# Parity doubling in two-color and two-flavor theory at high temperature 

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## Motivation - SU(2) gauge theory

Shares some nonperturbative properties with QCD, such as confinement and chiral symmetry breaking.

SU(2) gauge theory with even number of fundamental fermions
Finite density calculations are free from sign problem

## Pietro Guidice, Aleksandr Nikolaev [Poster]

Alford, Kapustin, Wilczek (1999)<br>Hands, Kogut, Lombardo, Morrison (1999)<br>Aloisio, Azcoiti, Di Carlo, Galante, Grillo (2000)

SU(2) gauge theory with two fundamental fermions
Technicolor/Composite Higgs/Dark Matter Jamo Rantaharju [Wed. 14:45]

Lewis, Pica, Sannino (2012) Hietanen, Lewis, Pica, Sannino (2014)
Finite $T$ calculations on an anisotropic lattice - finer temporal spacing

$$
T=\frac{1}{\sqrt{N} a}
$$

Lattice spacing $a$ is fixed
Change $T$ by changing $N_{\tau}$

## Model

SU(2) gauge theory with 2 Dirac fermions in fundamental representation

$$
\mathcal{L}=-\frac{1}{4} F_{\mu \nu}^{a} F_{a \mu \nu}+\bar{u}\left(i \gamma^{\mu} D_{\mu}-m\right) u+\bar{d}\left(i \gamma^{\mu} D_{\mu}-m\right) d
$$

Global symmetry:

$$
\begin{array}{rll}
\mathrm{SU}(4) \xrightarrow{\text { broken }} & \mathrm{Sp(4)} \\
\langle\bar{u} u+\bar{d} d\rangle \neq 0 & \text { at chiral limit } \\
\langle\bar{u} u+\bar{d} d\rangle \neq 0, m \bar{u} u, m \bar{d} d & \text { at non-zero mass }
\end{array}
$$

5 Goldstone bosons: 3 pseudoscalar mesons + 2 diquark baryons
Degenerate (two-point correlation functions are identical)
Aloisio, Azcoiti, Di Carlo, Galante, Grillo (2000)
Hands, Montvay, Morrison, Oevers, Scorzato, Skullerud (2000)
Observables: Isovector mesons $\mathcal{O}_{\bar{u} d}^{(\Gamma)} \equiv \bar{u}(x) \Gamma d(x)$,
where $\Gamma=1, \gamma^{5}, \gamma^{\mu}, \gamma^{\mu} \gamma^{5}$

## \& Anisotropic Lattice - Applications

Any lattice calculations require a fine temporal lattice spacing while keeping the moderate size of spacial lattice.

Reduce computational cost.

Heavy quark simulations

$$
\text { cut off effects } \sim\left(a_{t} m_{q}\right)^{n}
$$

Spectroscopy of excited states \& Glueballs
Limited number of data points, for large hadron mass $a_{t} m_{h}$

Finite Temperature calculations
e.g) spectral function from temporal correlators

## \% Anisotropic Lattice Action

Standard Wilson action on an anisotropic lattice

$$
\begin{aligned}
& S_{g}[U]=\frac{\beta}{\xi_{g}^{0}}\left[\sum_{i}\left(\xi_{g}^{0}\right)^{2}\left(1-\frac{1}{N} \operatorname{Re} \operatorname{tr} \mathcal{P}_{0 i}\right)+\sum_{i<j}\left(1-\frac{1}{N} \operatorname{Re} \operatorname{tr} \mathcal{P}_{i j}\right)\right] \\
& S_{f}[U, \bar{\psi}, \psi]= a_{s}^{3} a_{t} \sum \bar{\psi}(x) D_{m} \psi(x) \quad \text { Bare gauge anisotropy } \\
& D_{m} \psi(x) \equiv\left(D+m_{0}\right) \psi(x) \\
&= \frac{1}{a_{t}}\left[\left(a_{t} m_{0}+1+3 / \xi_{f}^{0}\left\llcorner\psi(x)-\frac{1}{2}\left(\left(1-\gamma_{0}\right) U_{0}(x) \psi(x+\hat{0})+\left(1+\gamma_{0}\right) U_{0}^{\dagger}(x-\hat{0}) \psi(x-\hat{0})\right)\right.\right.\right. \\
&\left.-\frac{1}{2 \xi_{f}^{0}} \frac{\sum_{j}}{j}\left(\left(1-\gamma_{j}\right) U_{j}(x) \psi(x+\hat{j})+\left(1+\gamma_{j}\right) U_{j}^{\dagger}(x-\hat{j}) \psi(x-\hat{j})\right)\right]
\end{aligned}
$$

Bare parameters need to be tuned in order that the renormalized gauge and fermion anisotropies are same for a given quark mass.

