Universität Bielefeld

Lattice constraints on the thermal dilepton and photon rate

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H-T.Ding, F.Meyer, OK

Thermal dilepton rates and electrical conductivity of the QGP from the lattice

[arXiv:1604.06712, to appear in PRD]

J.Ghiglieri, OK, M.Laine, F.Meyer

Lattice constraints on the thermal photon rate

[arXiv:1604.07544, to appear in PRD]

XQCD 2016, Plymouth

01.08.2016

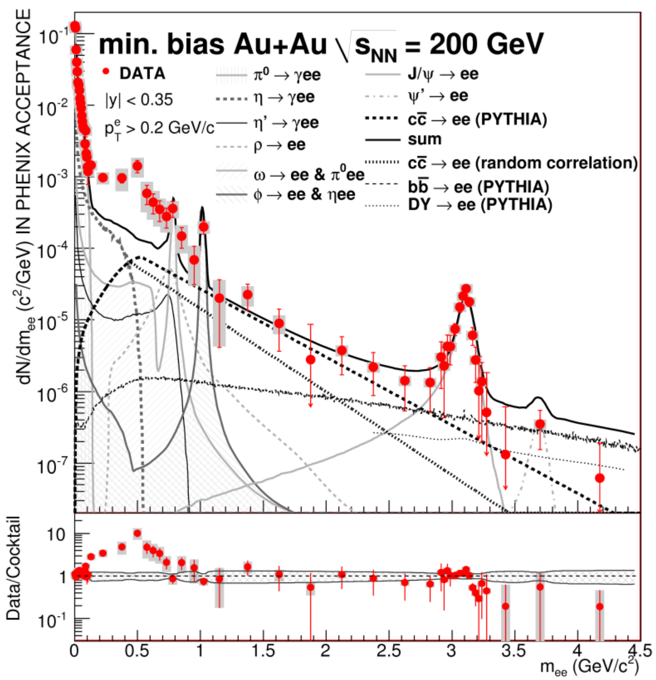
Motivation – PHENIX/STAR results for the low-mass dilepton rates

pp-data well understood by hadronic cocktail

large enhancement in Au+Au between 150-750 MeV

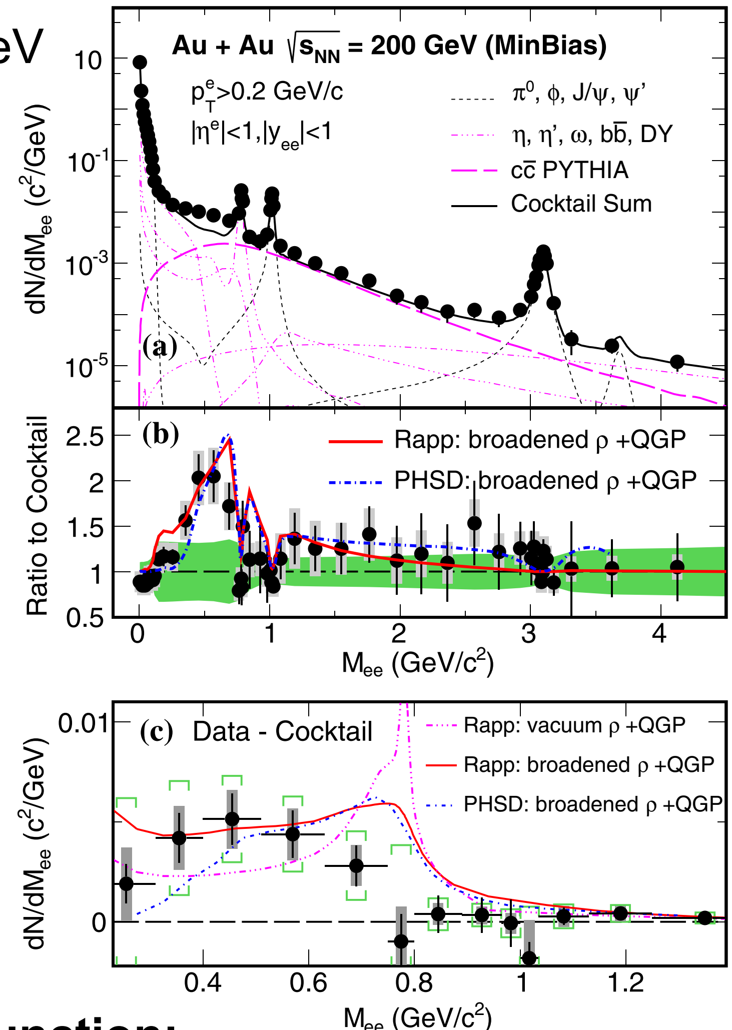
indications for thermal effects!?

Need to understand the contribution from QGP



[PHENIX PRC81, 034911 (2010)]

[STAR collaboration 2014]

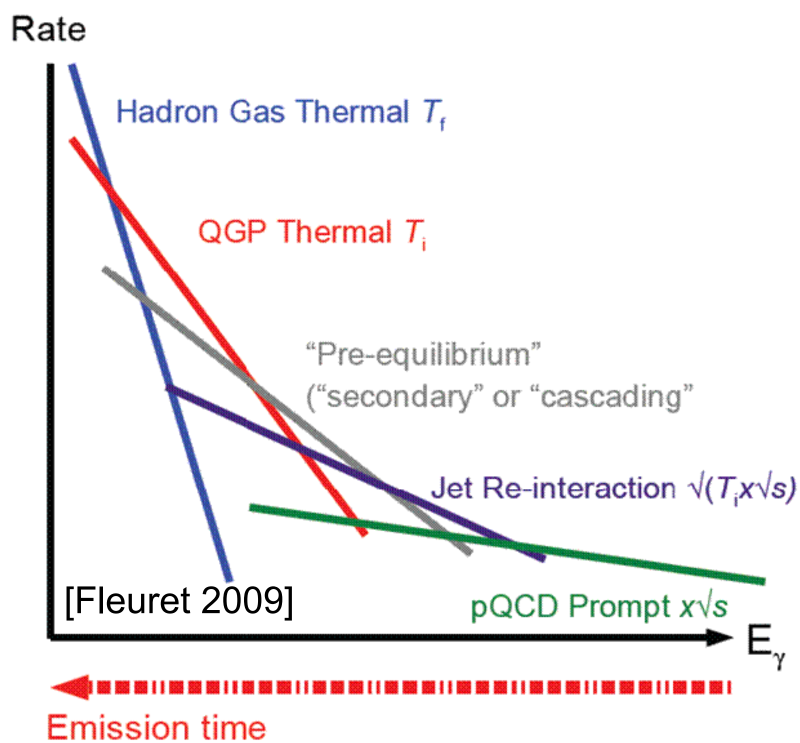


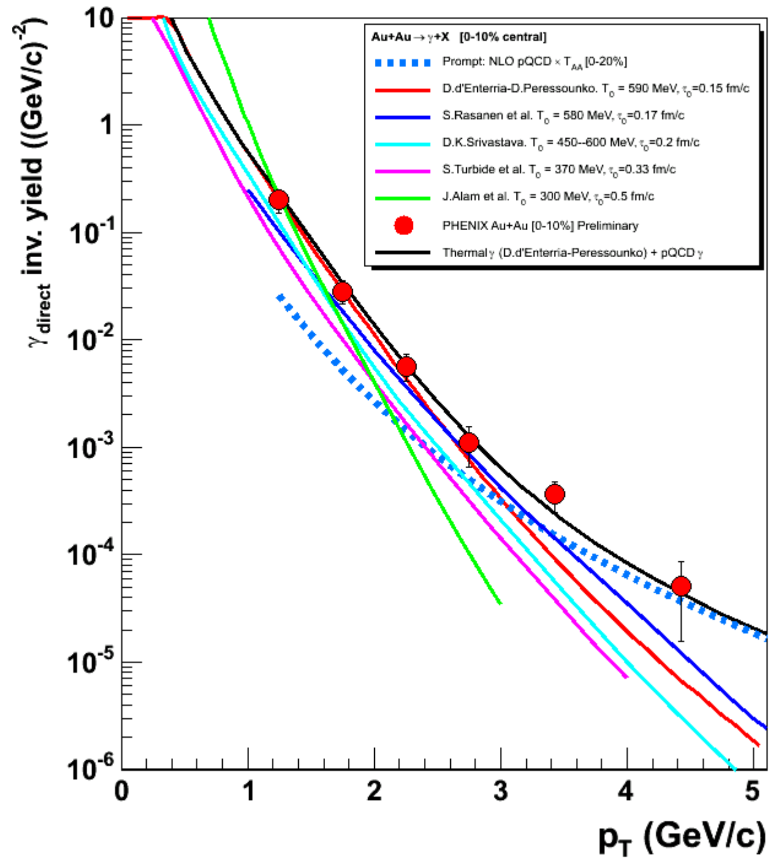
Dileptonrate directly related to vector spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3 p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)} \rho_{\mathbf{V}}(\omega, \vec{\mathbf{p}}, \mathbf{T})$$

Hard Probes in Heavy Ion Collisions - Photons

Direct and fragmentation photon relative contribution





Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{dN_\gamma}{d^4x\, d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \sigma_V(\omega = |\vec{p}|, T)$$

Motivation - Transport Coefficients

Transport Coefficients are important ingredients into **hydro/transport models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

Need to be determined from QCD using first principle lattice calculations!

for heavy flavour:

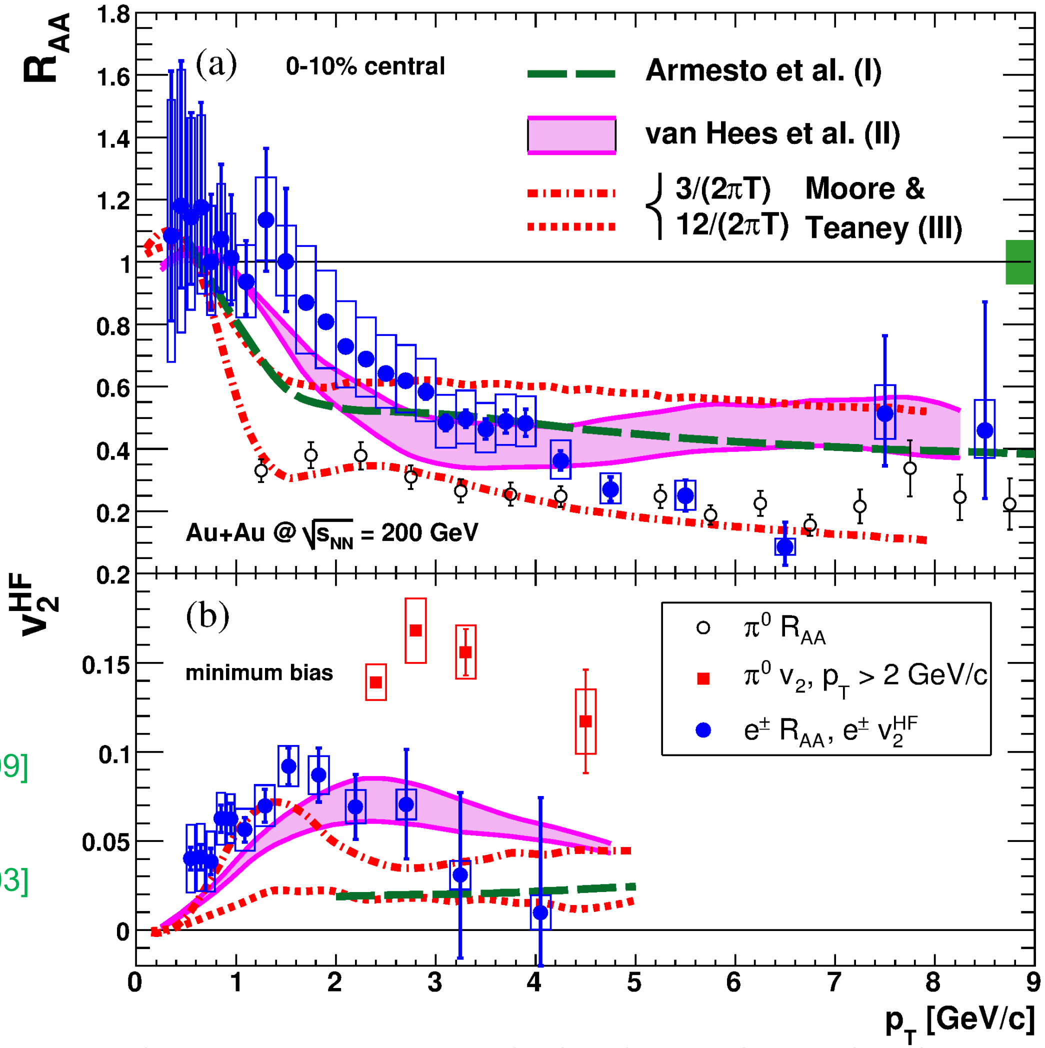
Heavy Quark Diffusion Constant D
[H.T.Ding, OK et al., PRD86(2012)014509]

Heavy Quark Momentum Diffusion κ
[A.Francis, OK, et al., PRD92(2015)116003]

for light quarks:

Light quark flavour diffusion

Electrical conductivity
[A.Francis, OK et al., PRD83(2011)034504
H-T.Ding, F.Meyer, OK, arXiv:1604.06712]



[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

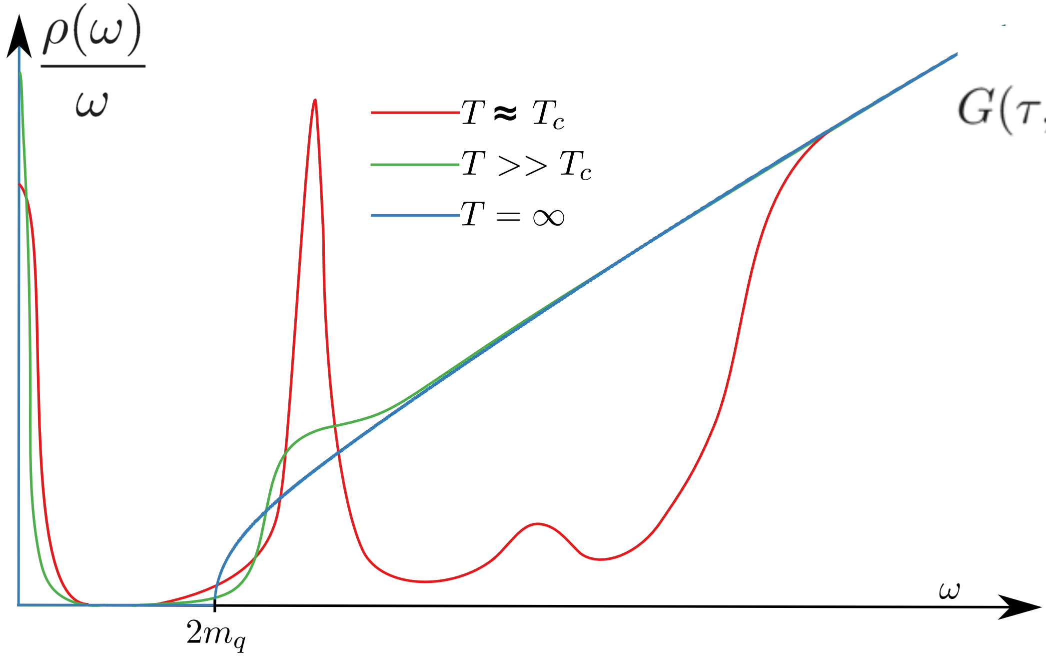
Vector meson spectral function – hard to separate different scales

