Complex spectrum of QCD at finite density

Hiromichi Nishimura

RIKEN BNL Research Center

Talk@XQCD2016

2 August 2016

<HN, M. Ogilvie, and K. Pangeni, PRD90 045039 (2014),
PRD91 054004 (2015), PRD93 094501 (2016)>

Overview

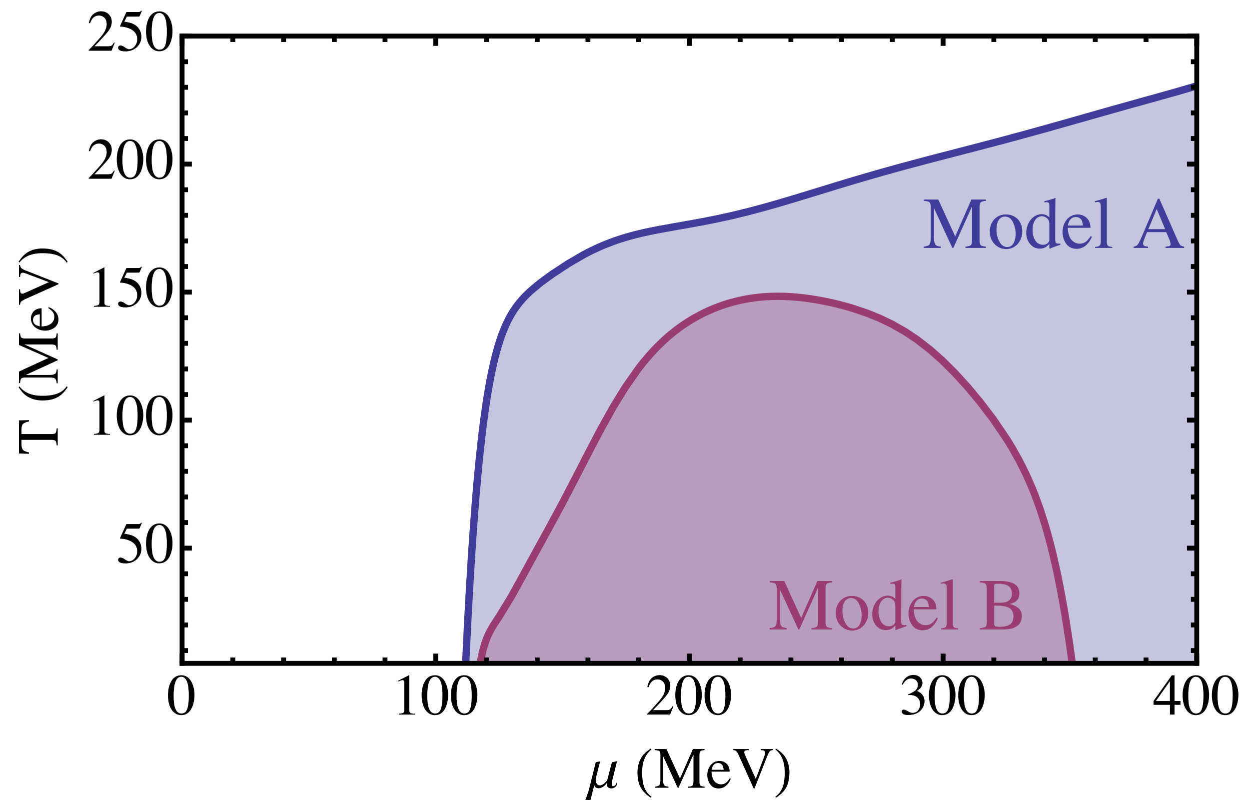
$$Z_{QCD} = \int DA\, e^{-S_{YM}} \det M(\mu)$$

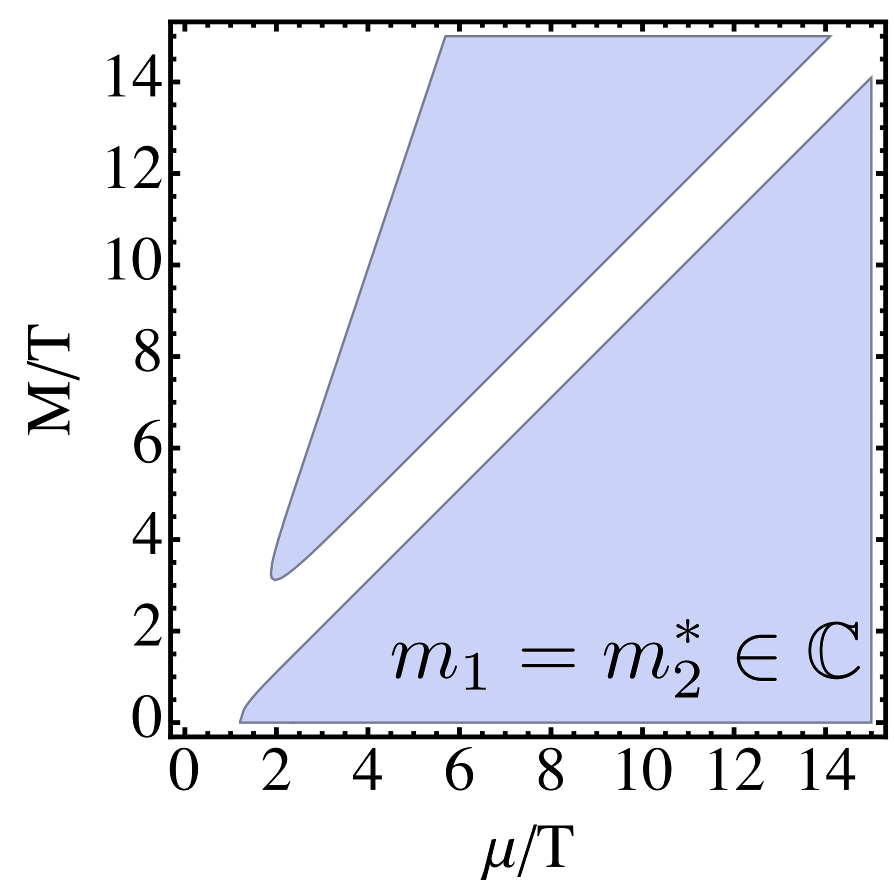
Weak-coupling

Strong-coupling

PNJL Model

SU(3) Spin Model





Main results:

