Heavy quarks from HERA to LHC

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HQ@LHC, IPPP, Durham, 20 *Apr 2016*

The ABMP16 ingredients

DATA:

- DIS NC/CC inclusive (HERA I+II added, no deuteron data included)
- DIS NC charm production (HERA)
- DIS CC charm production (HERA, NOMAD, CHORUS, NuTeV/CCFR)
- fixed-target DY

sa, et al. **hep-ph/1404.6469**

- LHC DY distributions (ATLAS, CMS, LHCb)
- t-quark data from the LHC and Tevatron

QCD:

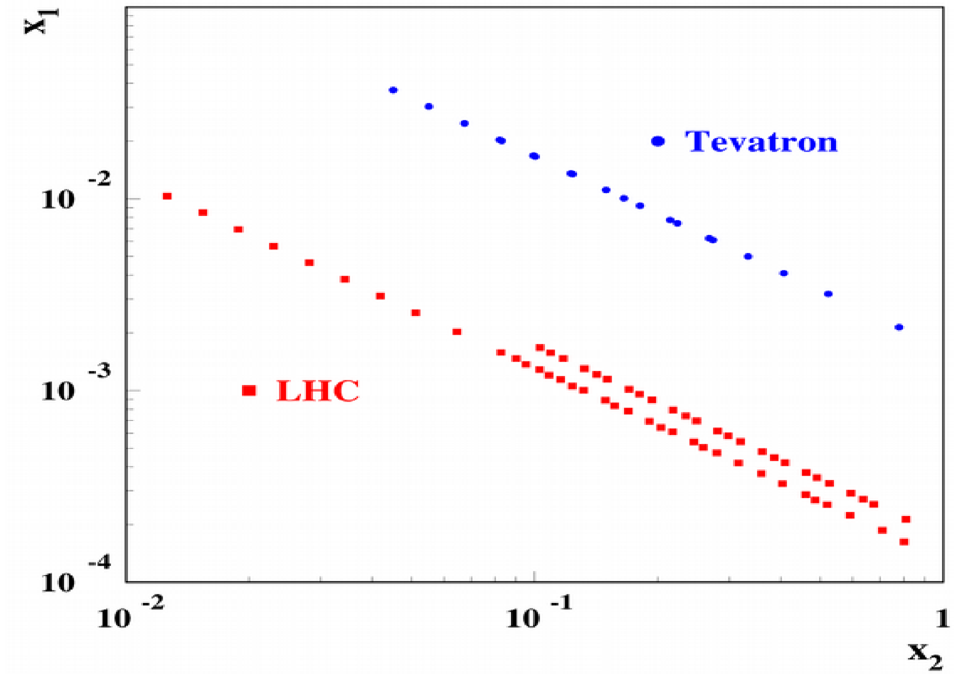
- NNLO evolution
- NNLO massless DIS and DY coefficient functions
- NLO+ massive DIS coefficient functions (**FFN scheme**)
  - – NLO + NNLO threshold corrections for NC
  - – NNLO CC at Q>> m_c
  - – running mass
- NNLO exclusive DY (FEWZ 3.1)
- NNLO inclusive ttbar production (pole / running mass)
- Relaxed form of (dbar-ubar) at small x

sa, Blümlein, Moch. Plačakytė hep-ph/1508.07923

Power corrections in DIS:

- target mass effects
- dynamical twist-4 terms

Collider W&Z data



In the forward region $x_2 >> x_1$

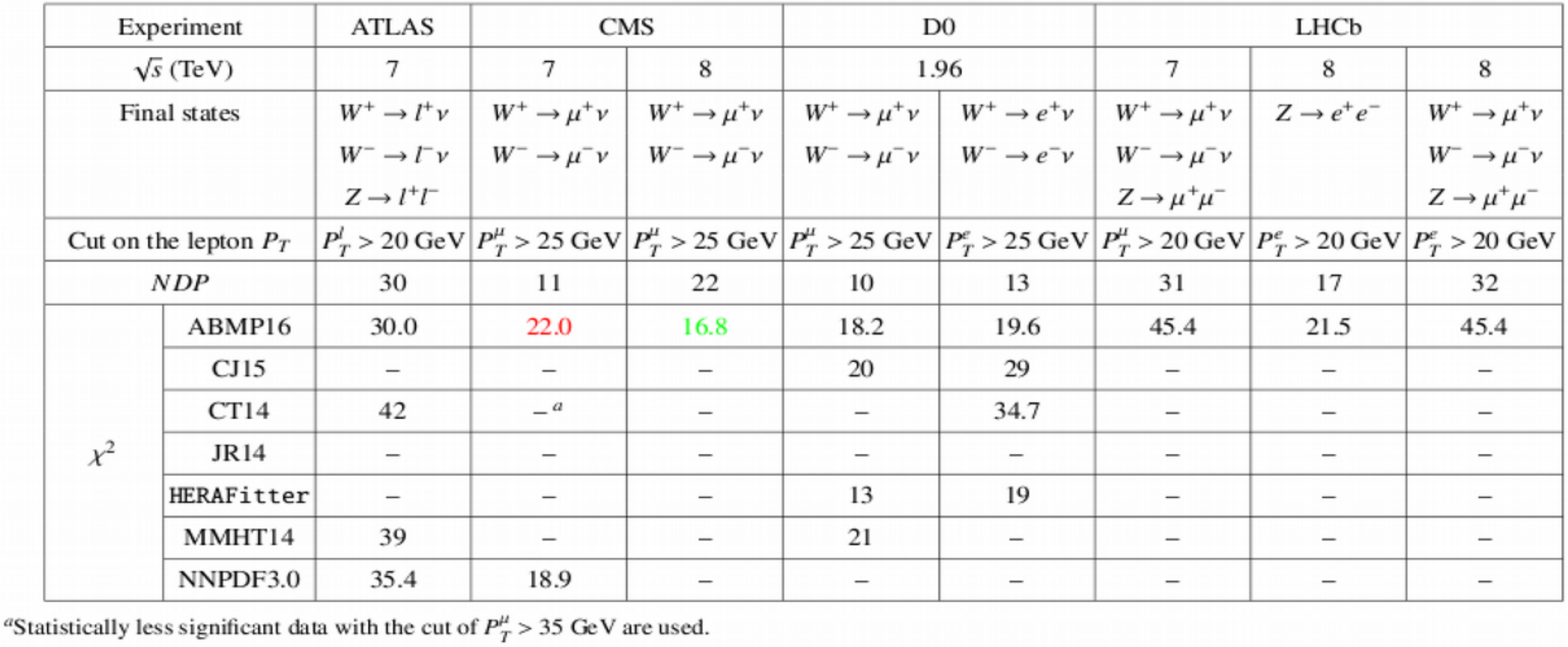
$\sigma(W^+) \sim u(x_2)\ \mathrm{dbar}\ (x_1)$

$\sigma(W^-) \sim d(x_2)\ \mathrm{ubar}(x_1)$

$\sigma(Z) \sim Q_U^2 u(x_2)\ \mathrm{ubar}\ (x_1) + Q_D^2 d(x_2)\ \mathrm{dbar}(x_1)$

$\sigma(DIS) \sim q_u^2 u(x_2) + q_d^2 d(x_2)$

Forward W&Z production probes small/large x and is complementary to the DIS → constraint on the quark iso-spin asymmetry

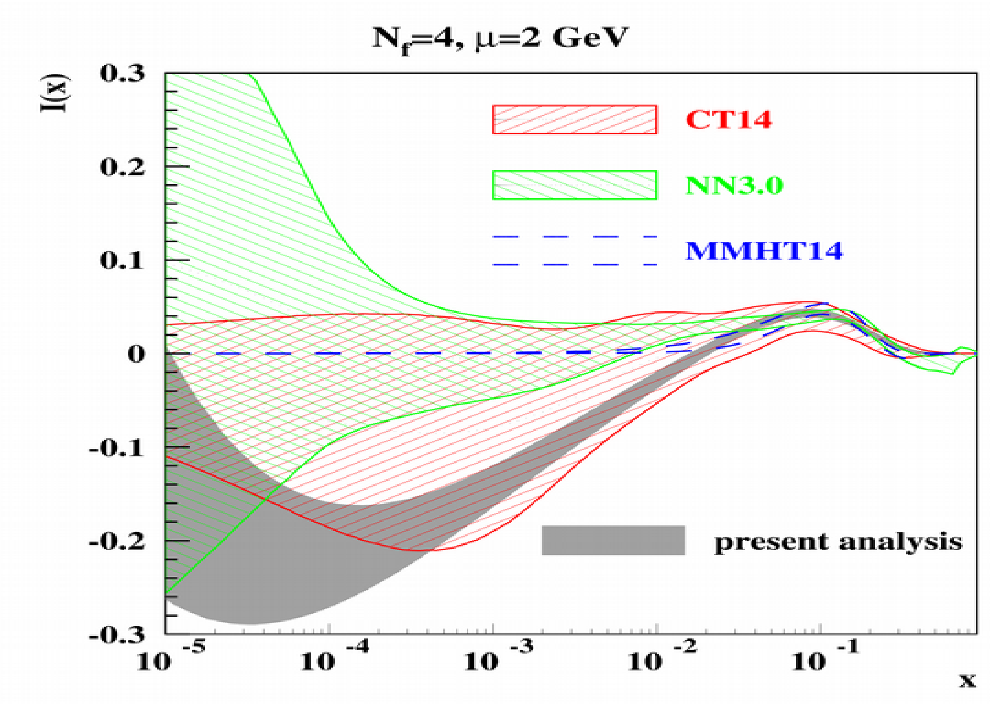


Experiment		ATLAS	CMS		D0		LHCb		
$\sqrt{s}$ (TeV)		7	7	8	1.96		7	8	8
Final states		$W^+ \to l^+\nu$ $W^- \to l^-\nu$ $Z \to l^+l^-$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$W^+ \to e^+\nu$ $W^- \to e^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$ $Z \to \mu^+\mu^-$	$Z \to e^+e^-$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$ $Z \to \mu^+\mu^-$
Cut on the lepton P_T		$P_T^l > 20$ GeV	$P_T^\mu > 25$ GeV	$P_T^\mu > 25$ GeV	$P_T^\mu > 25$ GeV	$P_T^e > 25$ GeV	$P_T^\mu > 20$ GeV	$P_T^e > 20$ GeV	$P_T^\mu > 20$ GeV
NDP		30	11	22	10	13	31	17	32
χ^2	ABMP16	30.0	22.0	16.8	18.2	19.6	45.4	21.5	45.4
	CJ15	–	–	–	20	29	–	–	–
	CT14	42	–^{a}	–	–	34.7	–	–	–
	JR14	–	–	–	–	–	–	–	–
	`HERAFitter`	–	–	–	13	19	–	–	–
	MMHT14	39	–	–	21	–	–	–	–
	NNPDF3.0	35.4	18.9	–	–	–	–	–	–

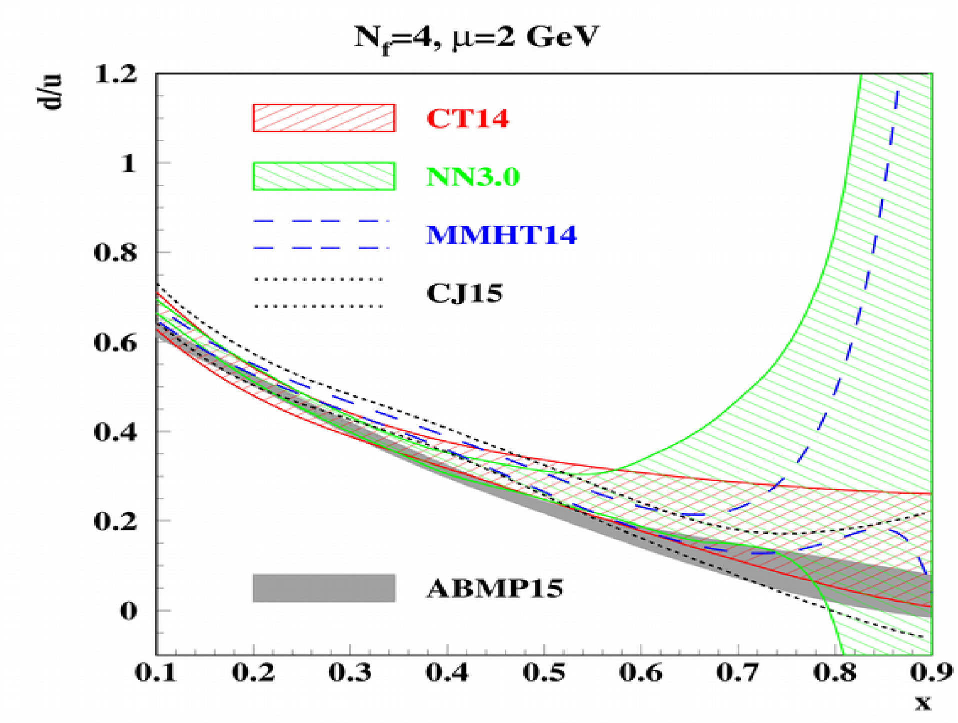
^{a}Statistically less significant data with the cut of $P_T^\mu > 35$ GeV are used.

