# Charmonium spectral functions from large quenched LQCD

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Abstract: We present our most updated results on the charmonium correlation and spectral functions. The correlation functions are obtained using the clover improved Wilson fermions on quenched lattices with spatial size fixed to 128. And temporal sizes of these lattices are chosen to be 96 and 48 corresponding to $0.73T_c$ and $1.5T_c$. We perform a detailed analysis of these charmonium correlators by using two different stochastic approaches, namely Stochastic Analytical Inference (SAI) and Stochastic Optimization Method (SOM) to extract the spectral functions. The systematic uncertainties of spectral function obtained from SOM and SAI are discussed. And the comparison with those from Maximum Entropy Method (MEM) is also discussed.

## I. Motivation

Quarkonium is proposed as a QGP thermometer[1][2].

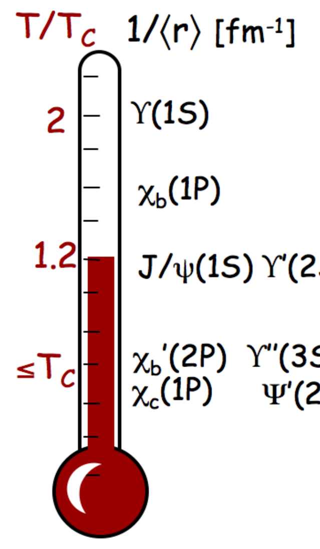


**Fig. 1** The QGP thermometer.

The dissociation temperatures of quarkonium can be read off from the corresponding spectral functions. The relation between spectral functions and Euclidean correlators is:

$$G_H(\tau,\mathbf{p}) = \int_0^\infty \frac{d\omega}{2\pi}\rho_H(\omega,\mathbf{p},T)K(\omega,\tau,T). \quad (1)$$

$\sim\mathcal{O}(10)$ $\quad\sim\mathcal{O}(1000)$

$\chi^2$ fitting fails without prior information !

**Previous work to solve this ill-posed problem:**

⇒ Maximum Entropy Method (MEM) [3]

⇒ A novel Bayesian approach [4]

⇒ The Backus-Gilbert Method(BGM) [5]

## II. Stochastic Analytic Inference

▶ A method based on Bayesian theorem:

$$\langle\langle n\rangle\rangle = \int d\alpha \langle n\rangle_\alpha P[\alpha|\bar{G}]. \quad (2)$$

▶ $n(x)$ relates spectral function to Default Model(DM):

$$n(x) = \frac{\rho(\omega)}{D(\omega)}, \quad x \equiv \phi(\omega) = \int_0^\omega D(\nu)d\nu. \quad (3)$$

**GOAL: find the distribution of $P[\alpha|\bar{G}]$ !**

▶ Field treatment of $n(x)$ [6]:

$$\langle n(x)\rangle_\alpha = \int \mathcal{D}n\; n(x)\mathbf{P}[\mathbf{n}|\alpha,\bar{\mathbf{G}}] = \int \mathcal{D}n\; n(x)\frac{1}{\mathbf{Z}(\alpha)}\mathbf{e}^{-\chi^2[\mathbf{n}(\mathbf{x})]/2\alpha}. \quad (4)$$

▶ The posterior probability $P[n|\alpha,\bar{G}]$:

$$P[n|\alpha,\bar{G}] = \frac{1}{P[\bar{G}|\alpha]}P[\bar{G}|\alpha,n]\; P[n|\alpha]. \quad (5)$$

likelihood function: $P[\bar{G}|n,\alpha] = \frac{1}{Z'}e^{-\chi^2[n]/2\alpha}$.

prior probability: $P[n|\alpha] = \Theta[n(x)]\delta(\int_0^{x_{max}} dx\; n(x) - 1)$.