1. Complex mass spectrum
2. Oscillation of Polyakov loop correlators

Overview

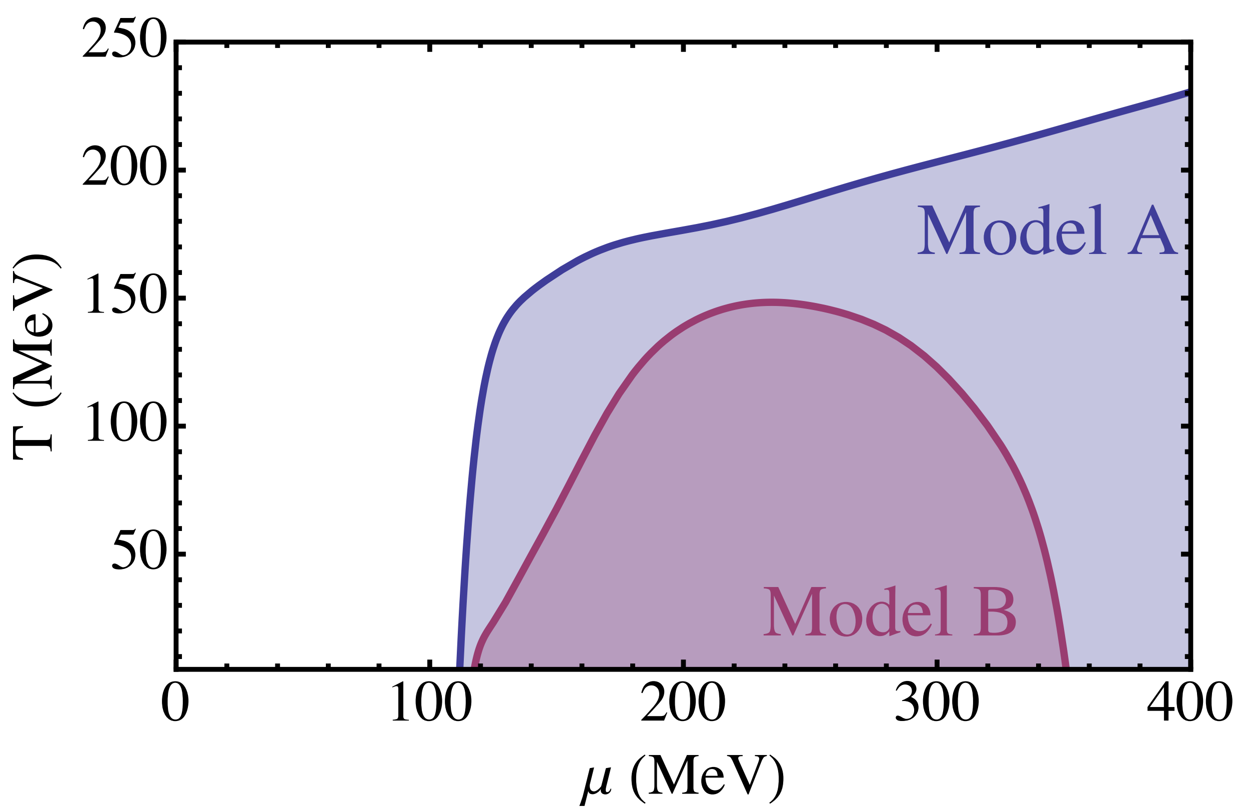
1. Introduction

$$Z_{QCD} = \int DA\, e^{-S_{YM}} \det M(\mu)$$

2. Spin model

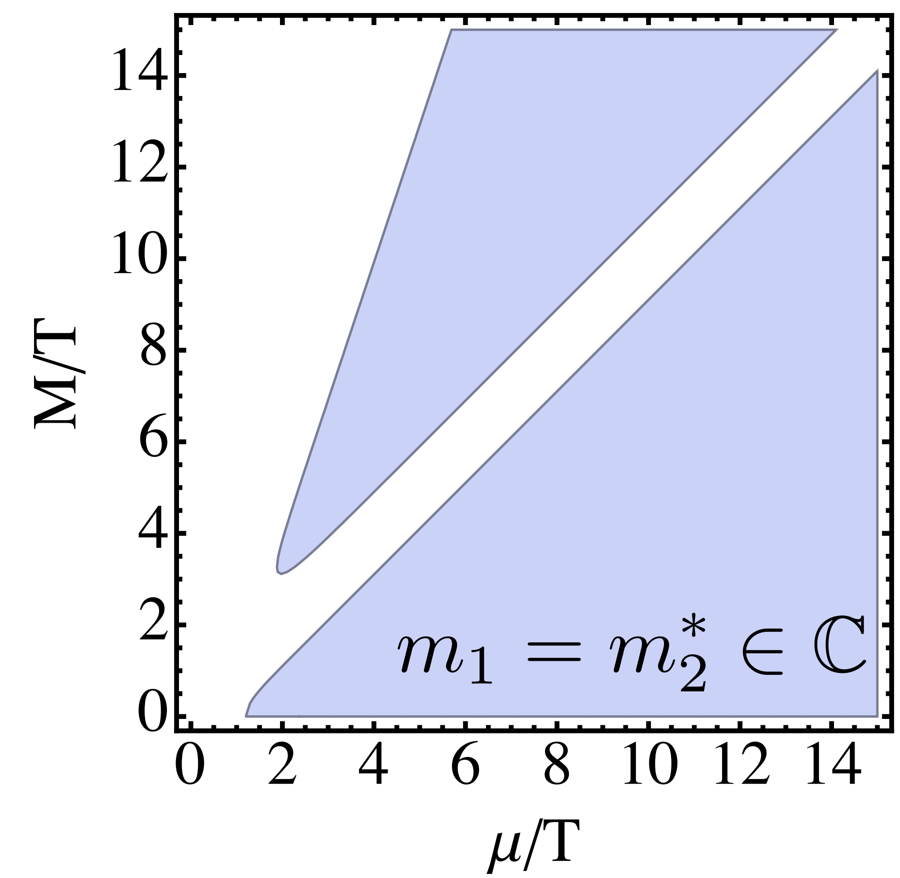
Weak-coupling

PNJL Model



Strong-coupling

SU(3) Spin Model



3. Discussions

Main results:

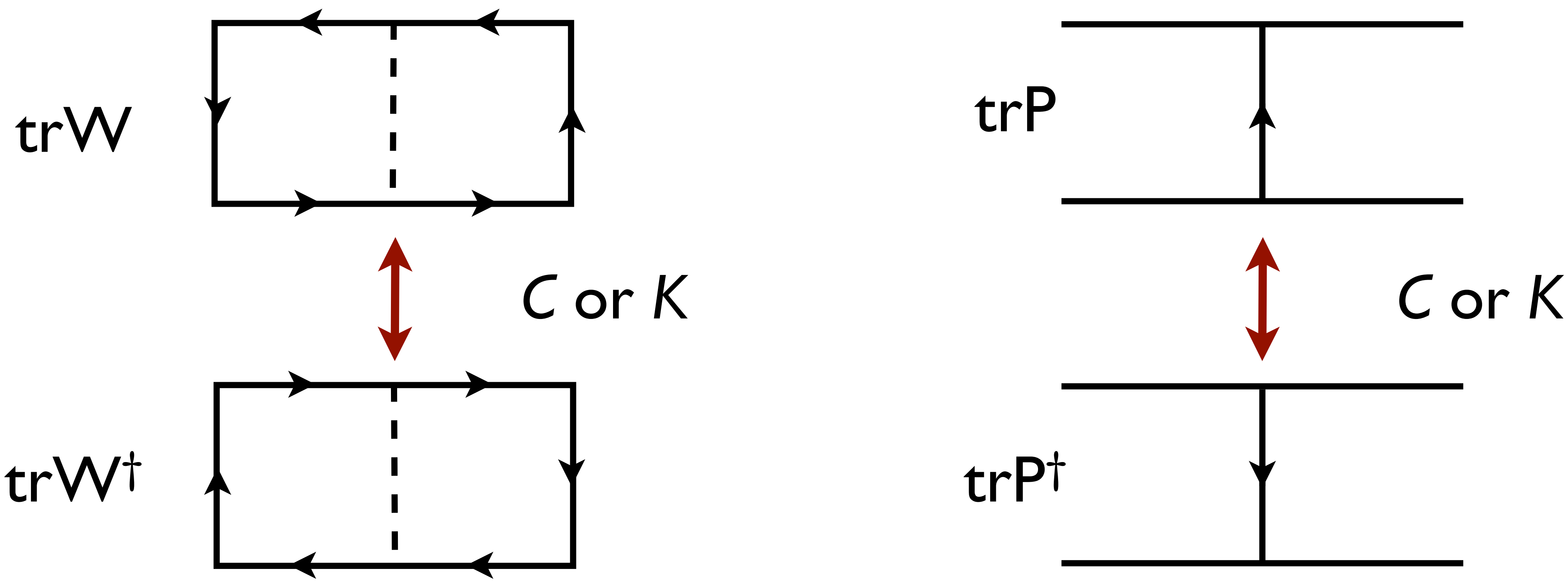
1. Complex mass spectrum
2. Oscillation of Polyakov loop correlators