Obsolete/superseded/low-accuracy Tevatron and LHC data are not used

Impact of the forward Drell-Yan data



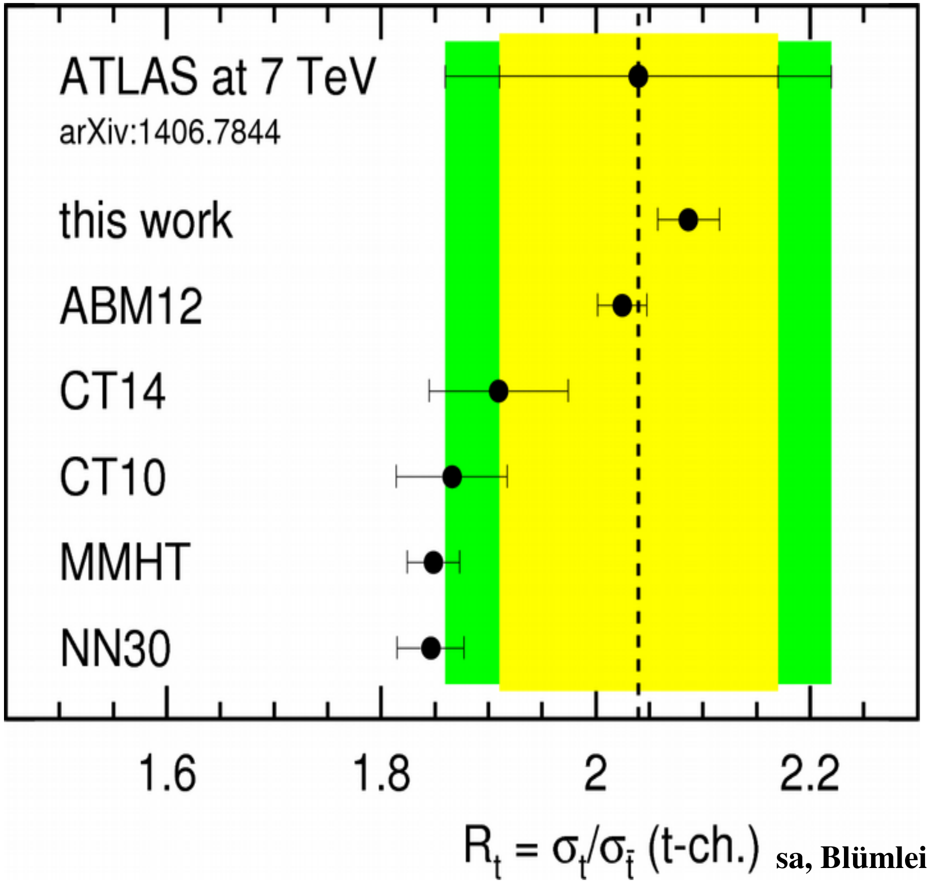
sa, Blümlein, Moch, Plačakytė, hep-ph/1508.07923

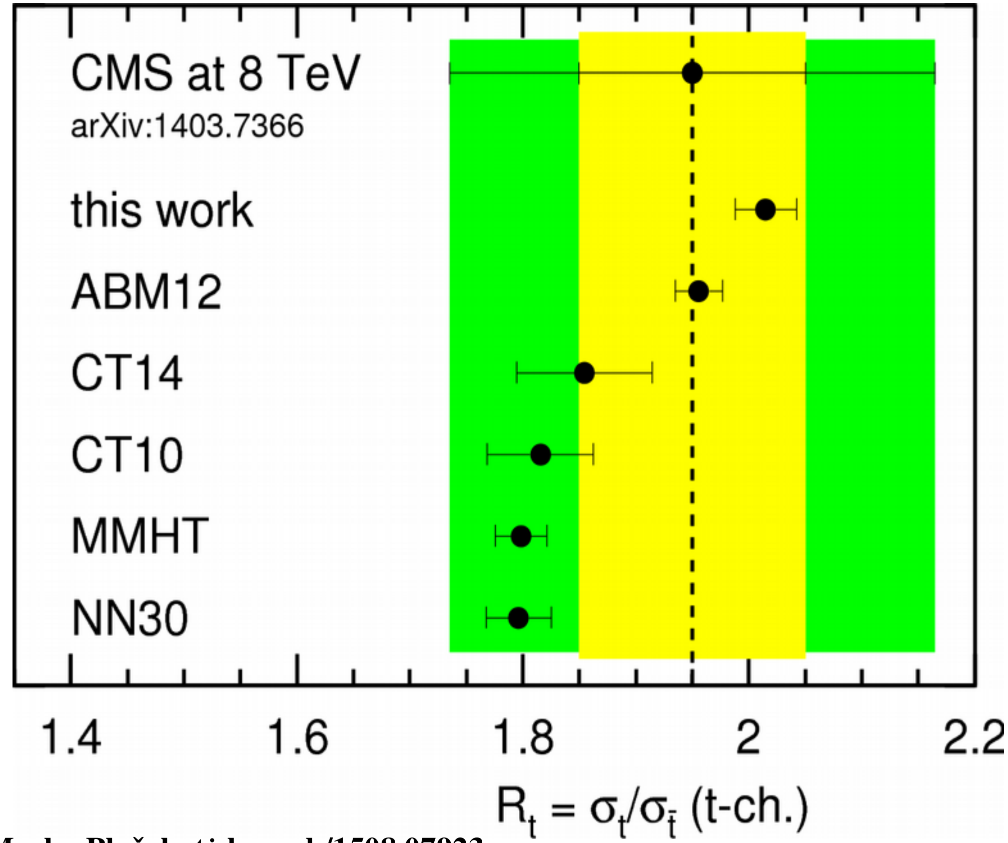


Accardi, et al. hep-ph/1603.08906

- Relaxed form of the sea iso-spin asymmetry I(x) at small x; Regge-like behaviour is recovered only at x~10^{-6}; at large x it is still defined by the phase-space constraint
- Good constraint on the d/u ratio w/o deuteron data → independent extraction of the deuteron corrections **Accardi, Brady, Melnitchouk, Owens, Sato hep-ph/1602.03154; talks by Accardi and Petti at DIS2016**
- Big spread between different PDF sets, up to factor of 30 at large x → PDF4LHC averaging is misleading in this part

Implication for(of) the single-top production





sa, Blümlein, Moch, Plačakytė hep-ph/1508.07923

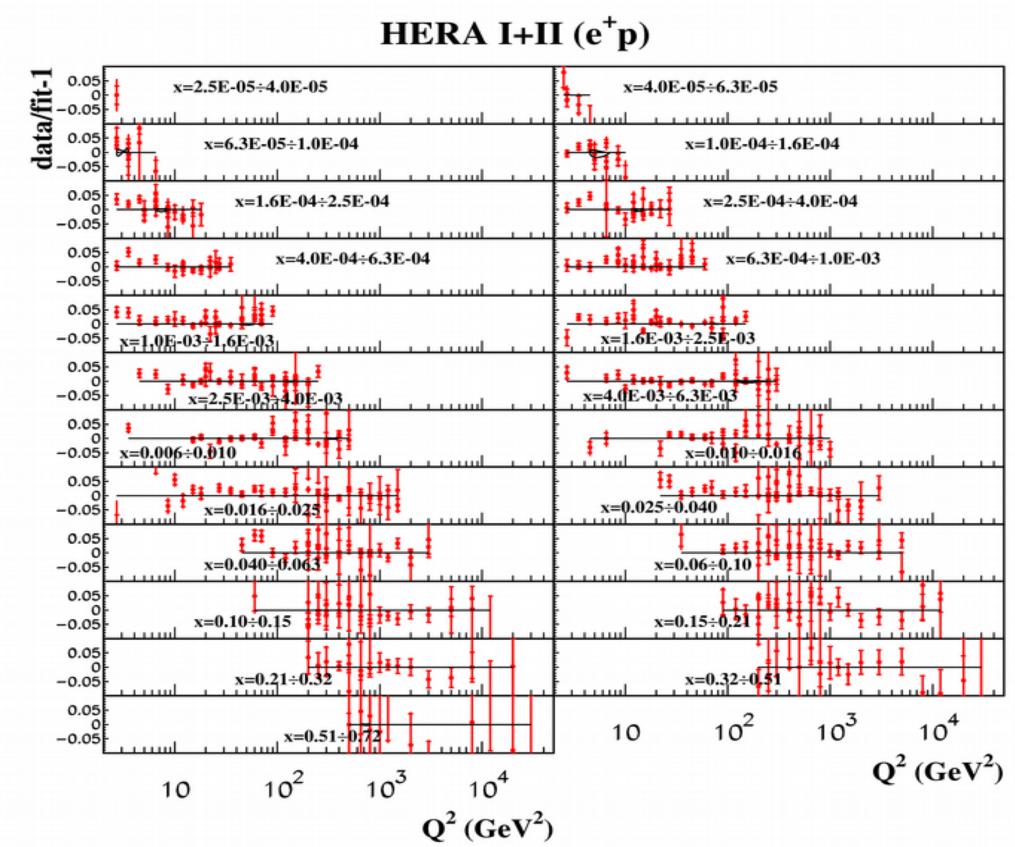
- ATLAS and CMS data on the ratio t/tbar are in a good agreement

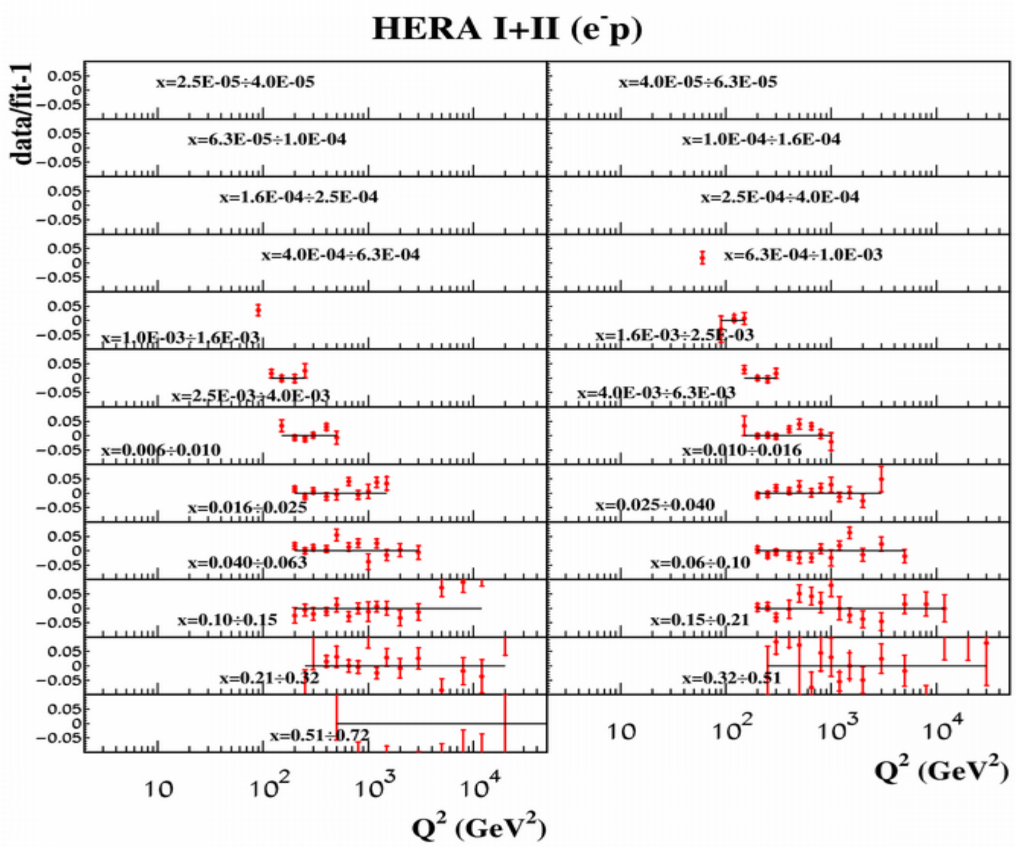
- The predictions driven by the froward DY data are in a good agreement with the single-top data (N.B.: ABM12 is based on the deuteron data → consistent deuteron correction was used **talk by Petti at DIS2016**)

Single-top production discriminate available PDF sets and can serve as a standard candle process

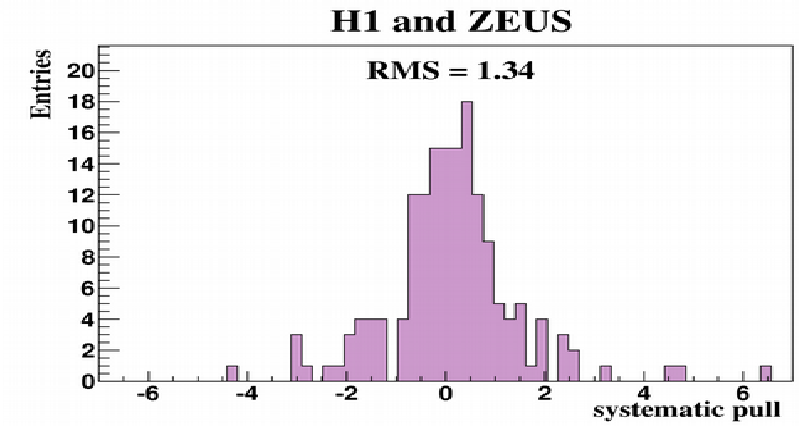
Inclusive HERA I+II data

H1 and ZEUS hep-ex/1506.06042





Q^2(HERA)	χ^2/NDP(HERA)
>2.5 GeV^2	1505/1168=1.29
>5 GeV^2	1350/1092=1.24
>10 GeV^2	1225/1007=1.22



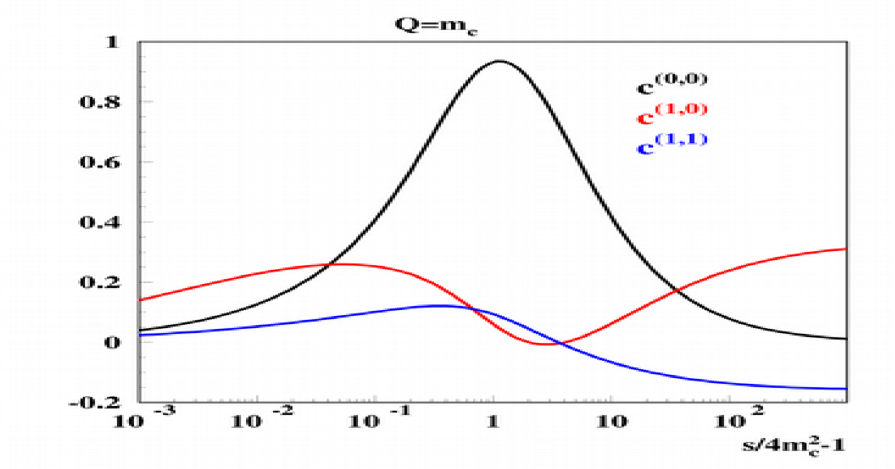
The value of χ^2/NDP is bigger than 1, however still comparable to the pull distribution width

Heavy-quark electro-production in the FFNS

- Only 3 light flavors appear in the initial state
- The dominant mechanism is photon-gluon fusion
- The coefficient functions are known up to the NLO

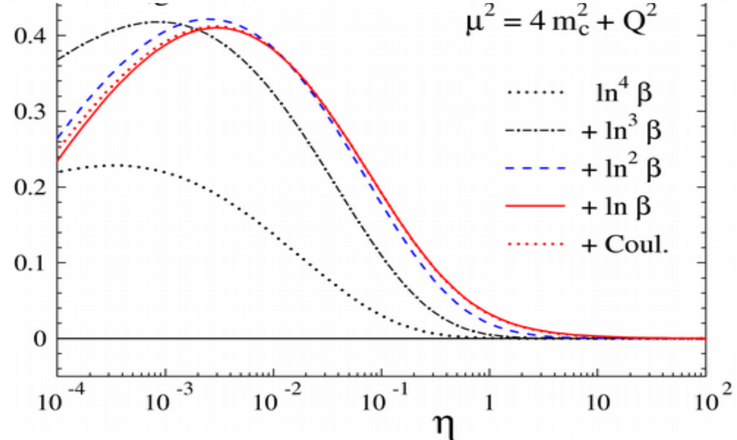
Witten NPB 104, 445 (1976)

Laenen, Riemersma, Smith, van Neerven NPB 392, 162 (199



- Involved high-order calculations:
  - NNLO terms due to threshold resummation

Laenen, Moch PRD 59, 034027 (1999)
Lo Presti, Kawamura, Moch, Vogt [hep-ph 1008.0951]



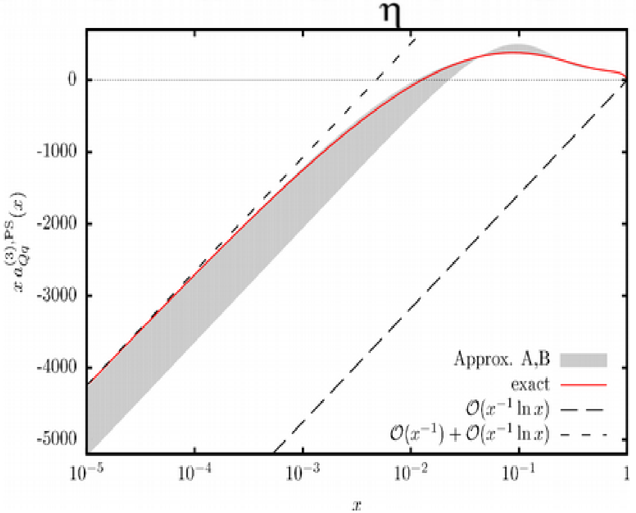
  - limited set of the NNLO Mellin moments

