Axionic suppression of plasma wakefield acceleration

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Axion-like particles: Theory and Experiment
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Wakefield acceleration in the lab

A sufficiently intense and short laser pulse with sufficiently high peak frequency propagating through a plasma creates a wave (wake behind the pulse).

- $E \sim \mathrm{TV/m}$, bubble diameter $\sim 10\,\mu\mathrm{m}$, peak electron number density $\sim 10^{23}\,\mathrm{cm}^{-3}$

Tajima et al, PRL (1979)

Mangles et al, Nature (2004)

Geddes et al, Nature (2004)

Faure et al, Nature (2004)

Particle beam-driven versions at SLAC and proposed at CERN

Blumenfeld et al, Nature (2007)

Assmann et al, Plasma Phys. Control. Fusion (2014)

Implications of ALPs?

What statements can we make in the absence of a suitable PIC code that includes ALPs?

Use units in which $\hbar = c = \varepsilon_0 = 1$ unless otherwise stated.

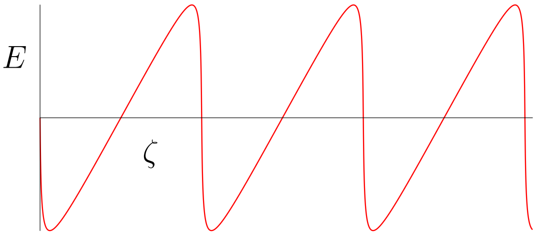
1D electrostatic waves in a cold plasma

Well-established analytical results exist (in the absence of vacuum polarisation and ALPs) going back to the 1950s.

$$\frac{dE}{d\zeta} = -e\, n_0\, \gamma^2 \left(\frac{v\, \bar{\mu}}{\sqrt{\bar{\mu}^2 - 1}} - 1 \right)$$

where $\zeta = z - vt$, $\gamma = 1/\sqrt{1 - v^2}$, n_0 is the plasma number density and

$$E = -\frac{1}{\gamma} \frac{m_\mathrm{e}}{e} \frac{d\bar{\mu}}{d\zeta}$$



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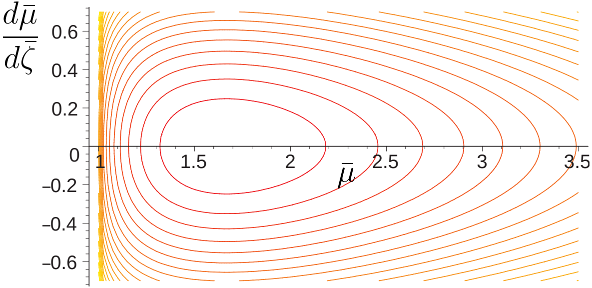
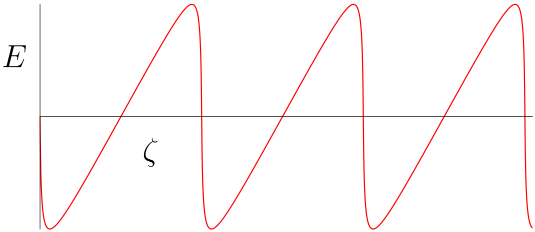


Maximum amplitude wave



- Amplitude of wave during which $\bar{\mu} = 1$:

$$E^{\max} = \sqrt{2 m_{\mathrm{e}} n_0 (\gamma - 1)}$$

Akheizer et al, Sov. Phys. JETP 3 (1956)

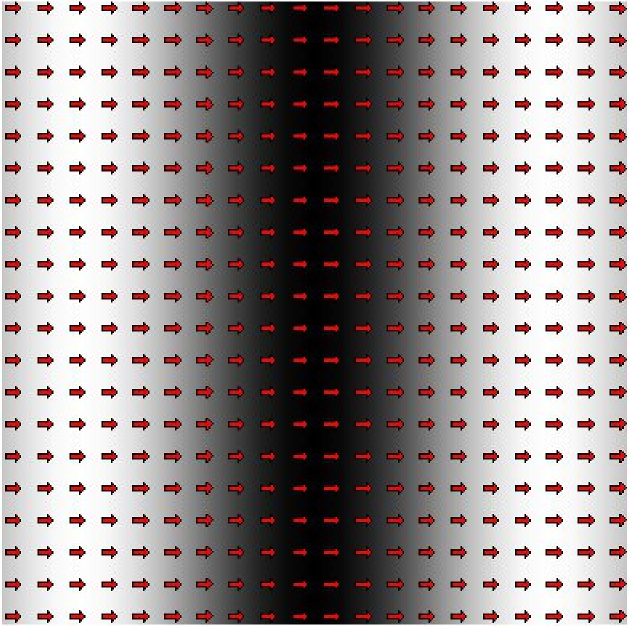
- Maximum energy gain of a test electron in the wave:

$$\boxed{W^{\max} = m_{\mathrm{e}} (4\gamma^3 - 3\gamma - 1)}$$

Esarey et al, Phys. Plas. (1995)

Aim of this talk

Generalise $W^{\max} = m_{\mathrm{e}}(4\gamma^3 - 3\gamma - 1)$ to include ALPs.