Different contributions and scales enter in the spectral function

- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies
- in addition cut-off effects on the lattice

notoriously difficult to extract from correlation functions

Spectral functions in the QGP



$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

$$\begin{aligned} G_{\mu\nu}(\tau,\vec{x}) &= \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle \\ J_\mu(\tau,\vec{x}) &= 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu \psi(\tau,\vec{x}) \end{aligned}$$

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

Spectral functions at high temperature

Free theory (massless case):

free non-interacting vector spectral function (infinite temperature):

$$\rho_{00}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega)$$
$$\rho_{ii}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\,\omega^2 \tanh(\omega/4T)$$

δ-functions exactly cancel in $\rho_V(\omega) = -\rho_{00}(\omega) + \rho_{ii}(\omega)$

With interactions (but without bound states):

while ρ_{00} is protected, the δ-funtion in ρ_{ii} gets smeared → transport peak:

Ansatz:

$$\kappa = \frac{\alpha_s}{\pi}$$

at leading order

$$\rho_{00}(\omega) = 2\pi \chi_q \omega \delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2 \tanh(\omega/4T)$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

→ electrical conductivity: $$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T} \qquad \frac{\sigma}{T} = \frac{2}{3}\,\frac{\chi_q}{T^2}\,\frac{T}{\Gamma}\,c_{BW}\,C_{em}$$

Vector correlation function on large & fine lattices

[H-T.Ding, F.Meyer, OK, arXiv:1604.06712, H.T.Ding, A.Francis, OK et al., PRD83(2011)034504]

quenched SU(3) gauge configurations (separated by 500 updates)

non-perturbatively O(a) clover improved Wilson fermion valence quarks

non-perturbative renormalization constants and quark masses close to the chiral limit

	N_τ	N_σ	β	κ	$T\sqrt{t_0}$	$T/T_c\|_{t_0}$	Tr_0	$T/T_c\|_{r_0}$	confs
1.1 T_c	32	96	7.192	0.13440	0.2796	1.12	0.8164	1.09	314
	48	144	7.544	0.13383	0.2843	1.14	0.8169	1.10	358
	64	192	7.793	0.13345	0.2862	1.15	0.8127	1.09	242
1.3 T_c	28	96	7.192	0.13440	0.3195	1.28	0.9330	1.25	232
	42	144	7.544	0.13383	0.3249	1.31	0.9336	1.25	417
	56	192	7.793	0.13345	0.3271	1.31	0.9288	1.25	273
1.5 T_c	24	128	7.192	0.13440	0.3728	1.50	1.0886	1.46	340
	32	128	7.457	0.13390	0.3846	1.55	1.1093	1.49	255
	48	128	7.793	0.13340	0.3817	1.53	1.0836	1.45	456

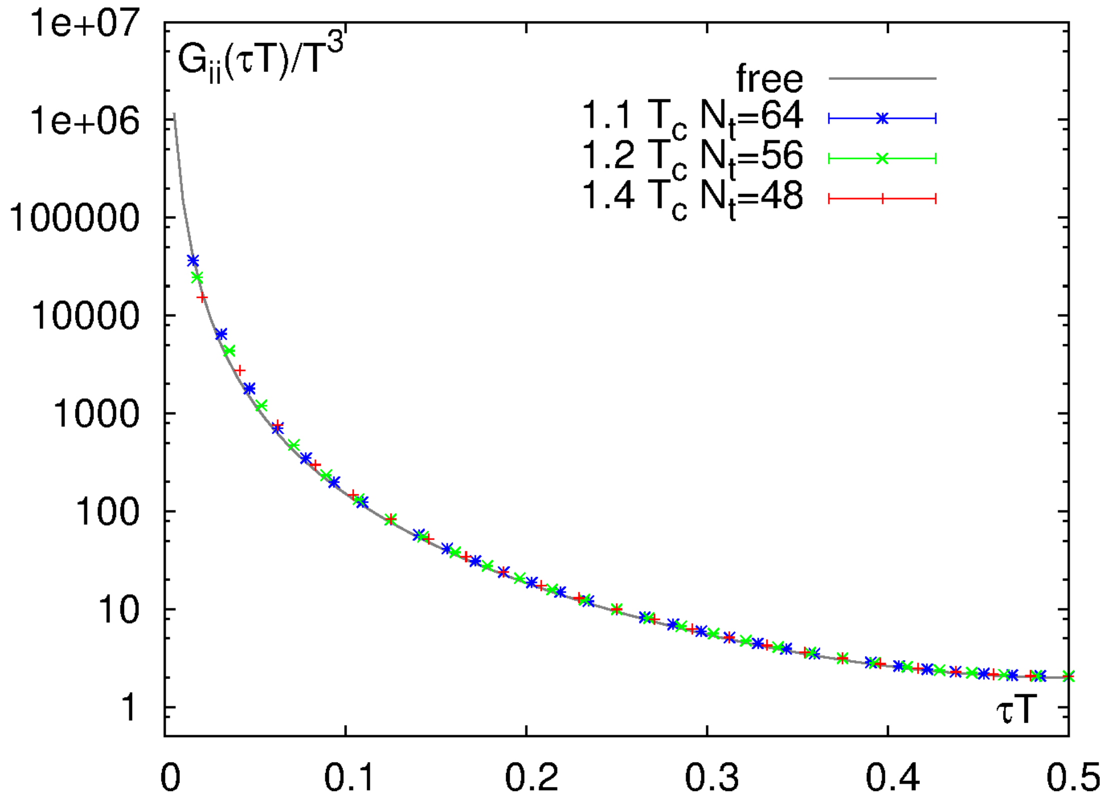
Scale setting using r_0 and t_0 [A.Francis, M.Laine, T.Neuhaus, H.Ohno PRD92(2015)116003]

fixed aspect ratio N_σ/N_τ = 3 and 3.43 to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

constant physical volume $(1.9\text{fm})^3$

Vector correlation function



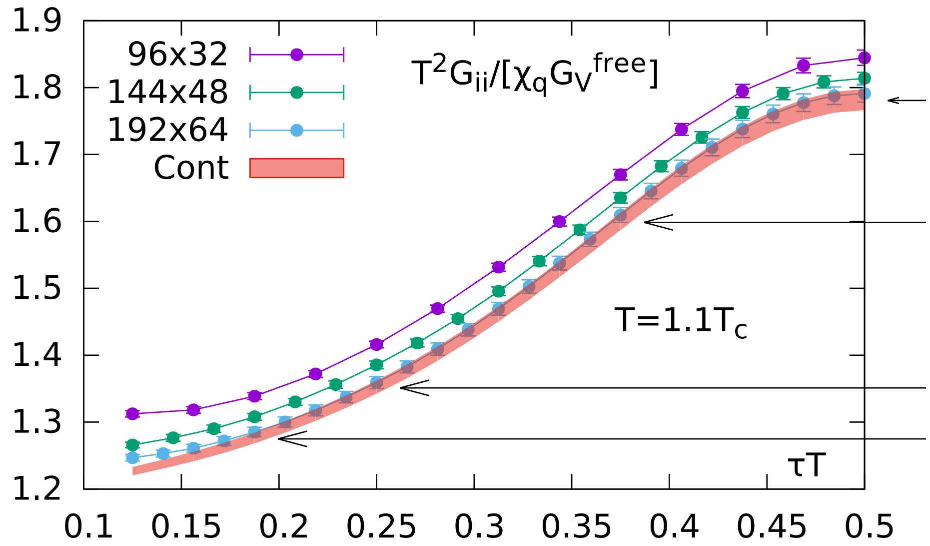
compared to free (non-interacting) correlator:

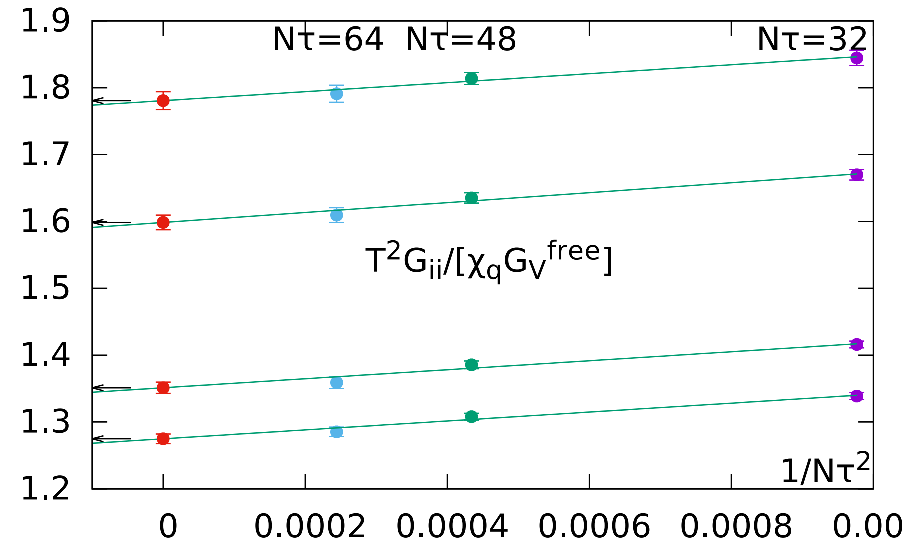
$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T) \frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

hard to distinguish differences due to different orders of magnitude in the correlator

→ in the following we will use $G_V^{free}(\tau)$ as a normalization

Continuum extrapolation





correlators normalized by quark number susceptibility χ_q independent of renormalization

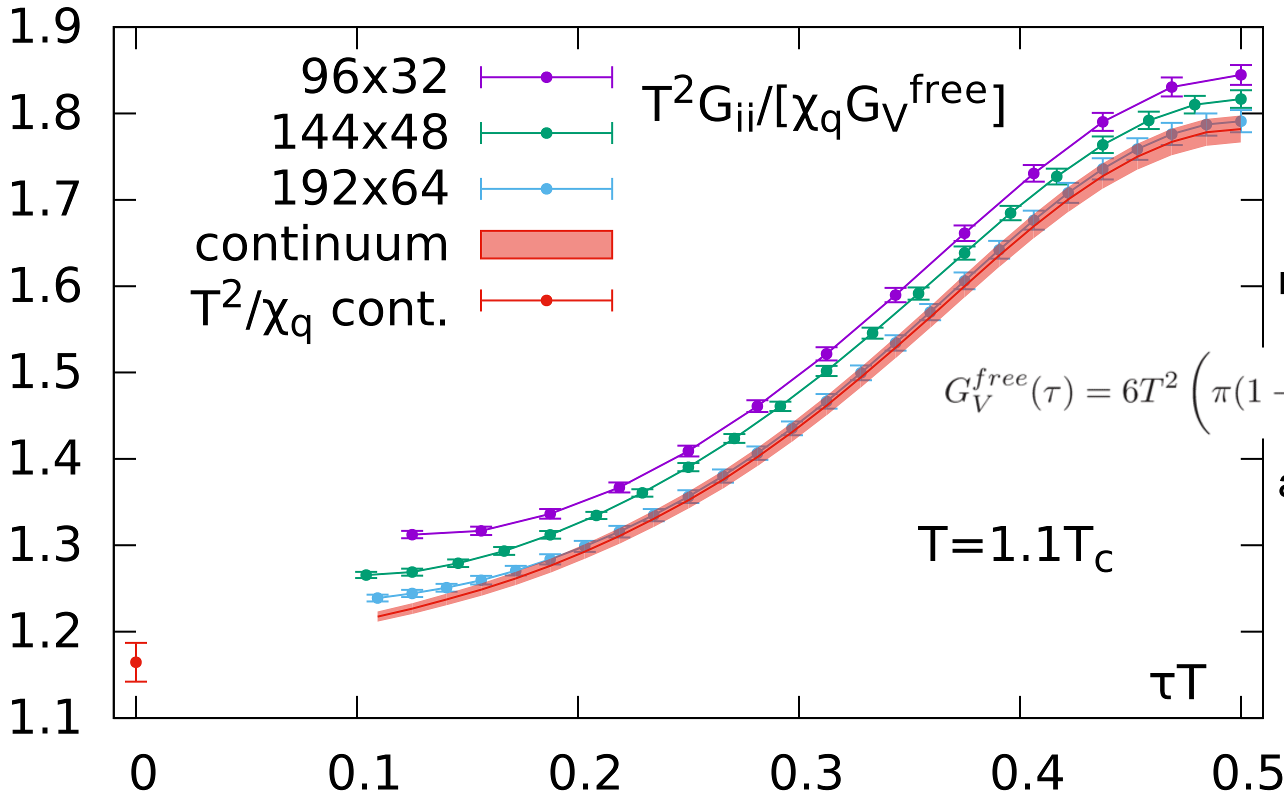
and by the free non-interacting correlator $G_V^{free}(\tau)$

we interpolate the correlator for each lattice spacing

and perform the continuum limit a→0 at each distance τT

cut-off effects are visible at all distances on finite lattices

Continuum extrapolation



normalized by free correlator

$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T)\frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