## \& Anisotropy - Gauge sector

Gauge anisotropy $\xi_{g}$ is determined by using Klassen's method Klassen (2000)


$$
R_{t}(x, t)=\frac{W_{s t}(x, t)}{W_{s t}(x+1, t)} \quad R_{s}(x, y)=\frac{W_{s s}(x, y)}{W_{s s}(x+1, y)}
$$

In practical, we minimize

$$
L\left(\xi_{g}\right)=\sum_{x, y} \frac{\left(R_{s s}(x, y)-R_{s t}\left(x, \xi_{g} y\right)\right)^{2}}{\left(\Delta R_{s}\right)^{2}+\left(\Delta R_{t}\right)^{2}}
$$

Umeda et. al. (CP-PACS) (2003)

## * Anisotropy - Gauge sector

Gauge anisotropy $\xi_{g}$ is determined by using Klassen's method Klassen (2000) $R_{t}\left(x, t=\xi_{g} y\right)=R_{s}(x, y)$ where $R_{s}(x, y)=\frac{W_{s s}(x, y)}{W_{s s}(x+1, y)}, R_{t}(x, t)=\frac{W_{s t}(x, t)}{W_{s t}(x+1, t)}$ In practical, we minimize $\quad L\left(\xi_{g}\right)=\sum_{x, y} \frac{\left(R_{s s}(x, y)-R_{s t}\left(x, \xi_{g} y\right)\right)^{2}}{\left(\Delta R_{s}\right)^{2}+\left(\Delta R_{t}\right)^{2}}$

Umeda et. al. (CP-PACS) (2003)
Small Wilson loops suffer from short range lattice artifacts, while large ones suffer from very large noise.

Fix $\max (x * y)$
Then, scan $\min (x * y)$
$\xi_{g}$ should approach the asymptotic value.


New method? Gradient flow Borsany et. al (2012)

## \& Anisotropy - Gauge sector

Gauge anisotropy $\xi_{g}$ is determined by using Klassen's method Klassen (2000)

$$
R_{t}\left(r, t=\xi_{g} y\right)=R_{s}(r, y) \text { where } R_{s}(r, y)=\frac{W_{s s}(r, y)}{W_{s s}(r+1, y)}, R_{t}(r, t)=\frac{W_{s t}(r, t)}{W_{s t}(r+1, t)}
$$

In practical, we minimize $\quad L\left(\xi_{g}\right)=\sum_{r, y} \frac{\left(R_{s s}(r, y)-R_{s t}\left(r, \xi_{g} y\right)\right)^{2}}{\left(\Delta R_{s}\right)^{2}+\left(\Delta R_{t}\right)^{2}}$


2D path in x-z plane(Bresenham algorithm)
 For 3D, Bolder et. al. (2001)

## \& Anisotropy - Fermion sector

Fermion anisotropy $\xi_{f}$ is determined by using meson dispersion relations

$$
E^{2}\left(\vec{p}^{2}\right)=m^{2}+\frac{\vec{p}^{2}}{\xi_{f}^{2}}, \vec{p}=2 \pi \vec{n} / L_{s}
$$ (Pseudo Goldstone Boson)

Point sources for both source and sink
We use the first four momentum vectors for fitting.

$$
\vec{n}=(0,0,0),(1,0,0),(0,1,0),(0,0,1)
$$

Higher momentum states are consistent with the fit result.



## \& Anisotropy - Fermion sector

Fermion anisotropy $\xi_{f}$ is determined by using meson dispersion relations

$$
E^{2}\left(\vec{p}^{2}\right)=m^{2}+\frac{\vec{p}^{2}}{\xi_{f}^{2}}, \vec{p}=2 \pi \vec{n} / L_{s}
$$ (Pseudo Goldstone Boson)

Point sources for both source and sink
We use the first four momentum vectors for fitting.

$$
\vec{n}=(0,0,0),(1,0,0),(0,1,0),(0,0,1)
$$

Also, $\xi_{f}$ from the dispersion relation for vector meson agrees.



