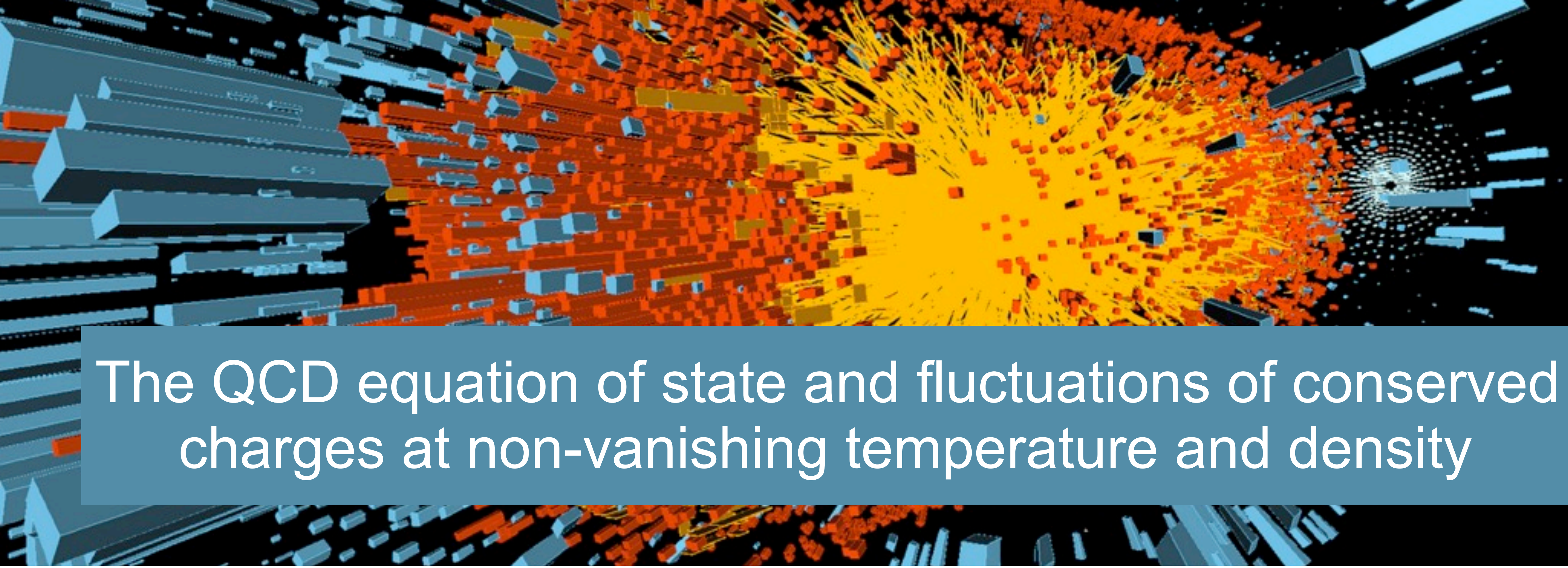

The QCD equation of state and fluctuations of conserved charges at non-vanishing temperature and density

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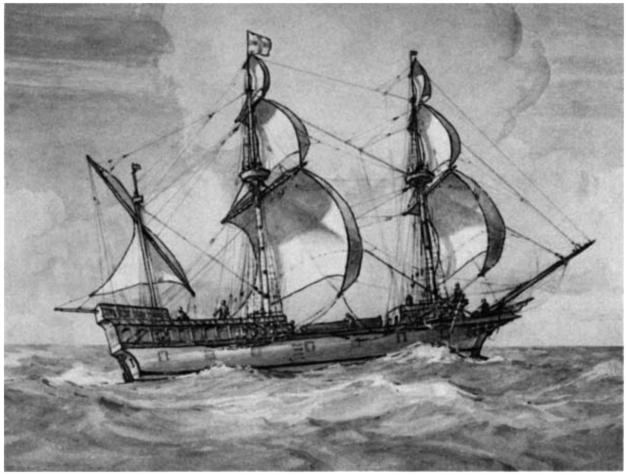
A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, S. Mukherjee, P. Petreczky, H. Sandmeyer, C. Schmidt, W. Soeldner, P. Steinbrecher, H. Ohno

Sir Francis Drake

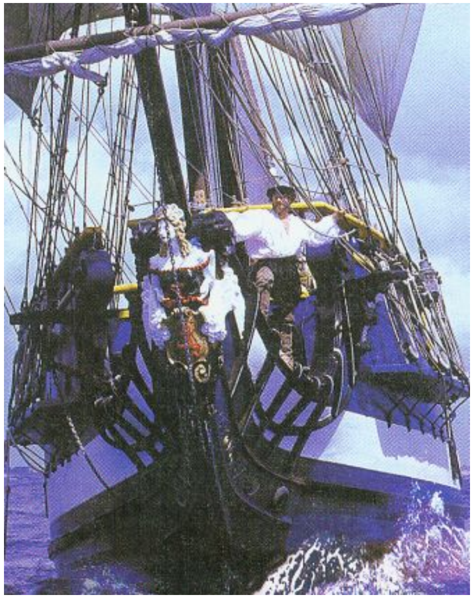
buccanneer (QE licensed pirate)