and quark number susceptibility

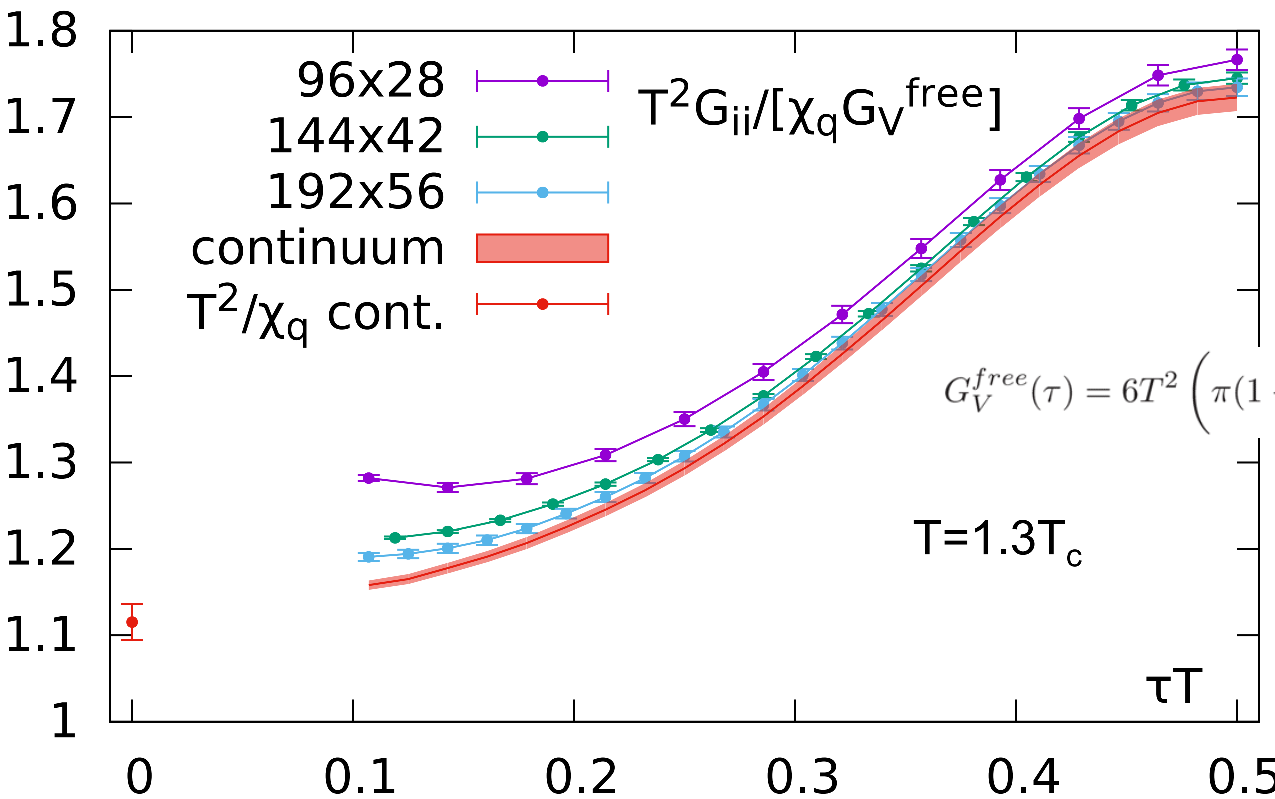
cut-off effects are visible at all distances on finite lattices but

well defined continuum limit on the correlator level

well behaved continuum correlator down to small distances

approaching the correct asymptotic limit for $\tau \to 0$

Continuum extrapolation



normalized by free correlator

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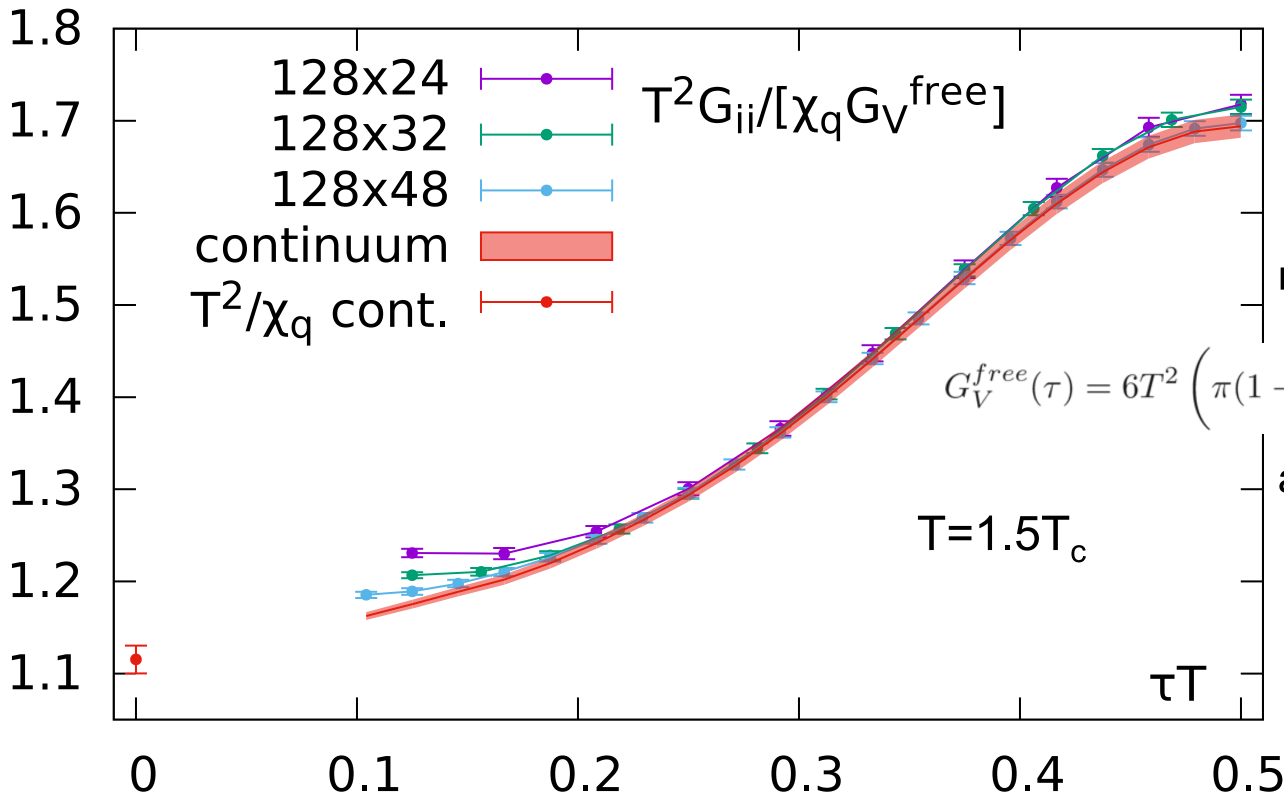

normalized by free correlator

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and quark number susceptibility

cut-off effects are visible at all distances on finite lattices but

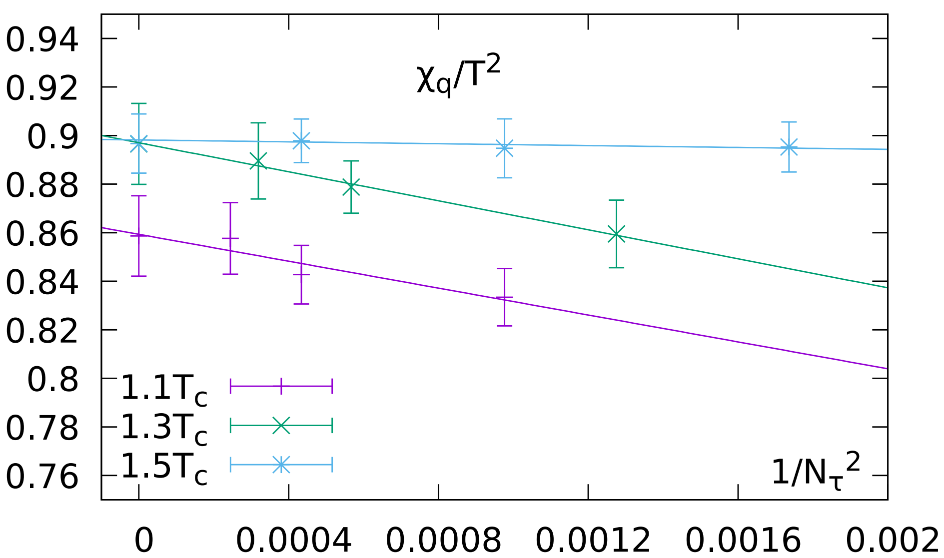
well defined continuum limit on the correlator level

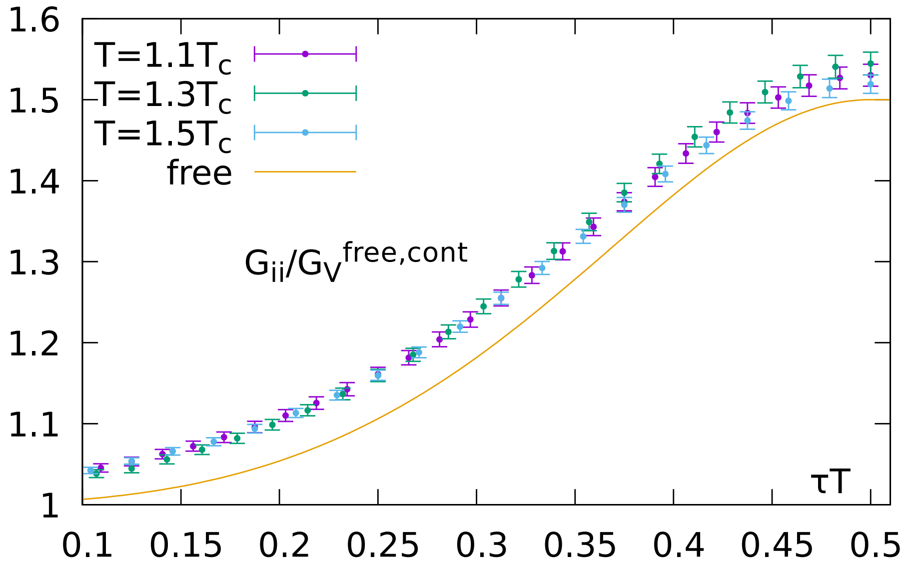
well behaved continuum correlator down to small distances

approaching the correct asymptotic limit for $\tau \to 0$

Continuum extrapolated vector correlation function

continuum extrapolated results available for three temperatures in the QGP





similar behavior in this temperature region

main difference due to different quark number susceptibility χ_q/T^2

→ indications for a weak T-dependence of the temperature scaled

electrical conductivity and thermal dilepton rates

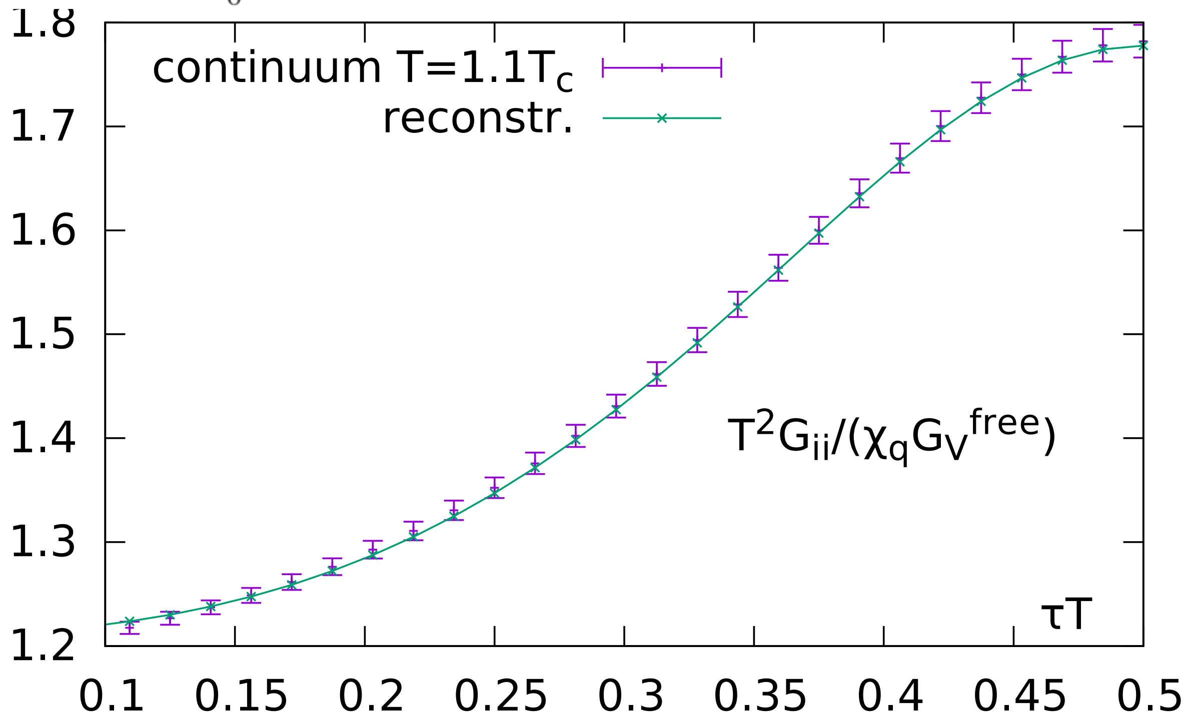
Spectral function and electrical conductivity

use our Ansatz for the spectral function and fit to the continuum extrapolated correlators

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)$$

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$



all three temperatures well described by this rather simple Ansatz

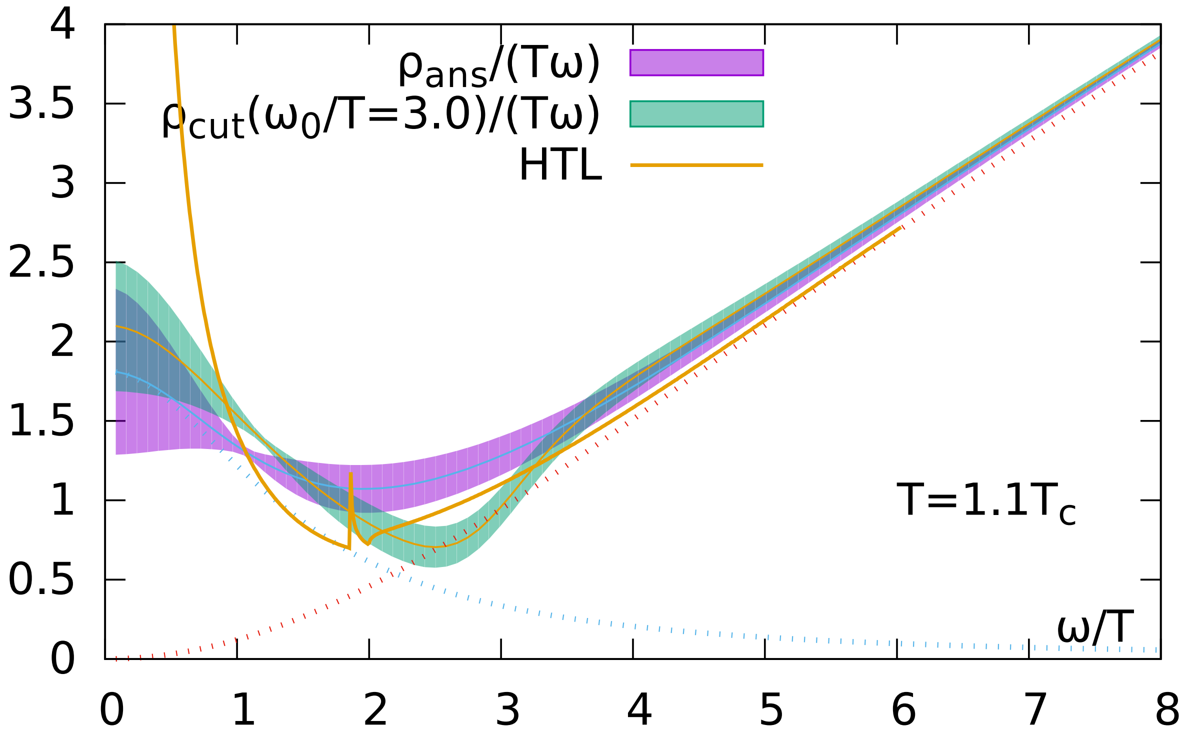
T	$\sigma/(C_{\rm em}T)$	Γ/T	$c_{BW}T/\Gamma$	k	χ^2/dof
$1.1T_c$	0.302(88)	2.86(1.16)	0.528(154)	0.038(8)	1.15
$1.3T_c$	0.254(51)	3.91(1.25)	0.425(85)	0.029(9)	0.52
$1.5T_c$	0.266(48)	3.33(89)	0.445(80)	0.040(7)	1.13