Ablinger at al. NPB 844, 26 (2011)
Bierenbaum, Blümlein, Klein NPB 829, 417 (2009)

- At large Q the leading-order coefficient → $\ln(Q/m_h)$ and may be quite big despite the suppression by factor of α_s and should be resummed

Shifman, Vainstein, Zakharov NPB 136, 157 (1978)

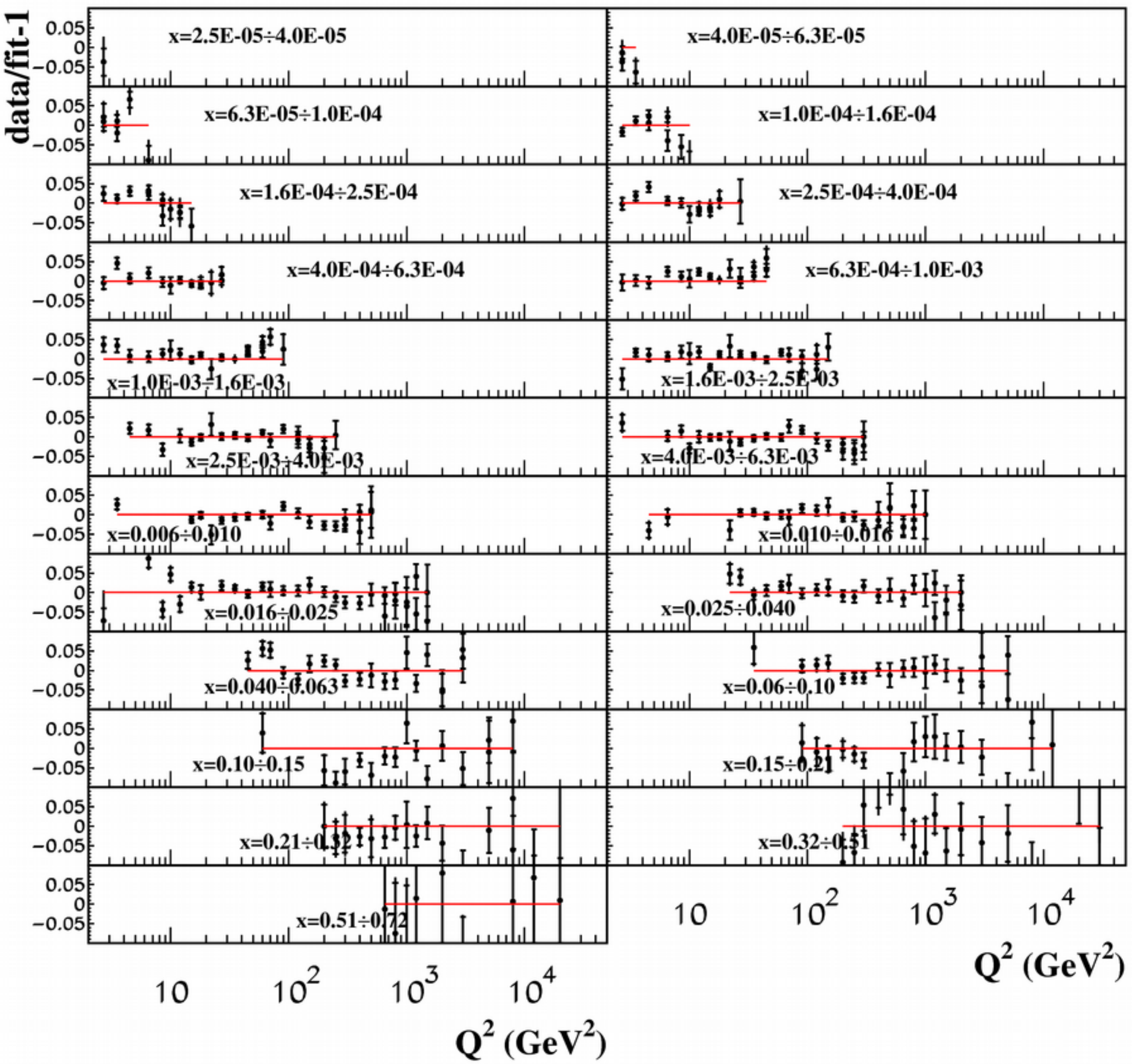
→ a motivation to derive the VFN scheme matched to the FFNS (ACOT...., RT..., FONLL....)



Ablinger et al. hep-ph/1409.1135

Statistical check of big-log impact in ABM12 fit

HERA-I $e^{+}p$

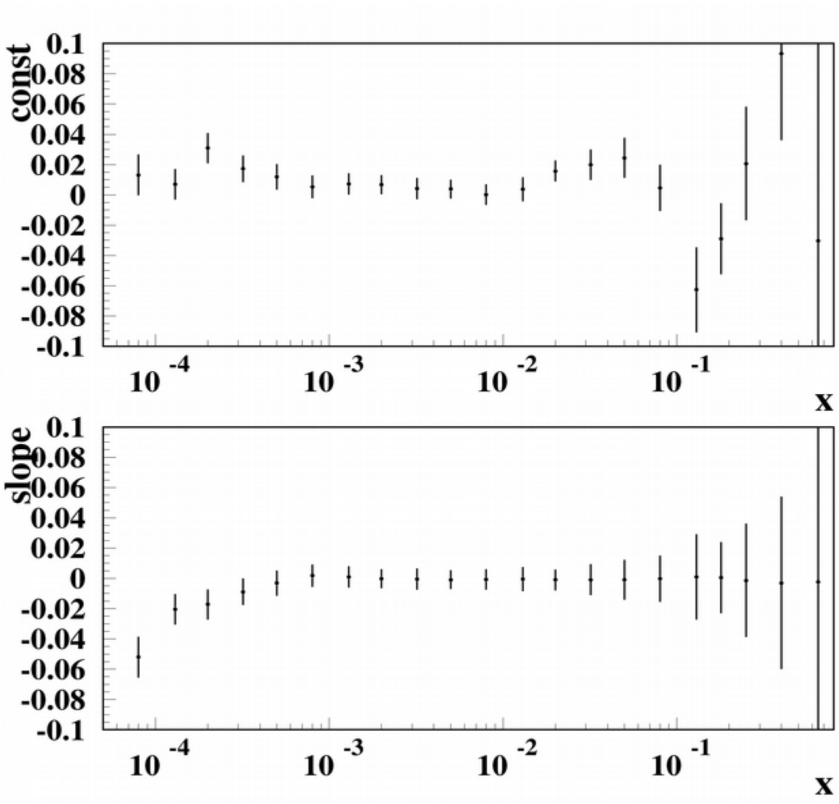


Q^2_{min} (GeV^2)	χ^2/NDP
10	366 / 324
100	193 / 201
1000	95 / 83

pulls=const + slope * log(Q^2/Q^2_0)

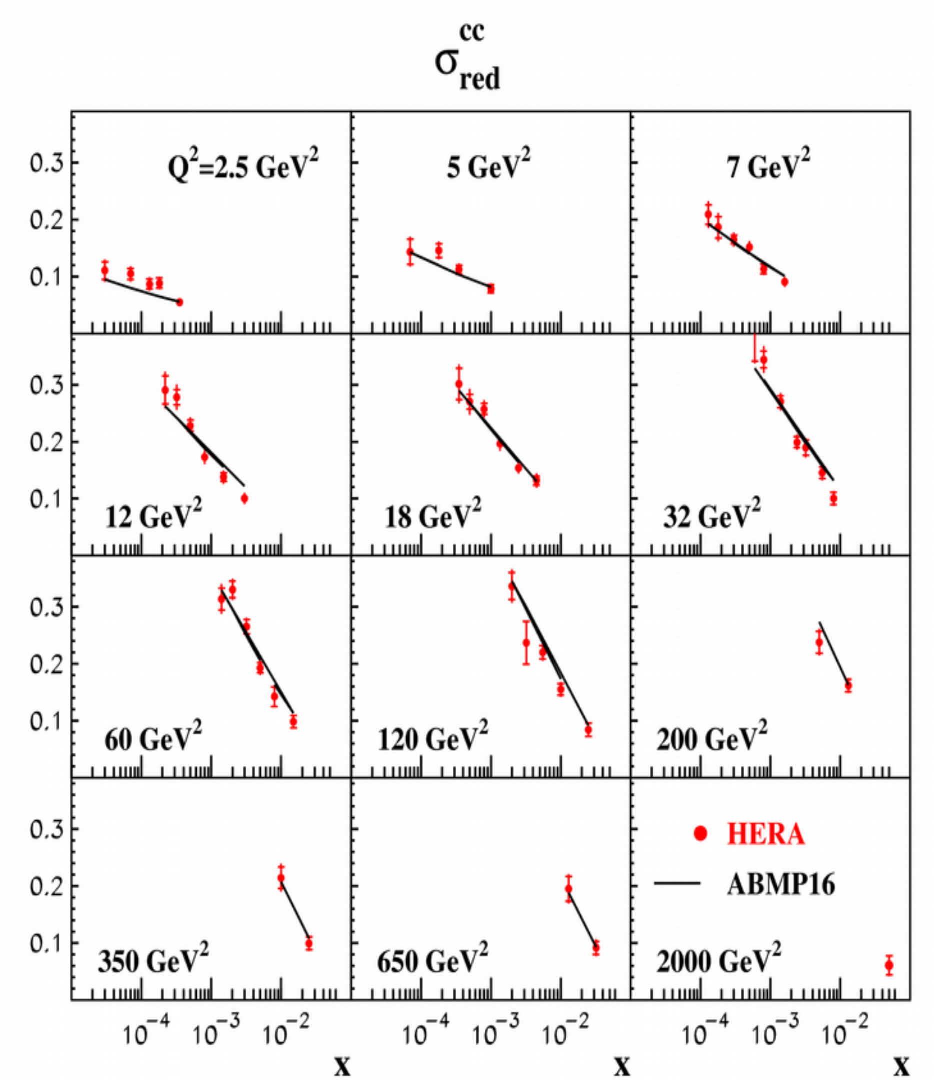
No traces of big logs

(cf. Extras and sa, Blümlein, Moch hep-ph/1307.7258)



HERA charm data and $m_c(m_c)$

H1/ZEUS PLB 718, 550 (2012)



- Approximate NNLO massive Wilson coefficients (combination of the threshold corrections, high-energy limit, and the NNLO massive OMEs)

Kawamura, Lo Presti, Moch, Vogt NPB 864, 399 (2012)

- Running-mass definition of m_c

X^2/NDP=61/52

$m_c(m_c)$=1.250±0.020(exp.) GeV ABMP16

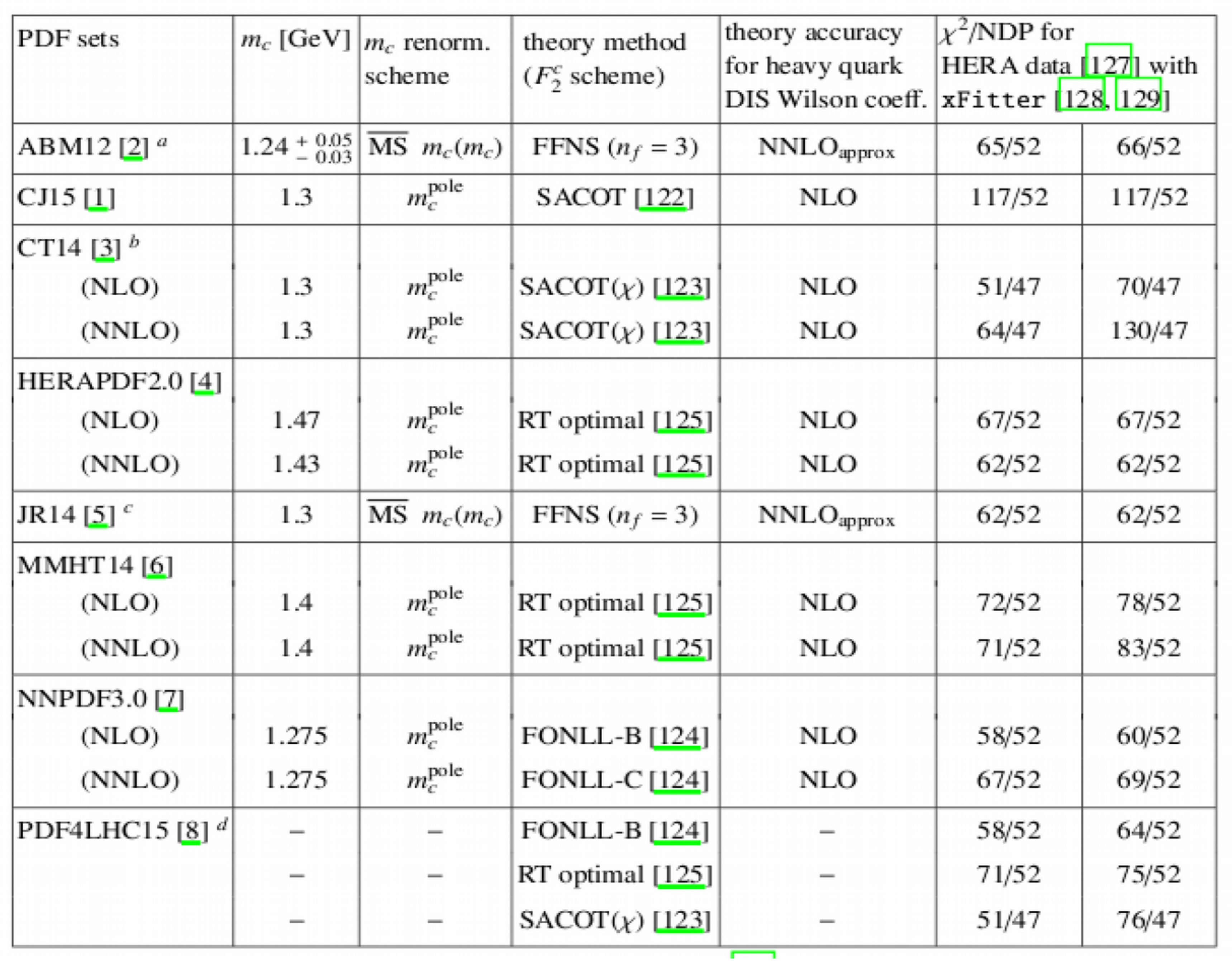
$m_c(m_c)$=1.24±0.03(exp.) GeV ABM12

Good agreement with the e+e- determinations → the FFN scheme nicely works for the existing data