4. Conclusions

Introduction

Sign problem and *CK* symmetry

- Hopping parameter expansion $\mu = 0 : \quad \det M(0) = [\det M(0)]^*$

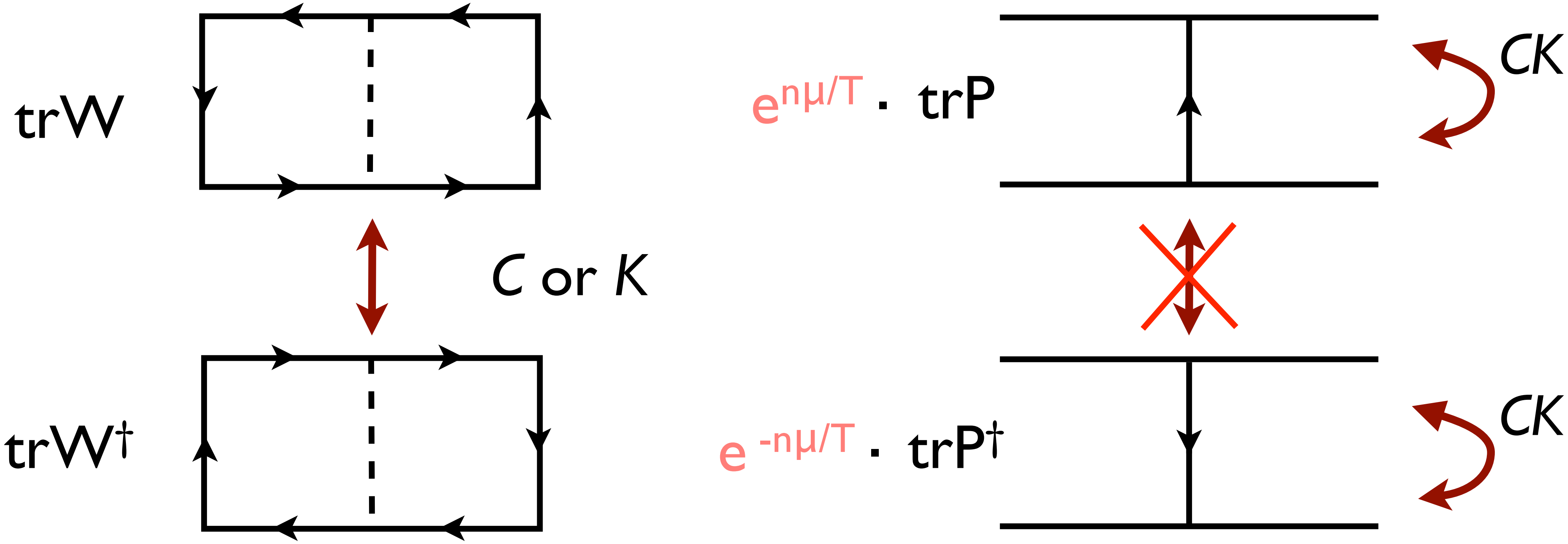


$$W = \mathcal{P} e^{ig \oint dx_\mu A_\mu} \qquad P = \mathcal{P} e^{ig \int_0^{1/T} dx_4 A_4}$$

Invariant under charge conjugation (*C*) or complex conjugation (*K*) at μ = 0.

Sign problem and *CK* symmetry

- Hopping parameter expansion $\quad \mu \neq 0 : \quad \det M(\mu) = [\det M(-\mu)]^*$



$$W = \mathcal{P} e^{ig \oint dx_\mu A_\mu} \qquad\qquad P = \mathcal{P} e^{ig \int_0^{1/T} dx_4 A_4}$$

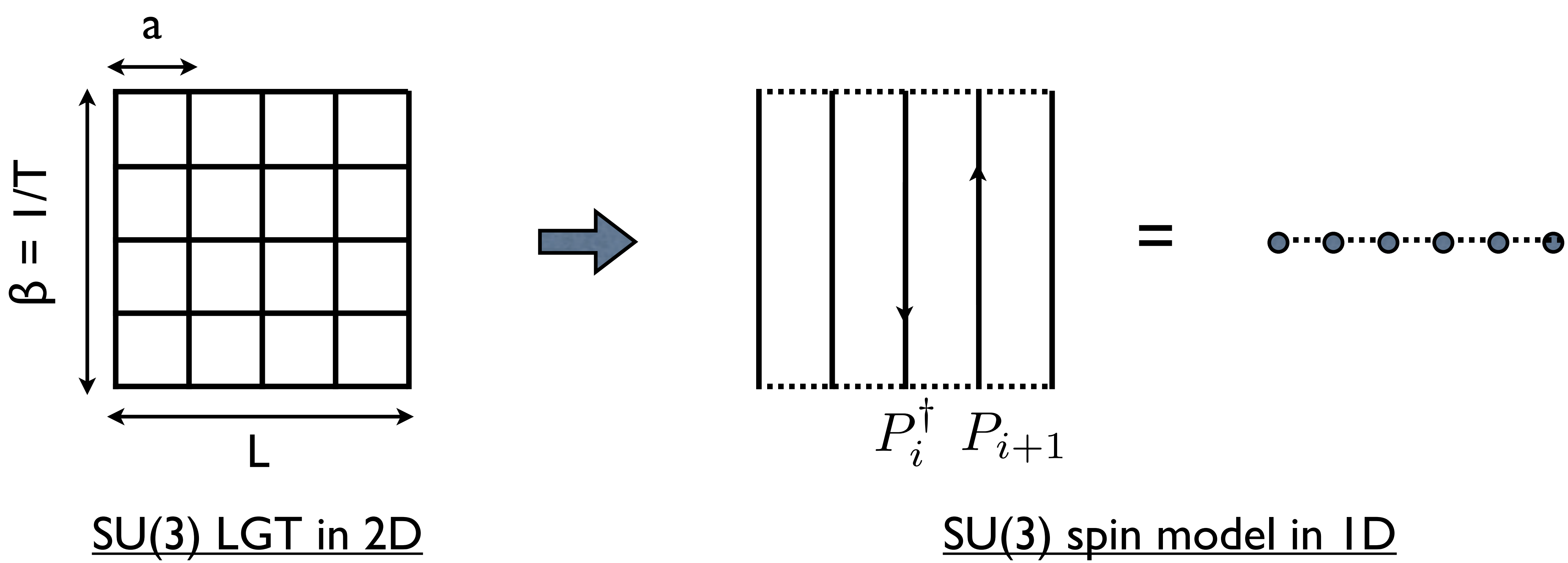
- *CK* symmetry is an exact symmetry of finite-density QCD.

<u>Any observables in any model should respect *CK* symmetry.</u>

SU(3) spin model

Yang-Mills in (1+1) dimensions

- Integrate out the spatial links using the character expansion.



- Construct the transfer matrix with the heat kernel action.

$$T_0 = \langle P_{i+1} | \, e^{-aH_0} \, | P_i \rangle \quad \text{where} \quad H_0 = \frac{g^2 \beta}{2} C$$

<Menotti and Onofri, 1981> etc

$$\to \langle r' | \, e^{-aH_0} \, | r \rangle = \mathrm{drag}(1, e^{-4a/3}, e^{-4a/3}, e^{-3a} \cdots)$$

Static quarks

- Inserting static quarks in the transfer matrix

$$T = \langle r' |\, e^{-aH_0/2} \det(1 + z_1 P) \det(1 + z_2 P^\dagger)\, e^{-aH_0/2}\, | r \rangle$$

$$\text{where} \quad z_1 = e^{(\mu - M)/T} \quad \text{and} \quad z_2 = e^{(-\mu - M)/T}$$

- Raising and lowering operators

$$\det(1 + z_1 P) = 1 \; + z_1 \, \square \; + z_1^2 \, \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \; + z_1^3$$

- Transfer matrix is non-Hermitian: A manifestation of the sign problem.