Field equations

- Homogeneous and isotropic background of fixed ions.
- Maxwell-ALP field equations:

$$dF = 0$$
$$d \star G = -q_{\mathrm{e}} n_{\mathrm{e}} \star \widetilde{V_{\mathrm{e}}} - q_0 n_0 \star \widetilde{V_0}$$
$$d \star d\alpha - m_\alpha^2 \alpha \star 1 = -\partial_\alpha \lambda \star 1$$

where $q_{\mathrm{e}} = -e$ and q_0 is the charge on an ion.

- Electromagnetic excitation 2-form G:

$$G = 2\big(\partial_X \lambda\, F - \partial_Y \lambda \star F\big)$$

where $\lambda(X, Y, \alpha) = \lambda_{\mathrm{EM}}(X, Y) + \frac{1}{2} g \alpha Y$ with

$$X = \star(F \wedge \star F), \quad Y = \star(F \wedge F)$$

- Electron fluid:

$$\nabla_{V_{\mathrm{e}}} \widetilde{V_{\mathrm{e}}} = \frac{q_{\mathrm{e}}}{m_{\mathrm{e}}} \iota_{V_{\mathrm{e}}} F, \qquad \eta(V_{\mathrm{e}}, V_{\mathrm{e}}) = -1$$

1D non-linear wave

- Seek solutions in $\zeta = z - v\,t$ with

$$F = E(\zeta)\,dt \wedge dz - B\,dx \wedge dy, \quad \alpha = \alpha(\zeta)$$
$$\widetilde{V}_{\mathrm{e}} = \mu\theta^1 - (\mu^2 - \gamma^2)^{1/2}\theta^2$$

  where $\mu = \mu(\zeta) > 0$ and the 1-form basis

$$\theta^1 = v\,dz - dt, \quad \theta^2 = dz - v\,dt = d\zeta$$

  is adapted to the "wave frame".
  - The Lorentz factor of the electron fluid is μ/γ in the wave frame
  - The electrons move slower than the wave
    - The wave is steepest for the solution in which $\mu = \gamma$

1D non-linear wave

The field equations lead to the ODEs

$$0 = \Bigg[2(\partial_X \lambda_{\mathrm{EM}}\, E^2 + \partial_Y \lambda_{\mathrm{EM}}\, EB) - \lambda_{\mathrm{EM}} - \frac{1}{2}\left(\frac{\alpha'^2}{\gamma^2} - m_\alpha^2 \alpha^2\right) + \\ + \frac{q_0}{q_{\mathrm{e}}} m_{\mathrm{e}} n_0 (v\sqrt{\mu^2 - \gamma^2} - \mu) \Bigg]'$$

and

$$\frac{\alpha''}{\gamma^2} - m_\alpha^2 \alpha = -gEB.$$

for μ and α, with g the ALP-photon coupling constant, $\mu' = d\mu/d\zeta$ and

$$E = \frac{m_{\mathrm{e}}}{q_{\mathrm{e}}} \frac{1}{\gamma^2} \mu'$$

Pathway to $W^{\max}$

Eliminate the ALP field α.

- Seek solutions whose ALP content is driven purely by the plasma wave.
- Focus on periodic solutions:

$$\mu(\zeta) = \sum_{n=-\infty}^{\infty} \mu_n \exp\left(2\pi i n \frac{\zeta}{l}\right),$$
$$\alpha(\zeta) = \sum_{n=-\infty}^{\infty} \alpha_n \exp\left(2\pi i n \frac{\zeta}{l}\right)$$

  where the period l must be determined as part of the analysis.
- The ALP field equation yields

$$\alpha_n = \frac{gBl}{4\pi^2 n^2 + m_\alpha^2 \gamma^2 l^2} \frac{m_{\mathrm{e}}}{q_{\mathrm{e}}} 2\pi i n \mu_n$$

Pathway to $W^{\max}$

We now have the non-local equation

$$\Big[2(\partial_X \lambda_{\mathrm{EM}}\, E^2 + \partial_Y \lambda_{\mathrm{EM}}\, EB) - \lambda_{\mathrm{EM}} + \varphi[\mu] + \frac{q_0}{q_{\mathrm{e}}} m_{\mathrm{e}} n_0 (v\sqrt{\mu^2 - \gamma^2} - \mu)\Big]' = 0$$