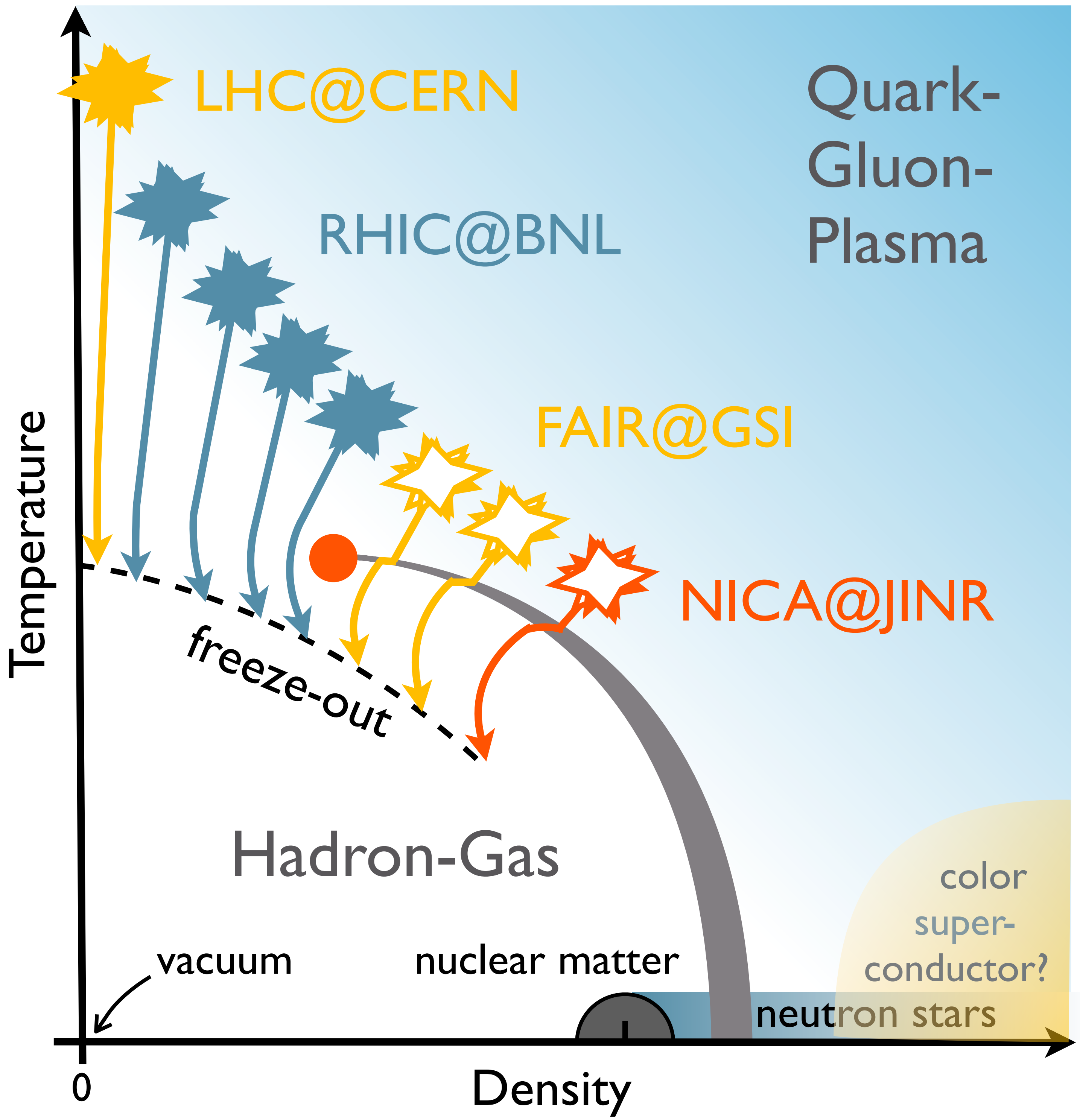
World
circum navigator

Admiral

with Sir Francis
to new shores



Motivation: The QCD phase diagram

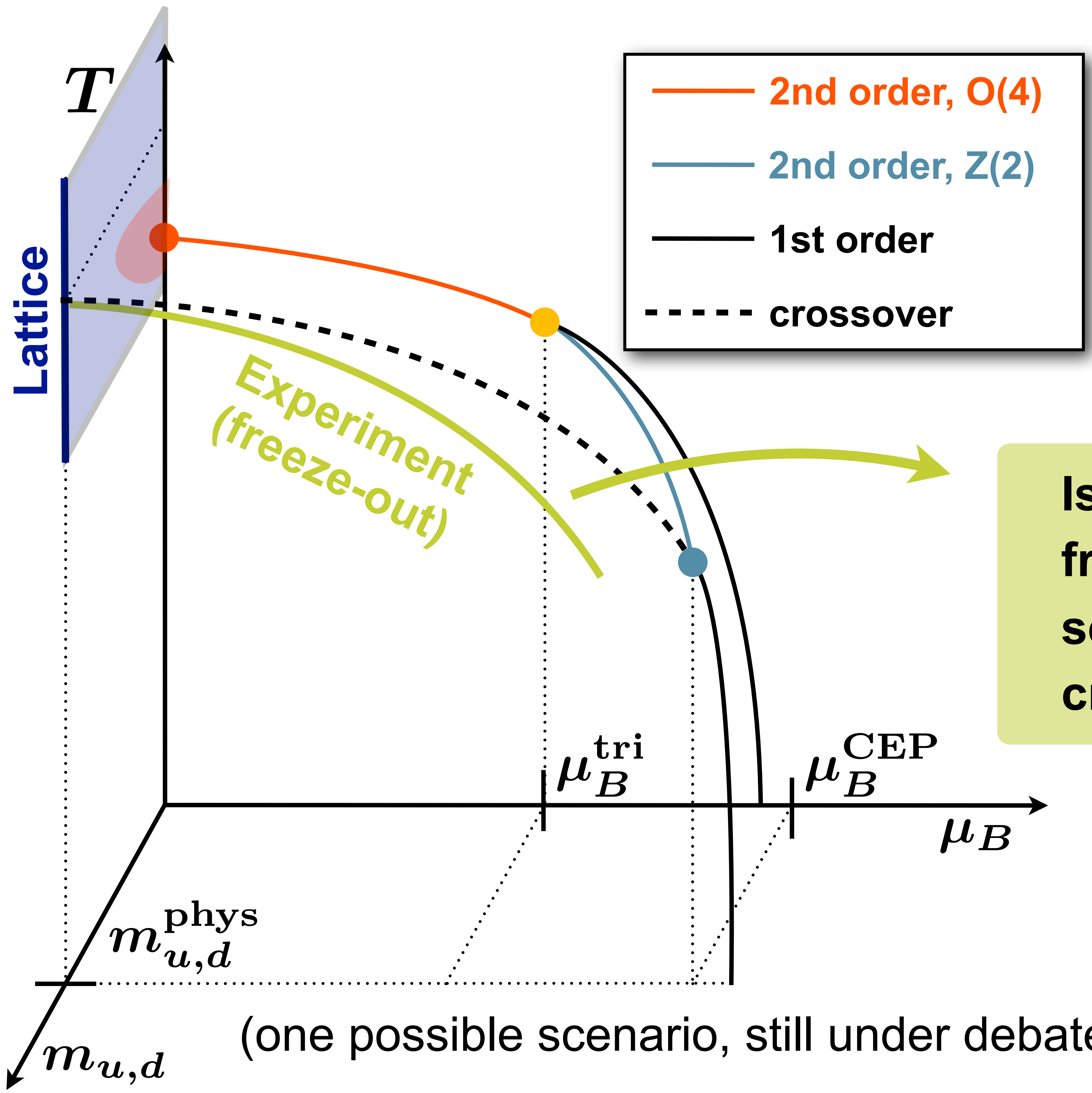


- The QCD phase diagram is extensively studied by heavy ion collisions (HIC)
- The QCD equation of state (EoS) is essential input to hydrodynamic modeling of HIC
- Hadronic abundances and fluctuations are measured at freeze-out

Lattice QCD can provide both, the EoS and fluctuations of conserved charges at small μ_B/T

Quark mass dependance of the phase diagram:

Not just one critical point!

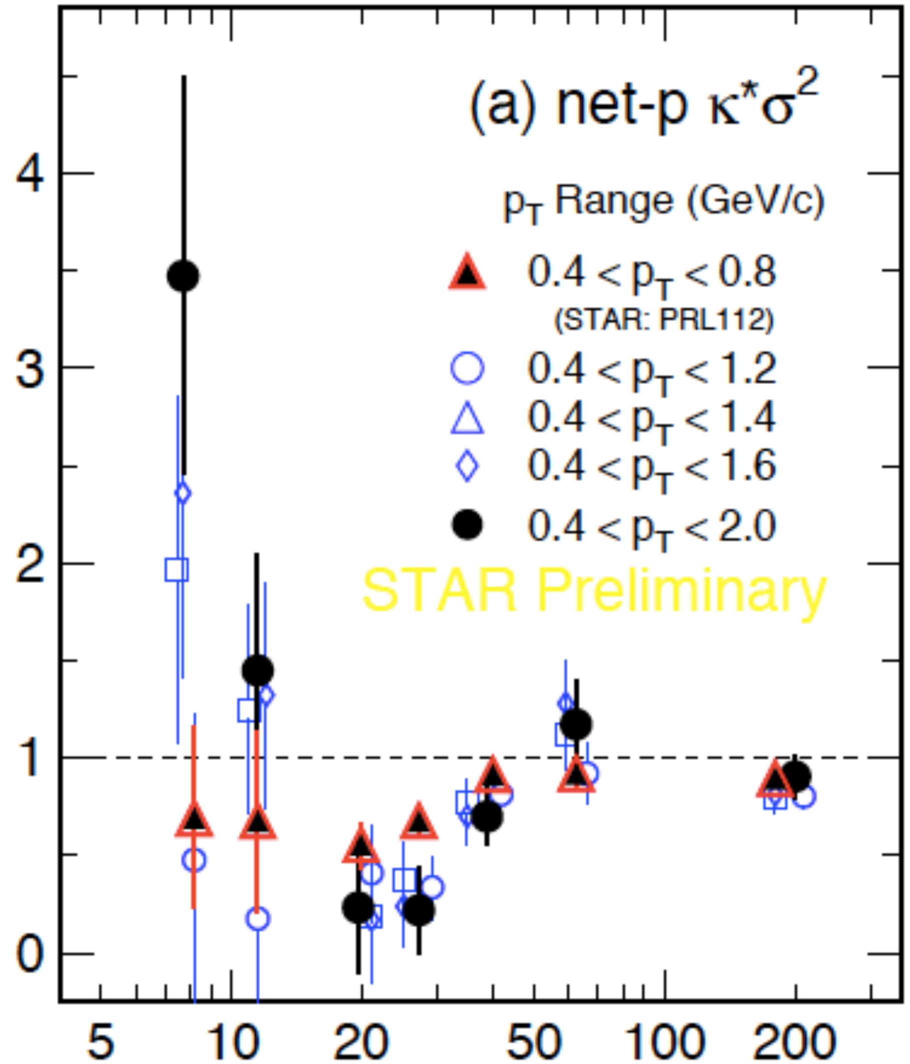


Is physics on the freeze-out line sensitive to QCD critical behavior?

(one possible scenario, still under debate)

Beam Energy Scan at RHIC

$$\kappa\sigma^2 = \chi_4/\chi_2$$



X. Luo, CPOD'14

$\Rightarrow$ intriguing non-monotonic behavior in the cumulant ratio of net-proton number fluctuations

Can this data be understood in terms of equilibrium thermodynamics?

Beam Energy Scan at RHIC

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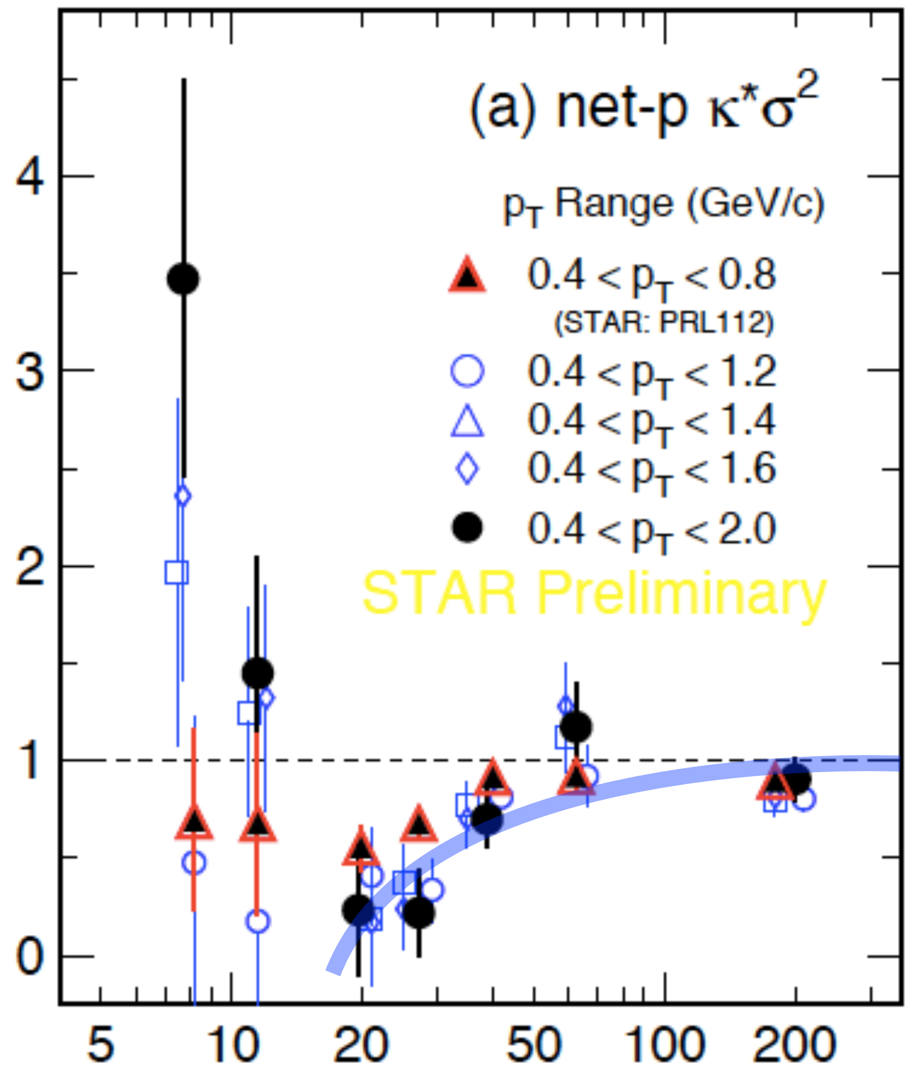


X. Luo, CPOD'14

$\Rightarrow$ intriguing non-monotonic behavior in the cumulant ratio of net-proton number fluctuations

Can this data be understood in terms of equilibrium thermodynamics?

How far do we get with a low order Taylor expansion?

Big obstacle on the lattice: the sign problem

- The QCD partition function

$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \exp\{-S_F(U,\psi,\bar{\psi}) - \beta S_G(U)\}$$

$$= \int \mathcal{D}U\ \det[M](U,\mu)\exp\{-\beta S_G(U)\}$$

complex for $\mu > 0$

can not be interpreted as probability

we find: $[\det M(\mu)]^* = \det M(-\mu^*)$

$\longrightarrow$ determinant is real only for

$\mu = 0$ or $\mu = i\mu_I$

Here we follow the Taylor expansion approach, all quantities are expanded in μ/T, at fixed T.