Pure SU(3) → With quarks ($z_2 = 0$)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{e^{4/3}} & 0 & 0 \\ 0 & 0 & \frac{1}{e^{4/3}} & 0 \\ 0 & 0 & 0 & \frac{1}{e^3} \end{pmatrix} \longrightarrow \begin{pmatrix} 1 + z_1^3 & \frac{z_1}{e^{2/3}} & \frac{z_1^2}{e^{2/3}} & 0 \\ \frac{z_1^2}{e^{2/3}} & \frac{1+z_1^3}{e^{4/3}} & \frac{z_1}{e^{4/3}} & \frac{z_1^2}{e^{13/6}} \\ \frac{z_1}{e^{2/3}} & \frac{z_1^2}{e^{4/3}} & \frac{1+z_1^3}{e^{4/3}} & \frac{z_1}{e^{13/6}} \\ 0 & \frac{z_1}{e^{13/6}} & \frac{z_1^2}{e^{13/6}} & \frac{1+z_1^3}{e^3} \end{pmatrix}$$

Mass spectrum

- Complex mass spectrum

$$T = \mathrm{diag}(e^{-m_0 a}, e^{-m_1 a}, \dots)$$

- Hermitian point at $z_1 = 1$.

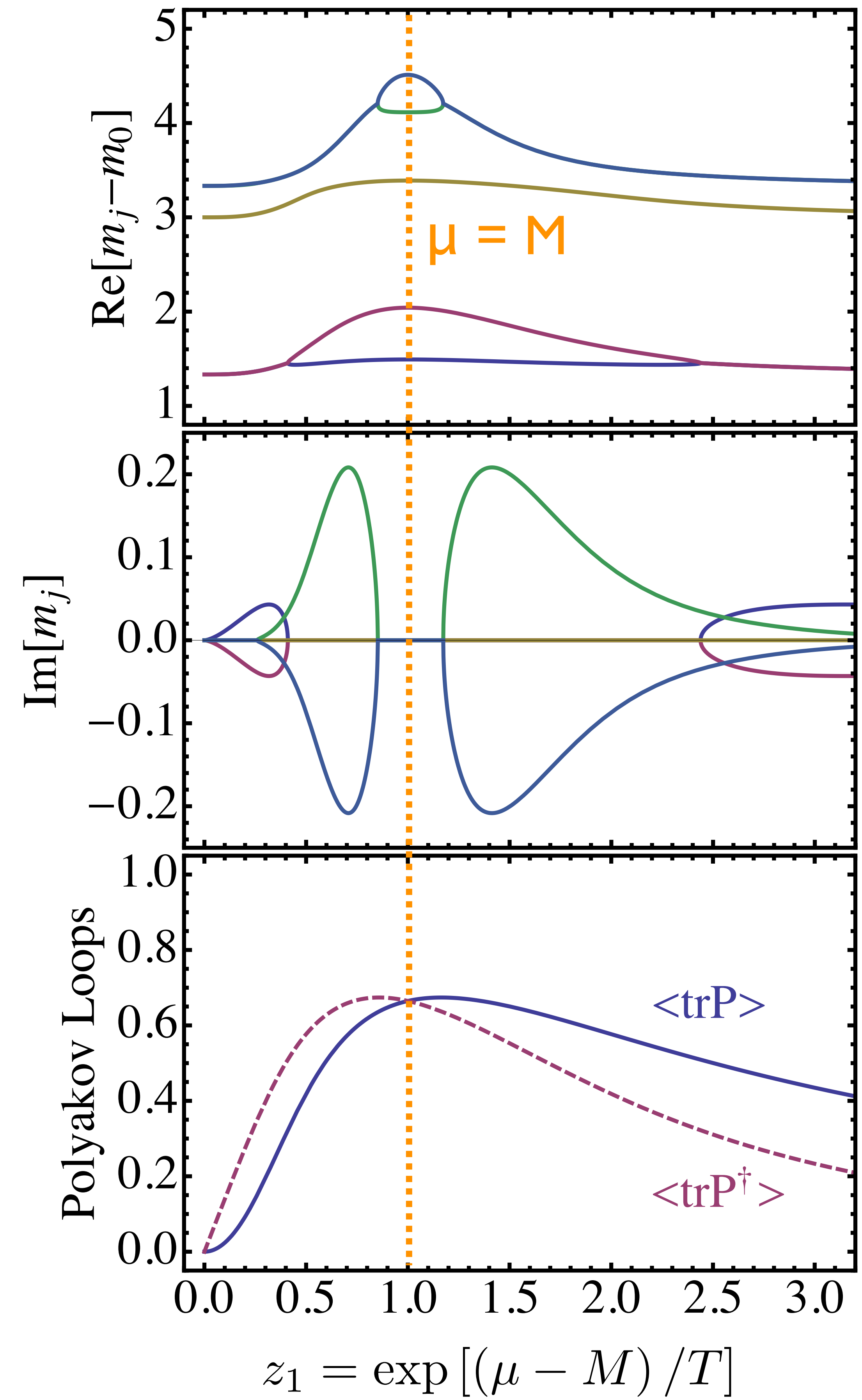
- Complex conjugate pairs due to *CK*.

- Invariant under $z_1 \to 1/z_1$

  - Particle-Antiparticle (*C*): $(z_1, z_2) \to (z_2, z_1)$

  - Particle-Hole (*K*): $(z_1, z_2) \to (1/z_1, 1/z_2)$

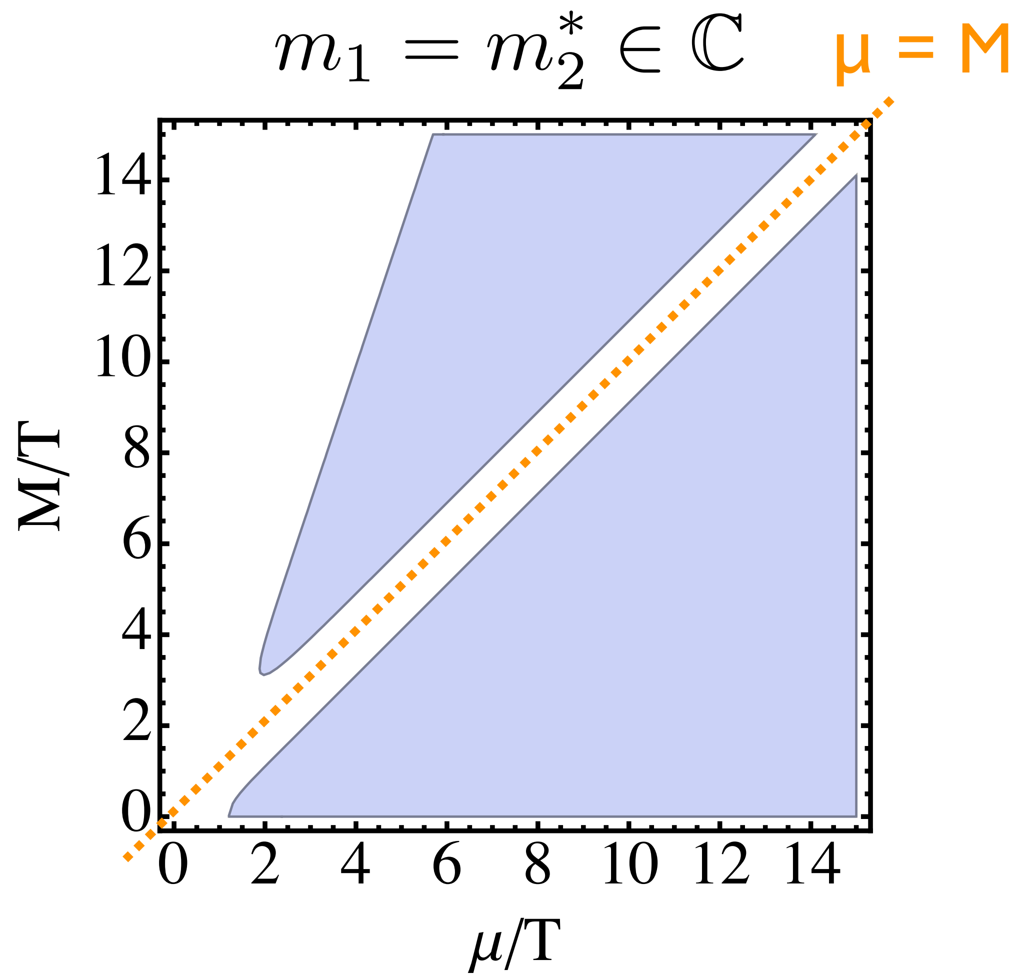
<u>*CK* symmetry at work</u>



Mass spectrum

- Complex mass spectrum

$$T = \mathrm{diag}(e^{-m_0 a}, e^{-m_1 a}, \dots)$$



- Hermitian point at $z_1 = 1$.