- RT optimal
  X^2/NDP=82/52 NNLO
  m_c(pole)=1.4 GeV **MMHT14 EPJC 75, 204 (2015)**
- F0NLL
  X^2/NDP=60/47 NNLO
  m_c(pole)=1.275 GeV **NNPDF3.0 JHEP 1504, 040 (2015)**
- S-ACOT-χ
  X^2/NDP=59/47 NNLO
  m_c(pole)=1.3 GeV **CT14 hep-ph 1506.07443**

$m_c(m_c)$=1.246±0.023 (h.o.) GeV NNLO

Kiyo, Mishima, Sumino hep-ph/1510.07072

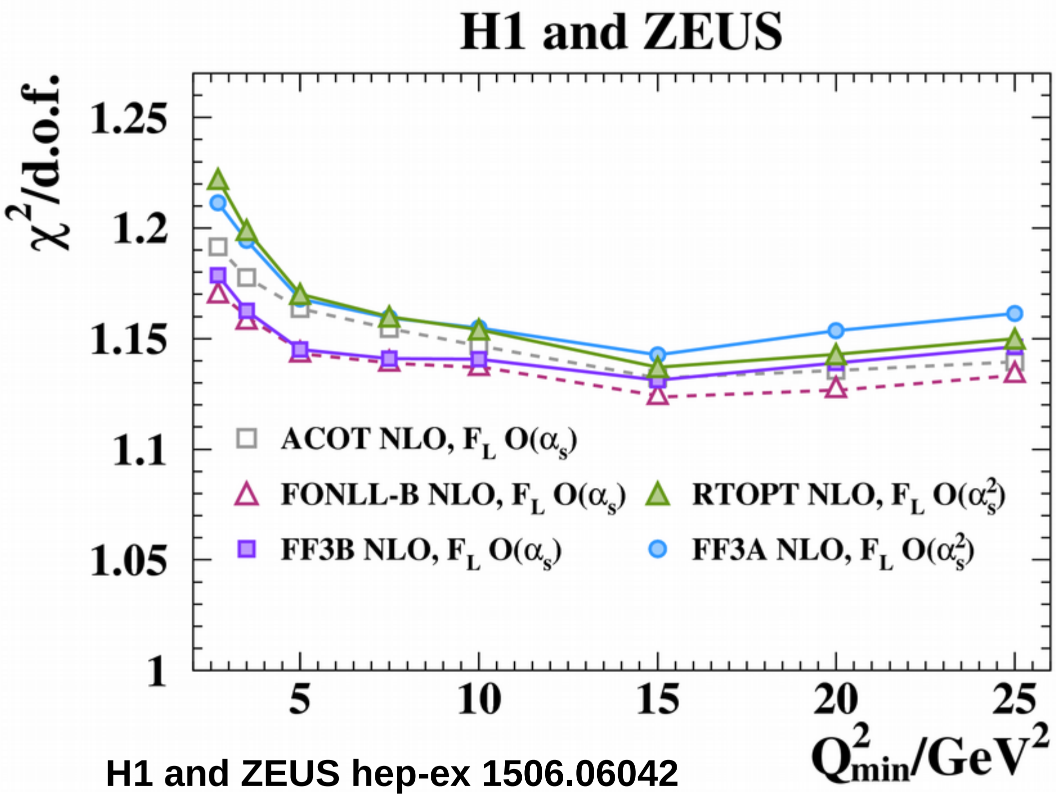


PDF sets	m_c [GeV]	m_c renorm. scheme	theory method (F_2^c scheme)	theory accuracy for heavy quark DIS Wilson coeff.	χ^2/NDP for HERA data [127] with `xFitter` [128, 129]	
ABM12 [2] [a]	$1.24\,^{+\,0.05}_{-\,0.03}$	$\overline{\text{MS}}$ $m_c(m_c)$	FFNS ($n_f = 3$)	NNLO$_{\text{approx}}$	65/52	66/52
CJ15 [1]	1.3	m_c^{pole}	SACOT [122]	NLO	117/52	117/52
CT14 [3] [b]						
(NLO)	1.3	m_c^{pole}	SACOT(χ) [123]	NLO	51/47	70/47
(NNLO)	1.3	m_c^{pole}	SACOT(χ) [123]	NLO	64/47	130/47
HERAPDF2.0 [4]						
(NLO)	1.47	m_c^{pole}	RT optimal [125]	NLO	67/52	67/52
(NNLO)	1.43	m_c^{pole}	RT optimal [125]	NLO	62/52	62/52
JR14 [5] [c]	1.3	$\overline{\text{MS}}$ $m_c(m_c)$	FFNS ($n_f = 3$)	NNLO$_{\text{approx}}$	62/52	62/52
MMHT14 [6]						
(NLO)	1.4	m_c^{pole}	RT optimal [125]	NLO	72/52	78/52
(NNLO)	1.4	m_c^{pole}	RT optimal [125]	NLO	71/52	83/52
NNPDF3.0 [7]						
(NLO)	1.275	m_c^{pole}	FONLL-B [124]	NLO	58/52	60/52
(NNLO)	1.275	m_c^{pole}	FONLL-C [124]	NLO	67/52	69/52
PDF4LHC15 [8] [d]	–	–	FONLL-B [124]	–	58/52	64/52
	–	–	RT optimal [125]	–	71/52	75/52
	–	–	SACOT(χ) [123]	–	51/47	76/47

Accardi, et al. hep-ph/1603.08906

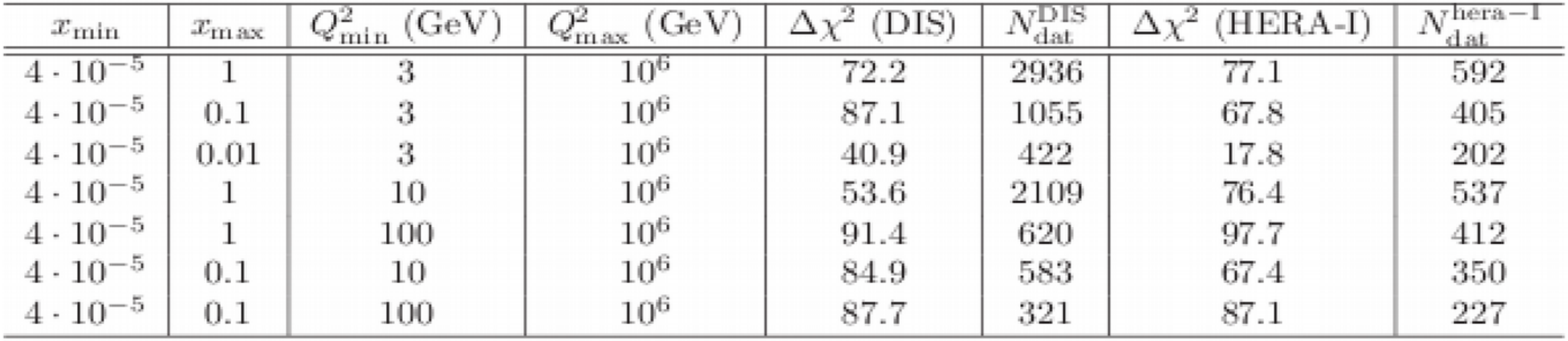
No advantage of the GMVFN schemes: the VFN χ^2 values are systematically bigger than the FFN ones

Factorization scheme benchmarking



H1 and ZEUS hep-ex 1506.06042

- Data allow to discriminate factorization schemes
- FFN scheme works very well in case of correct setting (running mass definition and correct value of m_c) → no traces of big logs due to resummation



$x_{\min}$	$x_{\max}$	$Q^2_{\min}$ (GeV)	$Q^2_{\max}$ (GeV)	$\Delta\chi^2$ (DIS)	$N_{\rm dat}^{\rm DIS}$	$\Delta\chi^2$ (HERA-I)	$N_{\rm dat}^{\rm hera-I}$
$4 \cdot 10^{-5}$	1	3	10^6	72.2	2936	77.1	592
$4 \cdot 10^{-5}$	0.1	3	10^6	87.1	1055	67.8	405
$4 \cdot 10^{-5}$	0.01	3	10^6	40.9	422	17.8	202
$4 \cdot 10^{-5}$	1	10	10^6	53.6	2109	76.4	537
$4 \cdot 10^{-5}$	1	100	10^6	91.4	620	97.7	412
$4 \cdot 10^{-5}$	0.1	10	10^6	84.9	583	67.4	350
$4 \cdot 10^{-5}$	0.1	100	10^6	87.7	321	87.1	227

We conclude that the FFN fit is actually based on a less precise theory, in that it does not include full resummation of the contribution of heavy quarks to perturbative PDF evolution, and thus provides a less accurate description of the data

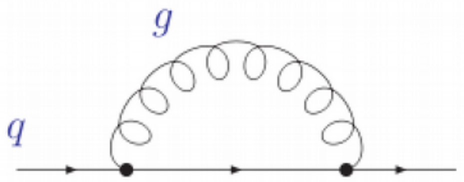
NNPDF PLB 723, 330 (2013)

Quark mass renormalization

Pole mass

- Based on (unphysical) concept of heavy-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2=m_q^2}$$



  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Renormalon ambiguity in definition of pole mass of $\mathcal{O}(\Lambda_{QCD})$
  Bigi, Shifman, Uraltsev, Vainshtein '94; Beneke, Braun '94; Smith, Willenbrock '97

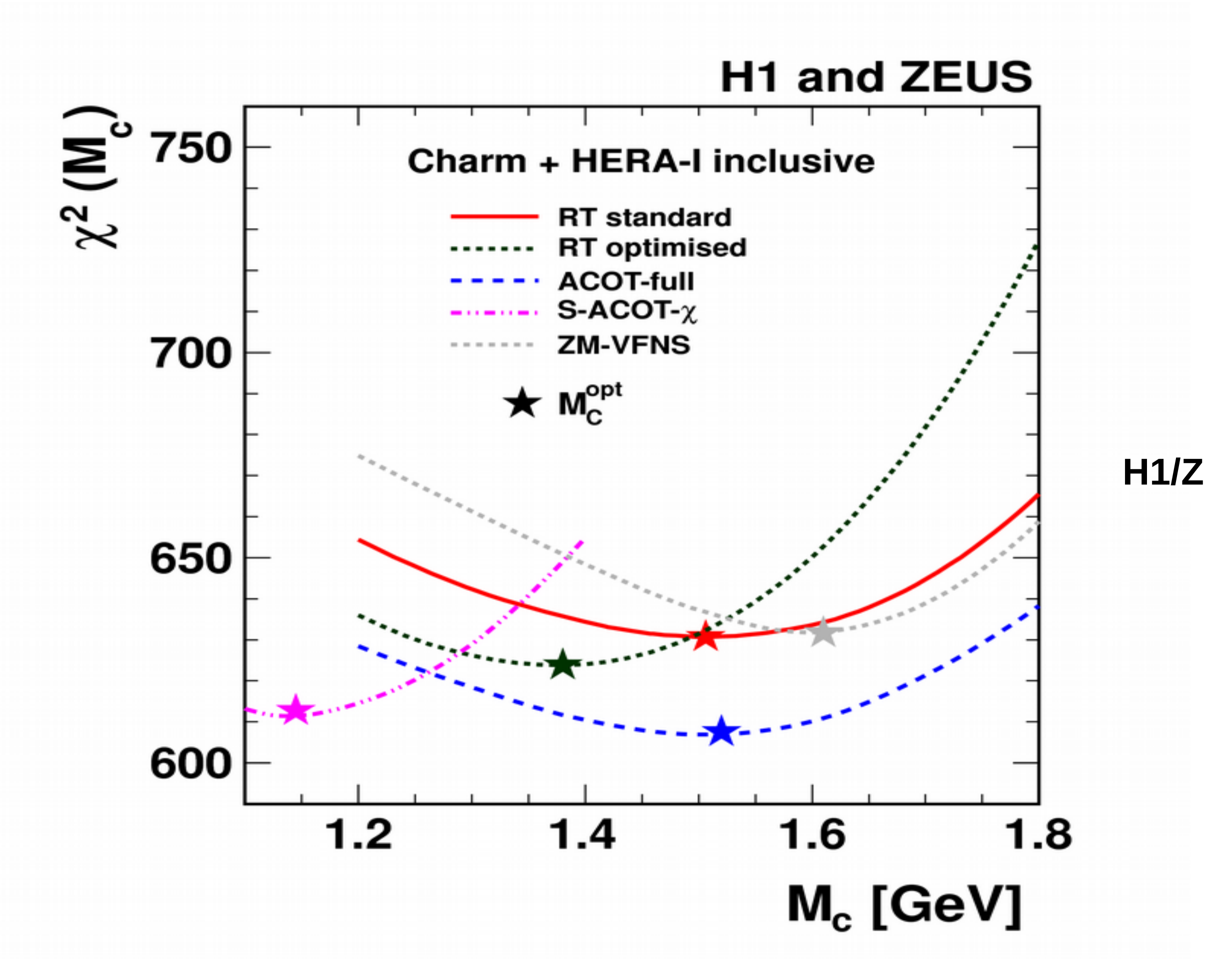
$\overline{MS}$ *mass*

- Free of infrared renormalon ambiguity
- Conversion between $m_{\rm pole}$ and $\overline{MS}$ mass $m(\mu_R)$ in perturbation theory known to four loops in QCD Marquard, Smirnov, Smirnov, Steinhauser '15
  - does not converge in case of charm quark

$$m_c(m_c) = 1.27 \text{ GeV} \longrightarrow m_c^{\rm pole} = 1.47 \text{ GeV (one loop)}$$
$$\longrightarrow m_c^{\rm pole} = 1.67 \text{ GeV (two loops)}$$
$$\longrightarrow m_c^{\rm pole} = 1.93 \text{ GeV (three loops)}$$
$$\longrightarrow m_c^{\rm pole} = 2.39 \text{ GeV (four loops)}$$