## \&- Anisotropy Tuning - Simulation details

Ensembles( $128 \times 12^{3}$ lattice, $\beta=2.0$ )

| $m_{0}$ | $\xi_{g}^{0}$ | $\xi_{f}^{0}$ | $N_{\text {traj }} / \ell_{\text {auto }}$ | $N_{\text {conf }}$ | $m_{\pi}$ | $m_{v}$ | $\xi_{g}$ | $\xi_{f}$ | $m_{\pi} / m_{\rho}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.195 | 4.7 | 4.7 | $1600 / 8$ | 200 | $0.1659(8)$ | $0.1823(10)$ | $6.19(7)$ | $6.34(10)$ | $0.910(7)$ |
| -0.195 | 4.9 | 4.7 | $2400 / 12$ | 200 | $0.1544(6)$ | $0.1709(13)$ | $6.33(8)$ | $6.33(9)$ | $0.903(8)$ |
| -0.2 | 4.5 | 4.7 | $2400 / 8$ | 300 | $0.1616(5)$ | $0.1784(8)$ | $6.03(6)$ | $6.28(7)$ | $0.906(5)$ |
| -0.2 | 4.7 | 4.5 | $2400 / 8$ | 300 | $0.1743(5)$ | $0.1910(7)$ | $6.07(7)$ | $6.12(6)$ | $0.913(4)$ |
| -0.2 | 4.7 | 4.7 | $2400 / 12$ | 200 | $0.1504(6)$ | $0.1678(10)$ | $6.13(6)$ | $6.41(11)$ | $0.896(6)$ |
| -0.2 | 4.9 | 4.7 | $3000 / 10$ | 300 | $0.1399(5)$ | $0.1589(7)$ | $6.42(6)$ | $6.35(7)$ | $0.880(5)$ |
| -0.2 | 5.1 | 4.7 | $2250 / 14$ | 160 | $0.1279(13)$ | $0.1479(19)$ | $6.58(9)$ | $6.34(17)$ | $0.865(14)$ |
| -0.209 | 4.7 | 4.5 | $2400 / 16$ | 150 | $0.1455(7)$ | $0.1643(11)$ | $6.10(6)$ | $6.04(10)$ | $0.885(7)$ |
| -0.209 | 4.7 | 4.7 | $3000 / 10$ | 300 | $0.1169(7)$ | $0.1392(13)$ | $6.22(6)$ | $6.35(12)$ | $0.840(10)$ |
| -0.209 | 4.9 | 4.5 | $3000 / 10$ | 300 | $0.1336(6)$ | $0.1533(9)$ | $6.34(7)$ | $6.11(9)$ | $0.872(6)$ |
| -0.209 | 4.9 | 4.7 | $2100 / 14$ | 150 | $0.1023(9)$ | $0.1243(15)$ | $6.35(6)$ | $6.25(12)$ | $0.823(12)$ |
| -0.215 | 4.7 | 4.7 | $1650 / 12$ | 138 | $0.0904(21)$ | $0.118(5)$ | $6.04(9)$ | . | $0.77(3)$ |

Configurations are generated using HMC algorithms(modified HiRep code). Del Debbio, Patella, Pica (2010)
Thermalization time is determined by monitoring Plaquette values.
Two adjacent configurations are separated by roughly one autocorrelation time(8~16 trajectories).
Implemented periodic boundary conditions for all directions.

## \% Anisotropy tuning - results

Linear Ansatz for renormalized parameters

$$
\begin{array}{r}
\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0} \\
\xi_{f}\left(\xi_{g}^{0}, \xi_{\xi}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0} \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0}
\end{array}
$$

Edwards, Joo, Lin (2008)

## \% Anisotropy tuning - results

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\xi_{f}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0} \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0}
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Edwards, Joo, Lin (2008)




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\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0}, \\
\xi_{f}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0} \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0} .
\end{array}
$$

Edwards, Joo, Lin (2008)




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\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0} \\
\xi_{f}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0} \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0}
\end{array}
$$

Edwards, Joo, Lin (2008)




## \% Anisotropy tuning - results

Linear Ansatz for renormalized parameters

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\begin{array}{r}
\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0},,_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0}, \\
\xi_{f}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0}, \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0} .
\end{array}
$$

Edwards, Joo, Lin (2008)




## \% Anisotropy tuning - results

Linear Ansatz for renormalized parameters

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\begin{array}{r}
\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0},,_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0}, \\
\xi_{f}\left(\xi_{\xi}^{0}, \xi_{f}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0}, \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0} .
\end{array}
$$

Edwards, Joo, Lin (2008)
mild dependence on $m_{0}$


## \% Anisotropy tuning - results

Linear Ansatz for renormalized parameters

$$
\begin{array}{r}
\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0} \\
\xi_{f}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0} \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0}
\end{array}
$$

Edwards, Joo, Lin (2008)
mild dependence on $\xi_{f}^{0}$



## \% Anisotropy tuning - results

Linear Ansatz for renormalized parameters

$$
\begin{array}{r}
\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0} \\
\xi_{f}\left(\xi_{g}^{0}, \xi_{\xi}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0} \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0}
\end{array}
$$