- Complex conjugate pairs due to *CK*.

- Invariant under $z_1 \to 1/z_1$

  - Particle-Antiparticle (*C*): $(z_1, z_2) \to (z_2, z_1)$

  - Particle-Hole (*K*): $(z_1, z_2) \to (1/z_1, 1/z_2)$

<u>*CK* symmetry at work</u>

Mass spectrum

- trP† - trP “measures” non-hermiticity

  Consistent with PNJL models.

  <HN, M. Ogilvie, K. Pangeni, 2015>

- The Polyakov loop goes to zero for large μ/M

  Consistent with

  - Strong-coupling

    <J. Langelage, M. Neuman, and O. Philipsen, 2014>

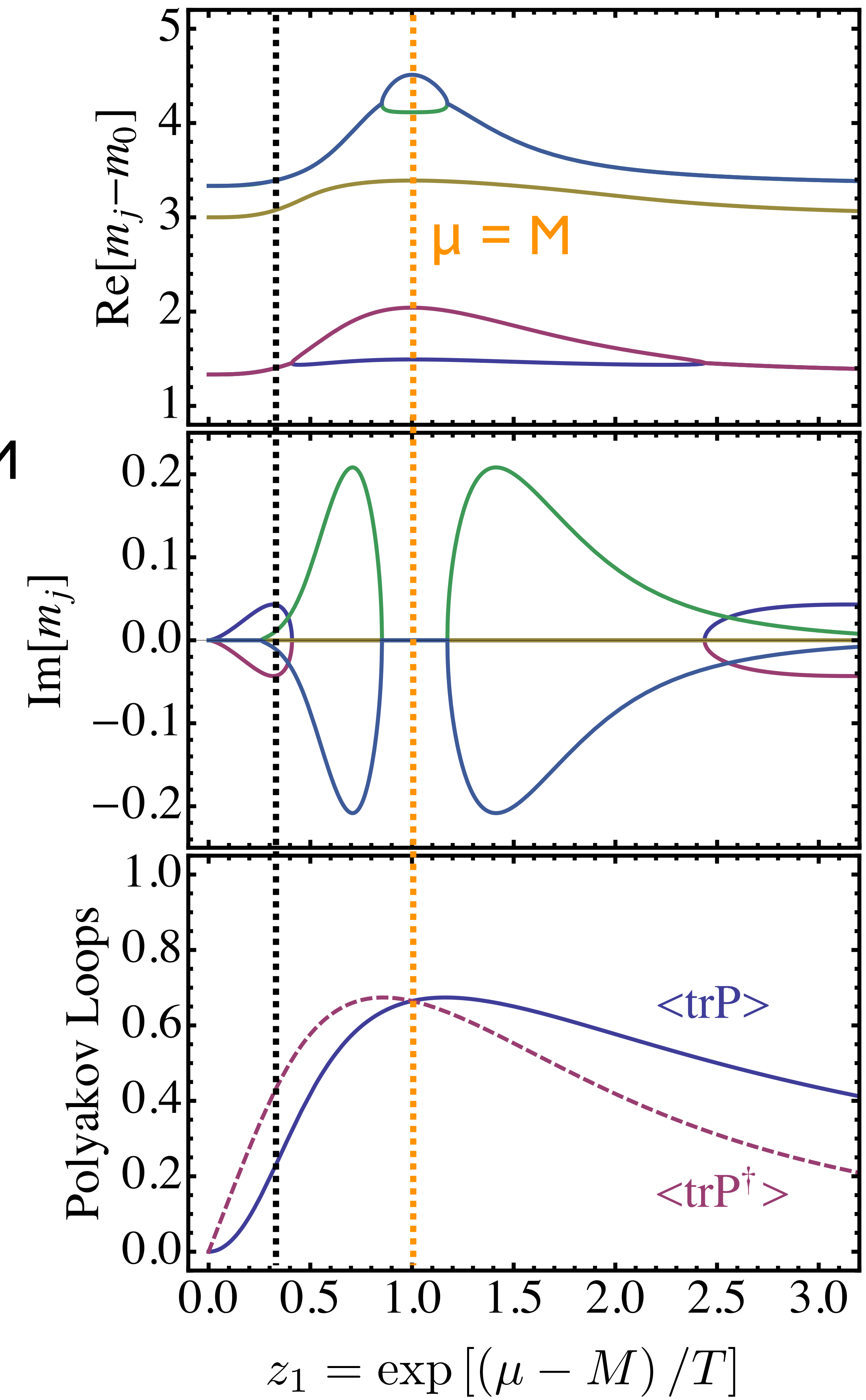
    <T. Rindlisbacher and P. de Forcrand, 2015>

