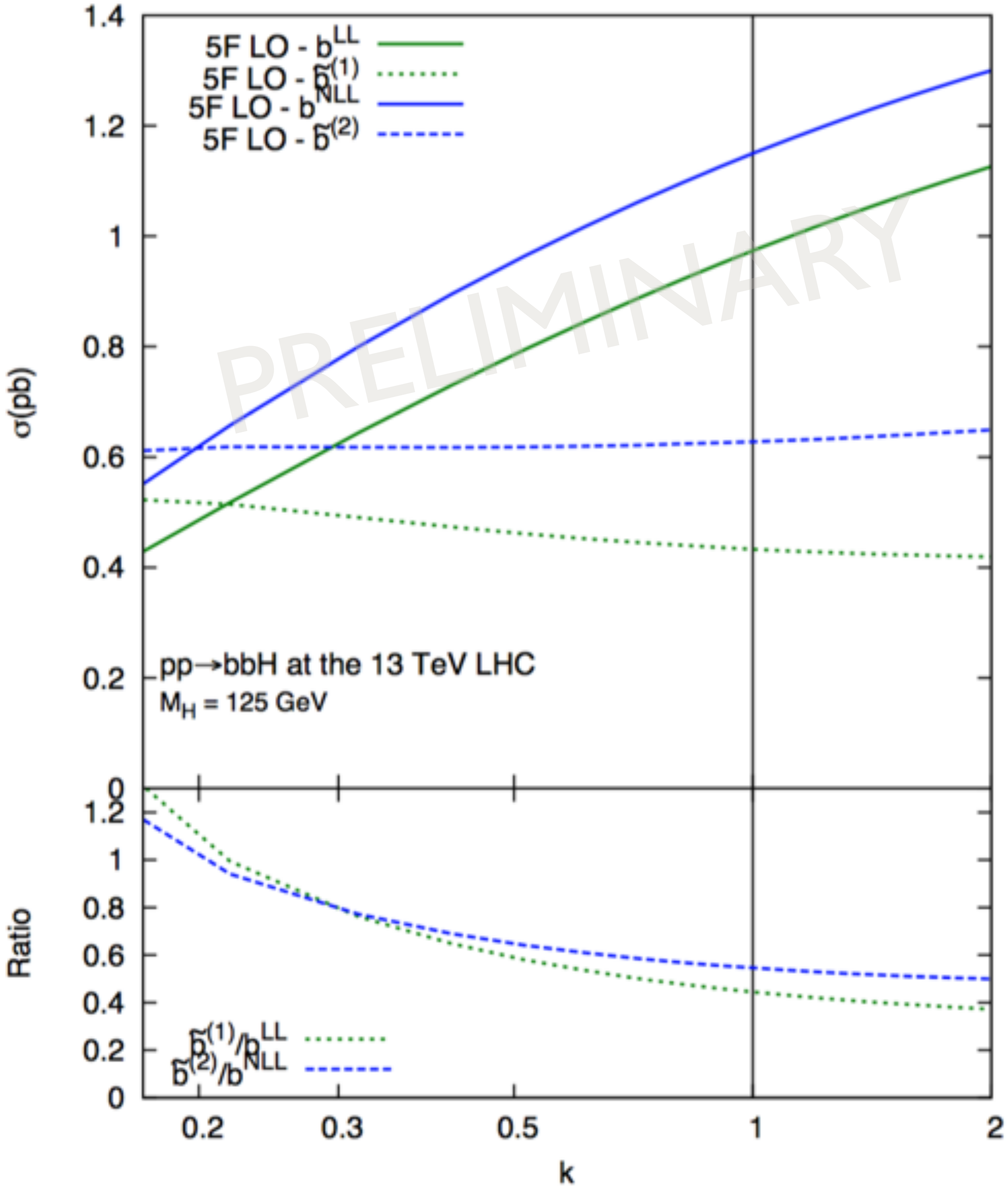
Interpretation(s)

