Finite temperature lattice QCD

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August 3 – 2016

In this review

$$T > 0$$

$$\mu = 0$$

$$B = 0$$

Outline

- Determinations of T_c
- Equation of state
- Cosmological application: axion

Determinations of T_c from $\chi_{\bar{\psi}\psi}$ peak

m_{phys}, continuum

- stout staggered: $T_c = 151(3)(3)\ MeV$ (WB hep-lat/0609068)

- HISQ staggered: $T_c = 154(9)\ MeV$ (HotQCD 1111.1710)

m_{phys}, $N_t = 8$

- domain wall: $T_c = 155(1)(8)\ MeV$ (HotQCD 1402.5175)

Note: T_c from Polyakov loop and/or $\chi_s \sim 20 - 25\ MeV$ higher

Determinations of T_c

Full agreement among different staggered discretizations

Lessons: fully controlled continuum limit, physical quark masses, balance between $T = 0$ and $T > 0$

Huge success of lattice QCD!

Note 1: Continuum Wilson results available but $m > m_{phys}$

Note 2: Domain wall $m = m_{phys}$ but single lattice spacing

Note 3: Flavor content is $2 + 1$ (okay around T_c)

Equation of state

Important input to hydrodynamical models of quark-gluon plasma
$\longrightarrow$ used by experimentalists/phenomenologists

Energy momentum tensor: $T_{\mu\nu}$

$$\frac{T_{\mu\mu}(T)}{T^4} = \frac{I(T)}{T^4} = \frac{\varepsilon - 3p}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right)$$

$$c_s^2 = \frac{dp}{d\varepsilon}$$

$$\frac{s}{T^3} = \frac{\varepsilon + p}{T^4}$$

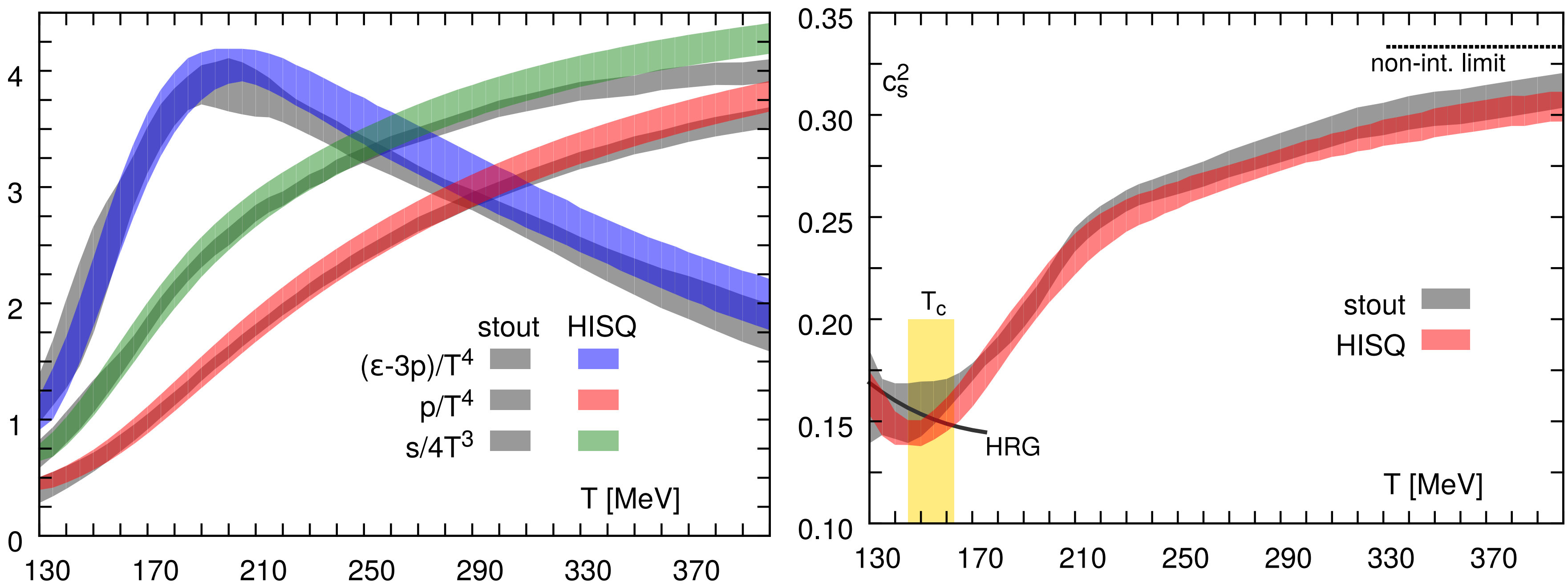
Equation of state

Typically calculated up to $2T_c, \;\; 3T_c, \;\; 4T_c, \;\; \ldots$

Flavor content $2+1$ or rather $2+1+1$

m_{phys} and continuum results are available

Equation of state, 2 + 1



Soltz DeTar Karsch Mukherjee Vranas 1502.02296

WB: 1309.5258

HotQCD: 1407.6387

Agreement another huge success of lattice QCD!