Edwards, Joo, Lin (2008)



## \% Anisotropy tuning - results

Linear Ansatz for renormalized parameters

$$
\begin{array}{r}
\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0} \\
\xi_{f}\left(\xi_{g}^{0}, \xi_{j}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0} \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0}
\end{array}
$$

Edwards, Joo, Lin (2008)
mild dependence on $m_{0}$


## \% Anisotropy tuning - results

## Linear Ansatz for renormalized parameters

$$
\begin{array}{r}
\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0} \\
\xi_{f}\left(\xi_{g}^{0}, \xi_{j}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0} \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0}
\end{array}
$$

Edwards, Joo, Lin (2008)
mild dependence on $\xi_{g}^{0}$



## \& Anisotropy tuning - results

Linear Ansatz for renormalized parameters

$$
\begin{gathered}
\xi_{g}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=a_{0}+a_{1} \xi_{g}^{0}+a_{2} \xi_{f}^{0}+a_{3} m_{0} \\
\xi_{f}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=b_{0}+b_{1} \xi_{g}^{0}+b_{2} \xi_{f}^{0}+b_{3} m_{0} \\
M_{p s}^{2}\left(\xi_{g}^{0}, \xi_{f}^{0}, m_{0}\right)=c_{0}+c_{1} \xi_{g}^{0}+c_{2} \xi_{f}^{0}+c_{3} m_{0}
\end{gathered}
$$

with
Edwards, Joo, Lin (2008)

$$
\begin{array}{r}
a_{0}=0.6(16), a_{1}=0.97(13), a_{2}=0.31(23), a_{3}=2(4), \\
b_{0}=1.8(24), b_{1}=0.06(18), b_{2}=1.1(3), b_{3}=4(7) \\
c_{0}=0.475(5), c_{1}=-0.0168(4), c_{2}=-0.0375(6), c_{3}=0.986(11)
\end{array}
$$

## \& Anisotropy tuning - results

Renormalized conditions

$$
\xi_{g}\left(\xi_{g}^{0 *}, \xi_{f}^{0 *}, m_{0}^{*}\right)=\xi_{f}\left(\xi_{g}^{0 *}, \xi_{f}^{0 *}, m_{0}^{*}\right)=\xi, \quad M_{p s}^{2}\left(\xi_{g}^{0 *}, \xi_{f}^{0 *}, m_{0}^{*}\right)=m_{p s}^{2} .
$$

Our target anisotropy is $\xi=6.3$ with $\left(m_{p s} a\right)^{2}=0.005$.

$$
\xi_{g}^{0 *}=4.84(8), \xi_{f}^{0 *}=4.72(12), m_{0}^{*}=-0.2148(37)
$$



Results of $128 \times 16^{3}$ Lattice

$$
\begin{gathered}
\xi_{g}=6.29(4), \xi_{f}=6.1(2) \\
\left(m_{p s} a\right)^{2}=0.00517(14)
\end{gathered}
$$

Consistent with our target parameters!

## * Finite T calculations

Ensembles (~200 configurations)

$$
\begin{aligned}
& N_{\tau} \times 16^{3} \text { lattice with } N_{\tau}=16,20,24,28,30,36,40,48,128 \\
& N_{\tau} \times 16^{2} \times 24 \text { lattice with } N_{\tau}=16,20,24,28,36,42,48,56
\end{aligned}
$$

Boundary conditions: antiperiodic temporal fermion boundary conditions periodic b.c. for all others

Critical temperature: deconfining transition
Rapid change of temporal Polyakov loop
$\Longrightarrow T^{c}=\frac{1}{N_{\tau}^{c} a}$

Temporal and spacial correlators of isovector mesons (Stochastic wall sources) pseudoscalar(PS), scalar(S), vector(V), and axial vector(AV) mesons Degeneracy between parity partners in hadron multiplets


Chiral symmetry restoration
$\mathrm{U}(1)_{\mathrm{A}}$ symmetry restoration
cf) two-flavor QCD Bastian Brandt [Poster]

## \% Renormalized Polyakov Loop

Multiplicative renormalization of Polyakov Loop

$$
\begin{aligned}
& \text { Free energy } \quad F_{R}=F+\Delta F \\
& \Delta \text { dditivo ranormalization }
\end{aligned}
$$

Renormalized Polyakov Loop

$$
L_{R}(T) \equiv Z_{L}^{N_{\tau}} L(T)
$$

Renormalization condition

$$
L_{R}\left(T_{R}\right) \equiv \mathrm{constant}
$$

Borsanyi et. al. (2012) Aarts et. al. (2014)