Content

1) **Introduction and Motivation**

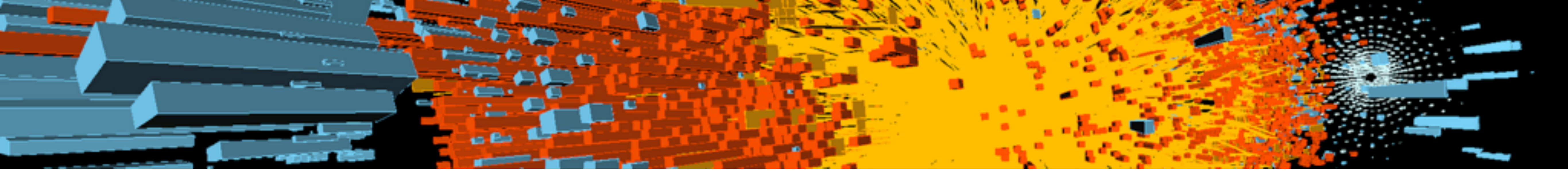
2) **Taylor expansion of the equation of state**

- definitions, state-of-the-art, convergence estimate
- constraints: strangeness neutrality, constant baryon number to electric charge ratio

2) **Cumulant ratios at nonzero baryon number density**

- expressing μ_B/T by M_B/σ_B^2
- RHIC data vs. QCD equilibrium thermodynamics

3) **Conclusions and Summary**



Taylor expansion of the pressure

Conserved charge fluctuations

Expansion of the pressure:

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk,0}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$X = B, Q, S$: conserved charges

Lattice

$$\chi_n^X = \left.\frac{\partial^n [p/T^4]}{\partial(\mu_X/T)^n}\right|_{\mu_X=0}$$

generalized susceptibilities

$\Rightarrow$ only at $\mu_X = 0$!

Experiment

$$\begin{aligned} VT^3\chi_2^X &= \left\langle(\delta N_X)^2\right\rangle \\ VT^3\chi_4^X &= \left\langle(\delta N_X)^4\right\rangle - 3\left\langle(\delta N_X)^2\right\rangle^2 \\ VT^3\chi_6^X &= \left\langle(\delta N_X)^4\right\rangle \\ &\quad -15\left\langle(\delta N_X)^4\right\rangle\left\langle(\delta N_X)^2\right\rangle \\ &\quad +30\left\langle(\delta N_X)^2\right\rangle^3 \end{aligned}$$

cumulants of net-charge fluctuations

$$\delta N_X \equiv N_X - \langle N_X \rangle$$

$\Rightarrow$ only at freeze-out $(\mu_f(\sqrt{s}), T_f(\sqrt{s}))$!

Conserved charge fluctuations

Expansion of the pressure:

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk,0}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$X = B, Q, S$: conserved charges

consider cumulant ratios to eliminate the freeze-out volume

Lattice		**Experiment**
$\frac{\chi_1^X(\mu_B,T)}{\chi_2^X(\mu_B,T)}$	$=$	$\frac{M_X}{\sigma_X^2}$
$\frac{\chi_3^X(\mu_B,T)}{\chi_2^X(\mu_B,T)}$	$=$	$S_X\sigma_X$
$\frac{\chi_4^X(\mu_B,T)}{\chi_2^X(\mu_B,T)}$	$=$	$\kappa_X\sigma_X^2$

$M :=$ mean
$\sigma^2 :=$ variance
$S :=$ skewness
$\kappa :=$ kurtosis

comparison with

HRG = Hadron Resonance Gas model: quite successful in describing hadron yields in HIC

$$\frac{p^{HRG}}{T^4} = \sum_{mesons} \ln Z_i^M(T, \mu_Q, \mu_S) + \sum_{baryons} \ln Z_i^B(T, \mu_B, \mu_Q, \mu_S)$$

$$\ln Z_i^{M/B} = \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} K_2(k\, m_i/T)\, \cosh(k(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S))$$

$$\simeq \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 K_2(m_i/T)\, \cosh(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S)$$

Boltzmann approximation: $k = 1$; recall $K_2(x) \sim e^{-x}$ $\to$ good except for pions

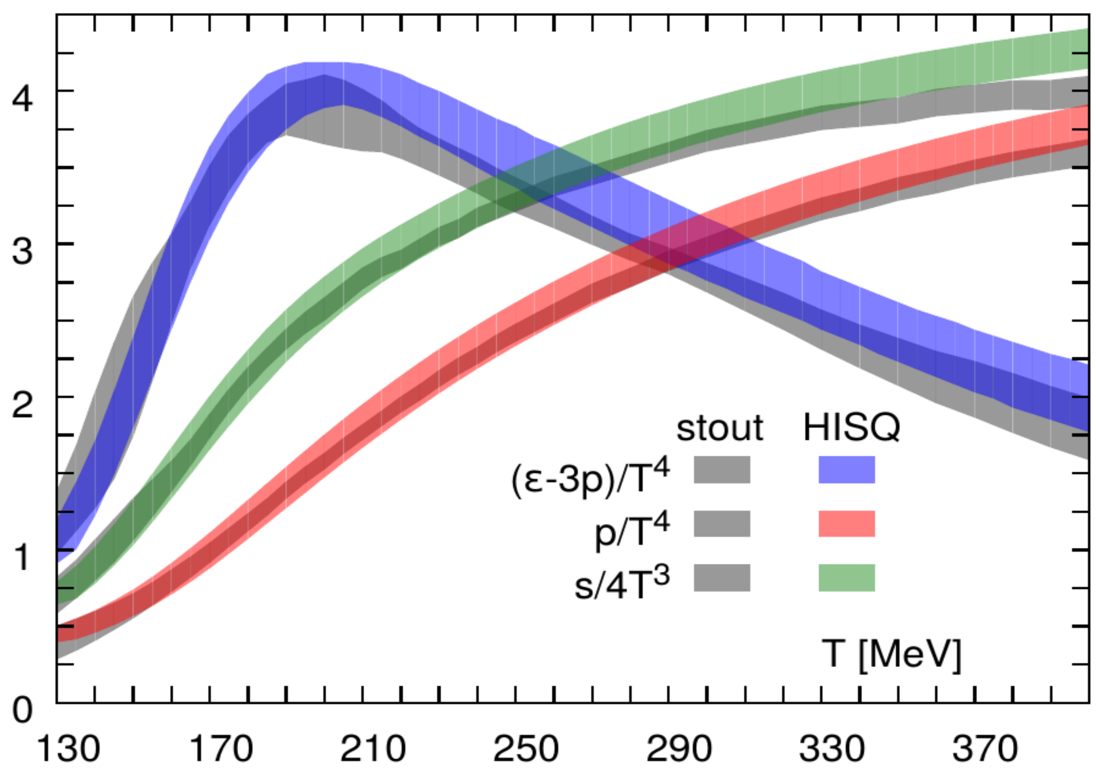
$\Rightarrow$ fluctuations as e.g.

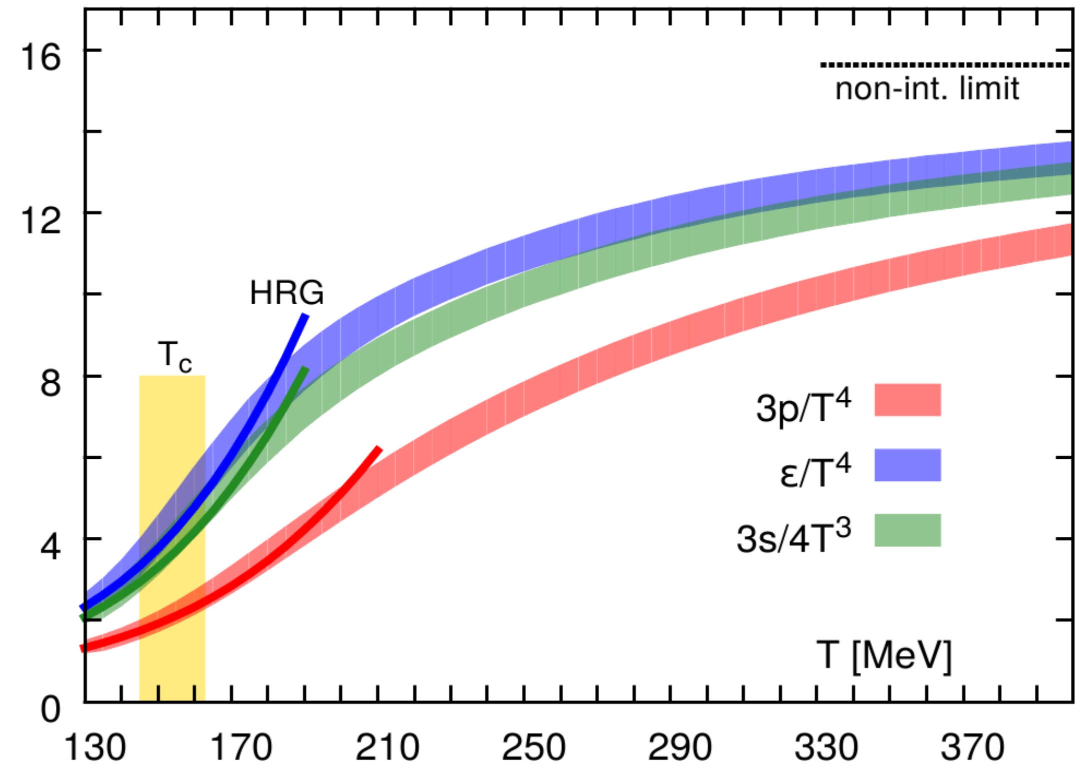
$$\chi_n^B|_{\vec{\mu}=0} \sim \sum_{baryons\, i} B_i^n\, K_2(m_i/T) \quad \simeq \quad K_2(m_i/T)$$

State-of-the-art equation of state for (2+1)-flavor

pressure p, energy density ε and entropy density s, at $\mu_B = \mu_Q = \mu_S = 0$:

Bazavov et al. [HotQCD], Phys. Rev. D90 (2014) 094503.





- improves over earlier HotQCD calculation Bazavov et al. [HotQCD], Phys. Rev. D80 (2009) 014504.
- consistent with results from Budapest-Wuppertal (stout) S. Borsanyi et al. [WB] Phys. Lett. B730 (2014) 99

- up to the crossover region the QCD EoS agrees well with the HRG EoS, however, QCD results are systematically above HRG

$\Rightarrow$ evidence for additional hadronic states?