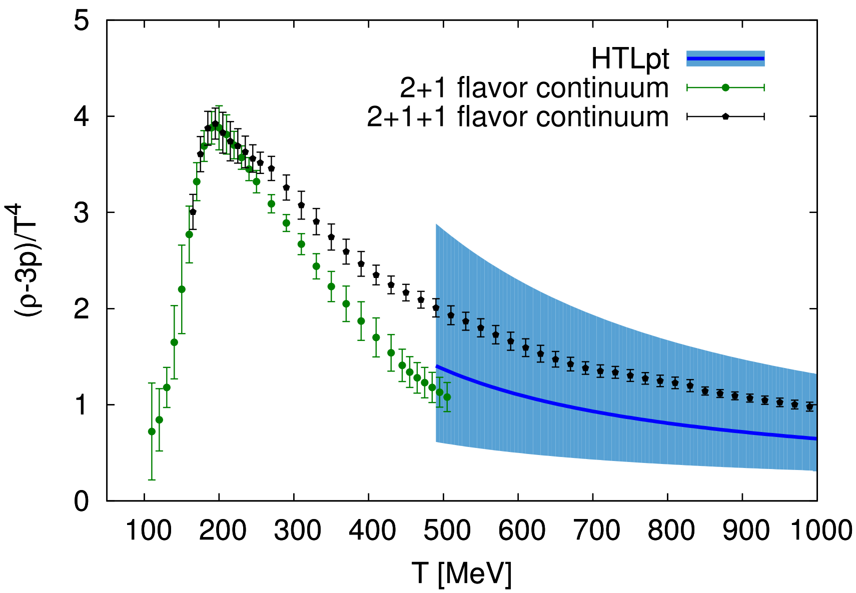
Note 1: Only staggered (but different) discretizations so far

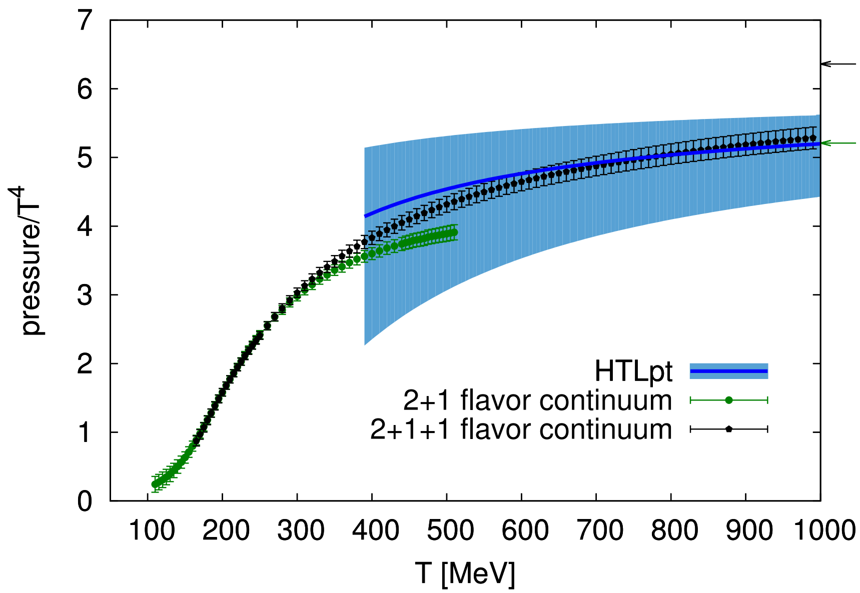
Note 2: These are $2+1$ flavor, but $m_c = 1.3\,GeV$

Equation of state, $2+1+1$

Perturbative expectation: at around 300 MeV charm becomes non-negligible

Borsanyi Fodor Kampert Katz Kawanai Kovacs Mages Pasztor Pittler Redondo Ringwald Szabo 1606.07494 (m_{phys}, continuum)

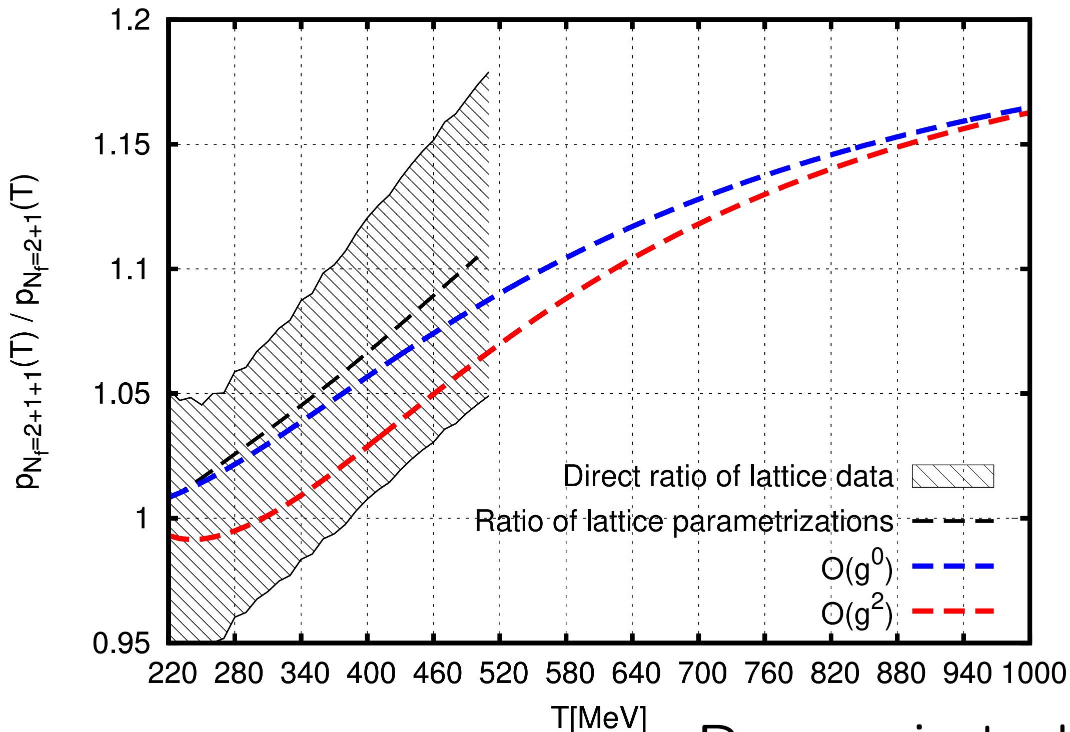




HTL: $2+1+1$ flavor NNLO

Equation of state, $2+1+1$

Apparently, charm contribution for ratio is well-described by leading order perturbation theory



Borsanyi et al. 1606.07494

$$\frac{p_{(2+1+1)}(T)}{p_{(2+1)}(T)} = \frac{SB(3) + F(m_c/T)}{SB(3)}$$

Equation of state, $2+1+1+1$?