  - Complex Langevin

    <G. Aarts, E. Seiler, D. Sexty, and I. Stamatescu, 2014>

    <G. Aarts, F. Attanasio, B. Jäger, and D. Sexty, 2016>

<u>One could see this as a consequence of *CK*.</u>



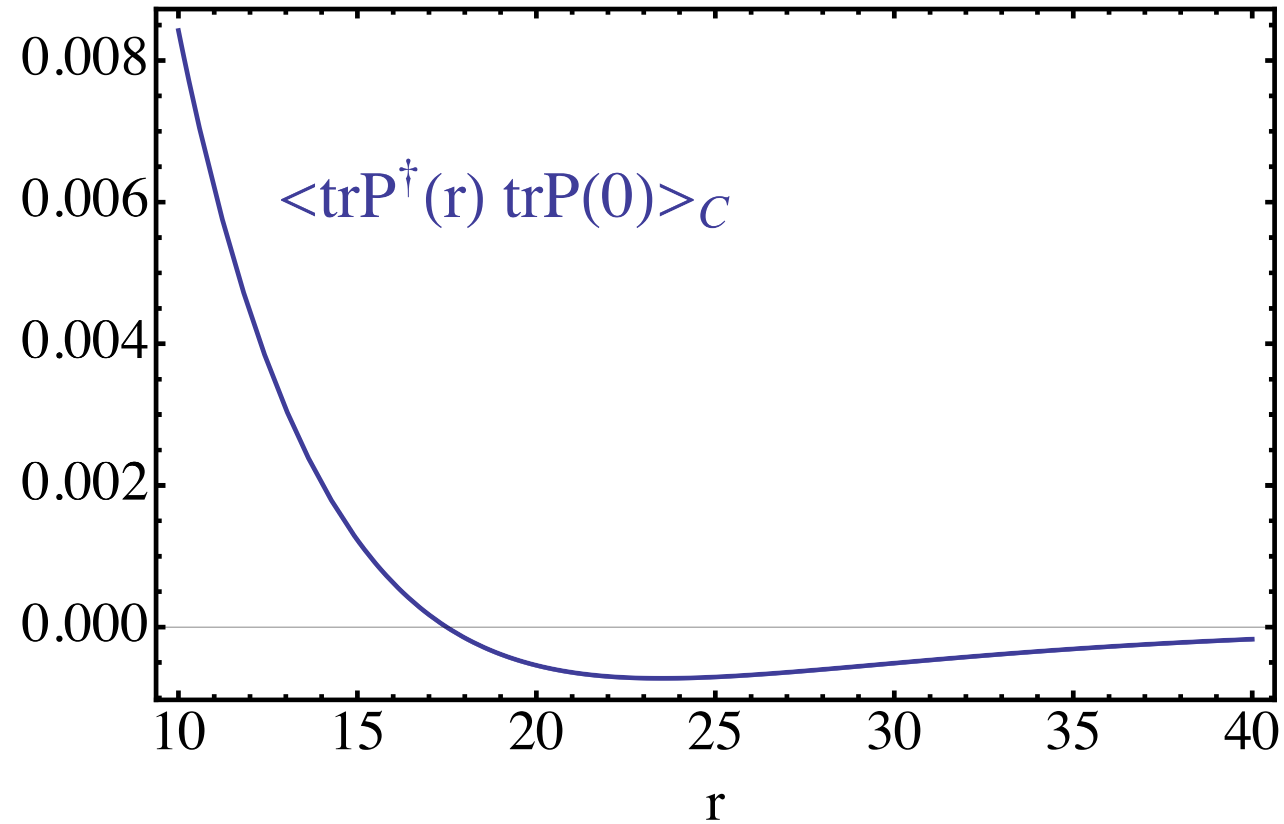
Polyakov loop correlator

- Sinusoidal exponential decay

$$\langle \mathrm{tr}_F P^\dagger(x)\, \mathrm{tr}_F P(0) \rangle_C$$

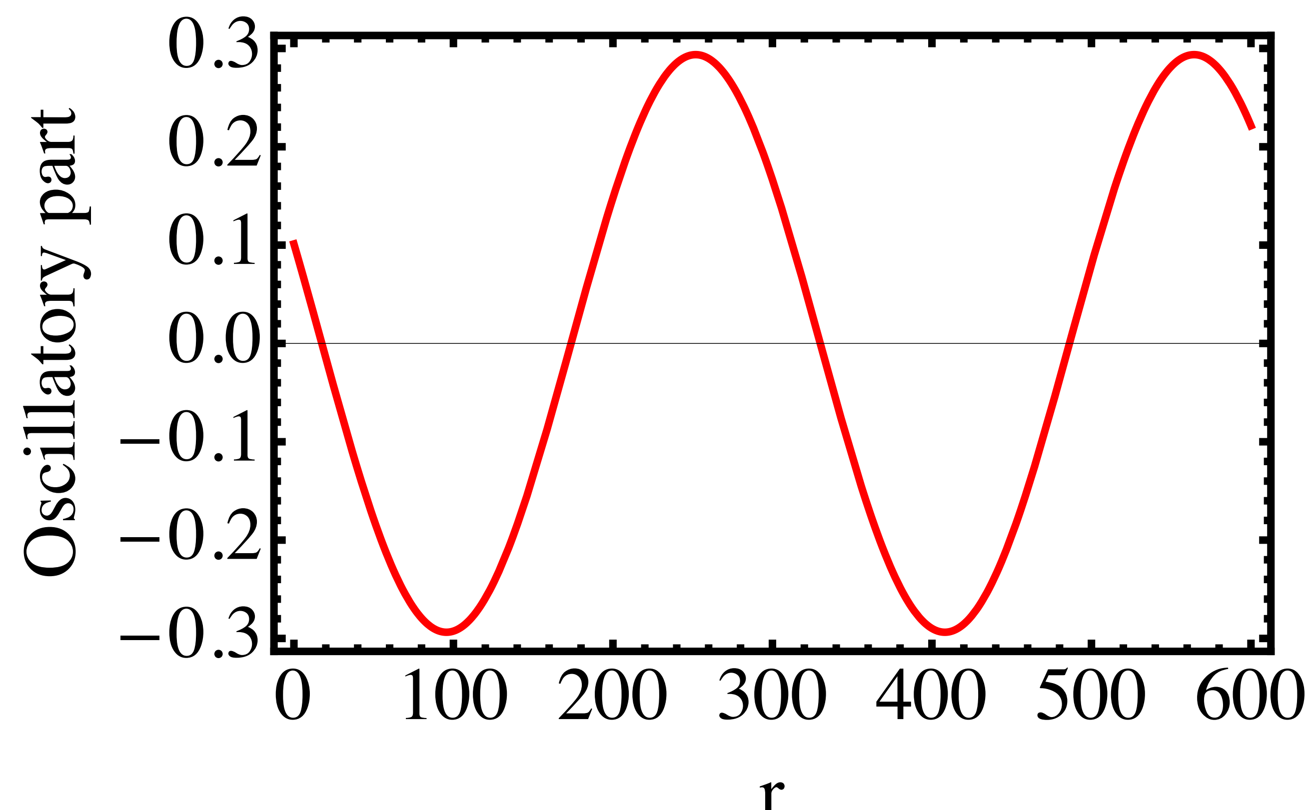
$$\sim \exp\left(-\mathrm{Re}[m_1 - m_0]\, x\right) \cos(\mathrm{Im}[m_1 - m_0]\, x)$$

See also:
<P. Meisinger and M. Ogilvie, 2014>
<O. Akerlund, P. de Forcrand, and T. Rindlisbacher, 2016>



- Small oscillation in the spin model.

- Oscillation remains in higher dimensions at strong coupling.



Discussions

PNJL model

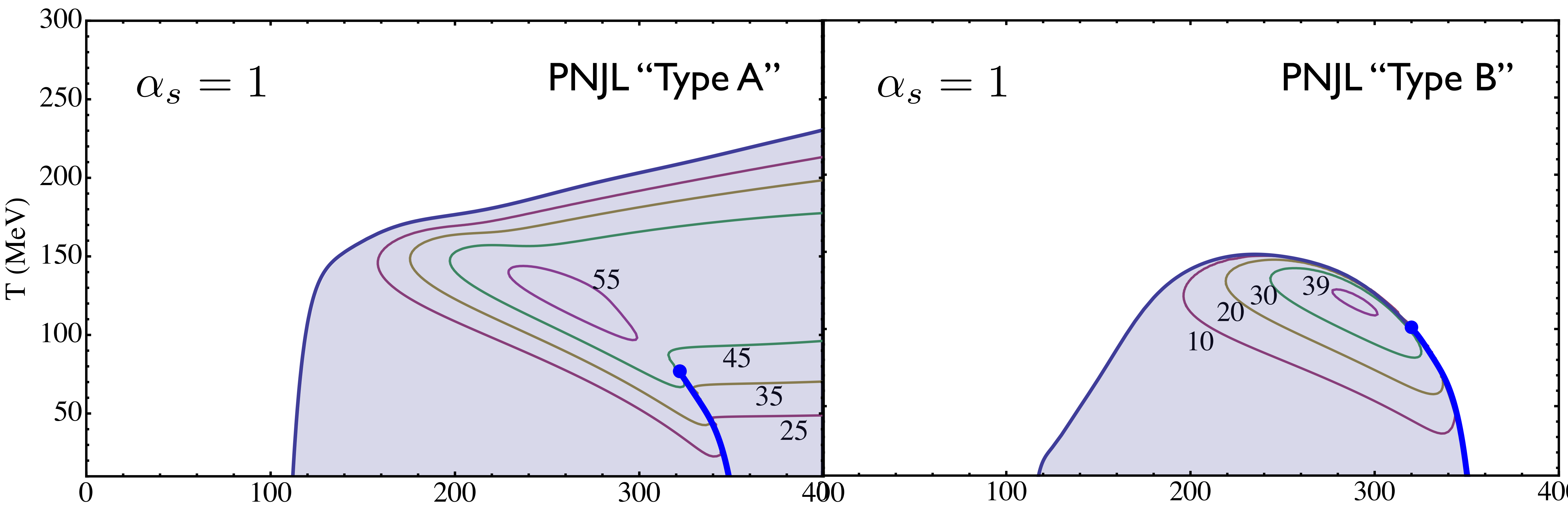
- Look for a complex *CK*-symmetric saddle point of the effective potential

<HN, M. Ogilvie, K. Pangeni, 2014>

- Analytic continuation of $A_4 = \theta_1 T_3 + \theta_2 T_8$ $\qquad \theta_1, \theta_2 \in \mathbb{C}$