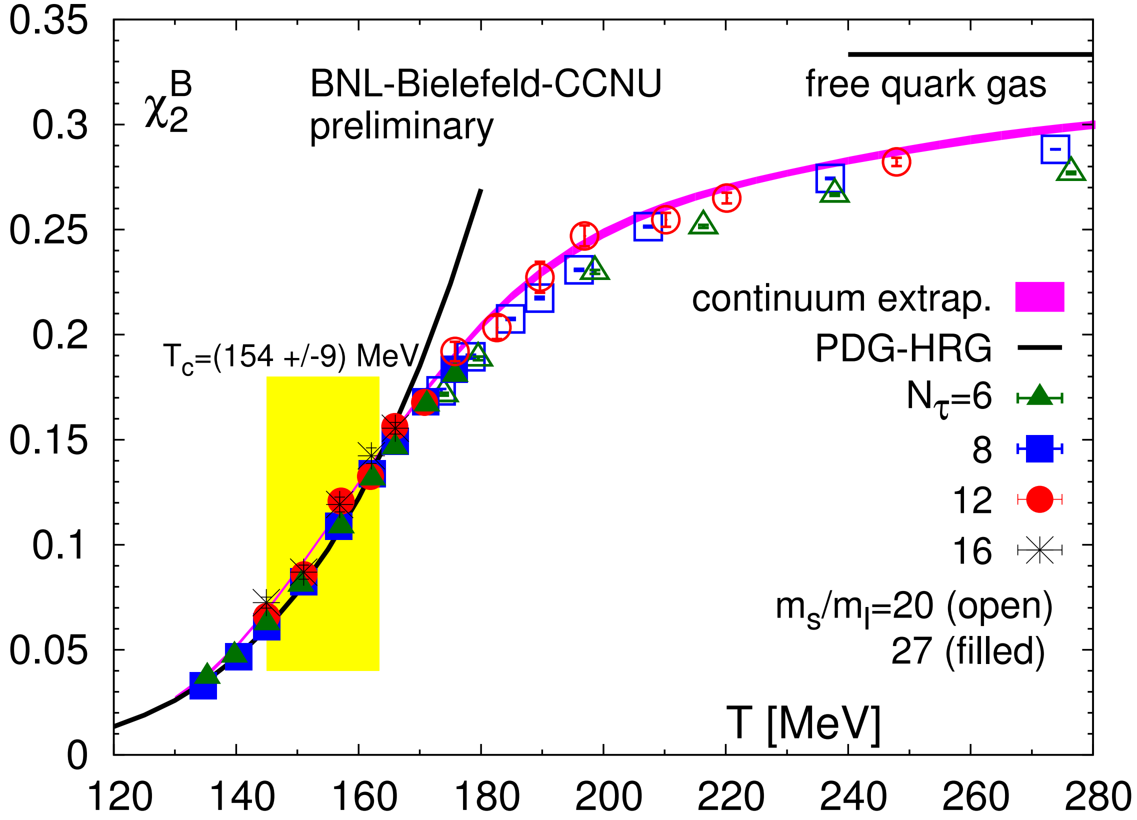
The equation of state at $\mu_B > 0$

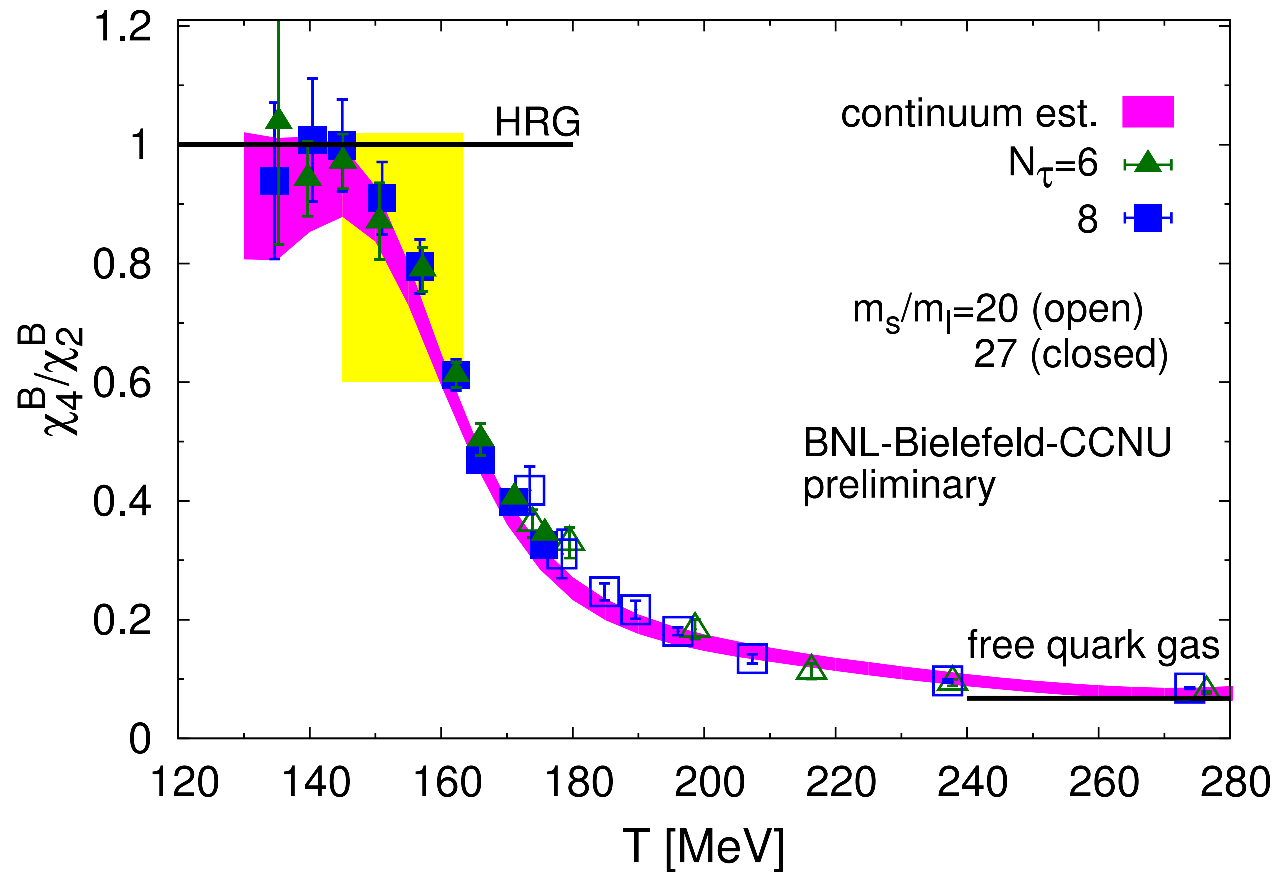
pressure correction due to non-vanishing baryon chemical potential:

$$\frac{\Delta p(T,\mu_B)}{T^4} = \frac{p(T,\mu_B) - p(T,0)}{T^4} = \frac{1}{2}\chi_2^B \left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2}\left(\frac{\mu_B}{T}\right)^2 + \cdots\right)$$