Bottom threshold: 2 additional steps

- Bottom mass dependence from leading order ratio

$$\frac{p_{(2+1+1+1)}(T)}{p_{(2+1+1)}(T)} = \frac{SB(4) + F(m_b/T)}{SB(4)}$$

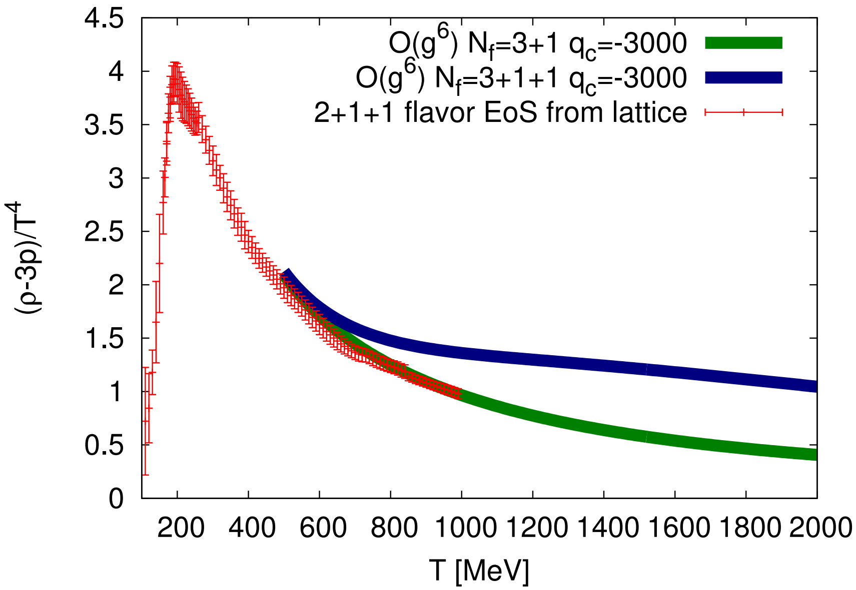
Hindmarsh Philipsen hep-ph/0501232

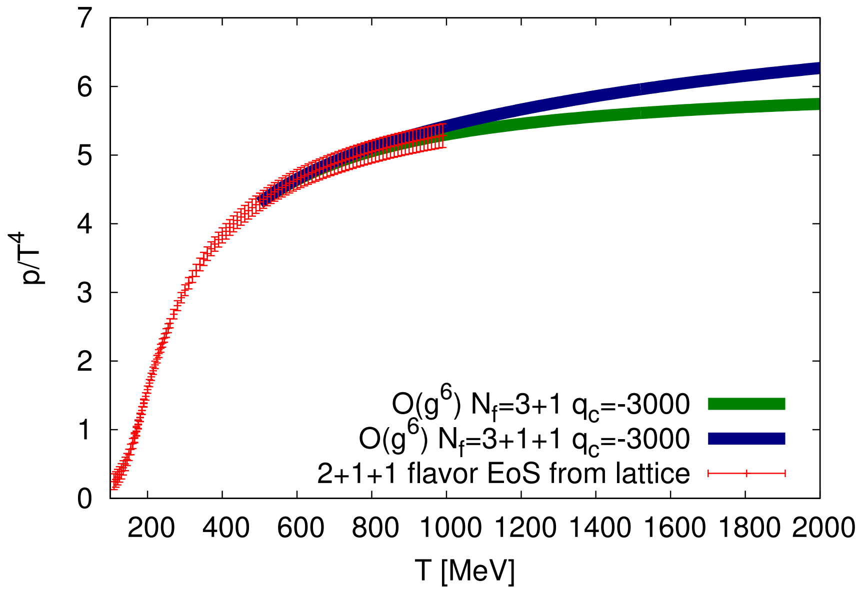
Laine Schroder hep-ph/0603048

- Use $2+1+1$, $O(g^6)$ perturbative formula, fit unknown coefficient q_c on $500\ldots1000$ MeV directly from continuum extrapolated lattice data