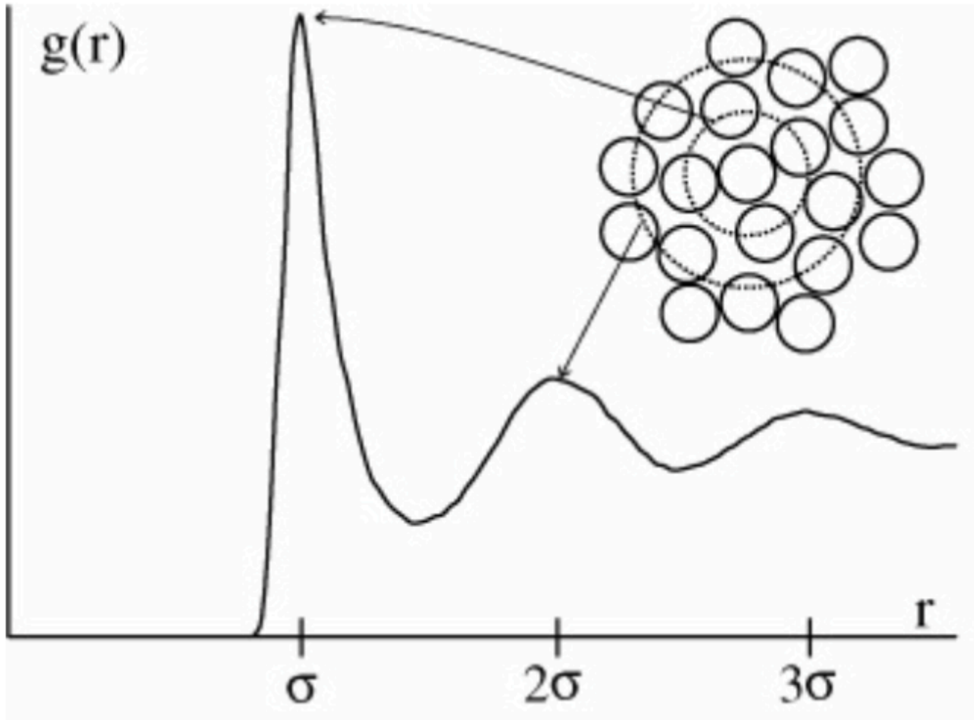
- Mass eigenvalues: $M_{ab} = \dfrac{\partial^2 V_{eff}}{\partial A_4^a \partial A_4^b} \longrightarrow m_{ev} = \kappa_R \pm i\kappa_I$

- Correlation function: $\langle A_4(r) A_4(0) \rangle \sim \dfrac{\mathrm{Exp}\left[-r\,\kappa_R\right]}{r}\left(\kappa_R \cos\left[r\,\kappa_I\right] + \kappa_I \sin\left[r\,\kappa_I\right]\right)$



<HN, M. Ogilvie, K. Pangeni, 2015>

QCD & Liquid-Gas

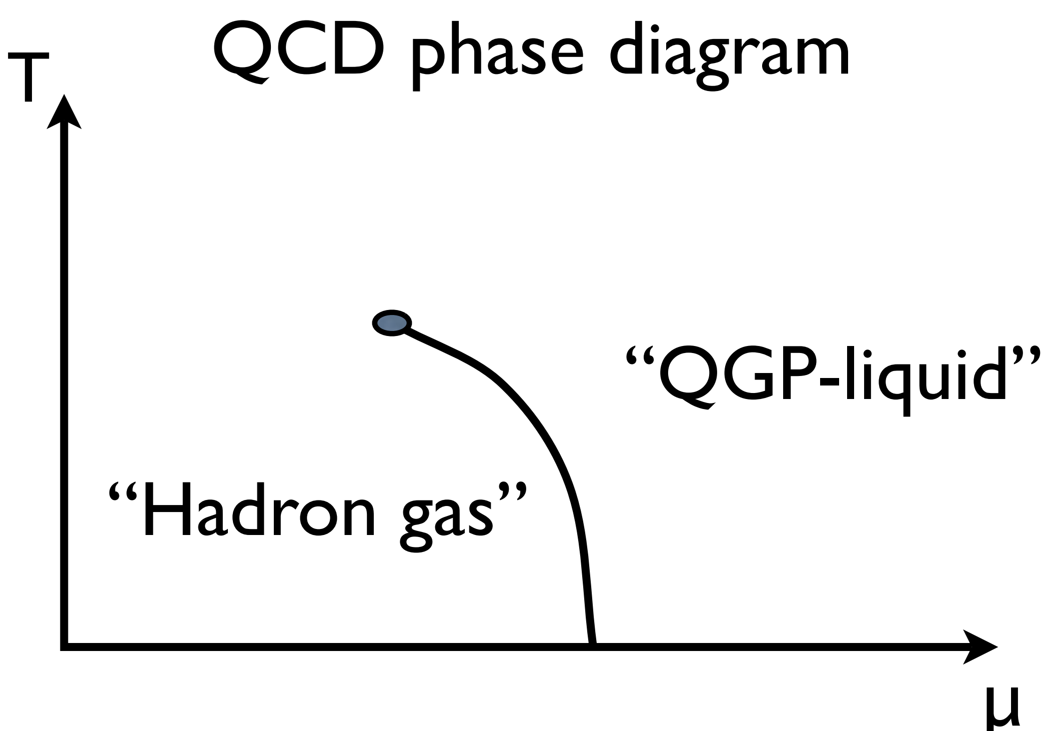


<Reichman and Charbonneau, 2005>

- Oscillation in density correlation function

  - Radial distribution g(r) for a liquid of size σ.

  - Often relevant near liquid-gas transition.