LO: variance of baryon number fluctuation

NLO: kurtosis x variance





The equation of state at $\mu_B > 0$

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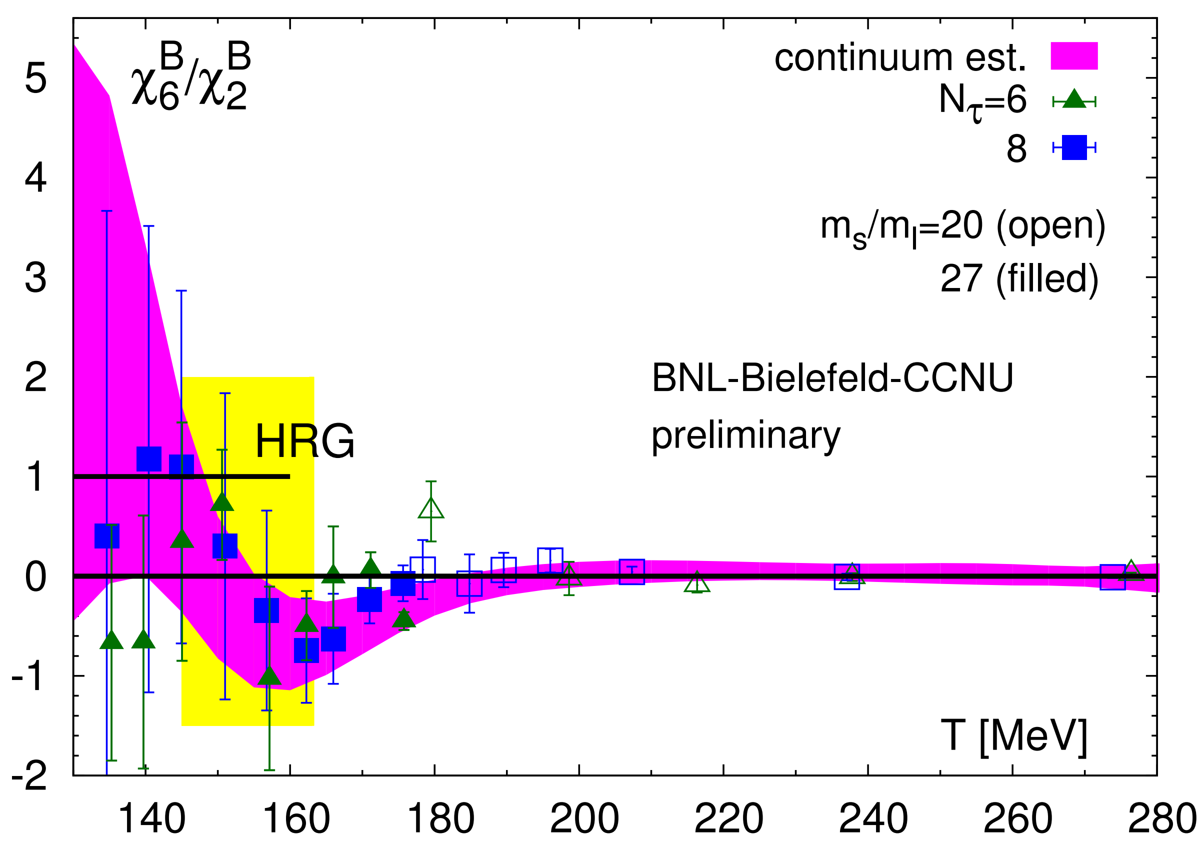
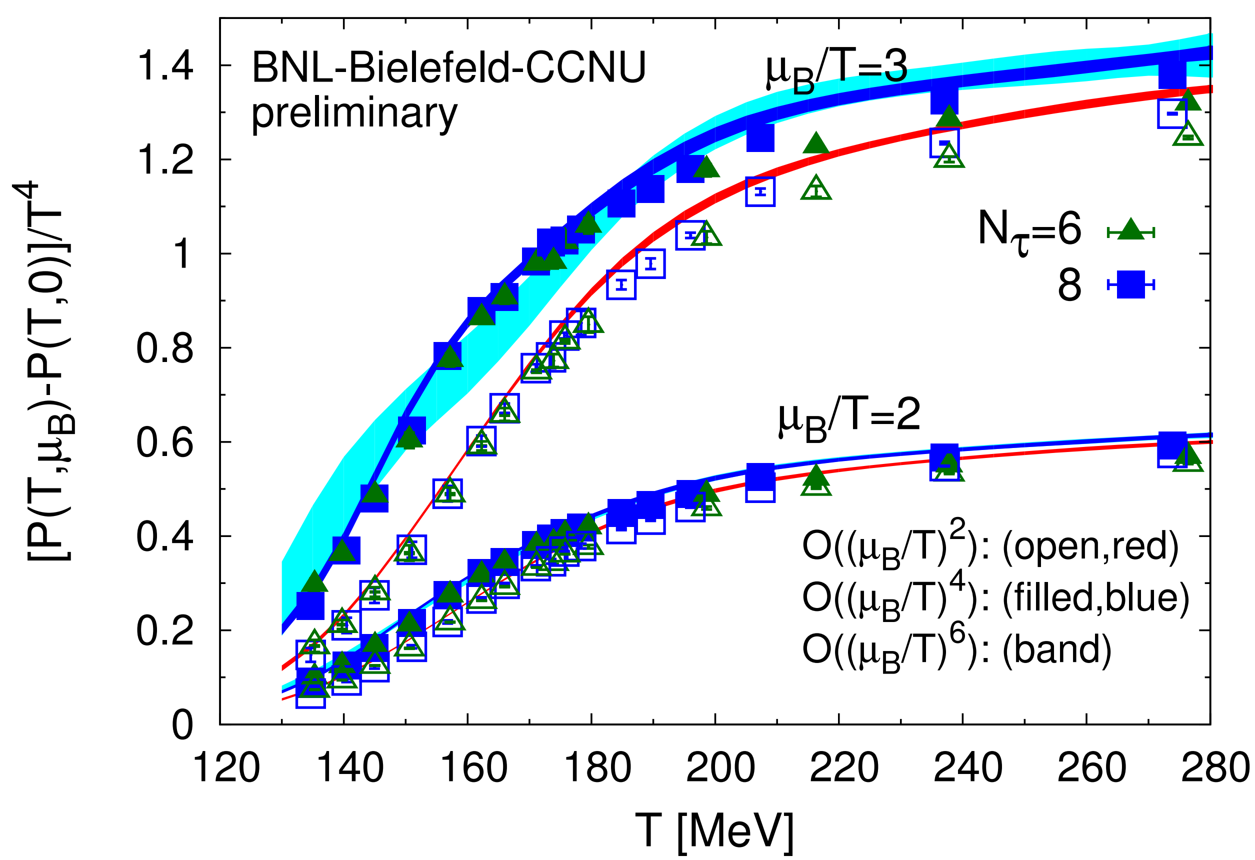
$$\frac{\Delta p(T,\mu_B)}{T^4} = \frac{1}{2}\chi_2^B \left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2}\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{360}\frac{\chi_6^B}{\chi_2}\left(\frac{\mu_B}{T}\right)^4 + \cdots\right)$$

NNLO

total pressure corrections:





$\Rightarrow$ pressure is well controlled for $\mu_B/T \leq 2$ or equivalently $\sqrt{s_{NN}} \geq 12$ GeV

Introducing $\mu_S > 0$ and $\mu_Q > 0$

Apply: initial conditions as in HIC

- **strangeness neutrality:** $\langle N_S \rangle = 0$
- **isospin assymetry:** $\langle N_Q \rangle = r \langle N_B \rangle$

$r \approx 0.4$
for Au-Au
and Pb-Pb

expand in powers of μ_B, μ_Q, μ_S
solve for μ_Q, μ_S

$$\mu_Q(T,\mu_B) = q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3$$
$$\mu_S(T,\mu_B) = s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3$$

LO NLO

$\hat{\mu}_B = \mu_B/T$

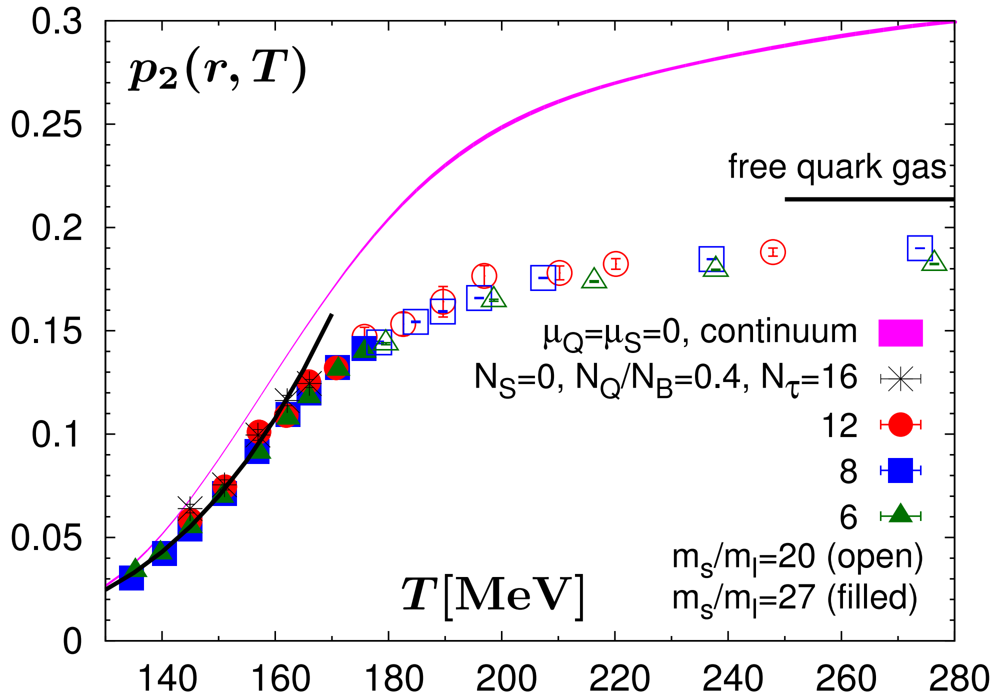
define strangeness neutral
coefficients p_n

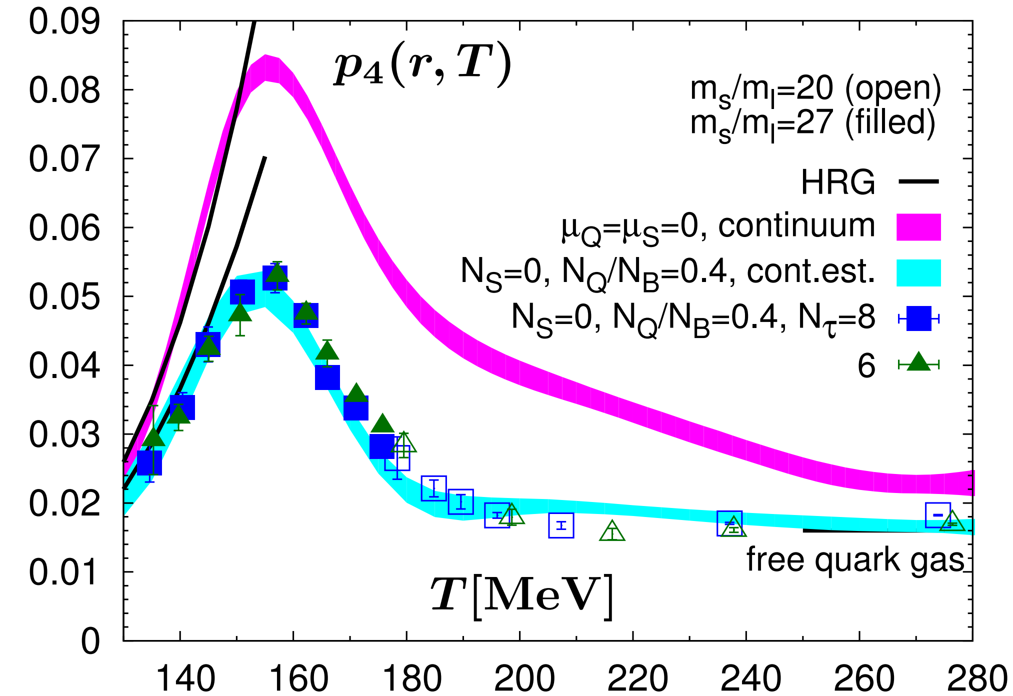
$$\frac{\Delta p}{T^4} = \frac{1}{2}\chi_2^B\hat{\mu}_B^2 + \frac{1}{2}\chi_2^Q\hat{\mu}_Q^2 + \frac{1}{2}\chi_2^S\hat{\mu}_S^2 + \chi_{11}^{BQ}\hat{\mu}_B\hat{\mu}_Q + \chi_{11}^{BS}\hat{\mu}_B\hat{\mu}_S + \chi_{11}^{QS}\hat{\mu}_Q\hat{\mu}_S + \cdots$$
$$= \underbrace{\frac{1}{2}\left(\chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2 + 2\chi_{11}^{BQ}q_1 + 2\chi_{11}^{BS}s_1 + 2\chi_{11}^{QS}q_1 s_1\right)}_{p_2}\hat{\mu}_B^2 + \cdots$$