Combine the 2 steps: extend EoS to include bottom up to 10 GeV

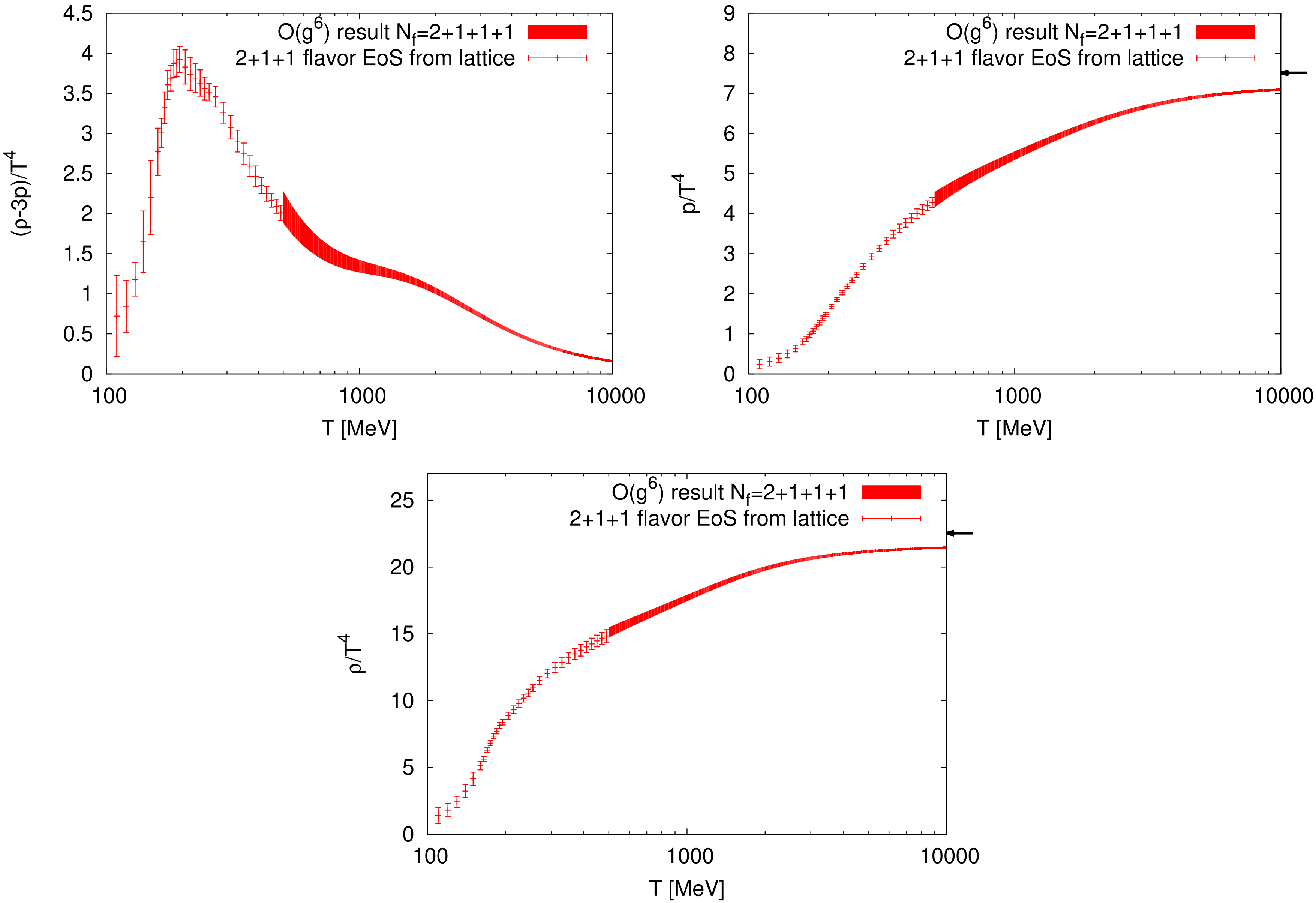
Equation of state, $2+1+1+1$





Borsanyi et al. 1606.07494

Equation of state, $2+1+1+1$, up to $T \sim 10\ GeV$



Borsanyi et al. 1606.07494

Equation of state, $2+1+1+1$, up to $T \sim 10\,GeV$

All very nice, would be even nicer:

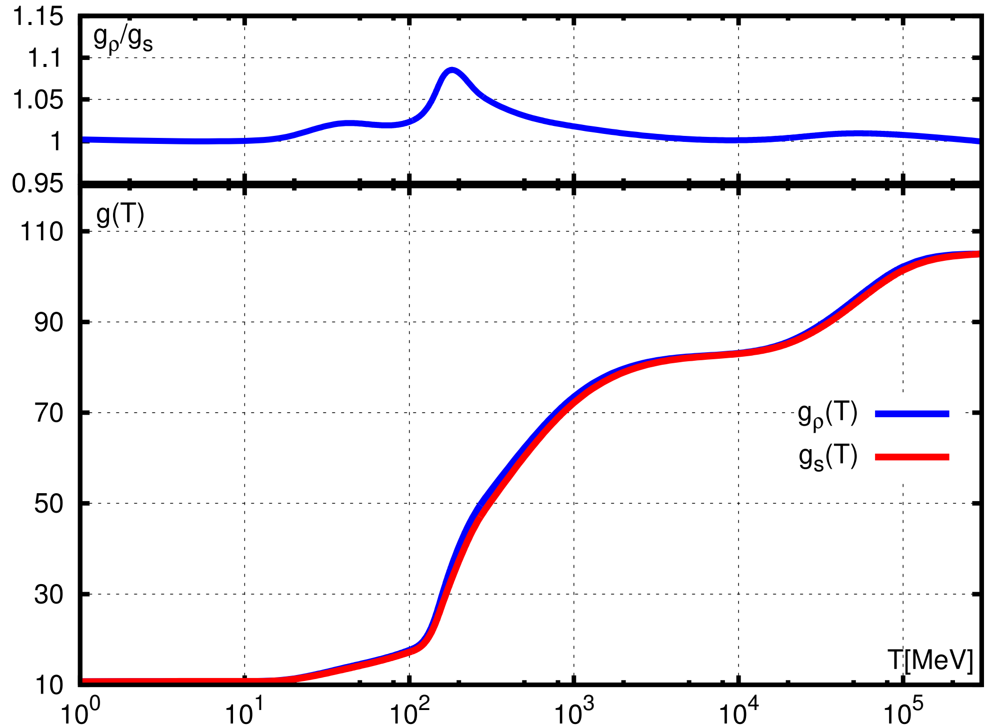
Independent cross-check of $2+1+1$ continuum m_{phys} EoS with different discretization

Twisted mass 1510.02262 (gluonic contribution only)

Equation of state, up to $T \sim 100\ GeV$

Rest of the Standard Model: added as before, Laine Schroder hep-ph/0603048, Laine Meyer 1503.04935

$$\frac{\varepsilon}{T^4} = \frac{\pi^2}{30} g_\rho \qquad\qquad \frac{s}{T^3} = \frac{2\pi^2}{45} g_s$$



Borsanyi et al. 1606.07494

Axion physics (very) briefly

Peccei-Quinn: solution to strong CP-problem

KSVZ variant:

$$\mathcal{L}(\phi,\Psi) = \partial_\mu\phi^*\partial_\mu\phi + V(\phi^*\phi) + \phi\bar{\Psi}_L\Psi_R + \phi^*\bar{\Psi}_R\Psi_L + \bar{\Psi}D(A)\Psi + \mathcal{L}_{QCD}(A)$$