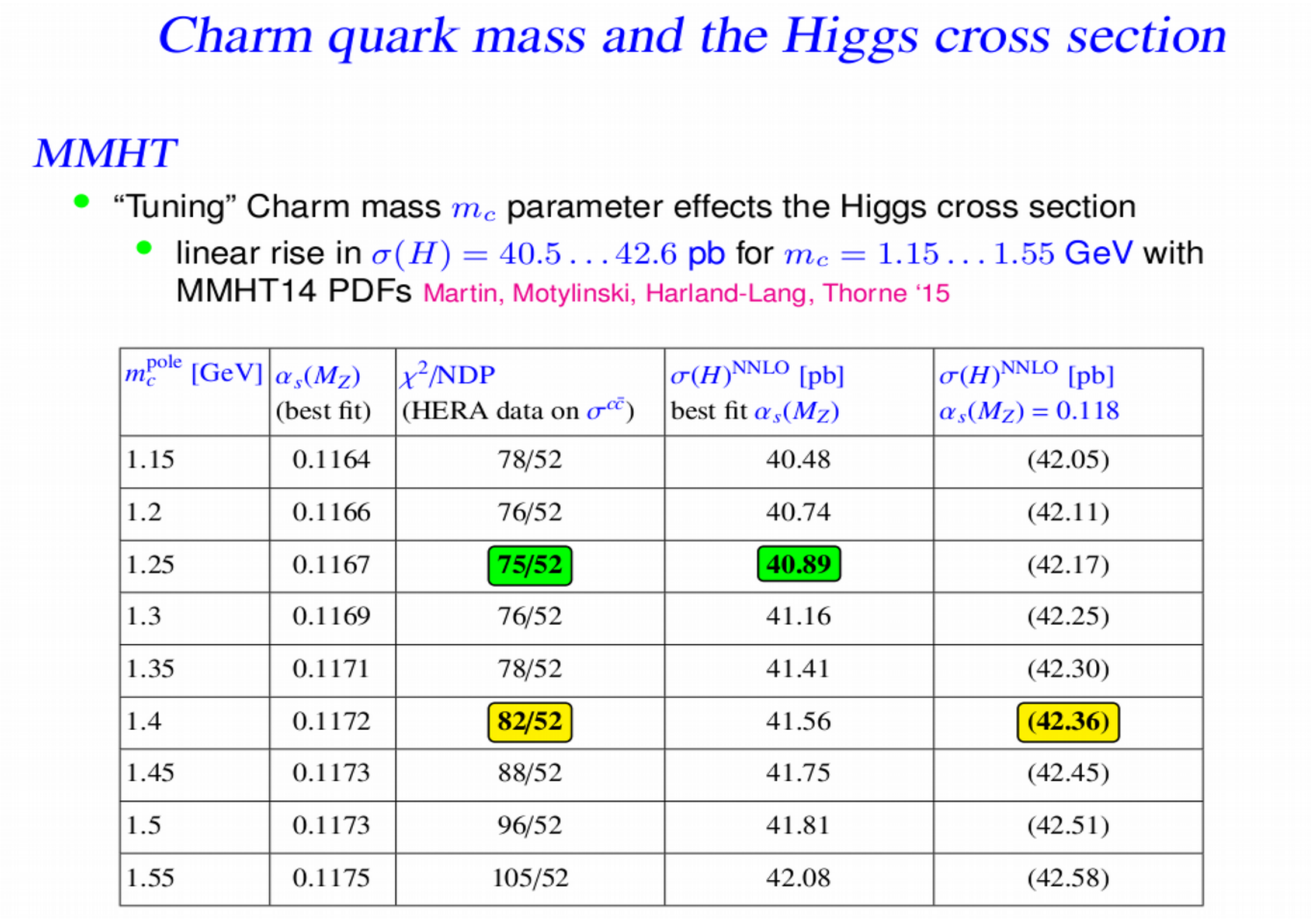
c-quark mass in the CMVFN schemes

The values of pole mass m_c used by different groups and preferred by the PDF fits are systematically lower than the PDG value



H1/ZEUS PLB 718, 550 (2012)

Wide spread of the m_c obtained in different version of the GMVFN schemes → quantitative illustration of the GMVFNS uncertainties



Charm quark mass and the Higgs cross section

MMHT

- "Tuning" Charm mass m_c parameter effects the Higgs cross section
  - linear rise in $\sigma(H) = 40.5 \ldots 42.6$ pb for $m_c = 1.15 \ldots 1.55$ GeV with MMHT14 PDFs Martin, Motylinski, Harland-Lang, Thorne '15

m_c^{pole} [GeV]	$\alpha_s(M_Z)$ (best fit)	χ^2/NDP (HERA data on $\sigma^{c\bar{c}}$)	$\sigma(H)^{\text{NNLO}}$ [pb] best fit $\alpha_s(M_Z)$	$\sigma(H)^{\text{NNLO}}$ [pb] $\alpha_s(M_Z) = 0.118$
1.15	0.1164	78/52	40.48	(42.05)
1.2	0.1166	76/52	40.74	(42.11)
1.25	0.1167	**75/52**	**40.89**	(42.17)
1.3	0.1169	76/52	41.16	(42.25)
1.35	0.1171	78/52	41.41	(42.30)
1.4	0.1172	**82/52**	41.56	**(42.36)**
1.45	0.1173	88/52	41.75	(42.45)
1.5	0.1173	96/52	41.81	(42.51)
1.55	0.1175	105/52	42.08	(42.58)

A spread of 41.0 ….. 42.3 pb was obtained by R.Thorne with α_s varied; the same trend is observed for MSTW08

Charm quark mass and the Higgs cross section

NNPDF

- Same trend: lighter charm mass implies smaller Higgs cross section
  - fit range for m_c too small and no correlation with value of $\alpha_s(M_Z)$
  - best fits with NNPDF2.1 and NNPDF30 give range $\sigma(H) = 42.6\ldots44.2$ pb

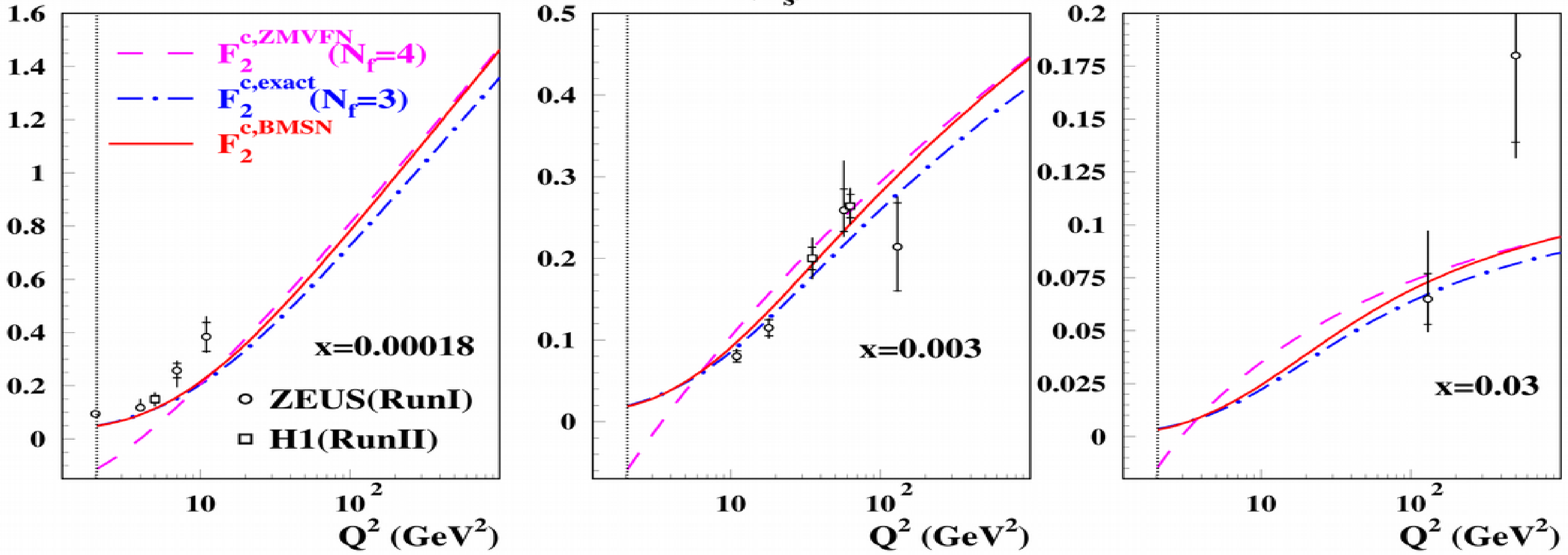
PDF sets	m_c^{pole} [GeV]	$\alpha_s(M_Z)$ (fixed)	χ^2/NDP (HERA data on $\sigma^{c\bar{c}}$)	$\sigma(H)^{\text{NNLO}}$ [pb] fixed $\alpha_s(M_Z)$
NNPDF2.1 [arXiv:1107.2652]	$\sqrt{2}$	0.119	**65/52**	44.18 ± 0.49
	1.5	0.119	78/52	44.54 ± 0.51
	1.6	0.119	92/52	44.74 ± 0.50
	1.7	0.119	110/52	44.95 ± 0.51
NNPDF2.3 [arXiv:1207.1303]	$\sqrt{2}$	0.118	71/52	43.77 ± 0.41
NNPDF3.0 [arXiv:1410.8849]	1.275	0.118	**67/52**	42.59 ± 0.80

BMSN prescription of GMVFNS

Buza, Matiounine, Smith, van Neerven EPJC 1, 301 (1998)

$$F_2^{h,\mathrm{BMSN}}(N_f+1,x,Q^2)=F_2^{h,\mathrm{exact}}(N_f,x,Q^2)+F_2^{h,\mathrm{ZMVFN}}(N_f+1,x,Q^2)-F_2^{h,\mathrm{asymp}}(N_f,x,Q^2)$$

$O(\alpha_s^2)$ **Cacciari, Greco, Nason JHEP 9805, 007 (1998)**

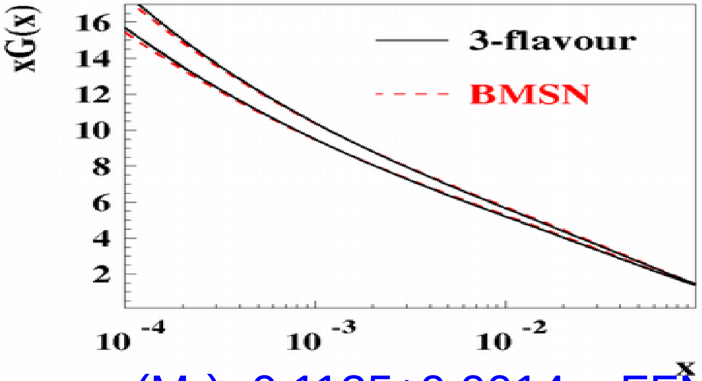


sa, Blümlein, Klein, Moch PRD 81, 014032 (2010)

- Very smooth matching with the FFNS at $Q \to m_h$
- Renormgroup invariance is conserved; the PDFs in MSbar scheme

In the $O(\alpha_s^2)$ the FFNS and GMVFNS are comparable at large scales since the big logs appear in the high order corrections to the massive coefficient functions

Glück, Reya, Stratmann NPB 422, 37 (1994)



The big-log resummation is important **NNPDF**

The value of $\alpha_s(M_z)$ is reduced in FFN **MSTW**

$\alpha_s(M_z)$=0.1135±0.0014 FFN

$\alpha_s(M_z)$=0.1129±0.0014 BMSN

FOPT PDFs and QCD evolution

$$c^{(1)}(x,\mu^2) = a_s(\mu^2)\int_x^1 \frac{dz}{z} A^{(1)}_{hg}(\frac{\mu^2}{m_c^2}, z) g\left(\frac{x}{z},\mu^2\right)$$

LO c-quark PDF (FOPT)

$$A^{(1)}_{hg}\left(\frac{\mu^2}{m_c^2},z\right) = \ln\left(\frac{\mu^2}{m_c^2}\right) P^{(0)}_{qg}(z)$$

LO massive OME

$$\dot{c}^{(1)}(x,\mu^2) \equiv \frac{dc^{(1)}(x,\mu^2)}{d\ln\mu^2} = a_s(\mu^2)\int_x^1 \frac{dz}{z} P^{(0)}_{qg}(z) g\left(\frac{x}{z},\mu^2\right)$$

c-quark evolution in LO, FOPT boundary condition at $\mu_0 \approx m_c$

$$\delta\dot{c}^{(1)}(x,\mu^2) = \frac{da_s}{d\mu^2}\frac{c^{(1)}(x,\mu^2)}{a_s}$$

(FOPT – evolved) in LO: =0 $\mu = m_c$

$$A^{(2)}_{hg,hq} = a^{(2,0)}_{hg,hq} + a^{(2,1)}_{hg,hq}\ln\left(\frac{\mu^2}{m_c}\right) + a^{(2,2)}_{hg,hq}\ln^2\left(\frac{\mu^2}{m_c}\right)$$

NLO massive OME

$$\delta\dot{c}^{(2)}(x,\mu^2) \sim a_s\frac{da_s}{d\mu^2} a^{(2,0)}_{hg}$$

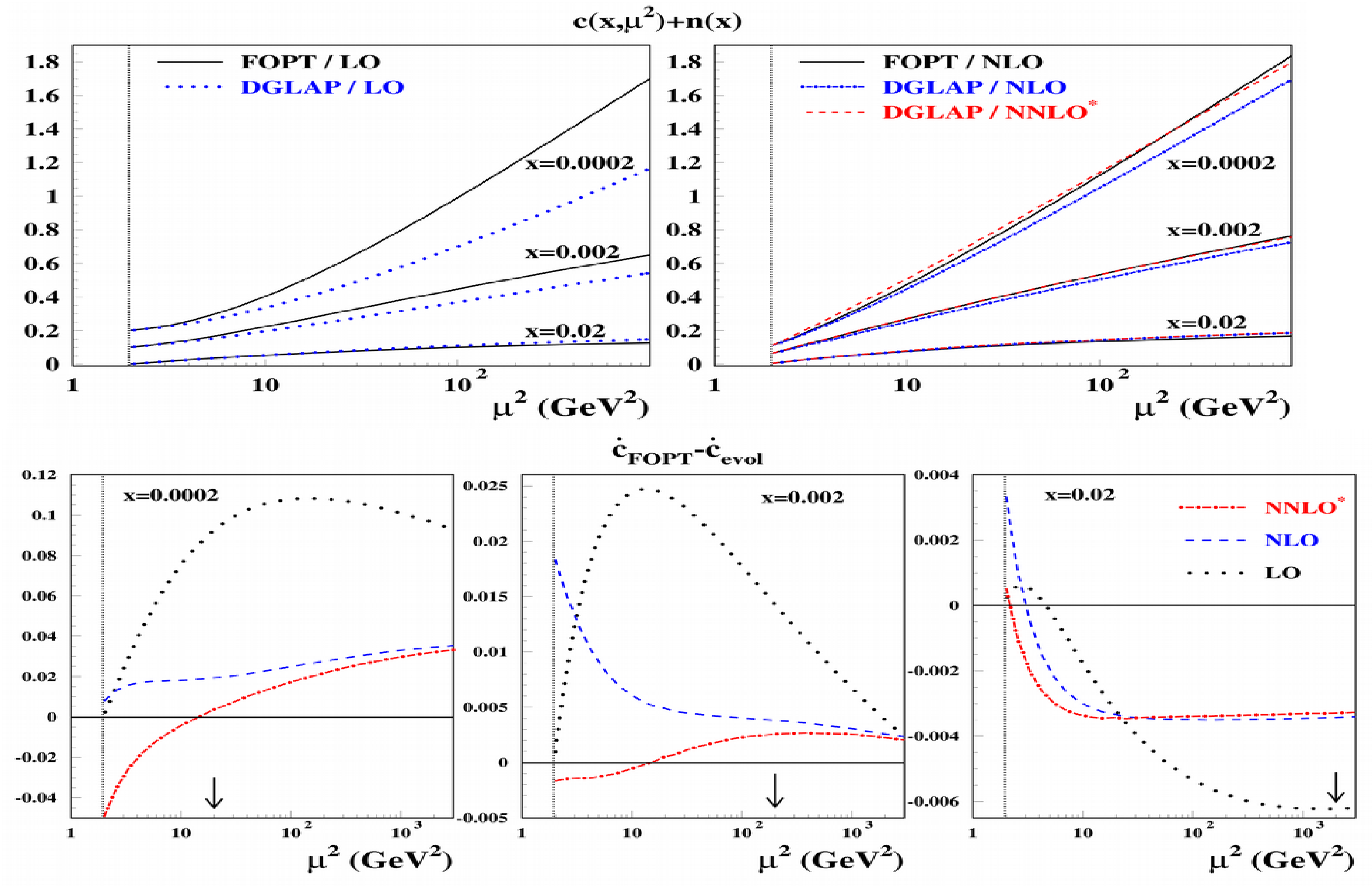
(FOPT – evolved) in NLO: ≠0 $\mu = m_c$

NLO: NLO evoultion with the FOPT boundary conditions in NLO

NNLO*: NNLO evoultion with the FOPT boundary conditions in NLO

Blümlein, Riemersma, Botje, Pascaud, Zomer, van Neerven, Vogt hep-ph/9609400

Comparison of the FOPT and evolved c-quark PDFs



The difference between FOPT and evolved PDFs is localized at small scales: uncertainties due to missing high-orders rather than impact of the big-log resummation