▶ Construct $P[\alpha|\bar{G}]$:

$$P[\alpha|\bar{G}] = \frac{P[\alpha]}{P[\bar{G}]}\int \mathcal{D}n\; P[\bar{G}|\alpha,n]\; P[n|\alpha] \sim P[\alpha]\;\alpha^{-N/2}Z(\alpha). \quad (6)$$

▶ Evaluate $Z(\alpha)$ using Wang-Landau Algorithm(WLA) [7].

**SAI v.s. MEM**

| *SAI* | *MEM* |
|---|---|
| Average: $\langle\langle n\rangle\rangle = \int d\alpha P[\alpha\|\bar{G}]\langle n\rangle_\alpha$ | Average: $\langle\langle\rho\rangle\rangle \approx \int d\alpha P[\alpha\|\bar{G}]\hat{\rho}_\alpha$ |
| $P[\alpha\|\bar{G}] \sim \int \mathcal{D}n\; P[\bar{G}\|\alpha,n]\; P[n\|\alpha]$ | $P[\alpha\|\bar{G}] \sim \int \mathcal{D}\rho\; P[\bar{G}\|\alpha,\rho]\; P[\rho\|\alpha]$ |
| Likelihood function: $P[\bar{G}\|n,\alpha] \sim e^{-\chi^2[n]/2\alpha}$ | Likelihood function: $P[\bar{G}\|\rho,\alpha] \sim e^{-\chi^2[\rho]/2}$ |
| Prior probability: $P[n\|\alpha] = \Theta[n(x)]\delta(\int_0^{x_{max}} dx\; n(x) - 1)$ | Prior probability: $P[\rho\|\alpha] \sim e^{\alpha S[\rho]}$ |
| Most likely solution: Integrate $\int \mathcal{D}n$ | Most likely solution: Minimize $F = \chi^2/2 - \alpha S$ |

**SAI to MEM:**

■ Mean field treatment in SAI:

$$S_{SAI}[n] \equiv \int \mathcal{D}n \ln\Omega(n) \approx \ln\Omega(\bar{n}) = -\int dx\; \bar{n}(x)\ln\bar{n}(x) = S_{MEM}[\bar{\rho}]. \quad (7)$$

■ SAI reduces to MEM at the **mean-field level !**

**SAI to SOM:**

■ SAI reduces to SOM when using **constant default model!**

## III. Analysis with mock data

We test the ability of SOM and SAI with mock data. Error in the mock data is of the form: $\sigma = \epsilon\cdot\bar{G}\cdot\tau$. Lattice spacing: a=1.