V: Mexican hat, SSB, vev $= f_A$, $\phi = f_A e^{i\theta}$, $f_A\theta$: axion

Chiral rotation of Ψ and $U(1)$ rotation of ϕ by $\theta(x)$:

$$\mathcal{L} = f_A^2\partial_\mu\theta\partial_\mu\theta + \partial_\mu f_A\partial_\mu f_A + V(f_A^2) + f_A\bar{\Psi}\Psi + \bar{\Psi}D(A)\Psi +$$

$$+\theta q(x) + \partial_\mu\theta\bar{\Psi}\gamma_5\gamma_\mu\Psi + \mathcal{L}_{QCD}(A)$$

Axion

$$\mathcal{L} = f_A^2 \partial_\mu \theta \partial_\mu \theta + \partial_\mu f_A \partial_\mu f_A + V(f_A^2) + f_A \bar{\Psi}\Psi + \bar{\Psi} D(A) \Psi +$$

$$+\theta q(x) + \partial_\mu \theta \bar{\Psi} \gamma_5 \gamma_\mu \Psi + \mathcal{L}_{QCD}(A)$$

f_A assumed to be large $\to$ radial excitation, Ψ can be integrated out, at low energy only θ + QCD

Low energy Lagrangian:

$$\mathcal{L} = f_A^2 \partial_\mu \theta \partial_\mu \theta + i\theta q + \partial_\mu \theta \cdot (\ldots) + \mathcal{L}_{QCD}$$

If originally θ_{QCD} present, $\theta \to \theta + \theta_{QCD}$

Effective potential is generated for constant θ

Axion

Effective potential is generated for constant θ

$$e^{-V_4 V_{eff}(\theta)} = \langle e^{iQ\theta} \rangle$$

At high temperature $V_{eff}(\theta, T) = \chi(T)(1 - \cos(\theta))$

Axion mass: $f_A^2 m_A^2(T) = \chi(T)$, purely QCD quantity

Mexican hat tilted by V_{eff}, degeneracy lifted, new minimum at $\theta = 0 \rightarrow$ misalignment or realignment

$\longrightarrow$ Strong CP solved

Axion

Key assumption: $\theta = const$, no axion strings, domain walls, etc.

For cosmological evolution: $\varepsilon(T)$ and $s(T)$ needed, beside $\chi(T)$

All 3 are purely QCD quantities

Axion, topological susceptibility

Expectation from 1-instanton

$$\chi(T) \sim \frac{1}{T^b}$$

$$b = 11 - 2/3N_f - 4 + N_f$$

$11 - 2/3N_f$: from β-function
4: dimension 4
N_f: from determinant with light non-zero mass