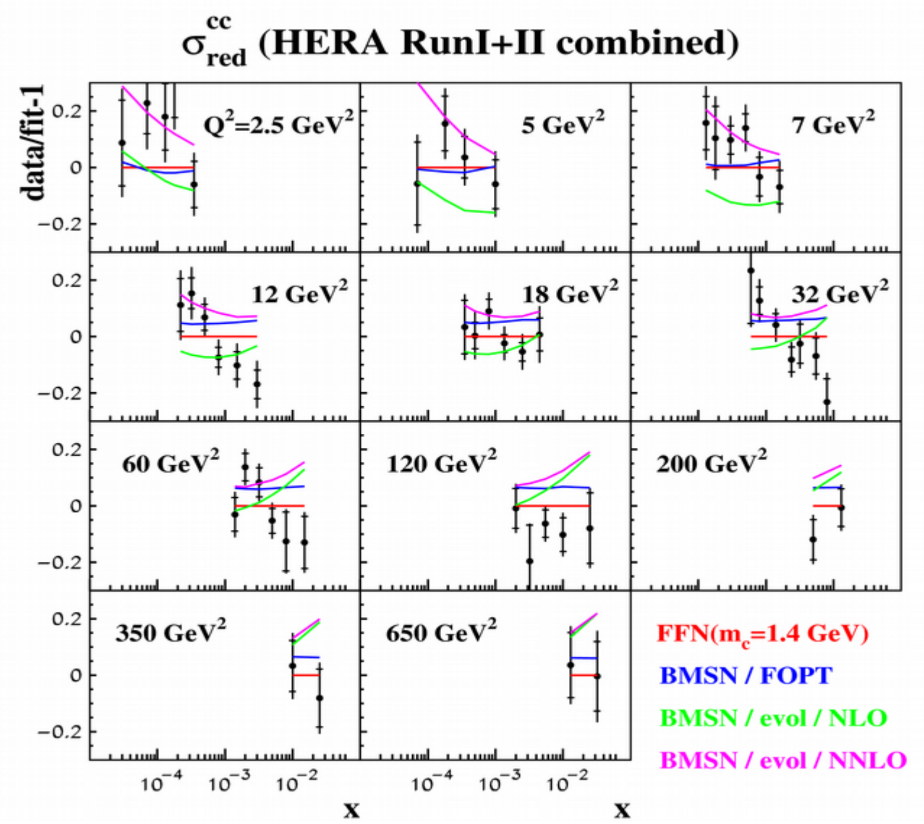
sa, Blümlein, Moch hep-ph/1307.7258)

BMSN with the evolved PDFs

- Combined HERA charm production data

H1/ZEUS PLB 718, 550 (2012)

- PDFs from variant of ABM11 fit with m_c=1.4 GeV (pole mass definition), option A of NNLO W.coef.



- Two variants of 4-flavor PDF evolution

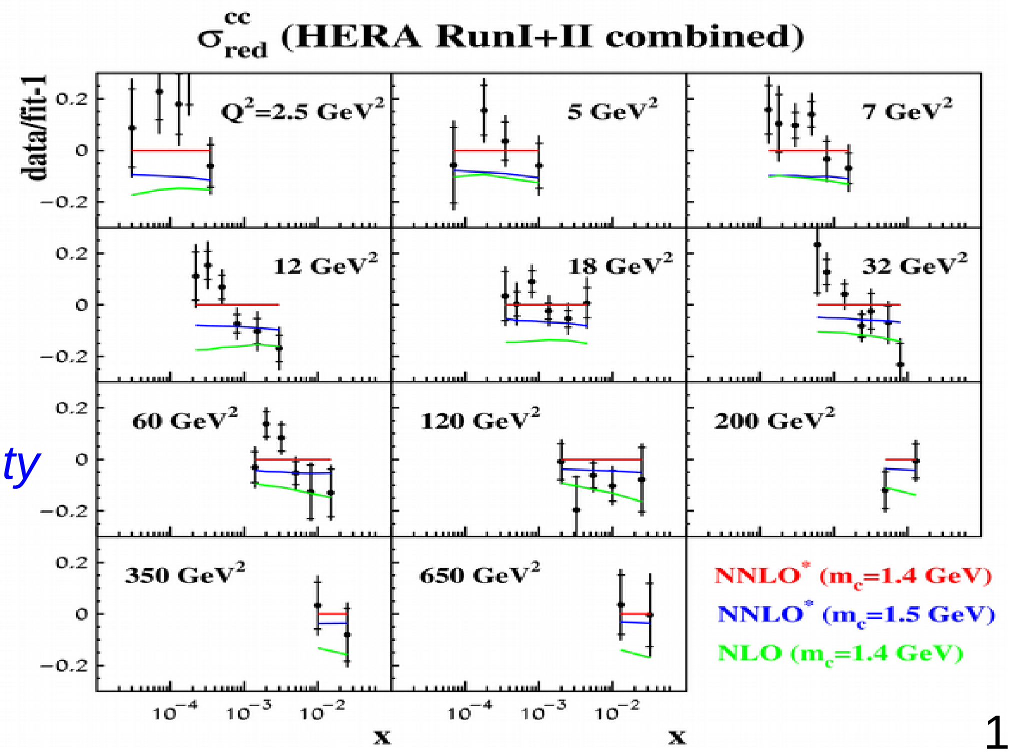
  NNLO (consistent with the light PDF evolution, inconsistent with the NLO matching) **

  NLO (inconsistent with the light PDF evolution, consistent with the NLO matching)

  ** commonly used in the VFN fits

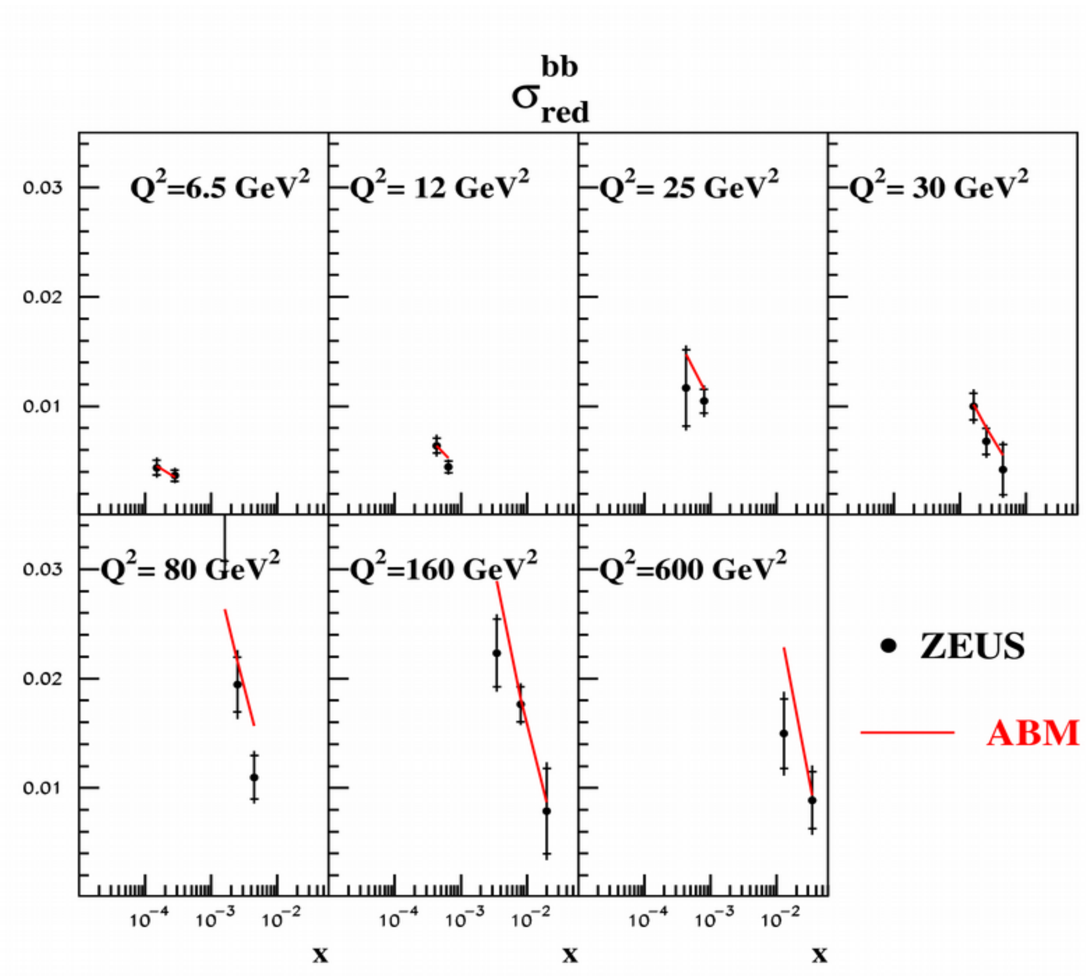
- Substantial difference between NLO and NNLO versions
- The evolved predictions demonstrate strong x-dependence and weak Q^2-dependence

The difference with FOPT appears rather due to inconsistent evolution than due to big-logs → should be considered as a theoretical uncertainty in the VFN predictions



ZEUS bottom data and $m_b(m_b)$

ZEUS hep-ex/1405.6915



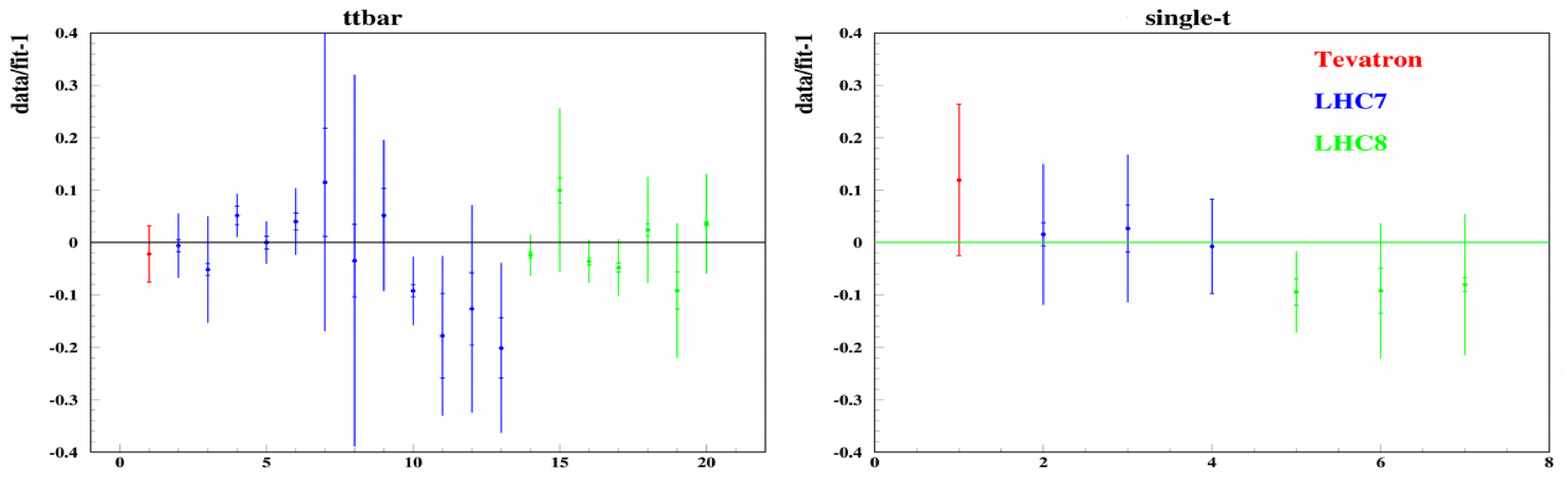
χ^2/NDP=16 / 17

$m_b(m_b)$=3.91±0.14(exp.) GeV ABMP16

$m_b(m_b)$=4.07±0.17(exp.) GeV

ZEUS JHEP 1409, 127 (2014)

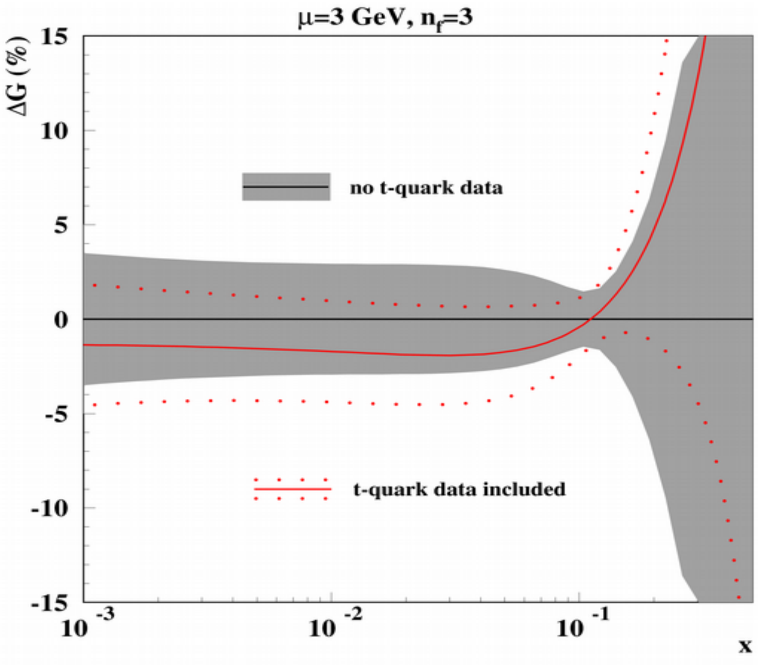
t-quark data from the LHC and Tevatron



talk by Thier at this conference

Running mass definition → better perturbative stability

sa, Blümlein, Moch PRD 86, 054009 (2012)



- $m_t(m_t)$=160.9±1.2(exp.) GeV NNLO
- $\alpha_s(M_Z)$=0.1145(9) → 0.1149(9) NNLO
- moderate change in the large-x gluon distribution

Summary

- The FFN scheme provides a nice description of the existing DIS data with a Consistent determination of the eavy-quark masses