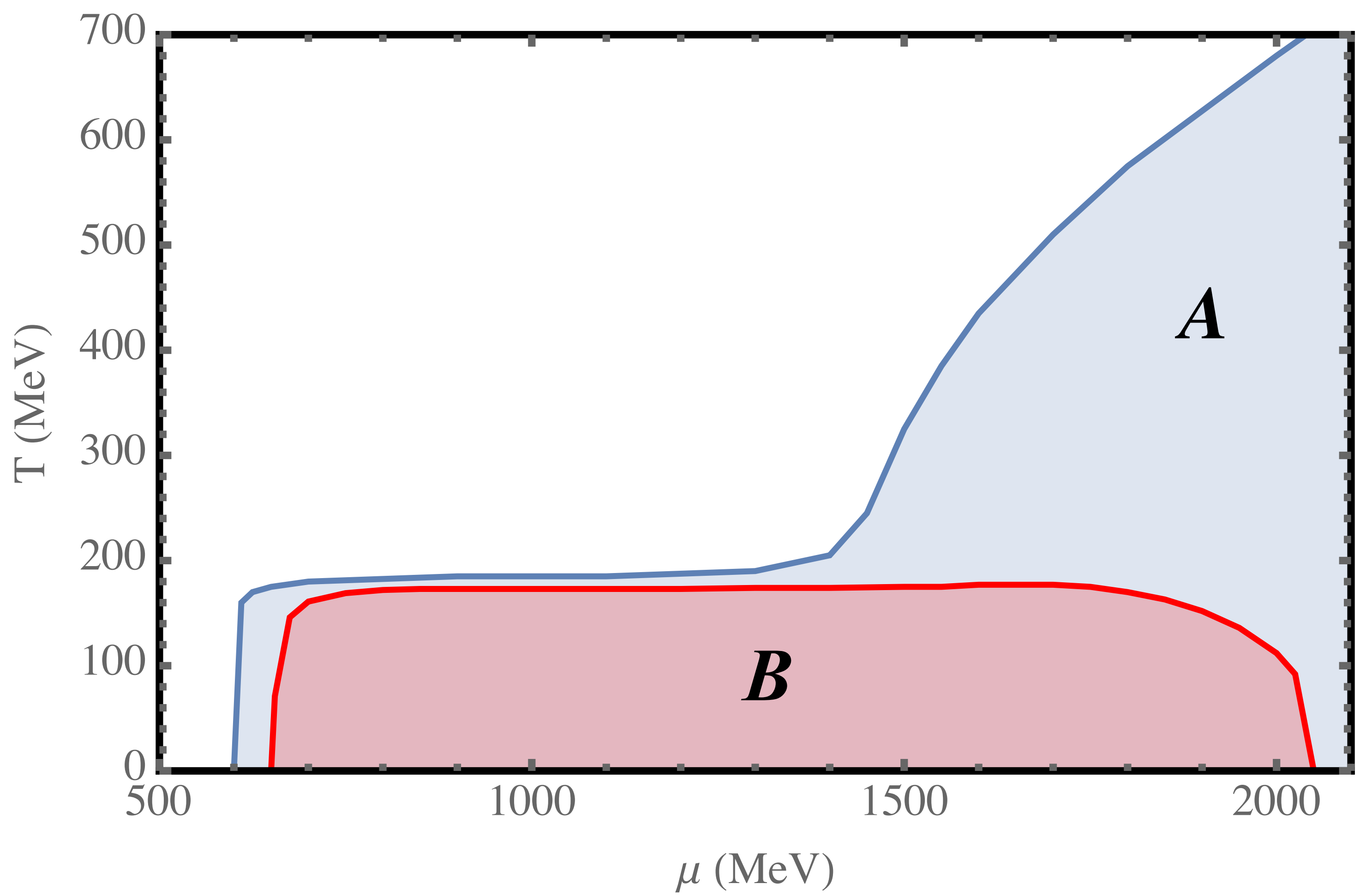
<Steinheimer et al, 2014>

- Oscillation in Polyakov-loop correlators

  - Due to screening between two quarks.

  - Oscillation of color-density correlator.

Heavy quarks



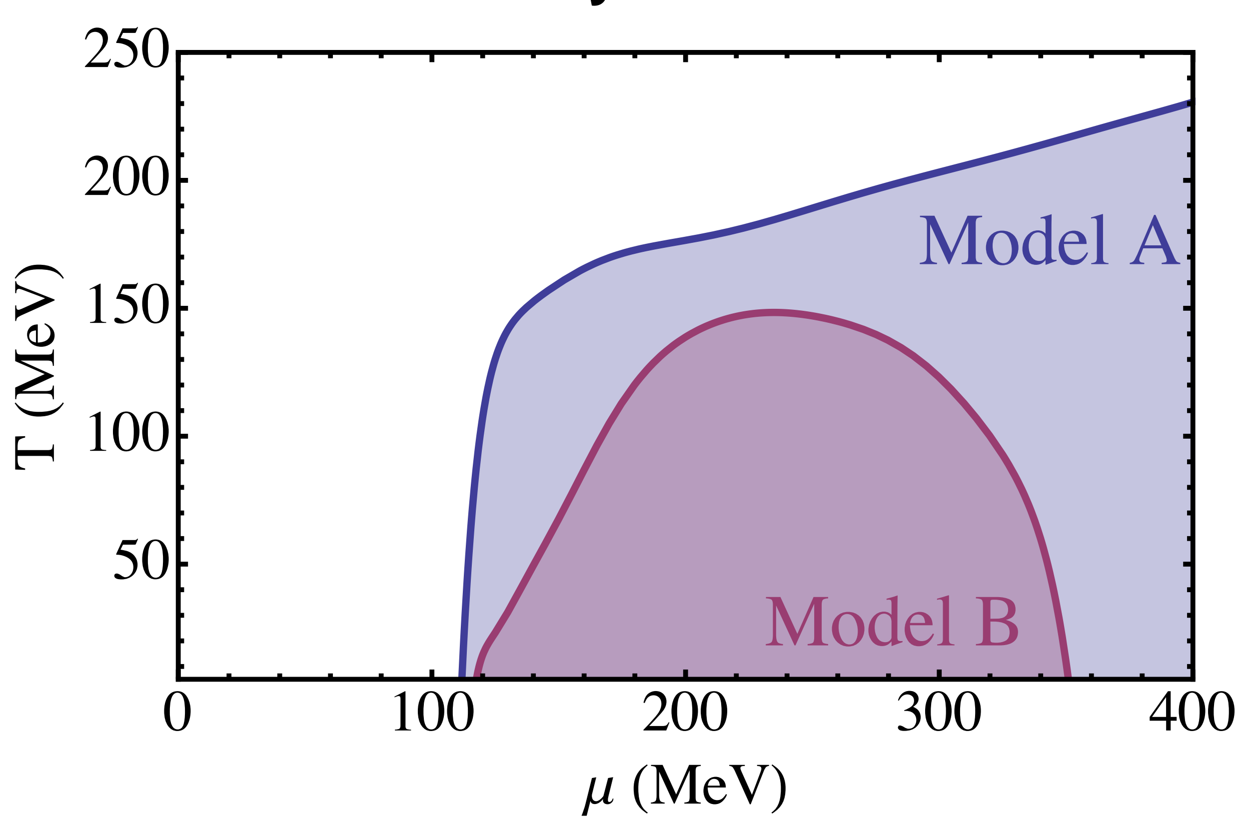
1. The disorder line serves as a benchmark for lattice simulations at finite μ.
2. Lattice simulations could differentiate the models of confinement.

Conclusions

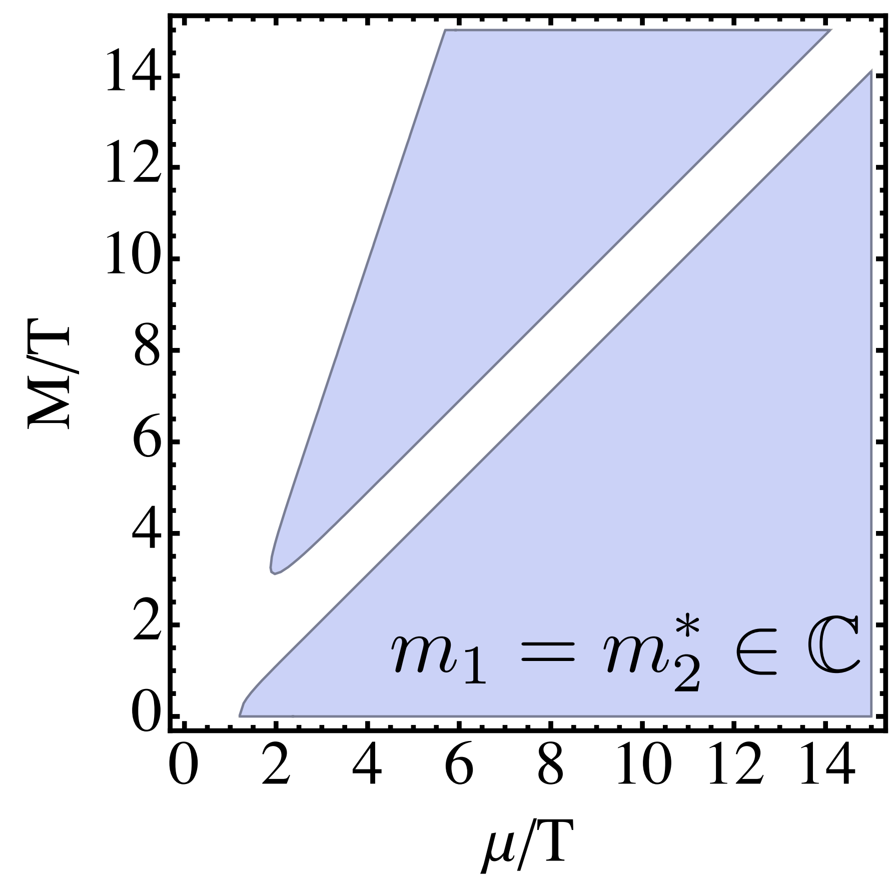
- Mass eigenvalues form complex conjugate pairs in models for finite-density QCD due to the sign problem and *CK* symmetry.

- Polyakov loop correlator oscillates. It should be observable in lattice simulations.

PNJL Model



SU(3) Spin Model



Appendix

Quark number density

- The number density saturates ($n_q \to 3$) at $\mu >> M$
- Particle-Hole symmetry if $\mu/T = M/T >> 1$ → Half-filling

See also <T. Rindlisbacher and P. de Forcrand, 2015>