Expected to work at high temperature, corrections from DIGA $\rightarrow b(T)$

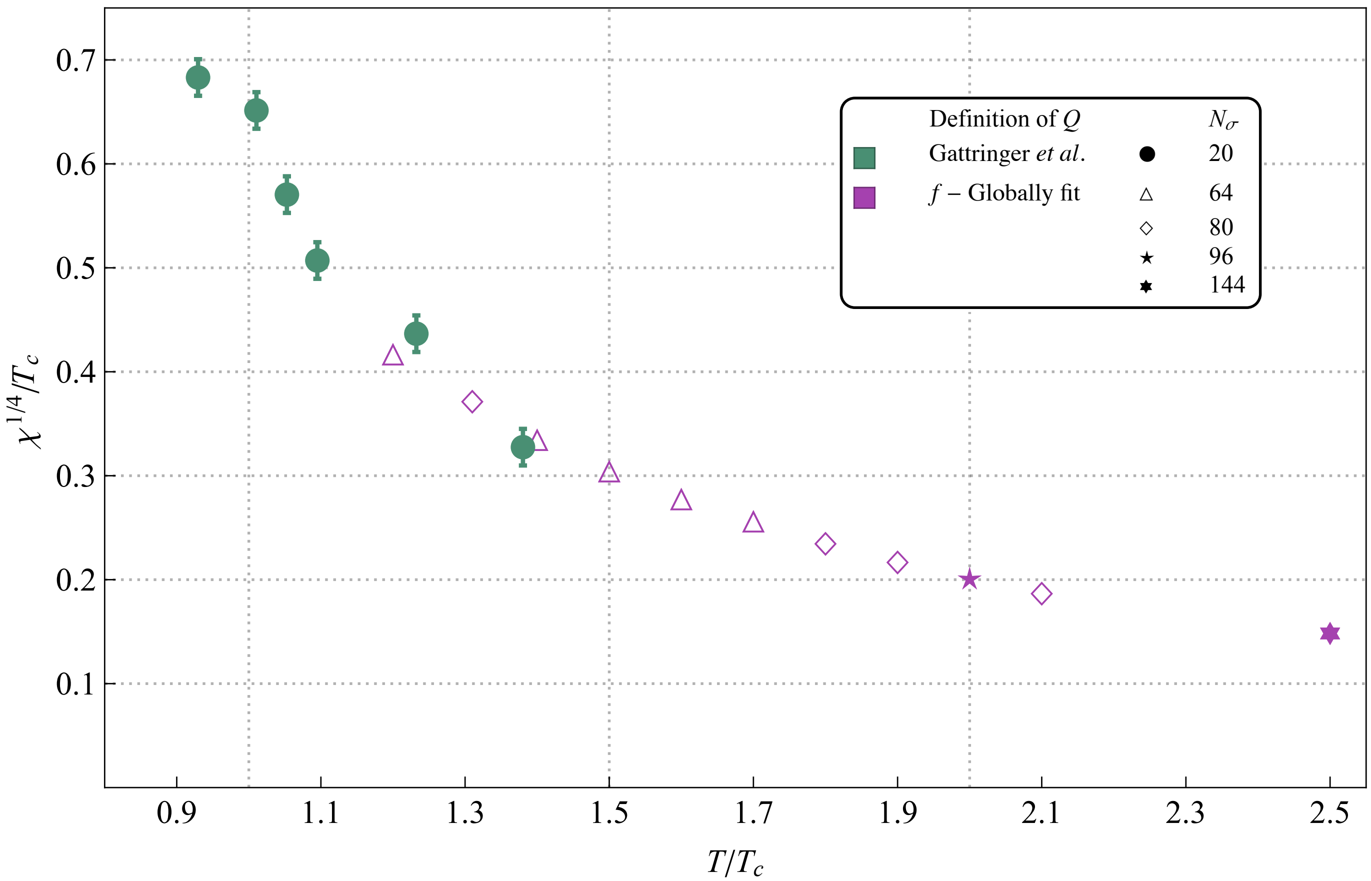
Axion, topological susceptibility

Even in pure gauge very difficult: $\chi(T)$ for high T

Because $\chi(T) = T\frac{\langle Q^2 \rangle}{V_3}$ is tiny and Q integer

Axion, topological susceptibility, pure gauge

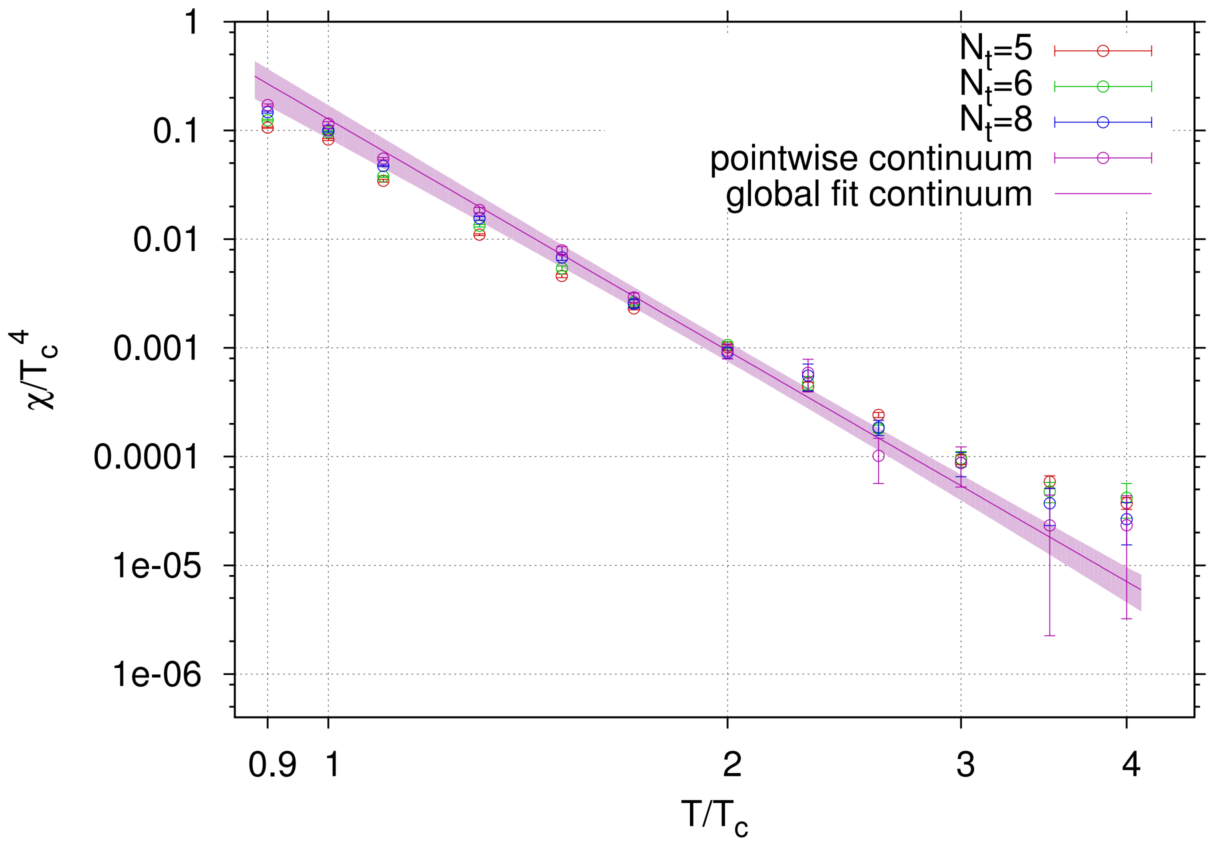
Berkowitz Buchoff Rinaldi 1505.07455



$N_t = 6$, $b = 5.64(4)$, no systematic errors, $T \sim 2.5T_c$

Axion, topological susceptibility, pure gauge

Borsanyi Dierigl Fodor Katz Mages N Redondo Ringwald Szabo 1508.06917