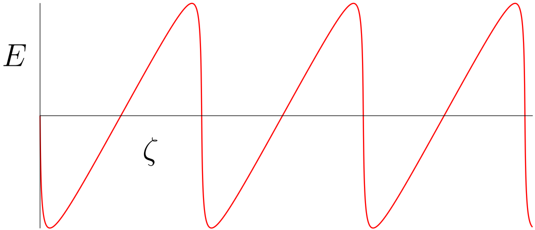
for μ where the functional φ is

$$\varphi[\mu] = -\frac{1}{2}\left(\frac{\alpha'^2}{\gamma^2} - m_\alpha^2 \alpha^2\right)$$

with α specified in terms of μ via Fourier decomposition.

Pathway to $W^{\max}$

Exploit a useful simplification.



- The *maximum* energy gain of a test particle occurs between two values of $E = 0$ (a maximum and minimum of μ) at $\zeta_{\rm I}$, $\zeta_{\rm II}$.

Pathway to $W^{\max}$

- Integrate the non-local equation for μ:

$$\left\{ \varphi[\mu] + \frac{q_0}{q_{\rm e}} m_{\rm e} n_0 (v\sqrt{\mu^2 - \gamma^2} - \mu) \right\}\Bigg|_{\zeta_{\rm I}}^{\zeta_{\rm II}} = 0$$

  where $\mu(\zeta_{\rm I}) = \gamma$
- Solve the above for $\mu(\zeta_{\rm II})$ and use it to construct $W^{\max}$
  - Work done on a test particle in the frame of the wave is proportional to $\mu(\zeta_{\rm II}) - \mu(\zeta_{\rm I})$
  - Use a perturbative approach to find $\mu(\zeta_{\rm II})$ to first order in g^2

The key result

$$W^{\max} \approx W^{\max}_{B=0}\left[1 - \frac{\hbar^3}{\mu_0 c}\frac{g^2 B^2}{m_\alpha^2}\left(1 - \frac{\tanh\sigma}{\sigma}\right)\right]$$

where $\hbar$, c, μ_0 have been reintroduced and

$$W^{\max}_{B=0} = Mc^2\left[2\Theta^2(\gamma^3 - \gamma) + 2\Theta(\gamma^2 - 1)\sqrt{1 + \Theta^2(\gamma^2 - 1)} + \gamma - 1\right],$$

$$\Theta = \sqrt{\frac{|Q|\, m_\mathrm{e}}{eM}},$$

$$\sigma = \frac{\sqrt{2}\, m_\alpha c^2}{\hbar\omega_p}\gamma^{3/2}$$

for a test particle with mass M and charge Q, and $\omega_p = \sqrt{e\, q_0 n_0/(\varepsilon_0 m_\mathrm{e})}$ is the plasma frequency.

- $M = m_\mathrm{e}$, $Q = -e$ leads to the well-known expression

$$W^{\max}_{B=0} = m_\mathrm{e} c^2(4\gamma^3 - 3\gamma - 1)$$

Estimates

Parameters associated with the environment of a neutron star:

$$\frac{\hbar^3}{\mu_0 c}\frac{g^2 B^2}{m_\alpha^2} = 3.8\times 10^{-2}\left(\frac{g}{10^{-7}\,\mathrm{GeV}^{-1}}\right)^2\left(\frac{B}{10^8\,\mathrm{T}}\right)^2\left(\frac{10^{-5}\,\mathrm{eV}}{m_\alpha c^2}\right)^2,$$

$$\sigma = 9.7\left(\frac{m_\alpha c^2}{10^{-5}\,\mathrm{eV}}\right)\left(\frac{2\pi\times 10^{18}\,\mathrm{rad\,s}^{-1}}{\omega_p}\right)\left(\frac{\gamma}{2\times 10^6}\right)^{3/2}.$$

- $\sim 3\%$ perturbation to $W_{B=0}^{\max}$ follows for the representative parameters
  - ALP mass m_α plays an important role:
    - $\sim 0.04\%$ perturbation to $W_{B=0}^{\max}$ for $m_\alpha c^2 = 10^{-4}\mathrm{eV}$
    - $\sim 90\%$ perturbation(!) to $W_{B=0}^{\max}$ for $m_\alpha c^2 = 10^{-6}\,\mathrm{eV}$

Summary

- We have calculated the maximum gain in energy of a test particle in a 1D non-linear ALP-plasma wave (in the absence of a background ALP field).
  - Our results suggest that ALPs should not be ignored when invoking plasma-based wakefield acceleration in the strongest magnetic fields in the universe.
- For further details:
  DAB, A Noble, TJ Walton, arXiv:1507.08858 [hep-ph]