Deconfining critical temperature

$$
T^{c}=1 / N_{\tau}^{c}=0.0254(14) \text { or } N_{\tau}^{c}=39.5(2)
$$

## \& Temporal correlators



Below the critical temperature, the temporal correlation
functions decay in the order of

$$
\log \left[\frac{C_{P S}(t)}{C_{P S}\left(N_{\tau} / 2\right)}\right]<\log \left[\frac{C_{V}(t)}{C_{V}\left(N_{\tau} / 2\right)}\right]<\log \left[\frac{C_{A V}(t)}{C_{A V}\left(N_{\tau} / 2\right)}\right]
$$

Onset of the critical temperature, the correlation functions for vector and axial-vector mesons are degenerate.


## \% Spacial correlators - Vector channel

Screening mass for vector and axial vector mesons $\quad M^{S}=m^{S} / \xi$


Black: $N_{\tau} \times 16^{3}$ lattice
Red: $N_{\tau} \times 16^{2} \times 24$ lattice


$$
R(T)=\frac{M_{A V}(T)-M_{V}(T)}{M_{A V}(T)+M_{V}(T)}
$$

V and AV mesons are degenerate at $T \gtrsim T_{c}$

## \% Spacial correlators - Vector channel

Screening mass for vector and axial vector mesons $\quad M^{S}=m^{S} / \xi$


Black: $N_{\tau} \times 16^{3}$ lattice
Red: $N_{\tau} \times 16^{2} \times 24$ lattice


Red dotted line $=2 \pi$ (free quark)

Above $2 \mathrm{~T}_{\mathrm{c}}$, screening mass of vector and axial-vector mesons begins to deviate from the plateau.

$$
\text { Lattice artifacts due to too small } N_{\tau}<T a=\frac{1}{N_{\tau}}
$$

## \% Spacial correlators - Scalar channel

Screening mass for scalar and pseudoscalar mesons $\quad M^{S}=m^{S} / \xi$



Black: $N_{\tau} \times 16^{3}$ lattice
Red: $N_{\tau} \times 16^{2} \times 24$ lattice

$$
R(T)=\frac{M_{S}(T)-M_{P S}(T)}{M_{S}(T)+M_{P S}(T)}
$$

S and PS mesons are degenerate at $T \gtrsim 1.5 T_{c}$

## \& Spacial correlators - Scalar channel

Screening mass for scalar and pseudoscalar mesons $\quad M^{S}=m^{S} / \xi$


Black: $N_{\tau} \times 16^{3}$ lattice
Red: $N_{\tau} \times 16^{2} \times 24$ lattice


$$
\text { Red dotted line }=2 \pi \text { (free quark) }
$$

Above $2 \mathrm{~T}_{\mathrm{c}}$, screening mass of pseudoscalar and scalar mesons begins to deviate from the plateau.

Lattice artifacts due to too small $N_{\tau}<T a=\frac{1}{N_{\tau}}$

## \& Limitation

Using the renormalized anisotropy for $\mathrm{N}_{\mathrm{t}}(1 / \mathrm{T})$


Black: $N_{\tau} \times 16^{3}$ lattice
Red: $N_{\tau} \times 16^{2} \times 24$ lattice


Green: isotropic $N_{\tau} \times 16^{2} \times 24$ lattice
Lattice artifacts due to too small $N_{\tau}<T a=\frac{1}{N_{\tau}}$

## \& Limitation

Using the bare anisotropy for $\mathrm{N}_{\mathrm{t}}(1 / \mathrm{T})$



No longer a nonperturbative world!

Green: isotropic $N_{\tau} \times 16^{2} \times 24$ lattice

## \% Conclusion and future work

SU(2) gauge theory with 2 fund. Wilson fermions on an anisotropic lattice Anisotropy tuning works!

Non-plain Wilson loops are helpful for determining gauge anisotropy.
Finite T results
Parity doubling in the temporal and spacial correlators for vector channel just above Tc
Parity doubling in the spacial correlators for scalar channel above 1.5Tc

## Systematic errors

cf) two-flavor QCD in the chiral limit Bastian Brandt [Poster]
For $T \leq T_{c}$ fitting errors in scalar and axial-vector screening mass due to very limited numbers of data points in the asymptotic region

Mistuned bare anisotropy generate up to ~3\% errors.
Anisotropy tuning breaks down at very high T.
Massless, infinite volume, and continuum limit needs to be investigated.
How does the meson spectrum change at finite chemical potential?
Pietro Guidice, Aleksandr Nikolaev [Poster]

## Thank you for your attention!