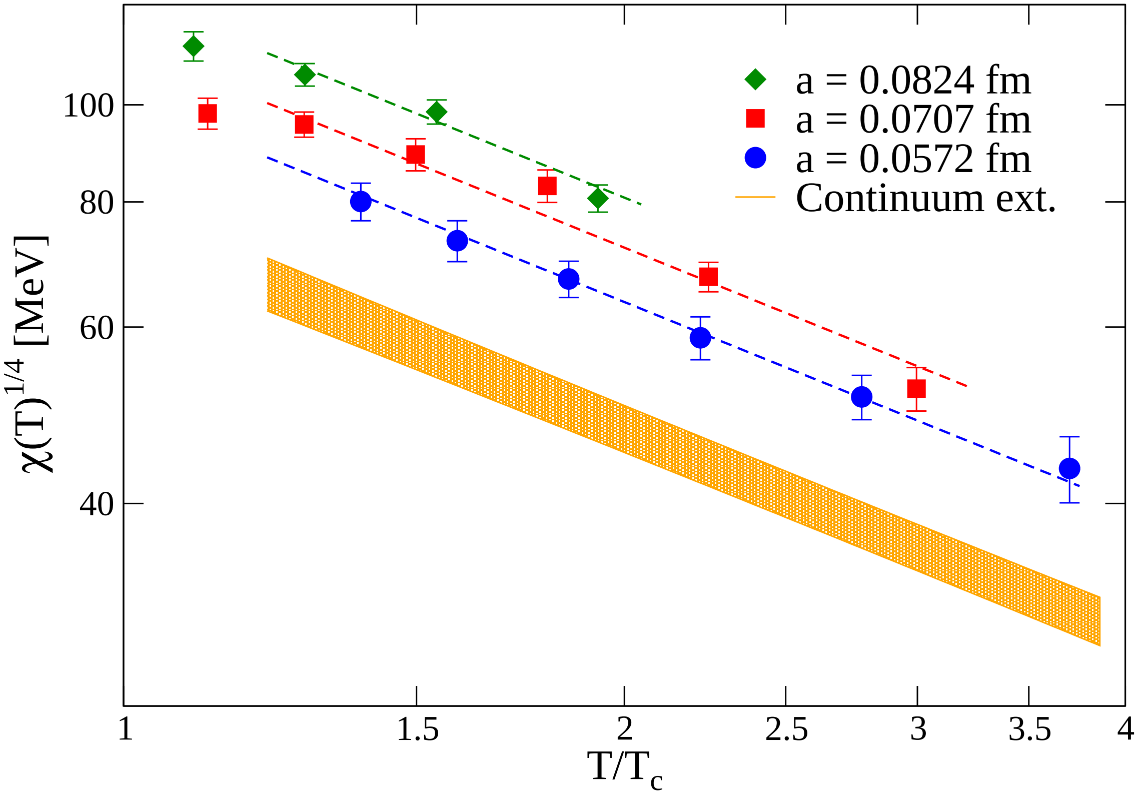


Continuum from $N_t = 5, 6, 8$, $b = 7.1(4)(2)$, $T \sim 4T_c$, over-all factor mismatch with DIGA

Axion, topological susceptibility

First $2+1$ dynamical result (staggered, stout, m_{phys})

Bonati D'Elia Mariti Martinelli Mesiti Negro Sanfilippo Villadoro
1512.06746



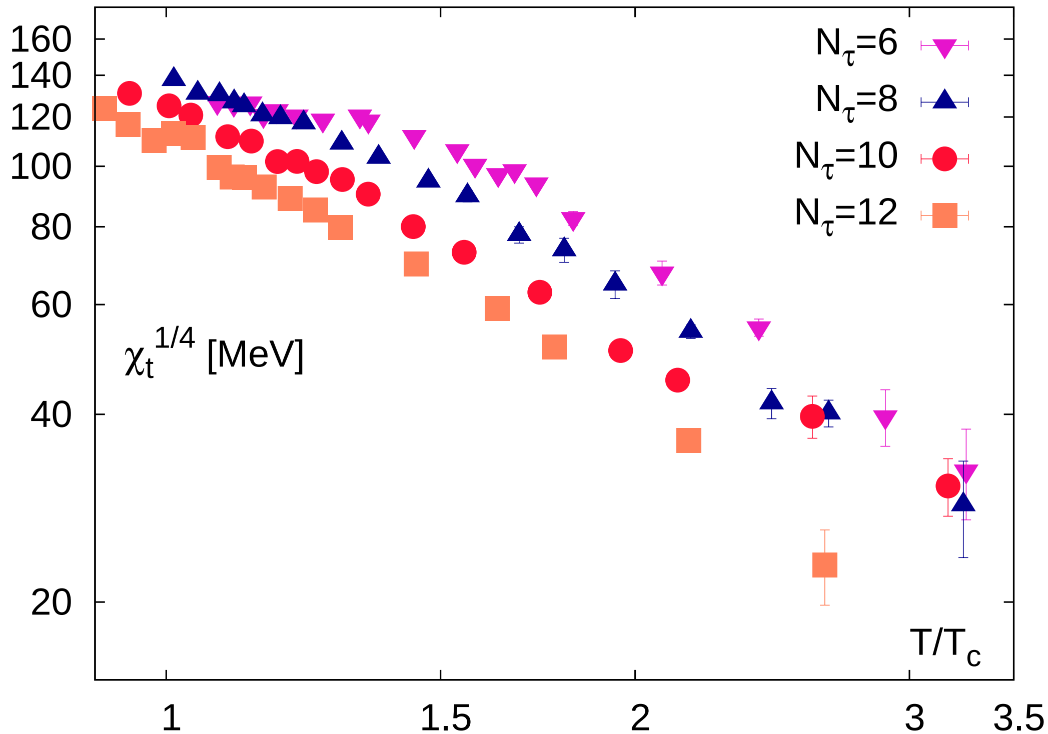
Fixed scale approach, $b = 2.90(65) \ll 8$, $T \sim 3.5 T_c$

Note: cut-off effects large

Axion, topological susceptibility

Another 2 + 1 dynamical staggered result (HISQ, $m_\pi = 160\ MeV$)