### 1. Different mock spectral functions

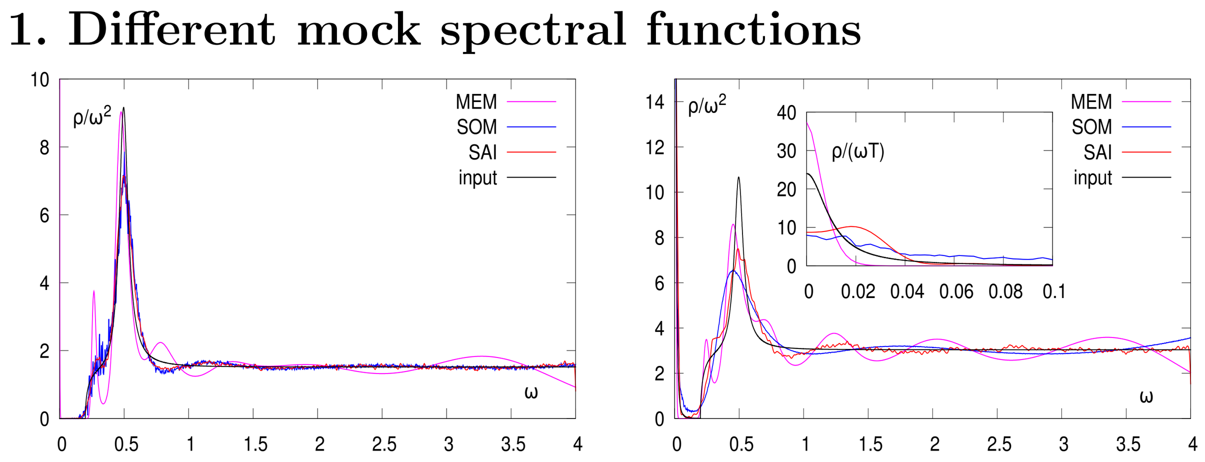

**Fig. 2** *Left*: resonance peak + free continuum, corresponding to $T < T_c$. *Right*: transport peak + resonance peak + free continuum, corresponding to $T > T_c$. $\epsilon = 10^{-4}$ and $N_\tau$=48 in both cases.

### 2. Dependence on $N_\tau$ and noise level $\epsilon$

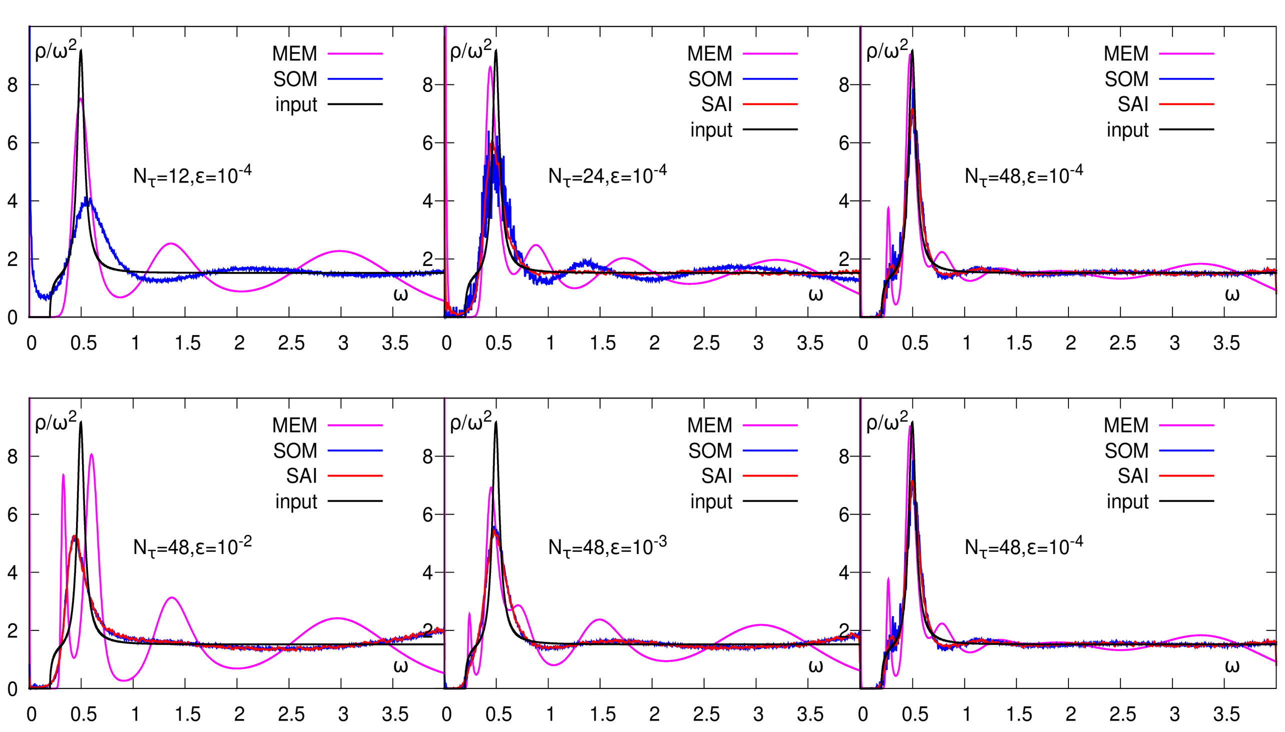

**Fig. 3** *Top:* $N_\tau$ dependence tests. Set $N_\tau$=12,24,48, respectively. Fix $\epsilon$ to $10^{-4}$. *Bottom:* Error dependence tests. Set $\epsilon = 10^{-2}, 10^{-3}, 10^{-4}$, respectively. Fix $N_\tau$ to 48.