Spectral function and electrical conductivity

Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$



Analysis of the systematic errors

using truncation of the large ω contribution

$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

systematic uncertainties (within this Ansatz) estimated by varying the truncation

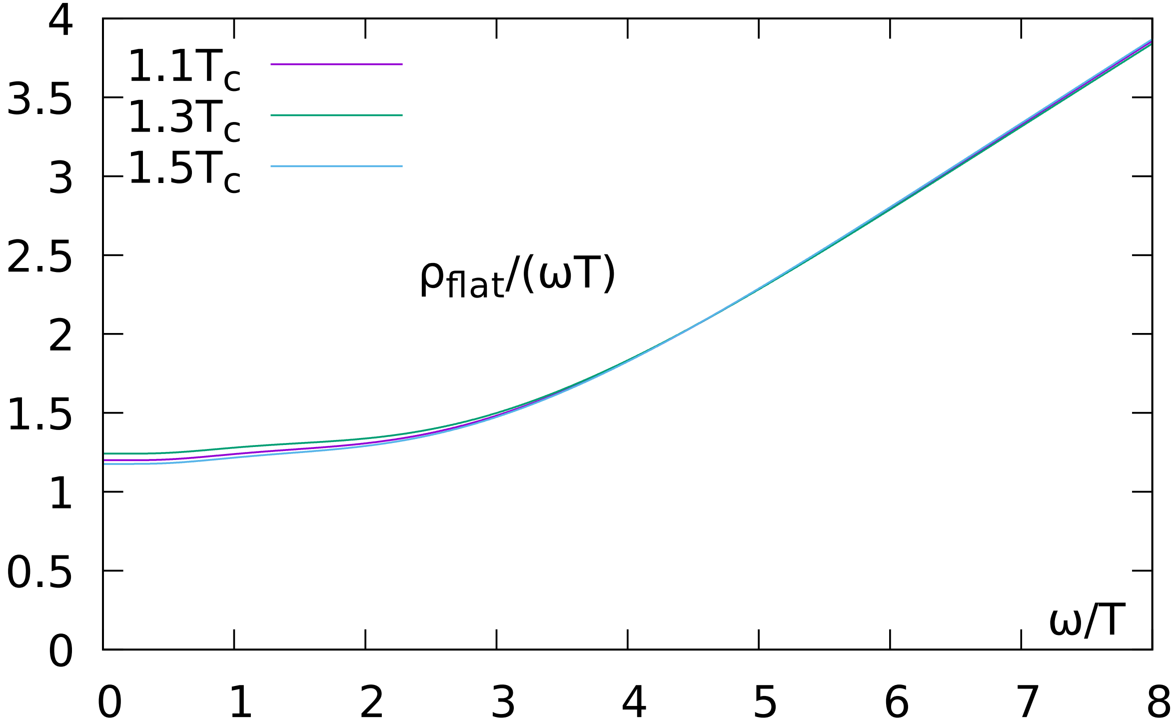
with similar χ^2/dof ~ 0.5-1.1

Spectral function and electrical conductivity

Use a flat transport Ansatz for the spectral function

$$\rho_{\text{flat}}(\omega) \quad = \quad a\chi_q\omega\left(1-\widetilde{\Theta}(\omega_0,\Delta_0)\right)+(1+k)\rho_{\text{free}}(\omega)\widetilde{\Theta}(\omega_1,\Delta_1)$$

$$\widetilde{\Theta}(\omega,\omega_i,\Delta_i)=\left(1+\exp\left(\frac{\omega_i^2-\omega^2}{\omega\Delta_i}\right)\right)^{-1}$$



Analysis of the systematic errors

using a flat behavior at small ω

$$\frac{\sigma}{T}=\frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

systematic uncertainties (within this Ansatz) estimated by varying the truncation

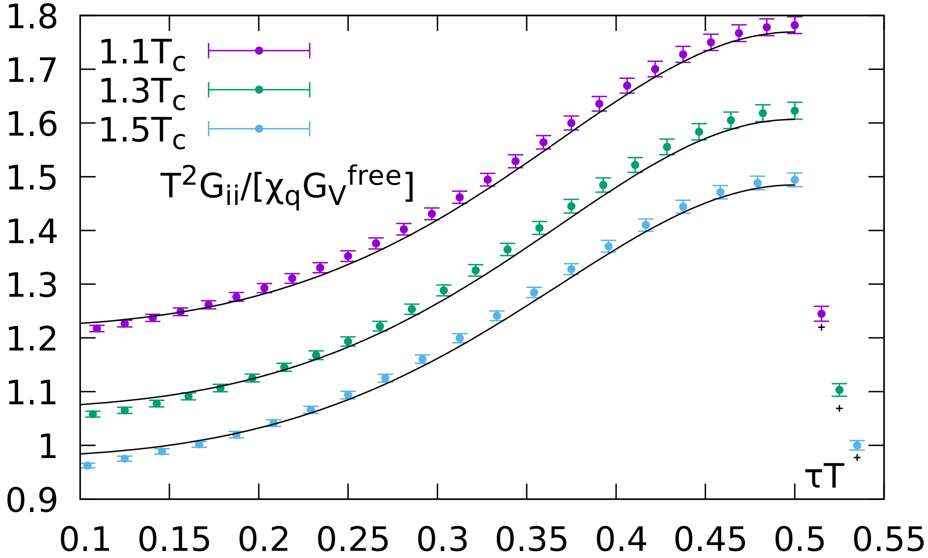
with similar χ^2/dof ~ 0.5-1.1 still consistent with our data → lower limit for σ/T

Spectral function and electrical conductivity

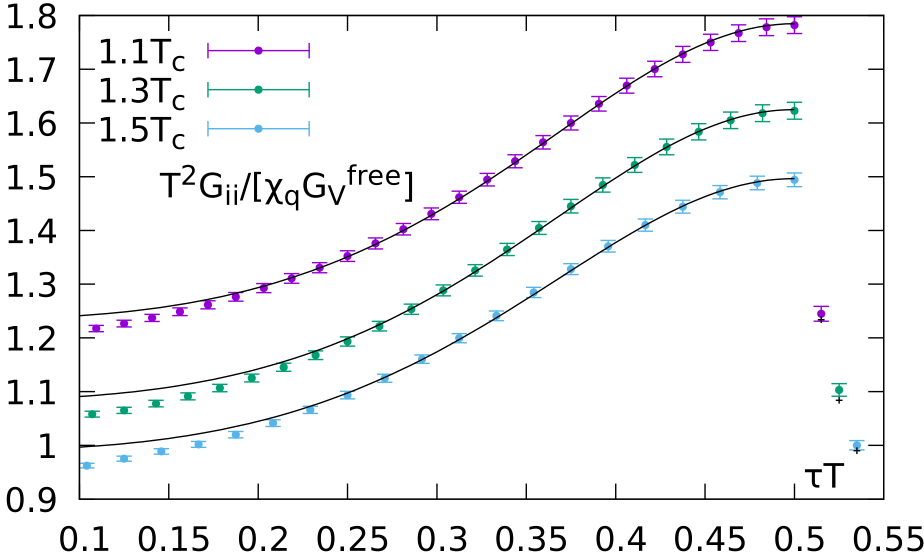
Can we rule out a very narrow transport peak?

$$\rho_\delta(\omega) \quad = \quad a\chi_q\omega\delta(\omega) + (1+k)\rho_{\mathrm{free}}(\omega)$$

with full covariance matrix:



with diagonal covariance matrix:



larger χ^2/dof ~ 1.5-2.5 and thermal moments not well reproduced

→ finite upper limit for σ/T from other Ansätze

→ non-zero lower limit for σ/T from other Ansätze

Using perturbative information

So far we have used a simple Ansatz for the spectral function:

$$\begin{aligned}\rho_{00}(\omega) &= 2\pi\chi_q\omega\delta(\omega)\\ \rho_{ii}(\omega) &= 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2}+\frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)\end{aligned}$$

including the leading order perturbative behavior at large ω w/o running coupling

Can we do better including more information from perturbation theory?

Problem: different scales contribute to the spectral function

different perturbative techniques required depending on the energy regimes

perturbation theory – vacuum spectral function

At very high energies, due to asymptotic freedom

→ perturbation should be working

→ thermal effects should be suppressed

→ “vacuum physics”

5-loop vacuum spectral function:

$$\rho_V(\omega) = \frac{3\omega^2}{4\pi} R(\omega^2)$$

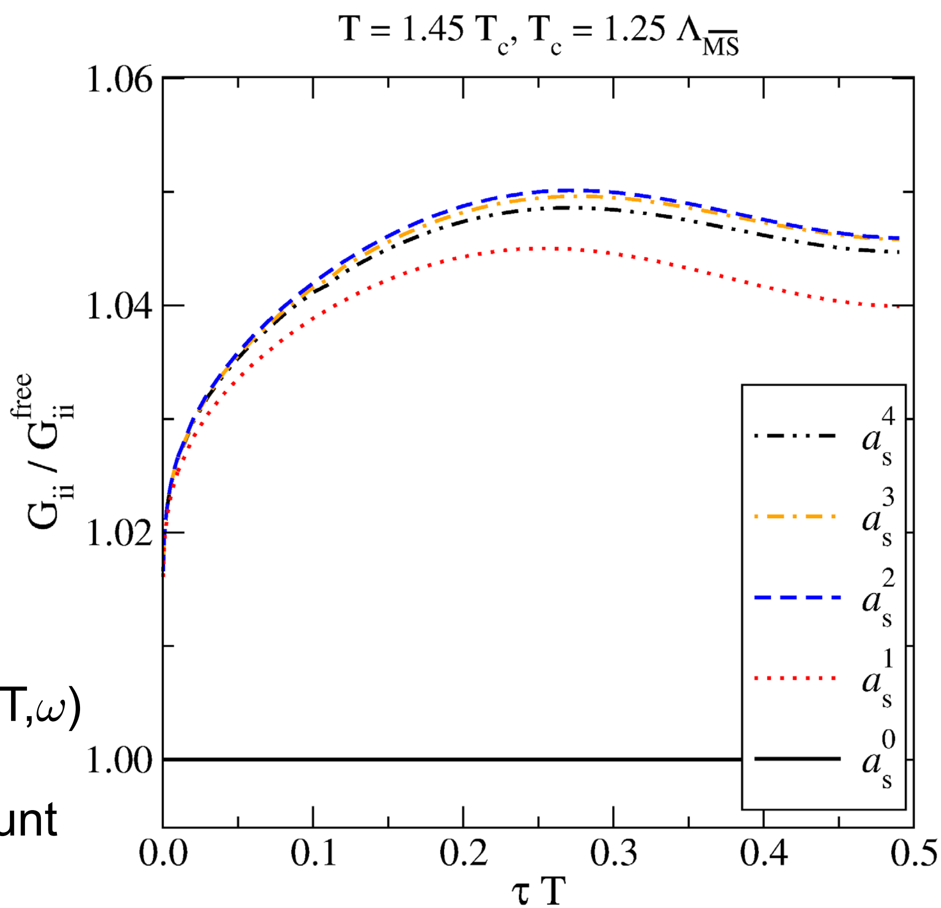
$$\begin{aligned} R(\omega^2) &= r_{0,0} + r_{1,0}\, a_s + (r_{2,0} + r_{2,1}\,\ell)\, a_s^2 \\ &+ (r_{3,0} + r_{3,1}\,\ell + r_{3,2}\,\ell^2)\, a_s^3 \\ &+ (r_{4,0} + r_{4,1}\,\ell + r_{4,2}\,\ell^2 + r_{4,3}\,\ell^3)\, a_s^4 + \mathcal{O}(a_s^5) \end{aligned}$$

using 3-loop α_s and l=log(μ^2/ω^2)

using a renormalization scale μ=(1..5)max(πT,ω)

taking leading order thermal effect into account

$$\rho_{ii}^{(T)}(\omega) \equiv \frac{3\omega^2}{4\pi}[1 - 2n_{\rm F}(\frac{\omega}{2})]R(\omega^2) + \pi\chi_{\rm q}^{\rm free}\omega\delta(\omega)$$