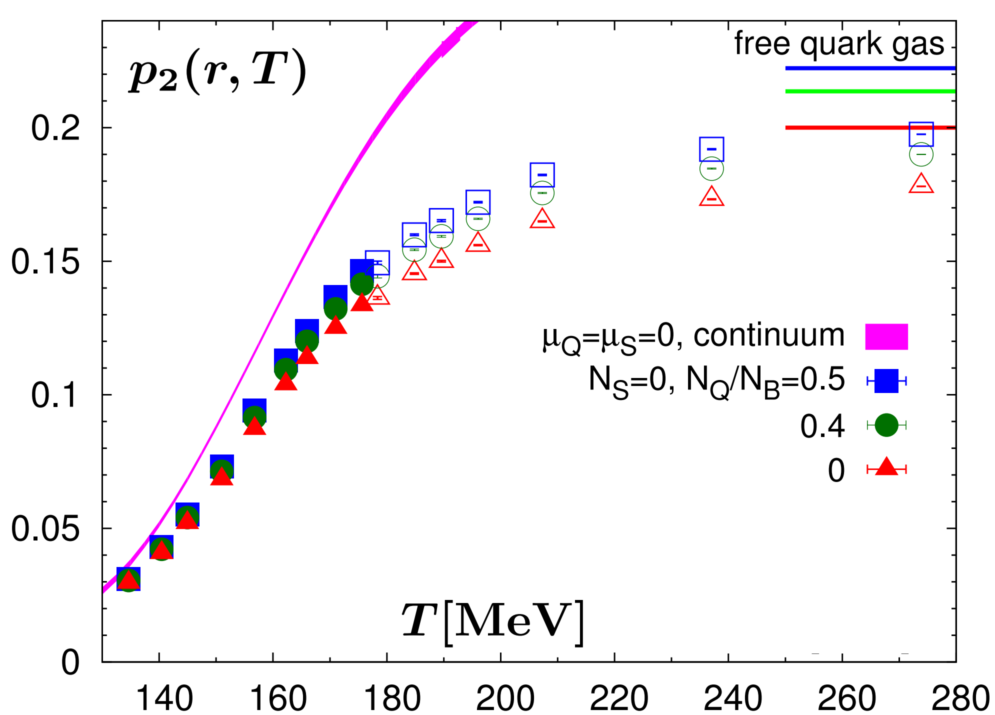
Introducing $\mu_S > 0$ and $\mu_Q > 0$

strangeness neutral pressure coefficients:

$$\frac{\Delta p}{T^4} = \frac{1}{2} p_2 \left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24} p_4 \left(\frac{\mu_B}{T}\right)^4 + \cdots$$





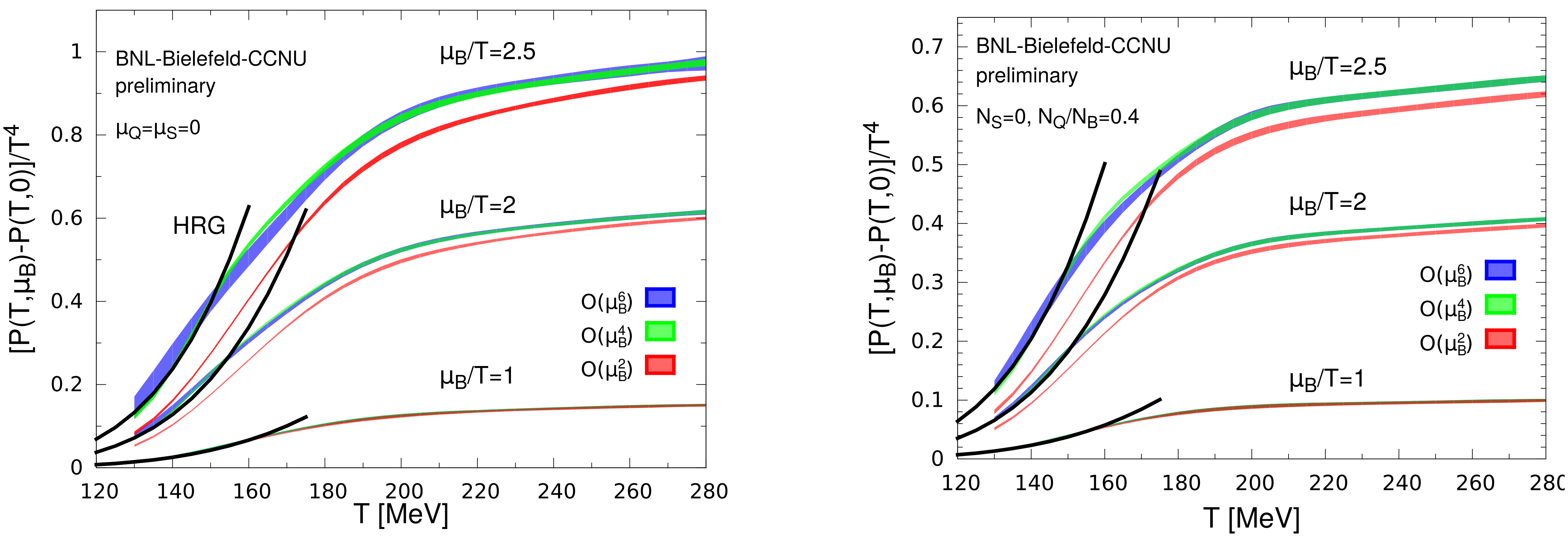


- considerable strangeness dependence
- mild isospin dependence

Introducing $\mu_S > 0$ and $\mu_Q > 0$

total pressure corrections (comparison):

$\mu_Q = \mu_S = 0$ $\qquad$ $N_S = 0,\ N_Q/N_B = 0.4$



- note scale difference

Cumulant ratios at $\mu_B > 0$

Conserved charge fluctutations and freeze-out

expanding ratios of baryon number fluctuations:

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{2}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

- in QCD all the ratios χ_{n+2}^B/χ_n^B are a functions of temperature, in HRG they are unity

current simplifications:

- on the left: $\mu_Q = \mu_S = 0$
- approximate freeze-out line by a constant

$$T^f(\mu_B) = T^f(0)\left(1 - \kappa_f\left(\mu_B/T\right)^2\right)$$

$$\kappa_f \approx 0$$

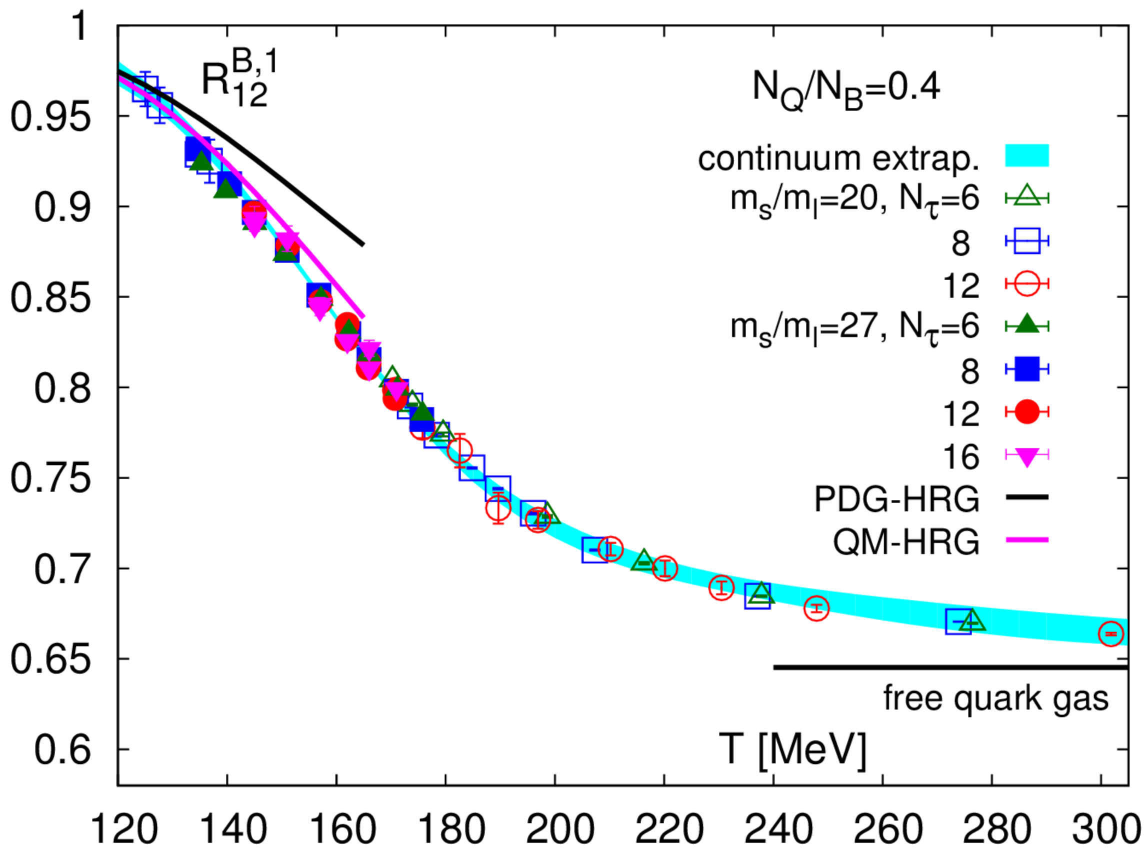
Conserved charge fluctutations and freeze-out

Aim for a comparison with RHIC data:

how to translate $\sqrt{s_{NN}}$ into μ_B without making further approximations?