### 3. Dependence on Default Model

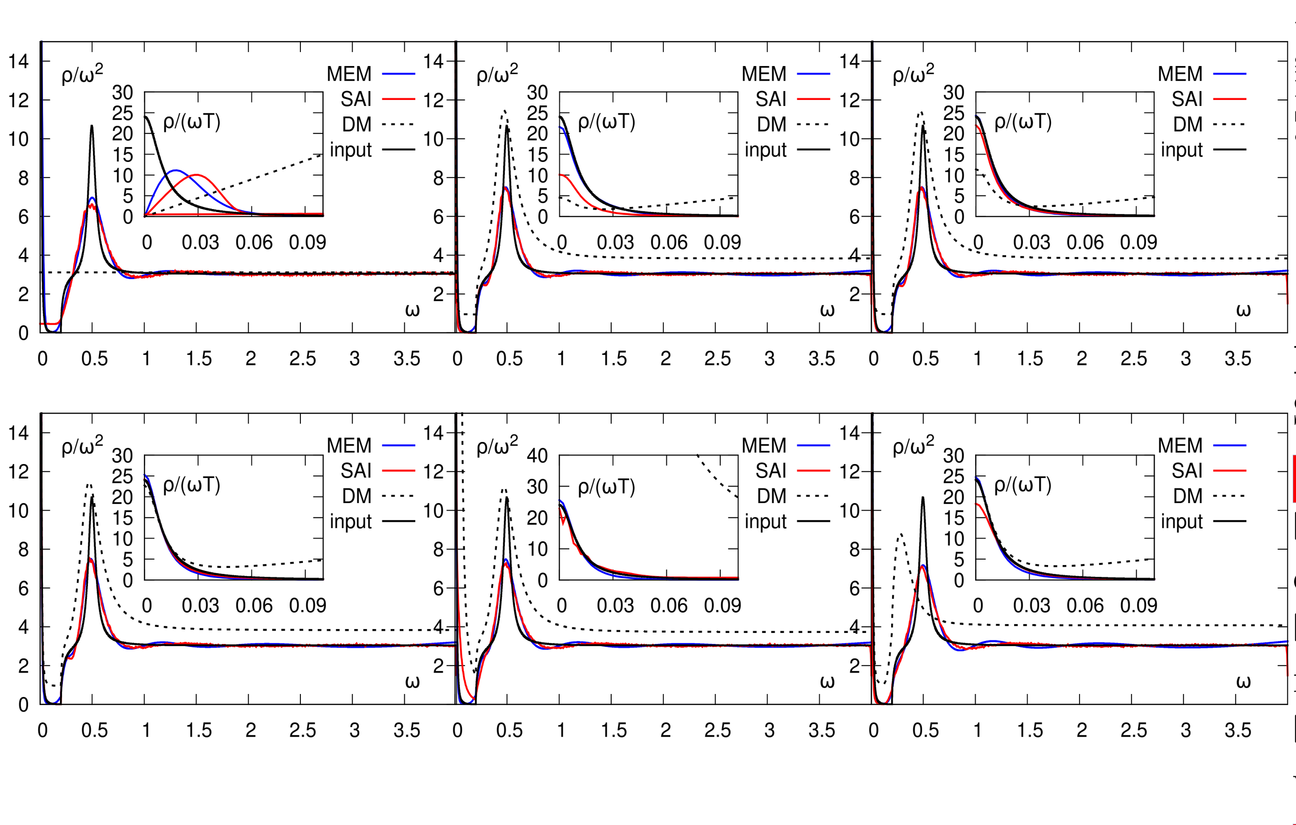

**Fig. 4** Dependence on DM of SAI&MEM. $\epsilon = 10^{-4}$ and $N_\tau$=48 in all cases .

## IV. Real lattice data results

### 1. Lattice setup

▶ Standard plaquette gauge action& $\mathcal{O}$(a)-improved Wilson valence quarks.