[Y.Burnier and M.Laine, arXiv 1201.1994]

perturbation theory – vacuum spectral function

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5-loop vacuum spectral function:

$$\rho_V(\omega) = \frac{3\omega^2}{4\pi} R(\omega^2)$$

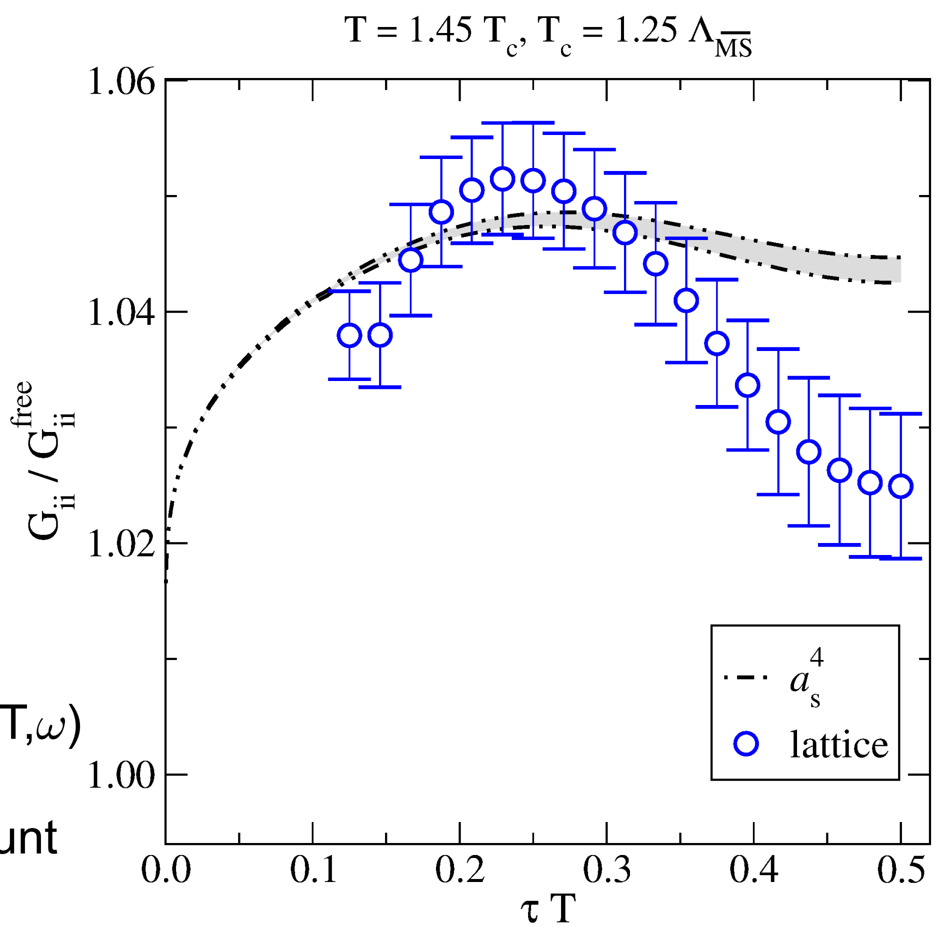
$$\begin{aligned} R(\omega^2) &= r_{0,0} + r_{1,0}\, a_s + \left(r_{2,0} + r_{2,1}\,\ell\right) a_s^2 \\ &+ \left(r_{3,0} + r_{3,1}\,\ell + r_{3,2}\,\ell^2\right) a_s^3 \\ &+ \left(r_{4,0} + r_{4,1}\,\ell + r_{4,2}\,\ell^2 + r_{4,3}\,\ell^3\right) a_s^4 + \mathcal{O}(a_s^5) \end{aligned}$$

using 3-loop α_s and l=log(μ^2/ω^2)

using a renormalization scale μ=(1..5)max(πT,ω)

taking leading order thermal effect into account

$$\rho_{ii}^{(T)}(\omega) \equiv \frac{3\omega^2}{4\pi}\left[1 - 2n_{\mathrm{F}}\left(\frac{\omega}{2}\right)\right] R(\omega^2) + \pi\chi_{\mathrm{q}}^{\mathrm{free}}\,\omega\,\delta(\omega)$$

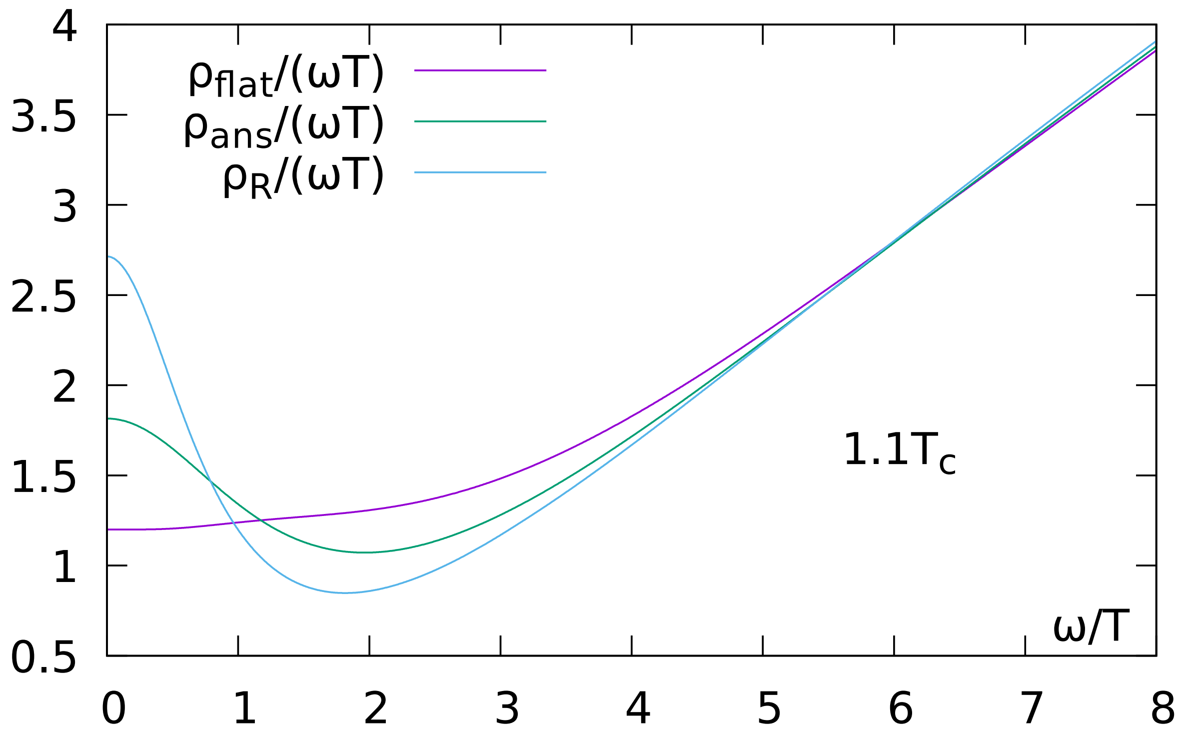


[Y.Burnier and M.Laine, arXiv 1201.1994]

Spectral function and electrical conductivity

include improved large ω behavior from vacuum perturbation theory

$$\begin{aligned}\rho_{\rm impr}(\omega,T) &= \rho_V^{\rm free}(\omega,T)R(\omega^2)\\ \rho_{\rm R}(\omega,T) &= \rho_{\rm BW}(\omega,T)+C\rho_{\rm impr}(\omega,T).\end{aligned}$$



T	$\sigma/(C_{\rm em}T)$	Γ/T	$c_{BW}T/\Gamma$	C	$\chi^2/{\rm dof}$
$1.1T_c$	0.452(251)	1.62(1.09)	0.790(438)	0.993(7)	1.11
$1.3T_c$	0.301(87)	2.89(1.18)	0.504(145)	0.984(8)	0.53
$1.5T_c$	0.326(87)	2.38(85)	0.548(146)	0.996(7)	1.12

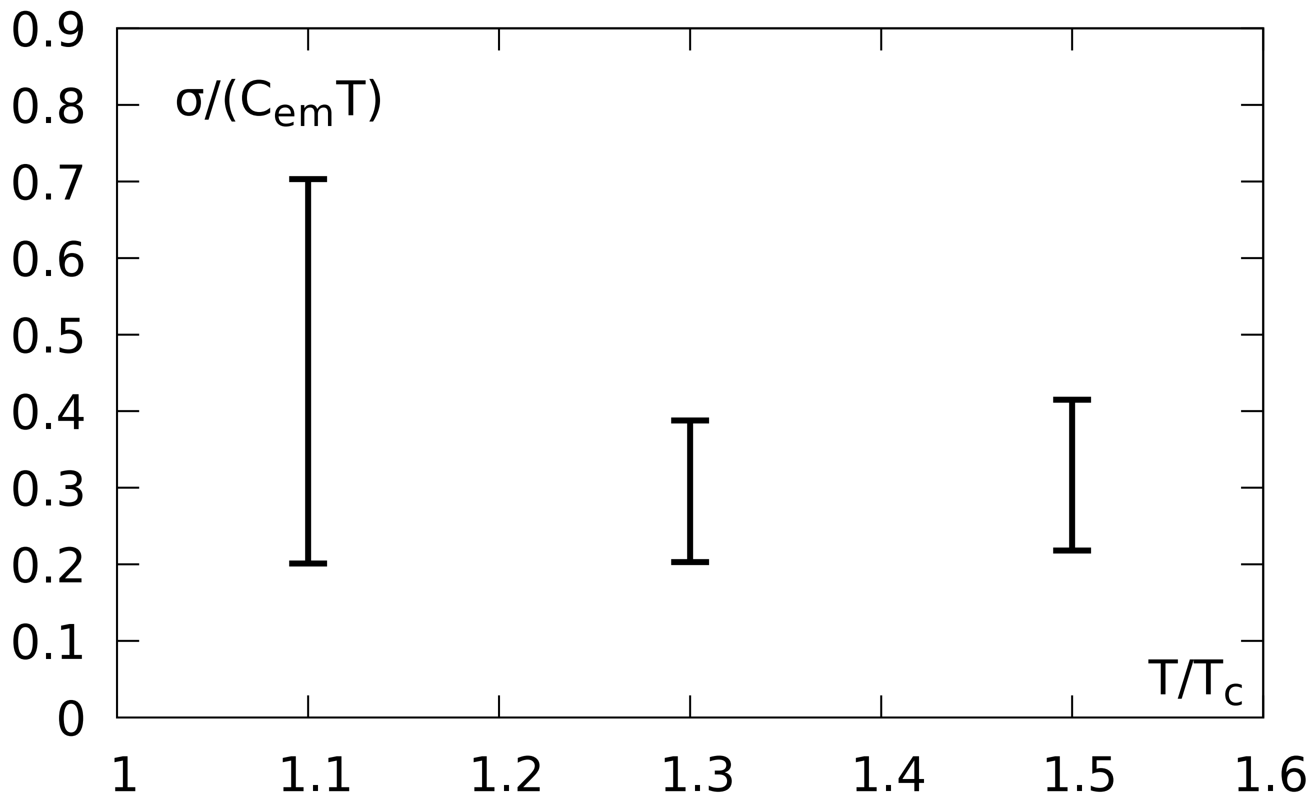
Electrical conductivity

continuum estimate for the

T-dependence of the electrical conductivity:

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

lower and upper limits from the systematic analysis of our classes of spectral functions:

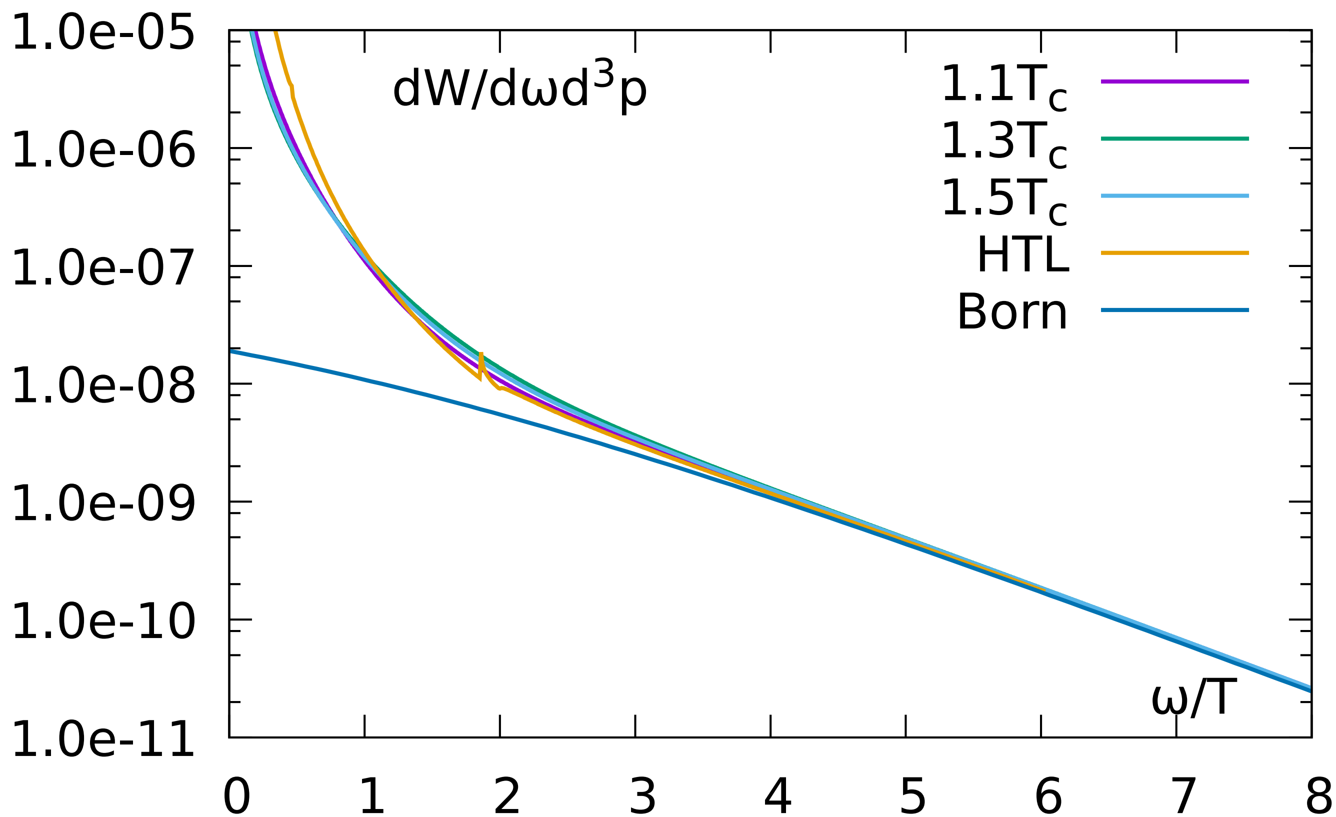


other studies using dynamical clover Wilson or staggered fermions (all w/o continuum limit):
A.Amato et al., arXiv:1307.6763, B.B.Brandt et al., JHEP 1303 (2013) 100, S.Gupta, PLB 597 (2004) 57
G.Aarts et al., PRL 99 (2007) 022002, Brandt et al., PRD93 (2016) 054510, G. Aarts et al., JHEP02 (2015)186

Dilepton rates

Dileptonrate directly related to vector spectral function:

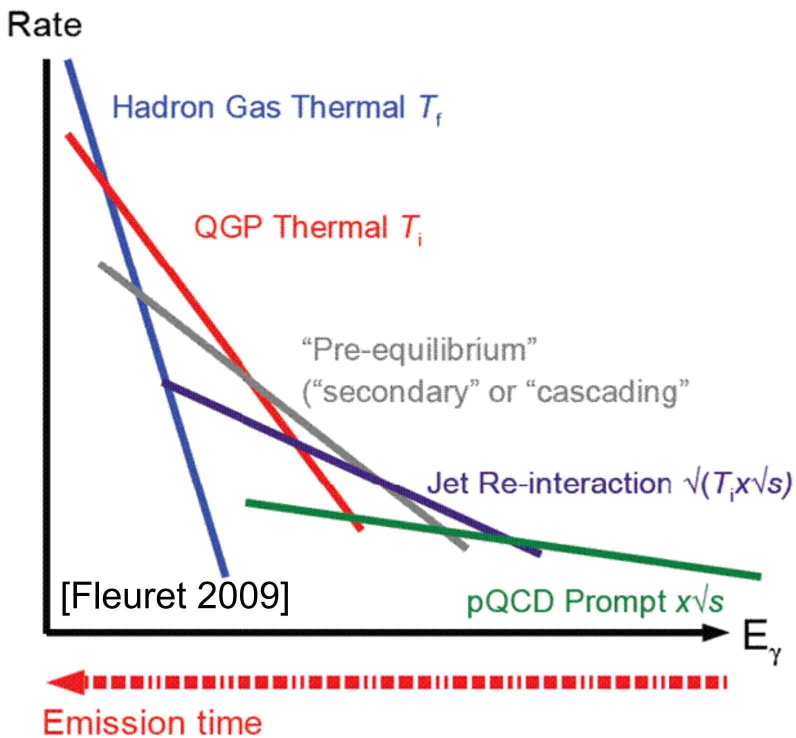
$$\frac{\mathrm{d}W}{\mathrm{d}\omega\mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\ \rho_{\mathbf{V}}(\omega,\mathbf{T})$$

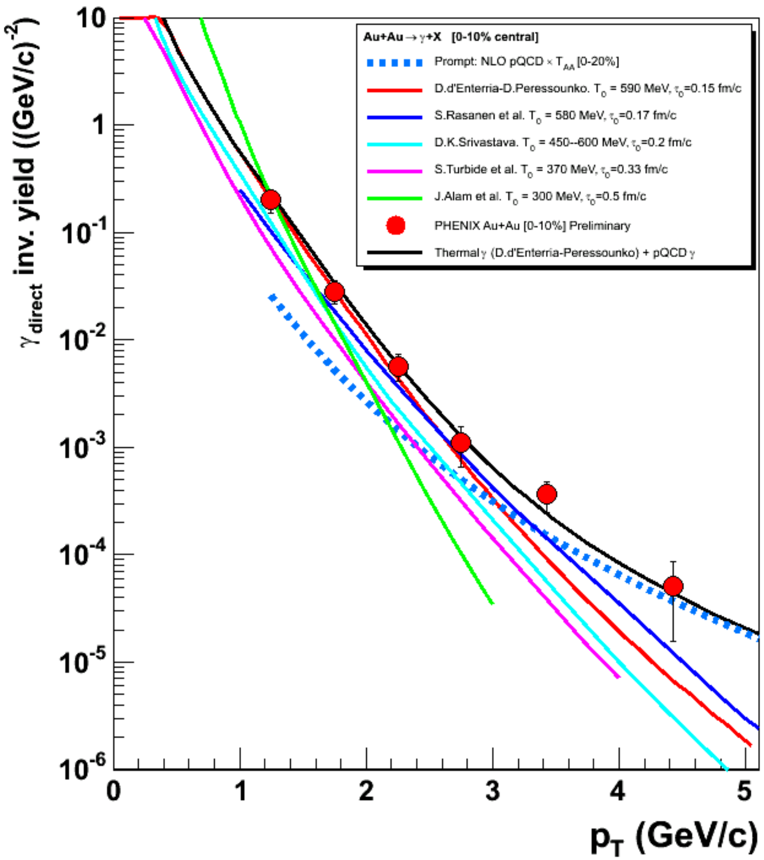


Hard thermal loop (HTL)
[E.Braaten, R.D.Pisarski, NP B337 (1990) 569]

Hard Probes in Heavy Ion Collisions - Photons

Direct and fragmentation photon relative contribution





Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{dN_\gamma}{d^4x d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \sigma_V(\omega = |\vec{k}|, T)$$

Perturbative knowledge on the vector spectral function

Non-interacting limit, “Born rate” for large invariant mass M>>πT, with $M^2=\omega^2+k^2$

$$\rho_V(\omega,\mathbf{k}) = \frac{N_c T M^2}{2\pi k}\left\{\ln\left[\frac{\cosh(\frac{\omega+k}{4T})}{\cosh(\frac{\omega-k}{4T})}\right] - \frac{\omega\,\theta(k-\omega)}{2T}\right\},$$

[G. Aarts and J.M. Martinez Resco, NPB 726 (2005) 93]

Leading-log order for invariant mass M=0: [J.I. Kapusta et al.,PRD44 (1991) 2774, R. Baier et al. Z.Phys.C53 (1992) 433]

$$\rho_V(k,\mathbf{k}) = \frac{\alpha_s N_c C_F T^2}{4}\ln\left(\frac{1}{\alpha_s}\right)\left[1-2n_F(k)\right] + \mathcal{O}(\alpha_s T^2),$$

Complete leading order for invariant mass M=0:
[Arnold, Moore, Yaffe, JHEP11(2001)57 and JHEP12(2001)9]

NLO at M = 0: [J.Ghiglieri et al., JHEP 1305 (2013) 010]

NLO at M $\sim$ gT: [J.Ghiglieri and G.D.Moore, JHEP 1412 (2014) 029]

NLO at M $\sim$ πT: [M.Laine, JHEP 1311 (2013) 120]

N^4LO at M >> πT: [S. Caron-Huot, PRD79 (2009) 125009, P.A.Baikov et al. PRL101 (2008) 012002]

Hydrodynamic regime

Vector spectral function in the hydrodynamic regime for $\omega, k \lesssim \alpha_s^2 T$:

$$\frac{\rho_{\mathrm{V}}(\omega,\mathbf{k})}{\omega} = \left(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2\right)\chi_{\mathrm{q}} D$$

with the quark number susceptibility: $$\chi_{\mathrm{q}} \equiv \int_0^\beta \mathrm{d}\tau \int_{\mathbf{x}} \langle V^0(\tau,\mathbf{x}) V^0(0)\rangle$$

and the diffusion coefficient: $$D \equiv \frac{1}{3\chi_{\mathrm{q}}} \lim_{\omega\to 0^+} \sum_{i=1}^{3} \frac{\rho^{ii}(\omega,\mathbf{0})}{\omega}$$

which relate to the electric conductivity: $$\sigma = e^2 \sum_{f=1}^{N_{\mathrm{f}}} Q_f^2 \chi_{\mathrm{q}} D$$

In this limit the (soft) photon rate becomes: $$\frac{\mathrm{d}\Gamma_\gamma(\mathbf{k})}{\mathrm{d}^3\mathbf{k}} \overset{k \lesssim \alpha_s^2 T}{\approx} \frac{2T\sigma}{(2\pi)^3 k}$$

AdS/CFT spectral functions

In the AdS/CFT framework the vector spectral function has the same infrared structure:

$$\frac{\rho_{\mathrm{V}}(\omega,\mathbf{k})}{\omega} = \left(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2\right)\chi_{\mathrm{q}} D$$

with the quark number susceptibility: $\chi_{\mathrm{q}} = \dfrac{N_c^2 T^2}{8}$

and the diffusion coefficient: $D = \dfrac{1}{2\pi T}$

it is close to the hydrodynamic form for k $\lesssim$ 0.5/D

and becomes negative for small ω and k $\lesssim$ 1.07/D

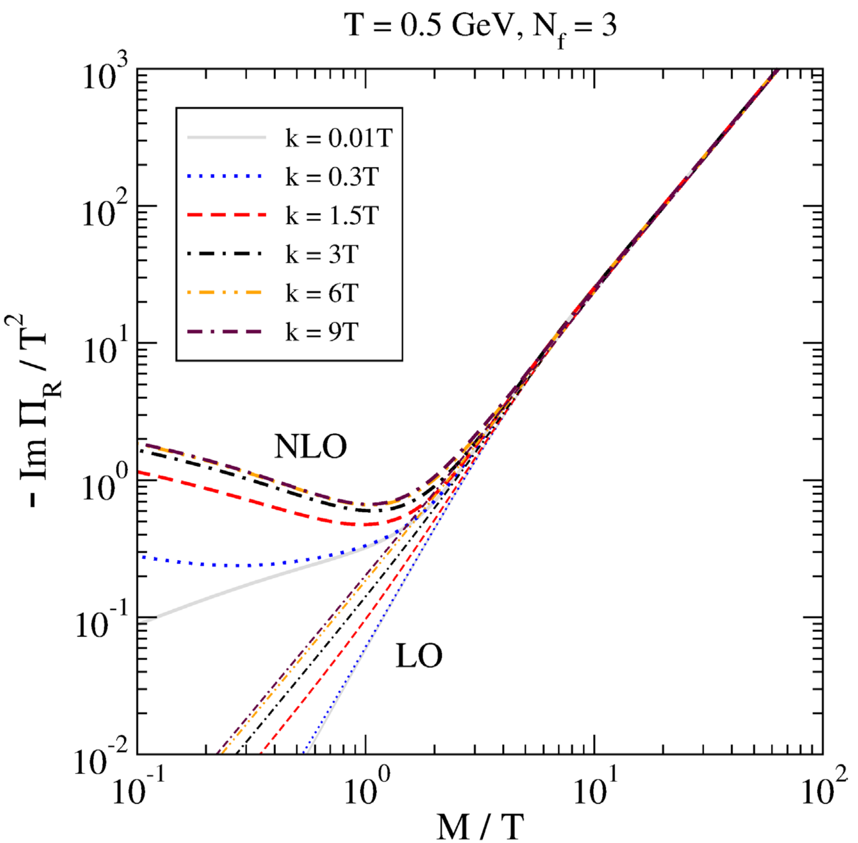
We make use of the numerical result that makes predictions beyond the hydro regime
[S.Caron-Huot, JHEP12 (2006) 015]

Although not QCD, it may give qualitative insight into the structures at small ω and k

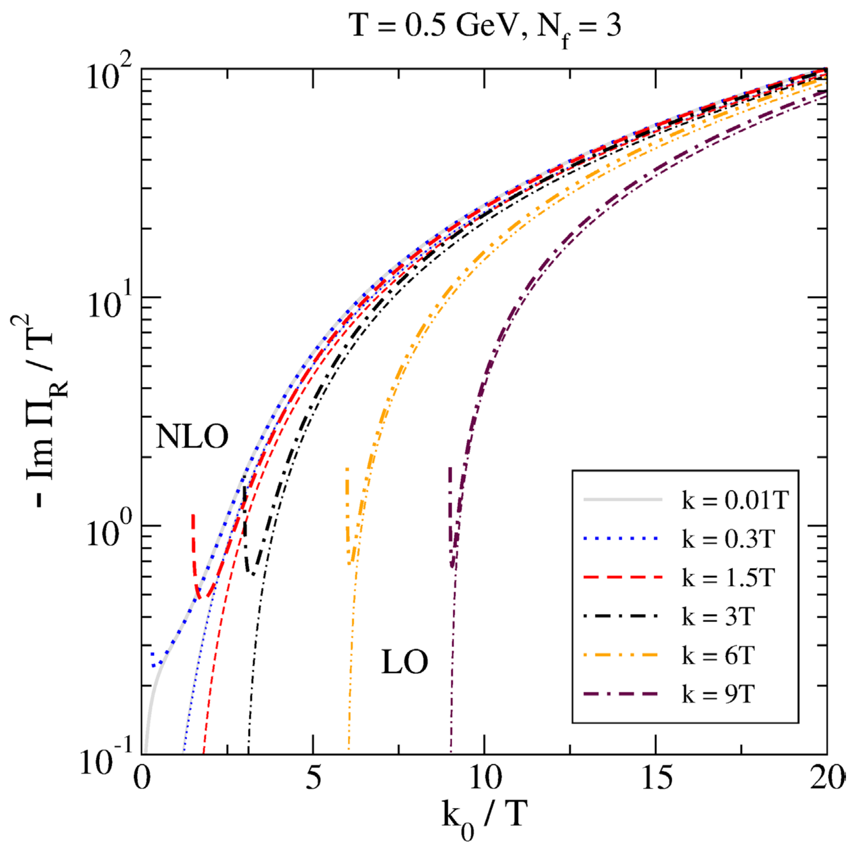
perturbation theory – thermal corrections

Thermal corrections in the intermediate frequency regime required [Altherr+Aurenche 1989]

[Moore+Robert 2006]