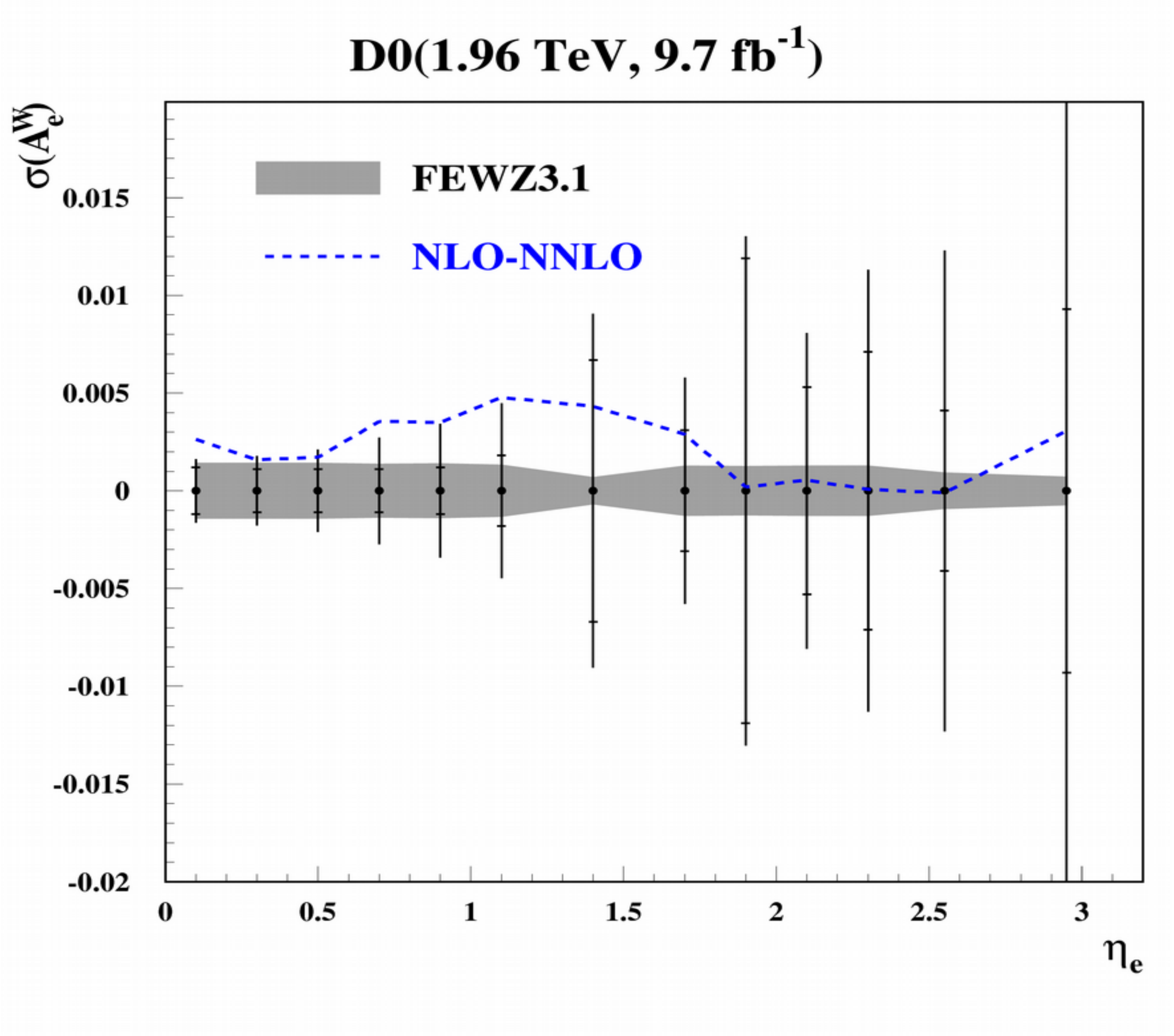
$m_c(m_c)$=1.250±0.020 GeV
$m_b(m_b)$=3.91±0.14 GeV

In constrast to the GMVFN schemes suffering from the uncertainties due to missing NNLO corrections to the OMEs and requiring tuning of m_c(pole)~1.3 GeV

- The ABMP16 PDF set has been :
  - HERA I+II data included → improved determination of $m_c(m_c)$; α_s increased by 1σ
  - deuteron data are replaced by the Drell-Yan ones from the LHC and Tevatron → reduced theoretical uncertainties in PDFs, in particular in d/u at large x; the small-x iso-spin sea asymmetry is relaxed and turns negative at $x\sim10^{-3}$ with onset of the Regge-like asymptotics at $x<10^{-5}$
  - moderate increase in the large-x gluon distribution due to impact of the ttbar data

EXTRAS

Computation accuracy



- Accuracy of O(1 ppm) is required to meet uncertainties in the experimental data → O(10^4 h) of running FEWZ 3.1 in NNLO

- An interpolation grid a la FASTNLO is used

NNLO DY corrections in the fit

The existing NNLO codes (DYNNLO, FEWZ) are quite time-consuming → fast tools are employed (FASTNLO, Applgrid,.....)

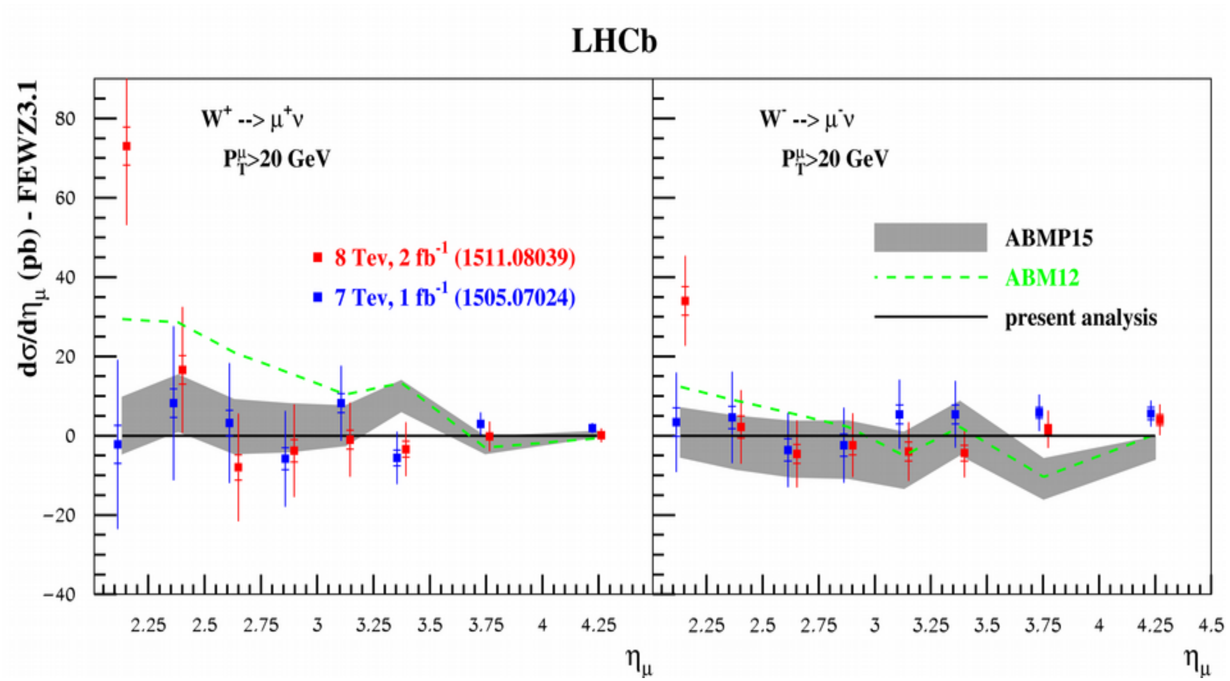
- the corrections for certain basis of PDFs are stored in the grid
- the fitted PDFs are expanded over the basis
- the NNLO c.s. in the PDF fit is calculated as a combination of expansion coefficients with the pre-prepared grids

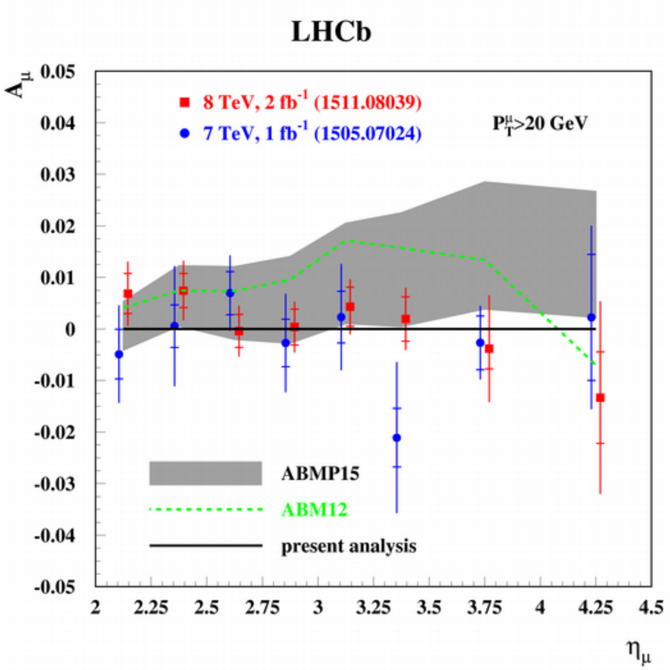
The general PDF basis is not necessary since the PDFs are already constrained by the data, which do not require involved computations → *use as a PDF basis the eigenvalue PDF sets obtained in the earlier version of the fit*

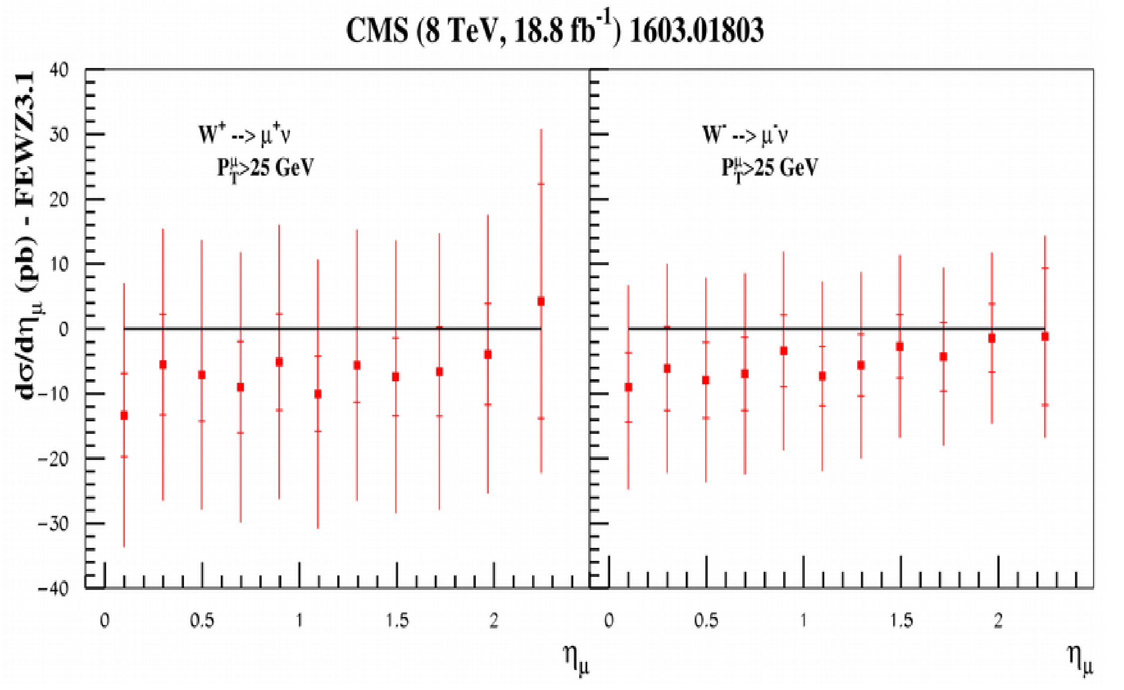
$\mathbf{P_0} \pm \Delta\mathbf{P}_0$ – vector of PDF parameters with errors obtained in the earlier fit
E – error matrix
P – current value of the PDF parameters in the fit

- store the DY NNLO c.s. for all PDF sets defined by the eigenvectors of **E**
- the variation of the fitted PDF parameters ($\mathbf{P} - \mathbf{P}_0$) is transformed into this eigenvector basis
- the NNLO c.s. in the PDF fit is calculated as a combination of transformed ($\mathbf{P} - \mathbf{P}_0$) with the stored eigenvector values

Most recent DY inputs







A filtering of the LHCb data has been performed:

– a bump at 7 Tev and Y=3.275 (not confirmed by the LHCb data at 8 TeV)

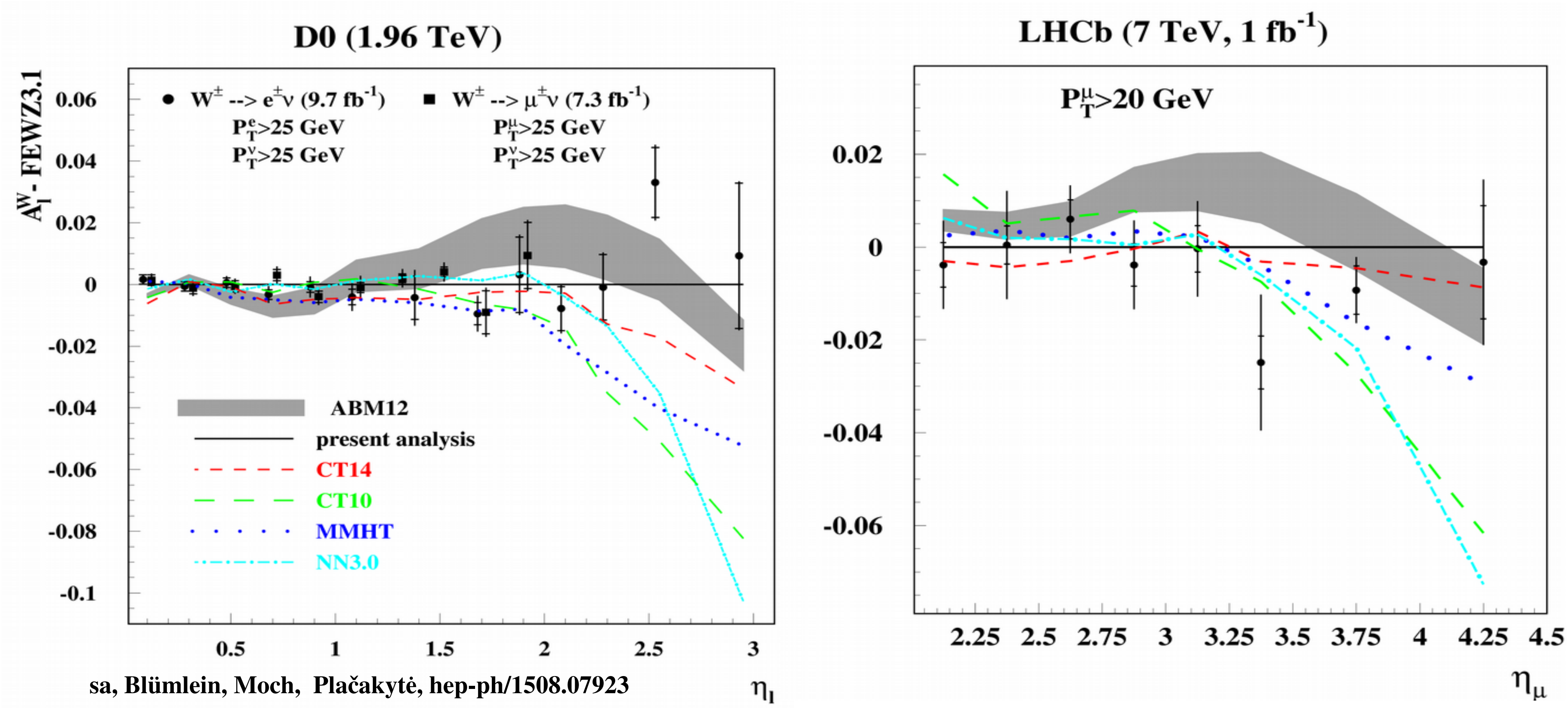
– and excess at 8 TeV and Y=2.125 (not confirmed by the CMS data at 8 TeV)