▶ In the quenched approximation.

▶ On fine and large isotropic lattices.

$-\beta = 7.793 \mapsto a = 0.009fm(a^{-1} = 21.8GeV)$.

–The scale has been set by $r_0 = 0.49fm$ and with an interpolation for $r_0/a$[8].

$-N_\sigma = 192, N_\tau = 96, 48 \mapsto T = 0.75T_c, 1.5T_c$.

$-\kappa = 0.13211 \mapsto m_V = 3.234(9)GeV$.

▶ Vector channel.

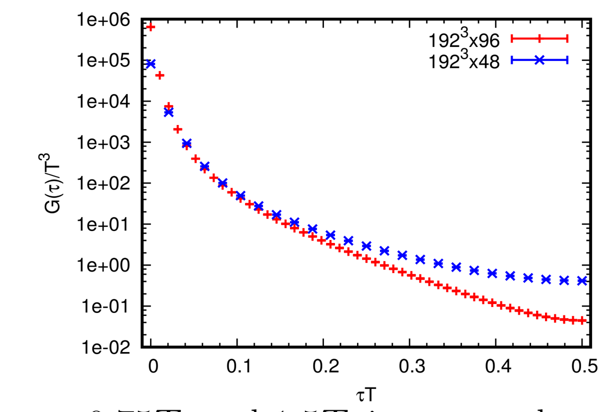

**Fig. 5** Correlators at $0.75T_c$ and $1.5T_c$ in vector channel.

### 2. Results at $0.75T_c$

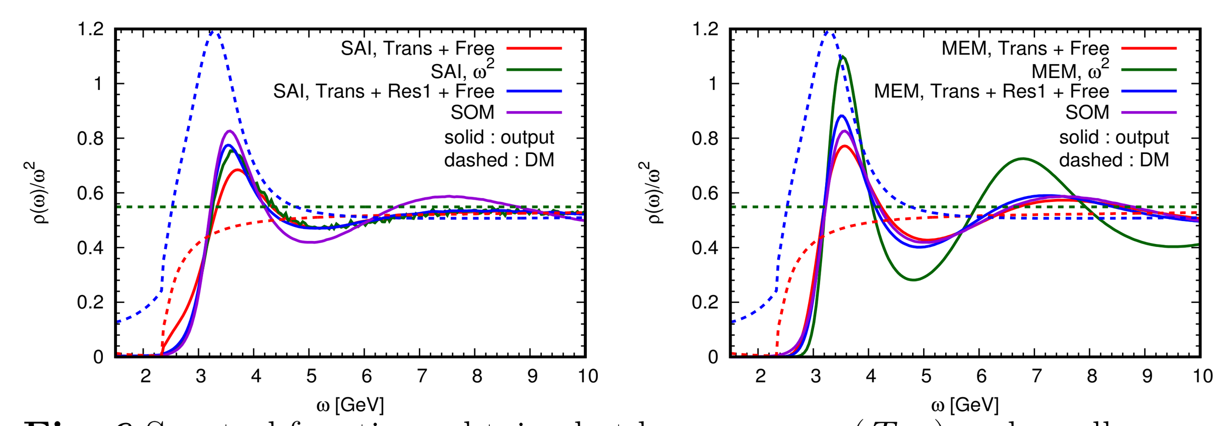

**Fig. 6** Spectral functions obtained at large $\omega$ range(*Top*) and small $\omega$ range (*Bottom*) by SAI/SOM(*Left*) and MEM/SOM(*Right*) at $0.75T_c$.