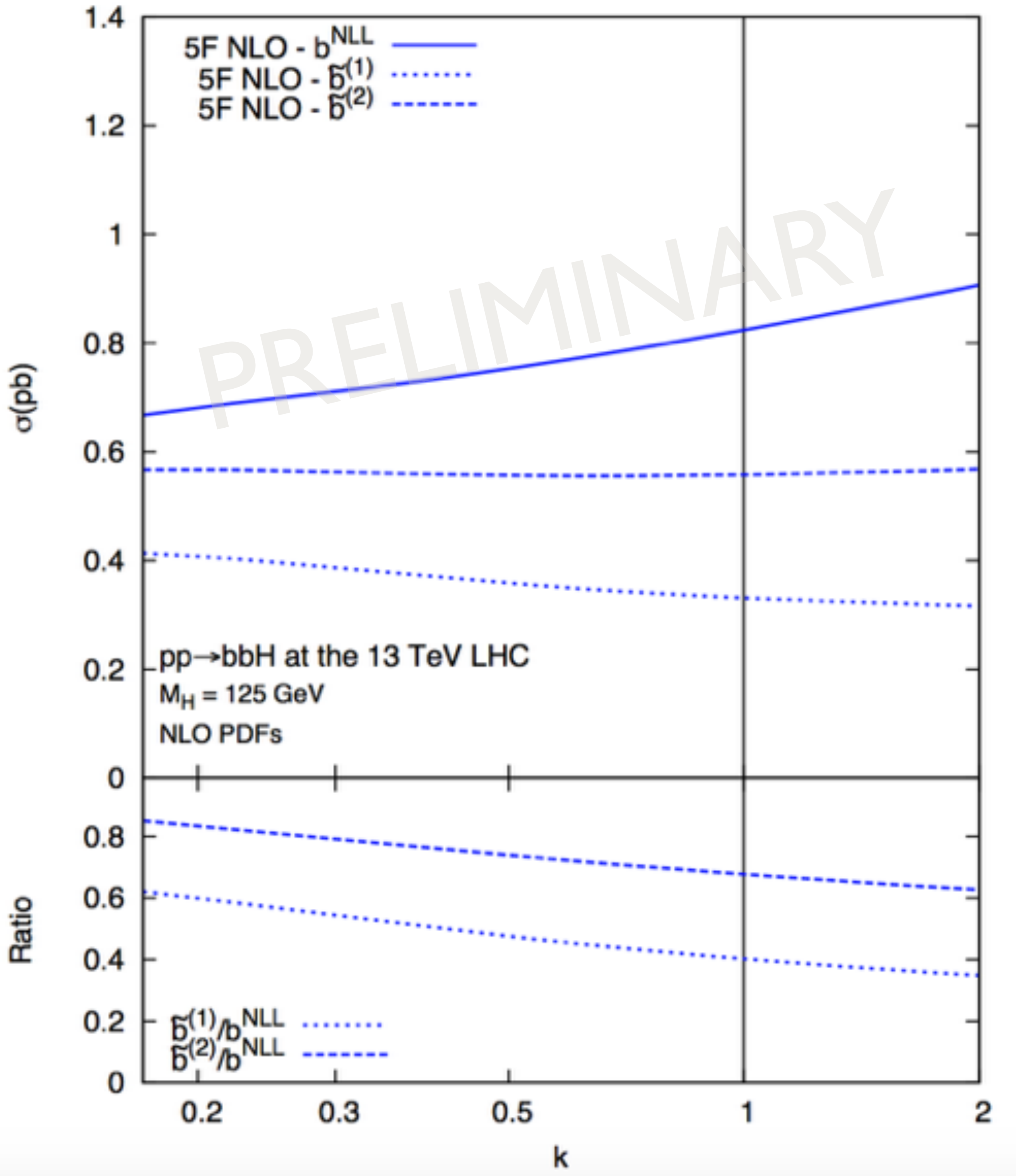


- Difference between 4F NLO and FONLL-B consistent with factor due to resummation of higher-order collinear logs in the b PDF

➔ Effect of resummation between 10% and 40% depending on the scale

Lim, Maltoni, Ridolfi, MU, in preparation

Interpretation(s)



- Difference between 4F NLO and FONLL-B consistent with factor due to resummation of higher-order collinear logs in the b PDF

➔ Effect of resummation between 10% and 40% depending on the scale

➔ Both in the LO and NLO 5FS cross section

Lim, Maltoni, Ridolfi, MU, in preparation

Conclusions

- Rich phenomenology of bottom-initiated processes
- For inclusive cross section a resummed calculation including all known mass effects is the most accurate
- Properly matched calculations clearly preferable to weighted average
- Shown that for bbH it is possible to extend the FONLL formalism
- 4FS NLO calculation matched with 5FS NNLO calculation (FONLL-B) shows that mass effects are moderate and that the matched cross section for SM Higgs masses close to 5FS result and with similar scale uncertainty
- Are collinear logs dominant? YES
- Do unresummed logs spoil perturbative expansion of the 4FS computation? NO, if a lower scale is used

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Thank you