The CMS data at 8 TeV are much smoother than the ones at 7 TeV:

χ^2=17/22 versus 22/11

cf. earlier data in sa, Blümlein, Moch, Plačakytė, hep-ph/1508.07923

DY at large rapidity

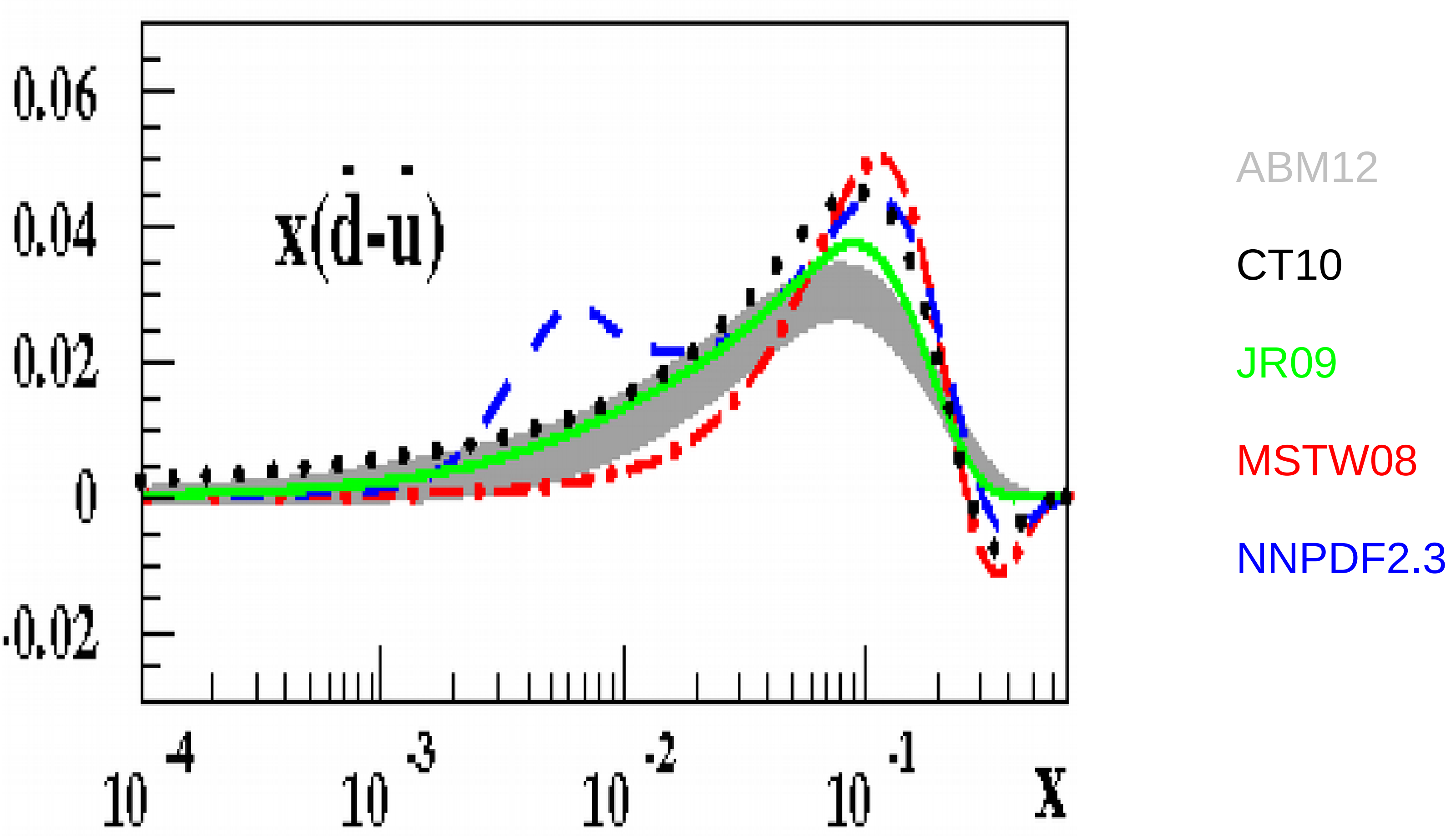


sa, Blümlein, Moch, Plačakytė, hep-ph/1508.07923

- The data can be evidently used for consolidation of the PDFs, however, unification of the theoretical accuracy is also needed

ABM	CT	MMHT	NNPDF
Interpolation of accurate NNLO grid (a la FASTNLO)	NNLL (ResBos)	NLO + NNLO K-factor	NLO + NNLO C-factors (y-dependent K-factors)

Sea quark iso-spin asymmetry



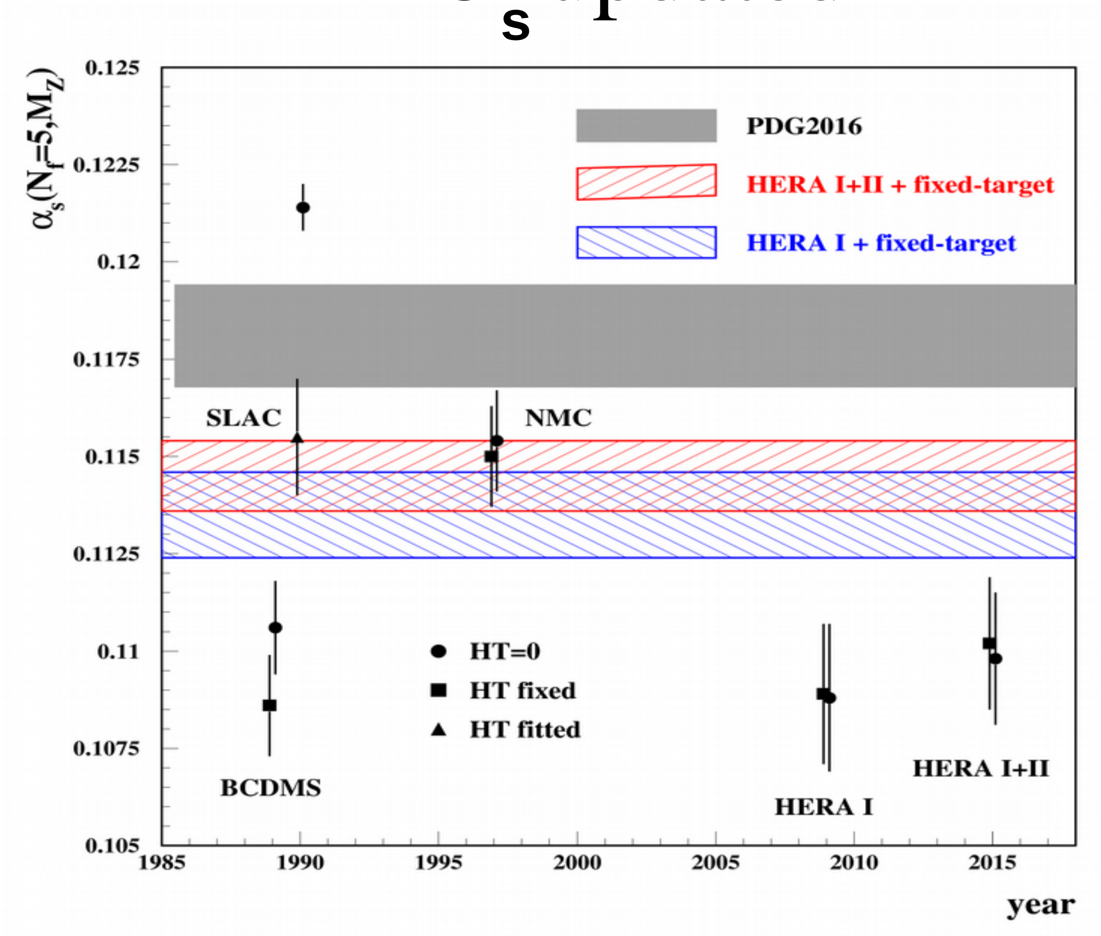
sa, Blümlein, Moch PRD 89, 054028 (2014)

- At x~0.1 the sea quark iso-spin asymmetry is controlled by the fixed-target DY data (E-866), weak constraint from the DIS (NMC)

- At x<0.01 Regge-like constraint like $x^{(a-1)}$, with a close to the meson trajectory intercept; the “unbiased” NNPDF fit follows the same trend

Onset of the Regge asymptotics is out of control

α_s updated



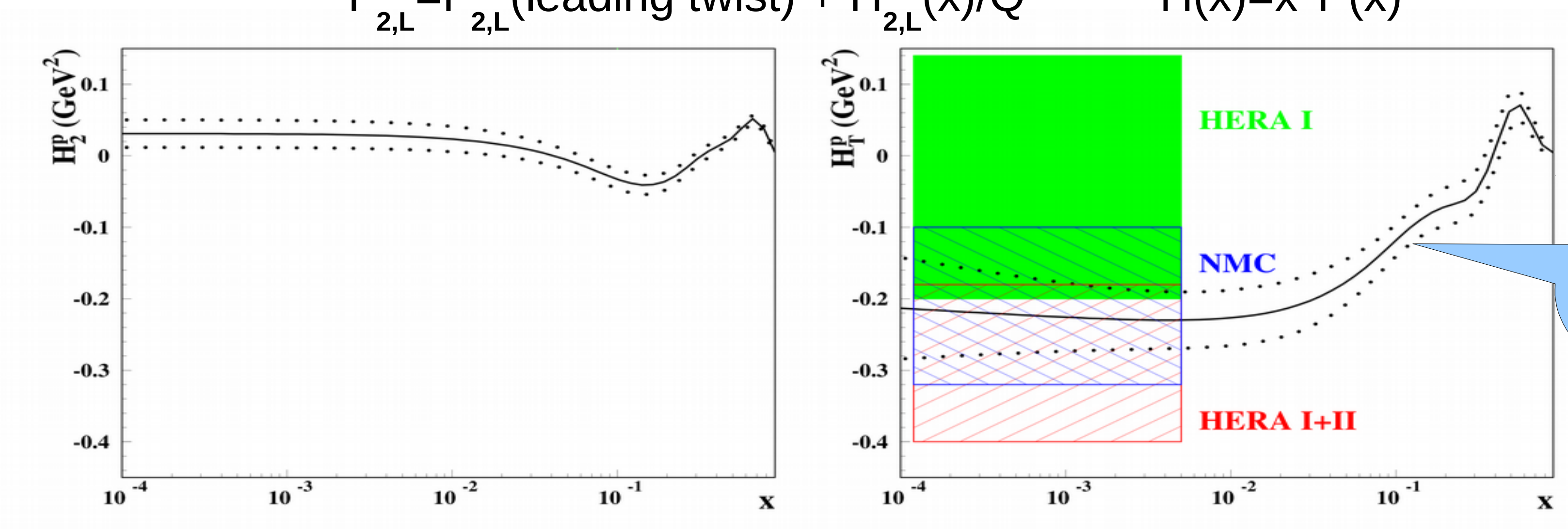
- α_s goes up by 1σ with HERA I+II data

- the value of α_s is still lower than the PDG one: pulled up by the SLAC and NMC data; pulled down by the BCDMS and HERA ones

- only SLAC determination overlap with the PDG band provided the high-twist terms are taken into account

High twists at small x

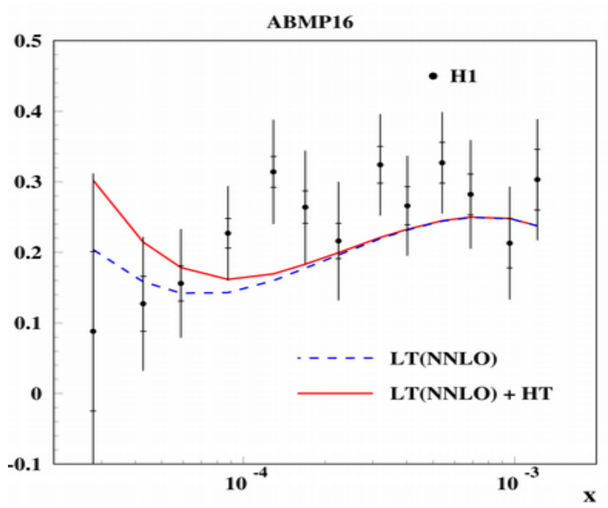
$F_{2,L}=F_{2,L}(\text{leading twist}) + H_{2,L}(x)/Q^2$ $H(x)=x^hP(x)$



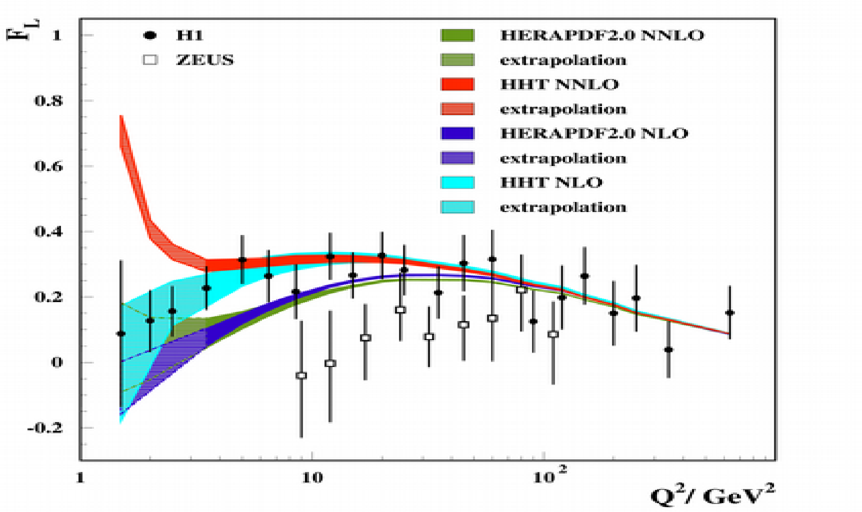
Controlled by SLAC and NMC data

sa, Blümlein, Moch
PRD 86, 054009 (2012)

- $H_T(x)$ continues a trend observed at larger x; $H_2(x)$ is comparable to 0 at small x
- $h_T=0.05\pm0.07$ → slow vanishing at x → 0
- $\Delta\chi^2 \sim -40$ **Harland-Lang, Martin, Motylinski, Thorne hep-ph/1601.03413**



No dramatic increase of F_L at small x



Abt, et al. hep-ex/1604.02299