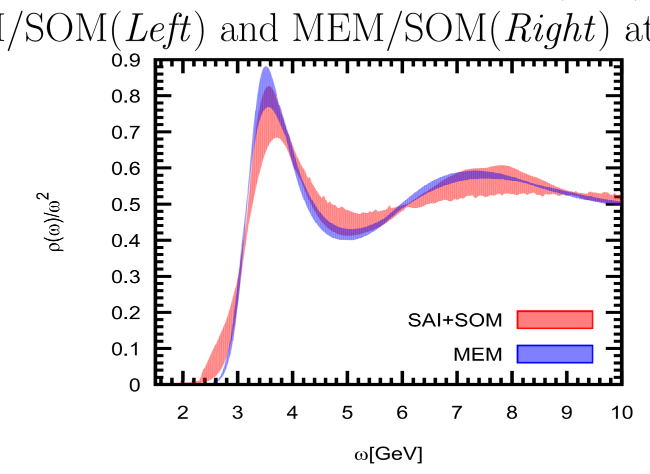

**Fig. 7** Systematic uncertainties of the SPFs obtained by SOM and SAI&MEM using different DMs at $0.75T_c$. The results suggest a stable resonance peak exsits.

### 3. Results at $1.5T_c$

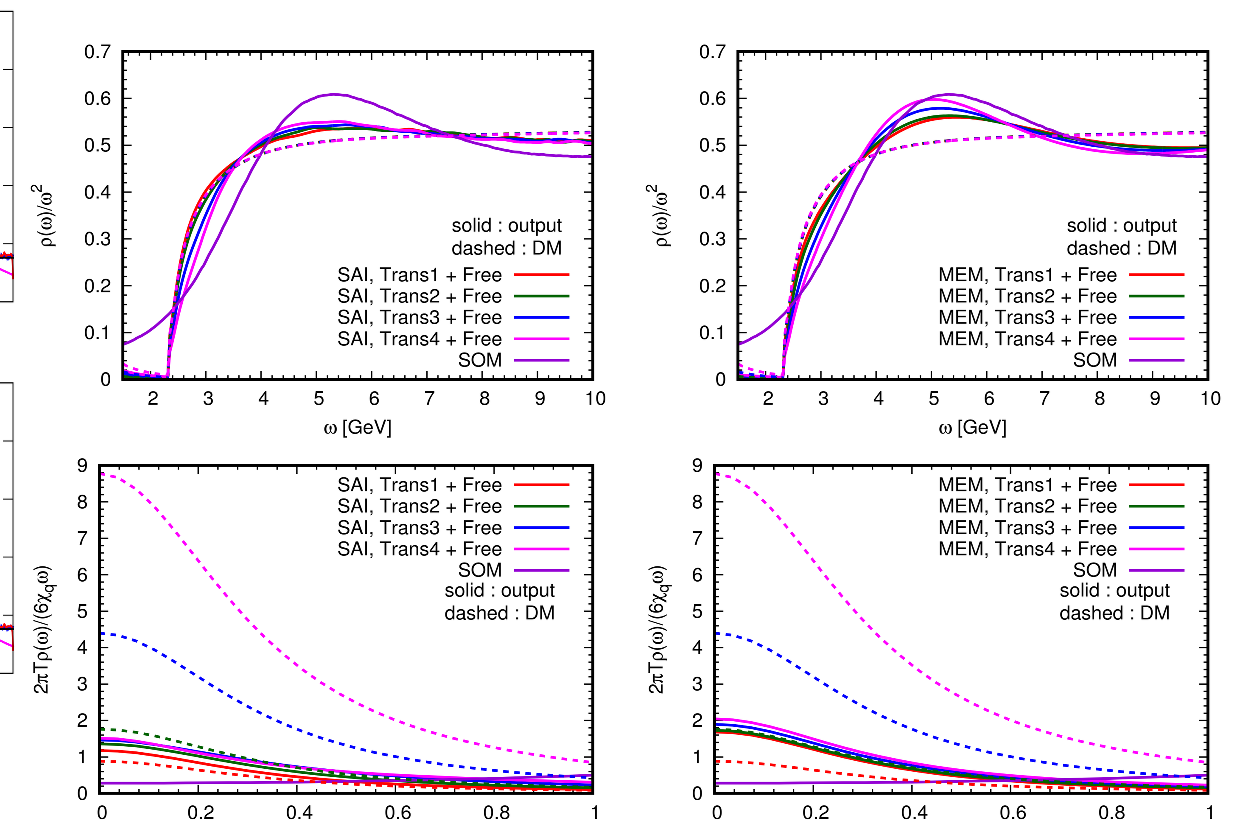

**Fig. 8** Spectral functions obtained at large $\omega$ range(*Top*) and small $\omega$ range (*Bottom*) by SAI/SOM(*Left*) and MEM/SOM(*Right*) at $1.5T_c$.

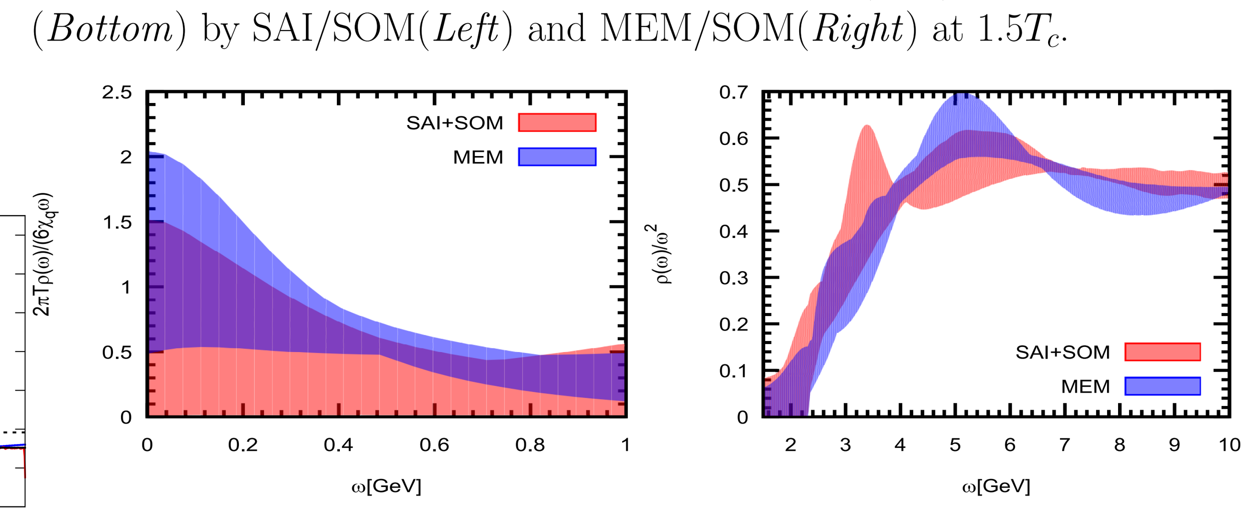

**Fig. 9** Systematic uncertainties of the SPFs obtained by SOM and SAI&MEM at $1.5T_c$ at small $\omega$ range (*Left*) and large $\omega$ range (*Right*).

## V. Conclusion

■ Stochastic methods gave almost DM-independent stable SPFs having a clear bound state peak at $0.75T_c$.

■ Most of the results suggest that $J/\psi$ may be melted around $1.5T_c$ but more detailed study is needed to conclude.

■ So far in this preliminary study, we observed an upper bound of $2\pi TD$ which is $1.5\sim2$ at $1.5T_c$, while a lower bound is not clear.

## VI. References

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