$M^2=\omega^2+k^2$



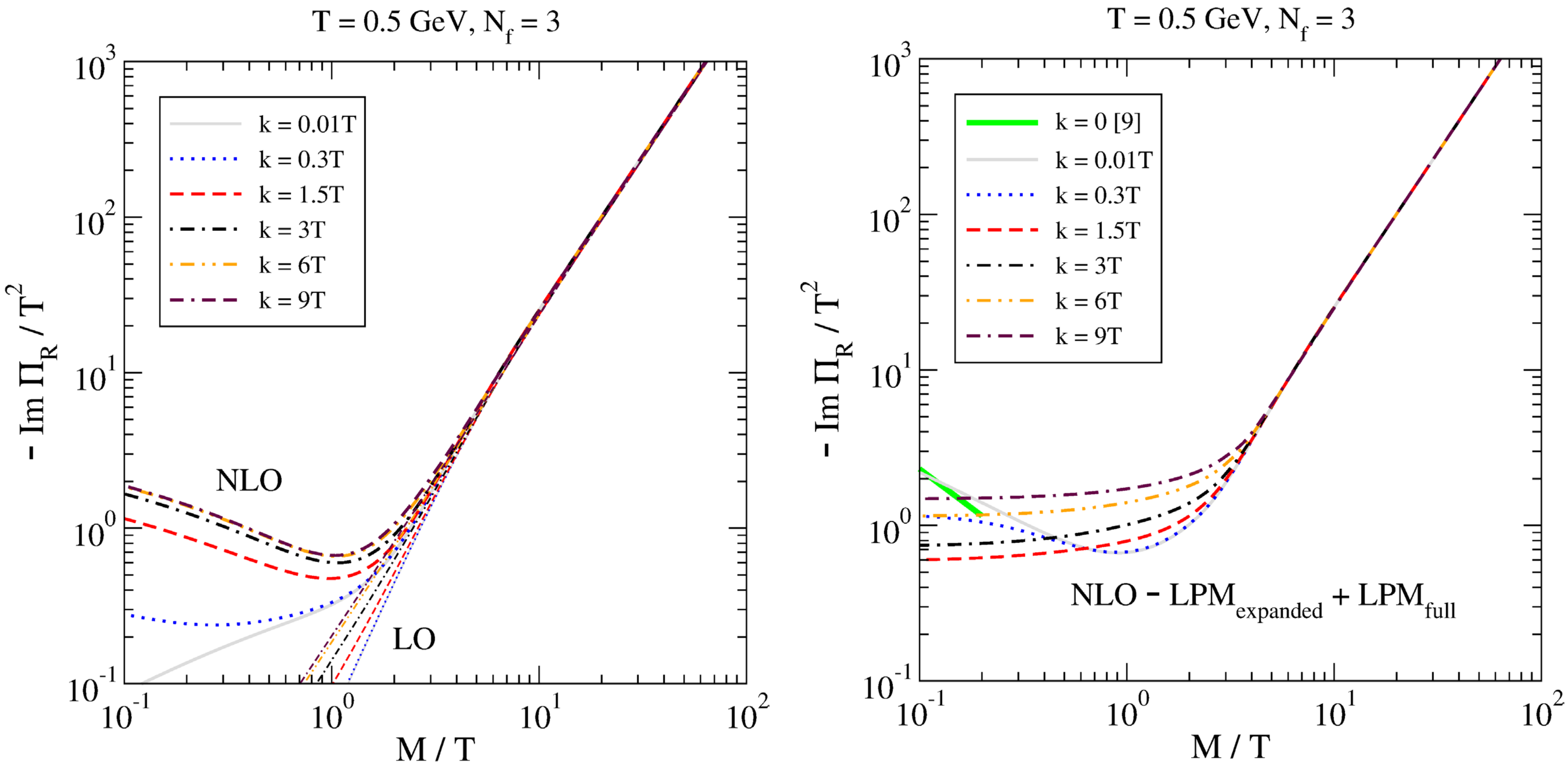
$k_0 = \omega$

[M.Laine, JHEP 11 (2013) 120, arXiv:1310.0164]

perturbation theory – thermal corrections

Thermal corrections in the intermediate frequency regime required [Altherr+Aurenche 1989]

and proper treatment of the small frequency regime [Ghiglieri+Moore 2014] [Moore+Robert 2006]



interpolation between different regimes [I. Ghisoiu and M.Laine, JHEP 10 (2014) 84, arXiv:1407.7955]

progress in perturbation theory in the past years → compare to lattice QCD results

pQCD spectral functions

pQCD spectral function used in our analysis

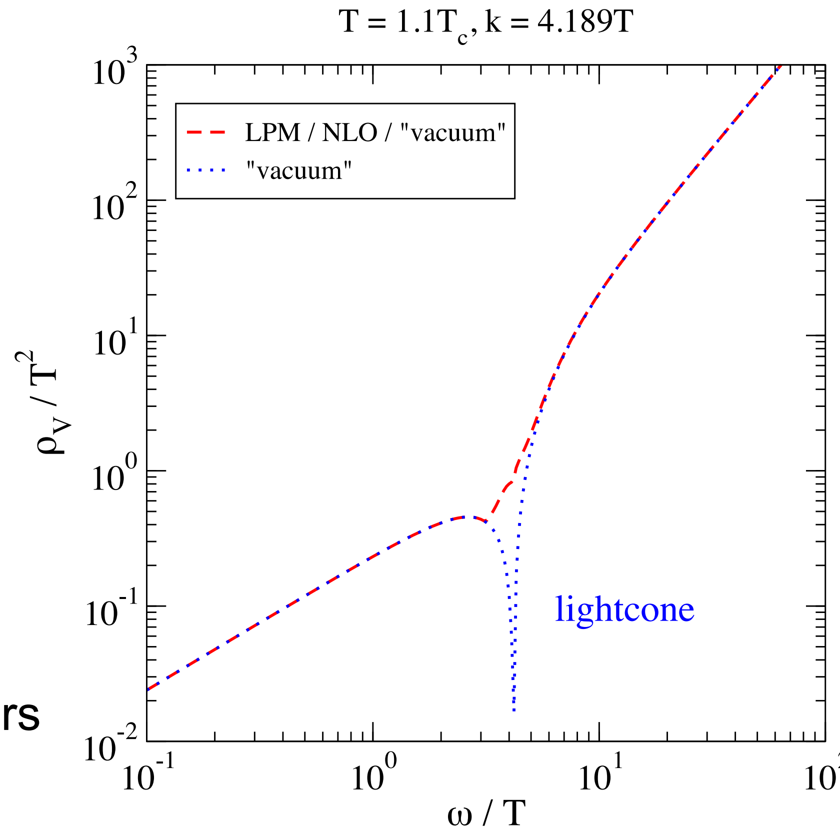
to allow for non-perturbative effects

and to analyze how far pQCD can be trusted

we model the infrared behavior

assuming smoothness at the light cone

and fit to continuum extrapolated lattice correlators



3T < ω < 10T: [J.Ghiglieri and G.D.Moore, JHEP 1412 (2014) 029]

ω > 10T: [I. Ghisoiu and M.Laine, JHEP 10 (2014) 84, arXiv:1407.7955]

ω >> 10T: [M.Laine, JHEP 1311 (2013) 120]

interpolation between the different regimes: www.laine.itp.unibe.ch/dilepton-lattice

Modeling the spectral function

$(5+2\,n_{max})^{th}$ order polynomial Ansatz at small ω:

$$\rho_{\rm fit} \equiv \frac{\beta\,\omega^3}{2\omega_0^3}\left(5-\frac{3\omega^2}{\omega_0^2}\right) - \frac{\gamma\,\omega^3}{2\omega_0^2}\left(1-\frac{\omega^2}{\omega_0^2}\right) + \sum_{n=0}^{n_{\rm max}} \frac{\delta_n \omega^{1+2n}}{\omega_0^{1+2n}}\left(1-\frac{\omega^2}{\omega_0^2}\right)^2$$

with the constraints to match smoothly with pQCD at ω_o

$$\rho_{\rm V}(\omega_0,\mathbf{k}) \equiv \beta\,, \quad \rho'_{\rm V}(\omega_0,\mathbf{k}) \equiv \gamma\,,$$

and n_{max}+1 free parameters

starting with a linear behavior at $\omega \ll$ T

smoothly matched to the perturbative spectral function at $\omega_0 \simeq \sqrt{k^2+(\pi T)^2}$

In the following we will use n_{max} = 0 and n_{max} = 1 for the fits to the lattice data

and to estimate the systematic uncertainties

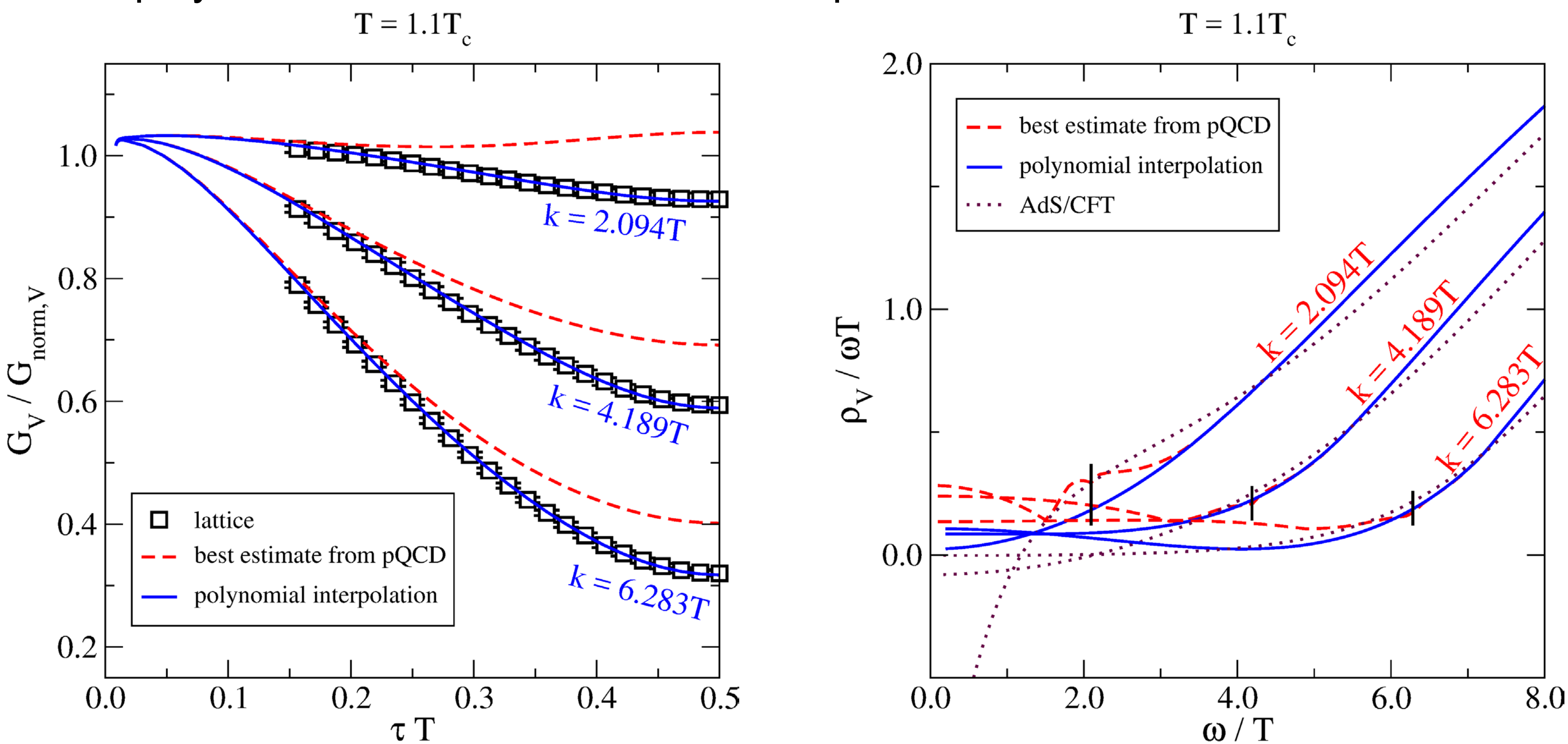
Lattice constraints on photon rates

Fixed aspect ratio used to perform continuum extrapolation at finite p

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}$	$T/T_c\vert_{t_0}$	Tr_0	$T/T_c\vert_{r_0}$
7.192	$96^3 \times 32$	314	0.2796	1.12	0.816	1.09
7.544	$144^3 \times 48$	358	0.2843	1.14	0.817	1.10
7.793	$192^3 \times 64$	242	0.2862	1.15	0.813	1.09
7.192	$96^3 \times 28$	232	0.3195	1.28	0.933	1.25
7.544	$144^3 \times 42$	417	0.3249	1.31	0.934	1.25
7.793	$192^3 \times 56$	273	0.3271	1.31	0.929	1.25

use perturbation theory at large ω and fit a polynomial at small ω to extract the spectral function



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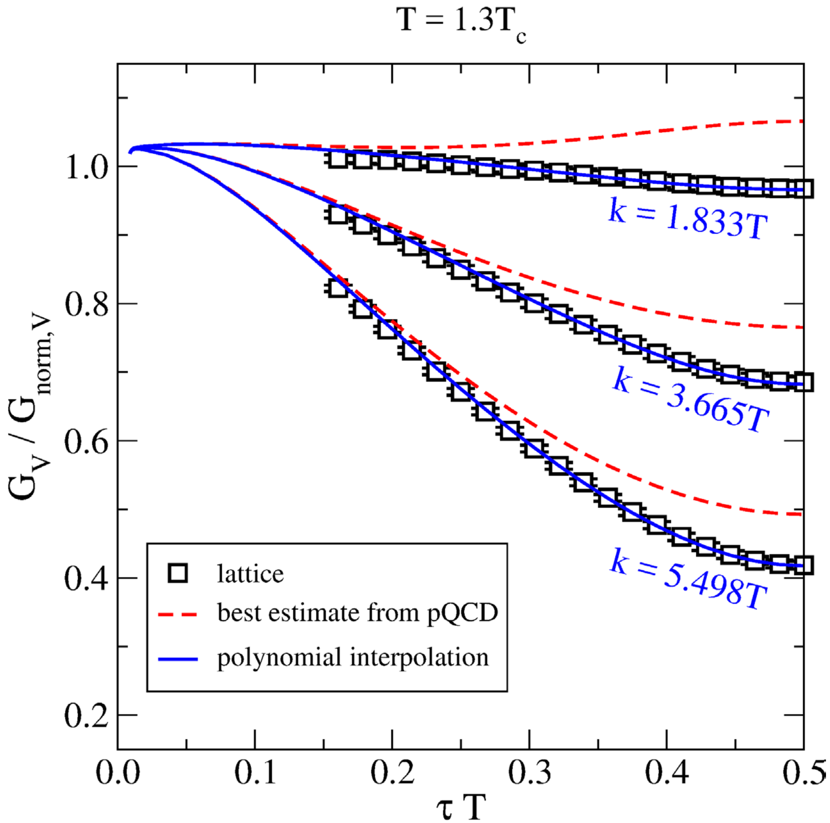
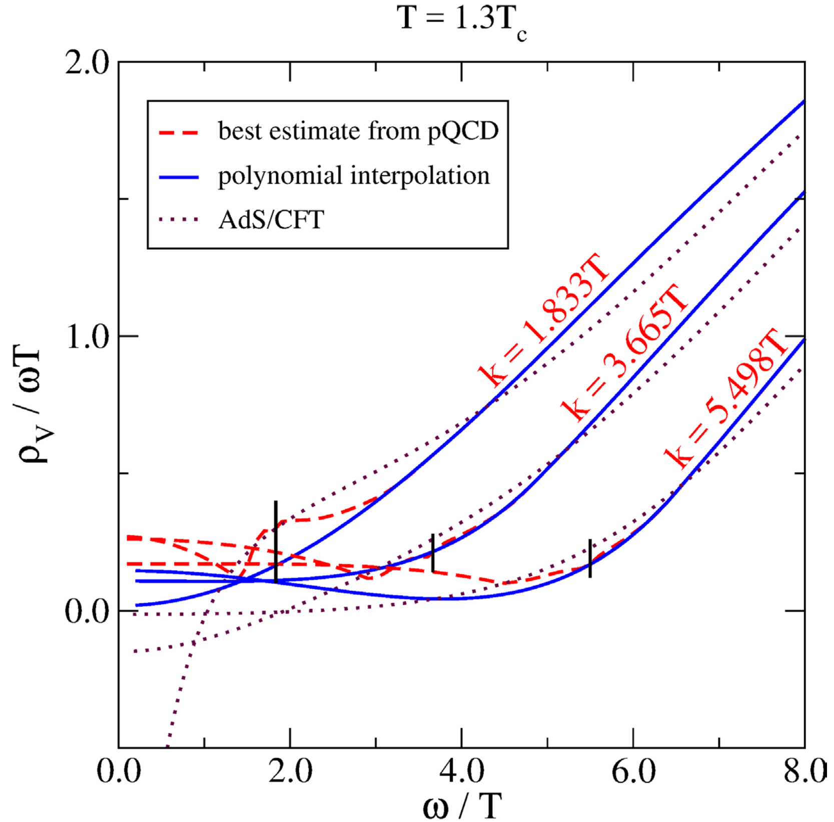