→ trick: express all ratios as function of $M_B/\sigma_B^2 = \chi_1^B/\chi_2^B = R_{12}^B$

- caution: RHIC measures net-proton and not net-baryon number fluctuations
- may use HRG motivated conversion factor:



HRG:

$$R_{12}^P = \tanh(\hat{\mu}_B + \hat{\mu}_Q) = \hat{\mu}_B + \mathcal{O}(\hat{\mu}_B^3)$$

no $\hat{\mu}_S$-dependence, neglect $\hat{\mu}_Q$-dependence

QCD:

$$R_{12}^B = R_{12}^{B,1}\hat{\mu}_B + \mathcal{O}(\hat{\mu}_B^3)$$

(strangeness neutral, r=0.4)

⟹ to leading order: $R_{12}^B/R_{12}^P = R_{12}^{B,1}$

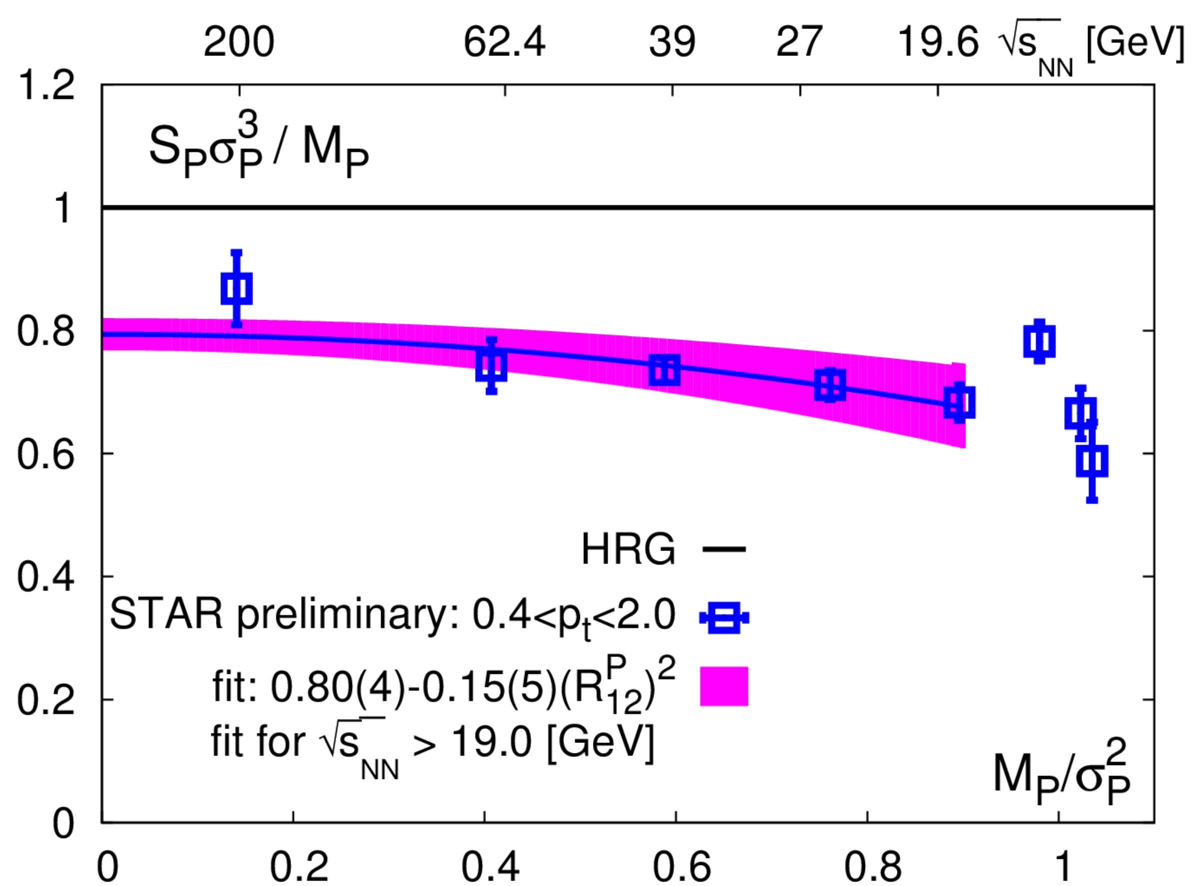
A. Bazavov et al., PRD 93 (2016) 014512

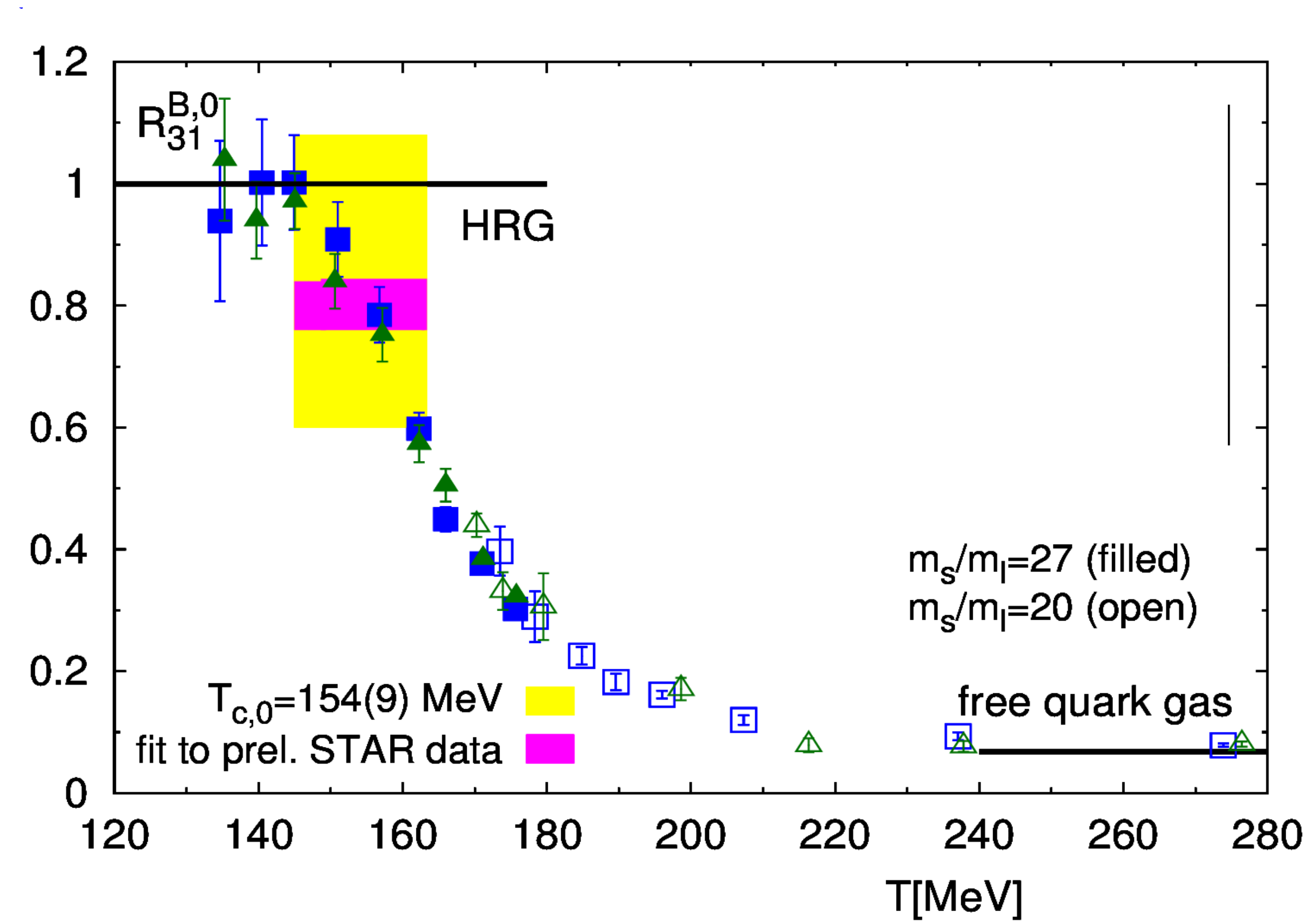
Skewness at $\mu_B > 0$

$$R_{31}^B \equiv \frac{S_B \sigma_B^3}{M_B} = R_{31}^{B,0} + \frac{R_{31}^{B,2}}{\left(R_{12}^{B,1}\right)^2} \left(\frac{M_B}{\sigma_B^2}\right)^2 + \cdots$$

(strangeness neutral, r=0.4)

F. Karsch et al., arXiv:1512.06987





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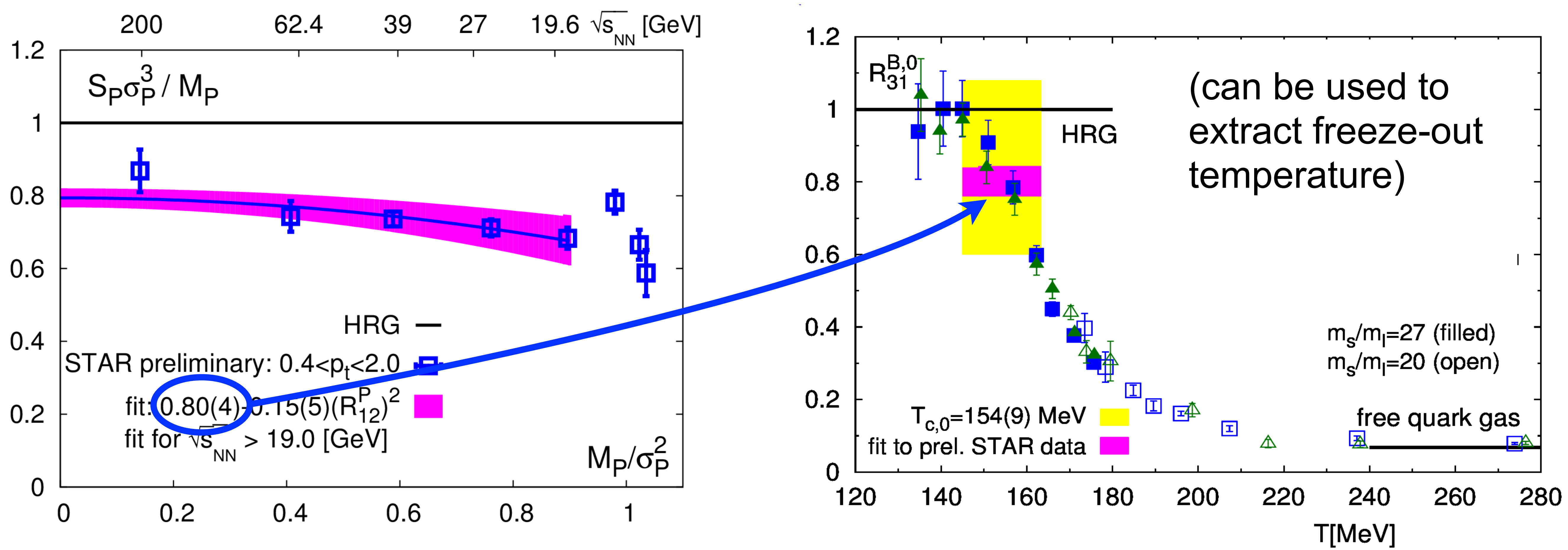