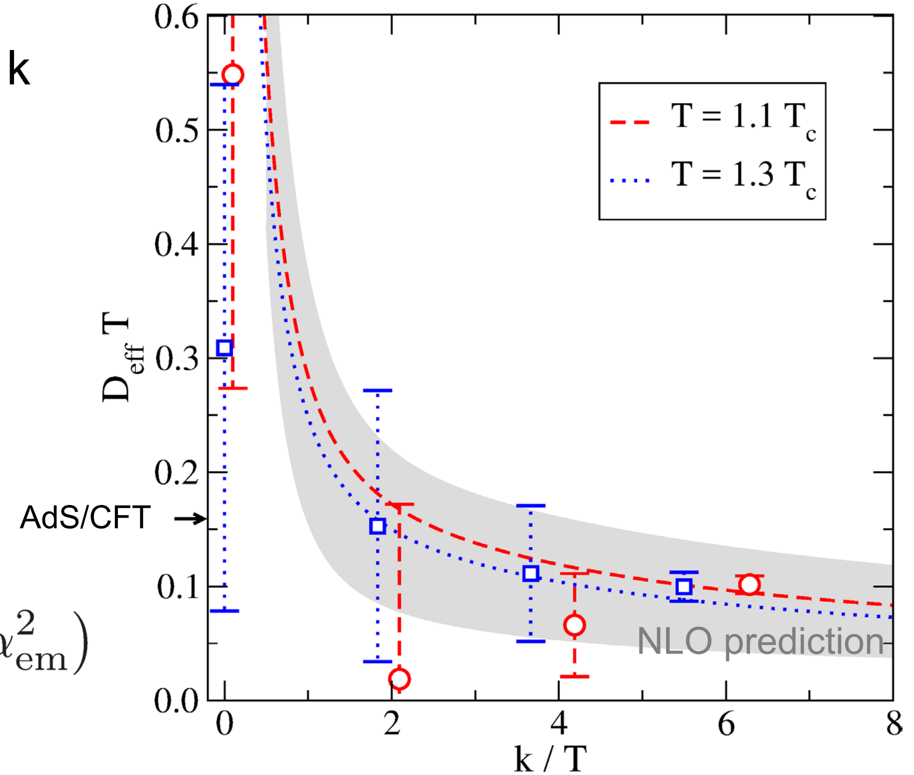
Lattice constraints on photon rates

The spectral function at the photon point ω = k

$$D_{\rm eff}(k) \equiv \begin{cases} \dfrac{\rho_{\rm V}(k,\mathbf{k})}{2\chi_{\rm q}k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_{\rm q}\omega} & , \quad k=0 \end{cases} .$$

can be used to calculate the photon rate

$$\frac{d\Gamma_\gamma(\mathbf{k})}{d^3\mathbf{k}} = \frac{2\alpha_{\rm em}\chi_{\rm q}}{3\pi^2} n_{\rm B}(k) D_{\rm eff}(k) + \mathcal{O}\left(\alpha_{\rm em}^2\right)$$



becomes more perturbative at larger k, approaching the NLO prediction (valid for k>>gT)

[J. Ghiglieri, G.D. Moore, JHEP12 (2014) 029]

but non-perturbative for k/T < 3

AdS/CFT: $DT = \dfrac{1}{2\pi}$

[G. Policastro, D.T. Son and A.O. Starinets, JHEP09 (2002) 043]

LO perturbation theory using lattice value for χ_q/T^2 : $DT = 2.9 - 3.1$

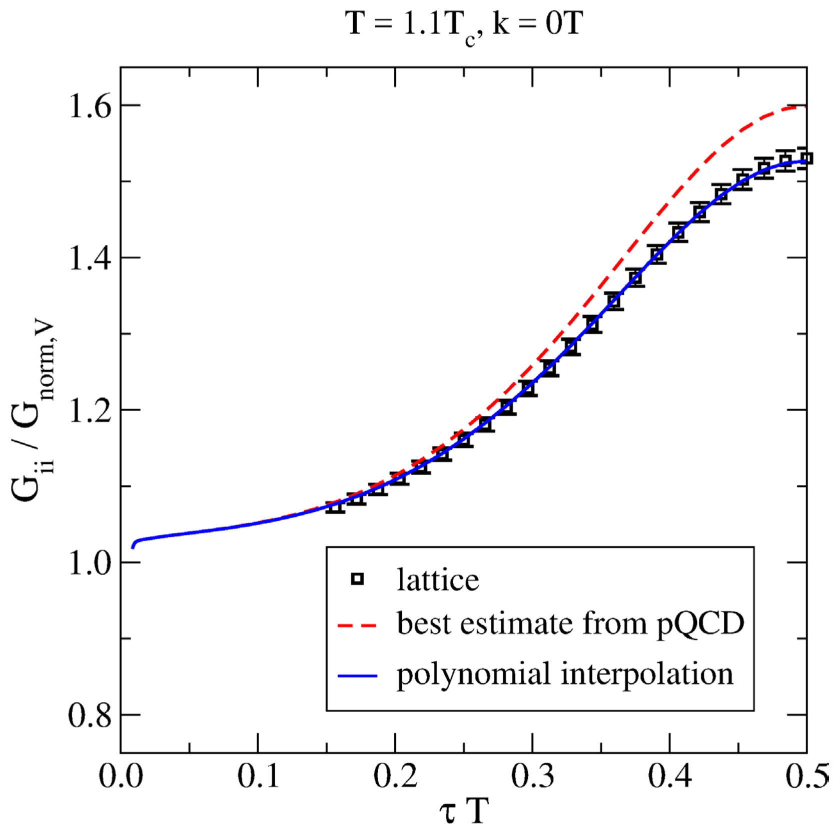
[Arnold, Moore Yaffe, JHEP 05 (2003)]

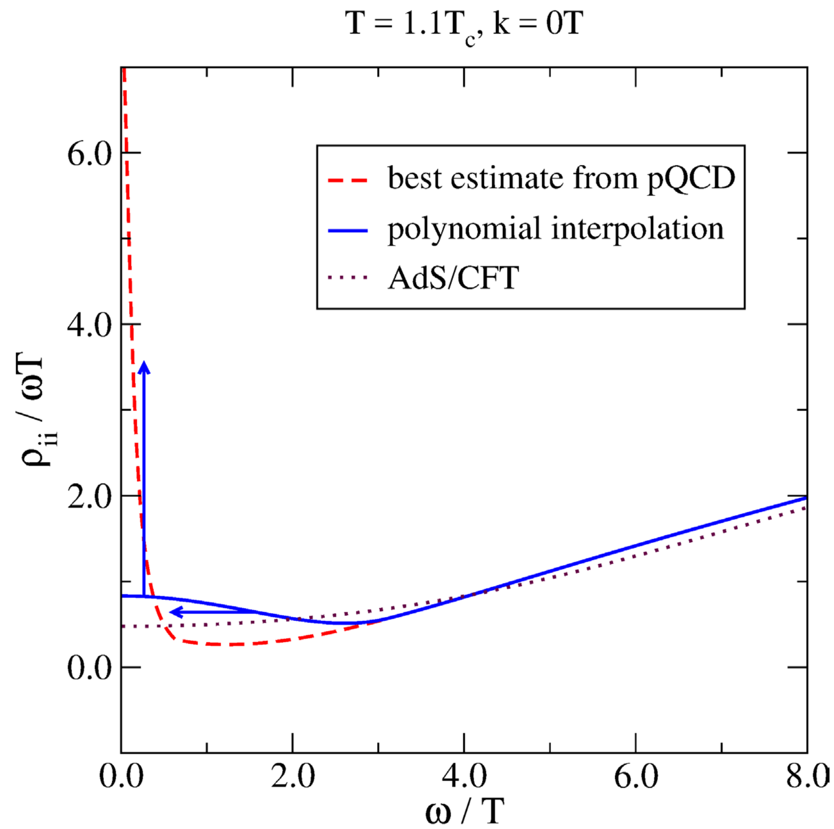
Cross-check with our k=0 calculation

At k=0 the narrowness of the transport peak poses a formidable challenge

The analysis using the polynomial Ansatz and pQCD shows large uncertainties

but gives consistent results with our previous estimates at k=0





→ measurements at non-zero momenta may offer an alternative way to estimate the diffusion coefficient or other tranport coefficients, e.g. shear and bulk viscosity

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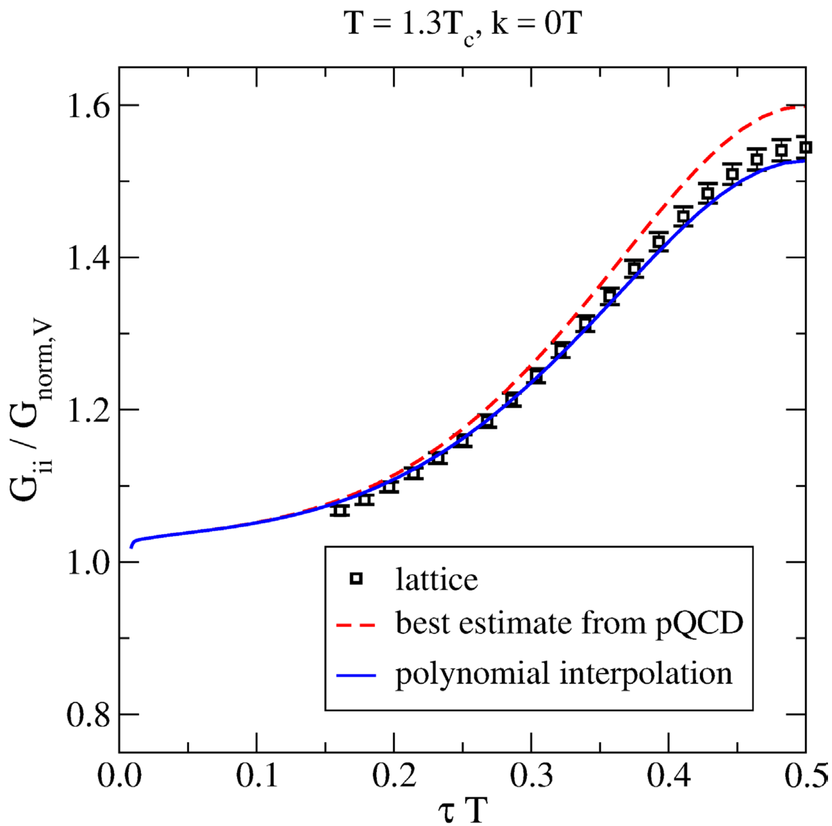

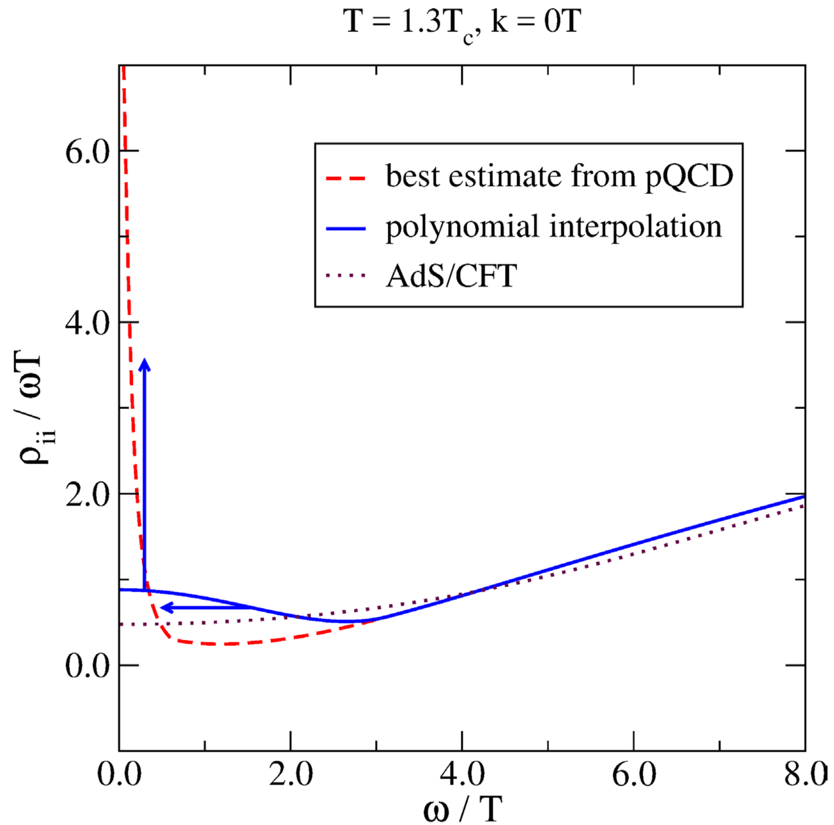

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Conclusions and Outlook

using continuum extrapolated correlation functions from Lattice QCD and

using phenomenologically inspired and perturbatively improved Ansätze

allows to extracted transport properties and spectral properties

These results for the

→ Electrical conductivity / Flavor diffusion coefficients

→ Thermal dilepton rates

→ Thermal photon rates

should be included in hydro models for the evolution of the medium

all parameters and pQCD spectral functions are available from
[arXiv:1604.06712, arXiv:1604.07544, www.laine.itp.unibe.ch/dilepton-lattice]

The methodology developed in this studies within the quenched approximation

shall be extended to full QCD calculations for a realistic QGP medium

as close to T_c dynamical fermion degrees of freedom will become important