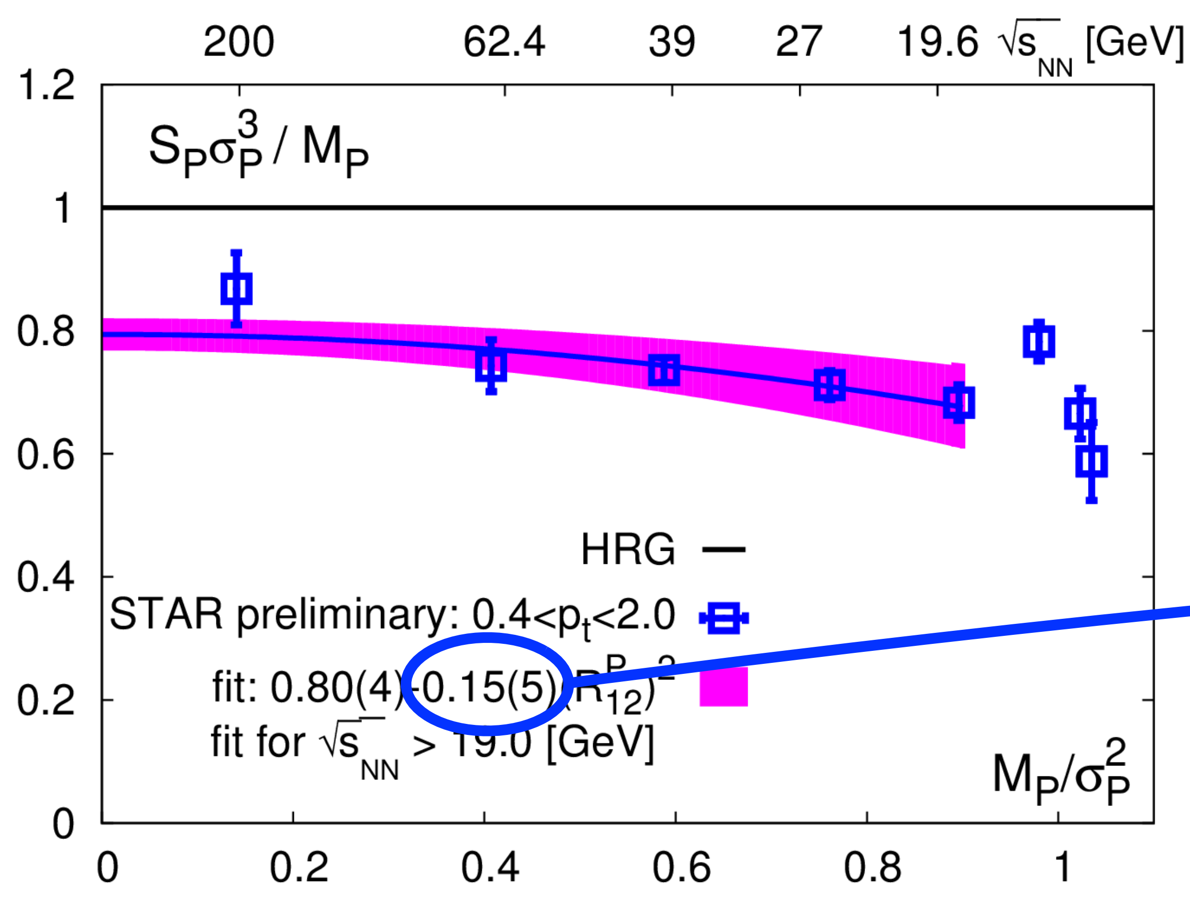
- intercept consistent with QCD result

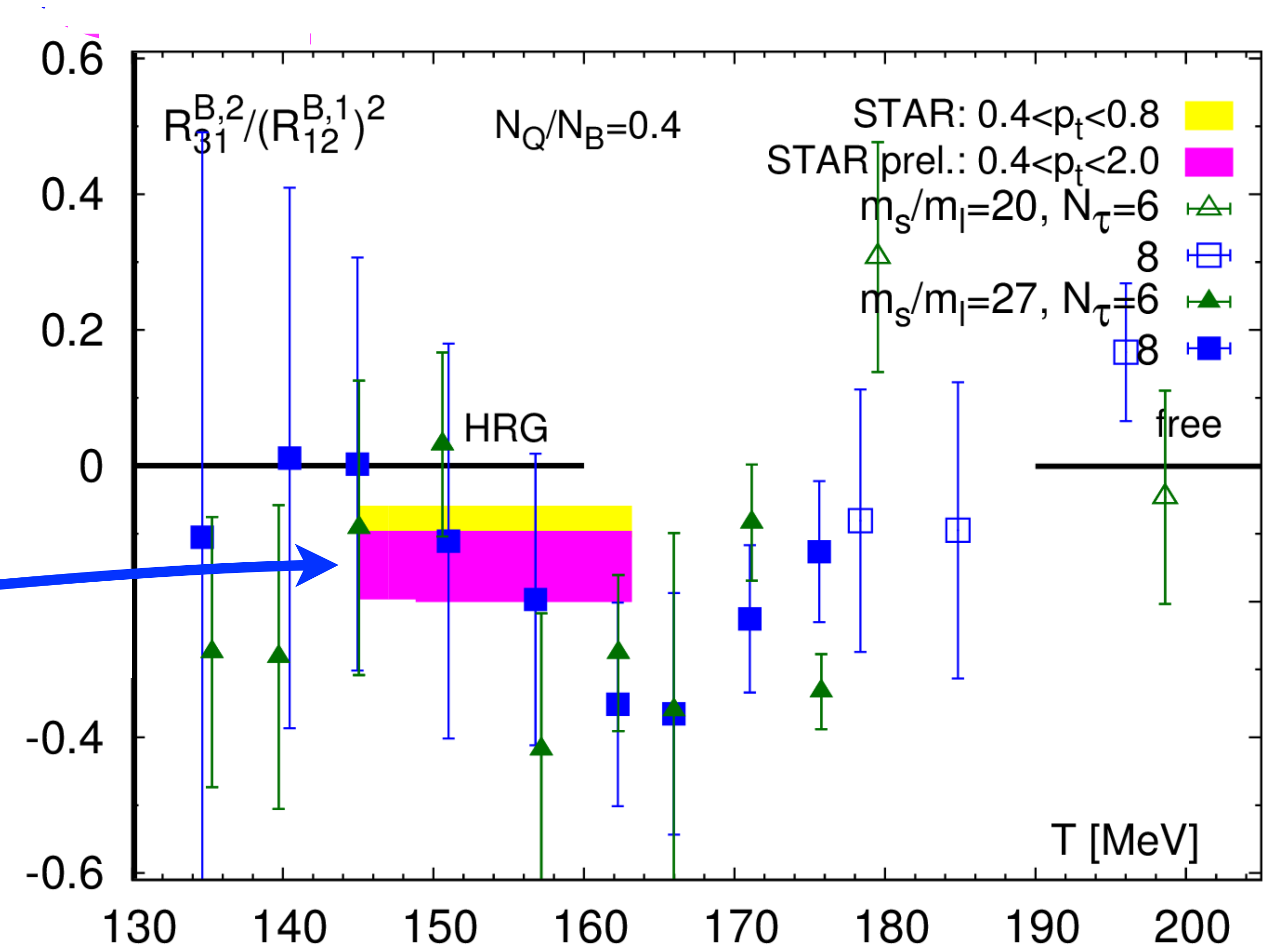
Skewness at $\mu_B > 0$

$$R_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = R_{31}^{B,0} + \frac{R_{31}^{B,2}}{\left(R_{12}^{B,1}\right)^2}\left(\frac{M_B}{\sigma_B^2}\right)^2 + \cdots$$

(strangeness neutral, r=0.4)

F. Karsch et al., arXiv:1512.06987





- intercept consistent with QCD result
- curvature consistent with QCD result (still large statistical error)

Kurtosis at $\mu_B > 0$

$$R_{42}^B \equiv \kappa_B \sigma_B^2 = R_{42}^{B,0} + \frac{R_{42}^{B,2}}{\left(R_{12}^{B,1}\right)^2} \left(\frac{M_B}{\sigma_B^2}\right)^2 + \cdots$$

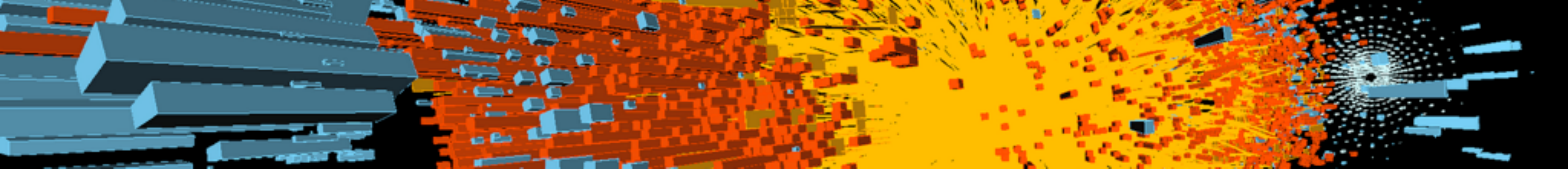
(strangeness neutral, r=0.4)

F. Karsch et al., arXiv:1512.06987

- find similar results for the kurtosis, i.e. intercept and curvature are in agreement with QCD results
- especially we find $\chi_{42}^{B,0} \simeq \chi_{31}^{B,0}$ and $\chi_{42}^{B,2} \simeq 3\chi_{31}^{B,2}$ (exact for $\mu_Q = \mu_S = 0$)

  which is also supported by the RHIC data

- in general: need to understand systematics
  - non-equilibrium effects (S. Mukherjee et al., arXiv:1506.00645)
  - proton vs. baryon number fluctuations (M. Kitazawa et al., arXiv:1205.3292, arXiv:1303.3338)
  - acceptance and pt-cuts (P.Garg et al. arXiv:1304.7133, F. Karsch et al., arXiv:1508.02614, A.Bazdak et al., arXiv:1206.4286)



Conclusions and Summary

Conclusions and Summary

- Cumulants of conserved charge fluctuations can be obtained on the lattice and are measured in heavy ion collision. They can be used to infer freeze-out parameter.

- Results on bulk thermodynamics based on Taylor expansion of the QCD partition function are currently well controlled for $\mu_B/T \leq 2$, i.e. for $\sqrt{s} \gtrsim 20\ \mathrm{GeV}$.

- in the range $20\ \mathrm{GeV} \leq \sqrt{s_{NN}} \leq 200\ \mathrm{GeV}$ the pattern seen in the beam energy dependance of up to 4th order cumulants of net-proton (baryon) number and electric charge fluctuations can be understood in terms of **QCD equilibrium thermodynamics**.

- QCD equilibrium thermodynamics sets a baseline for the discussion of the systematic effects which have to be taken into account for a more quantitative comparison.