Petreczky Schadler Sharma 1606.03145

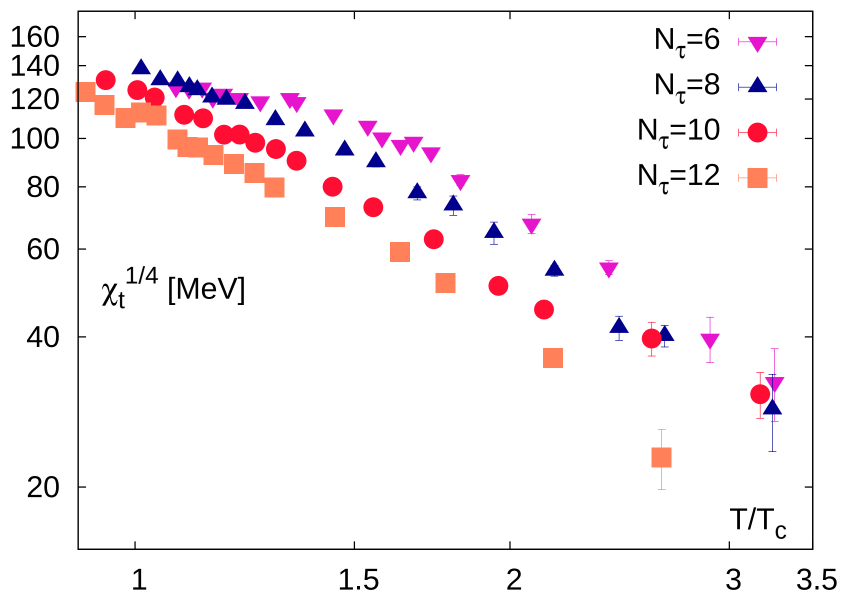


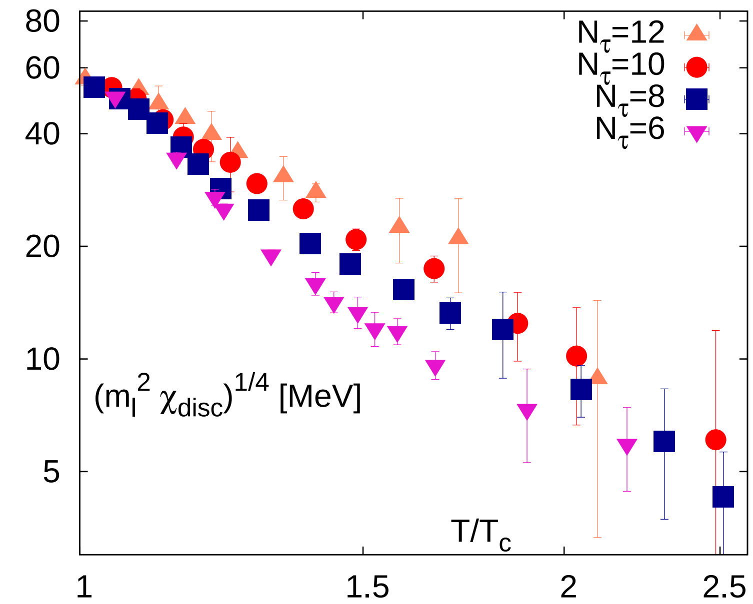
Note: cut-off effects large, gluonic definition for Q (as everybody else)

Axion, topological susceptibility

Petreczky Schadler Sharma 1606.03145

Use fermionic definition $Q = m\,\mathrm{Tr}\,D^{-1}\gamma_5 \to \chi = m^2\chi_{5,disc}$

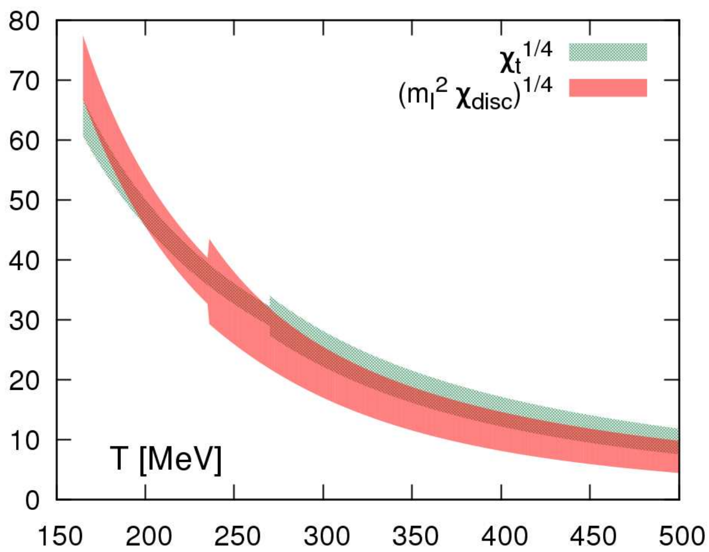


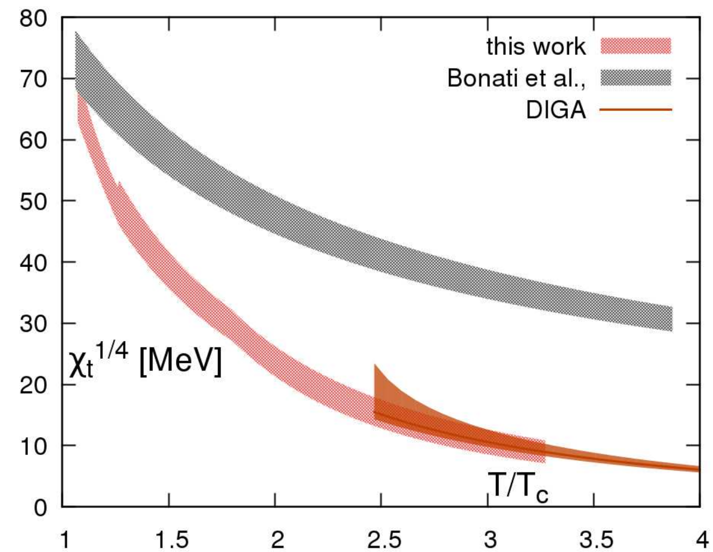


Note: direction of continuum limit opposite (good!) but still large

Axion, topological susceptibility

Petreczky Schadler Sharma 1606.03145





Note 1: $b = 7.4(6)$ for $T > 1.5T_c$

Note 2: Bonati et al. reproduced from $N_t = 6, 8$

Due to large cut-off effects, final result has large errors, even for $\chi^{1/4}$

Axion, topological susceptibility

Q1: Why are there so large cut-off effects on fine lattices with staggered fermions?

Q2: Is there a more efficient way of calculating χ at high T in general?

A1: Due to lack of exact zero-modes $\rightarrow$ not sufficient suppression of $Q \neq 0$ sectors $\rightarrow$ measured Q^2 too large $\rightarrow$ measured b too small (Borsanyi et al. 1606.07494)

A2: Integral method at fixed-Q (pure gauge: Frison Kitano Matsufuru Mori Yamada 1606.07175, dynamical: Borsanyi et al. 1606.07494, same day submission to arXiv)

See Julien Frison's poster!

Q1: Why are there so large cut-off effects on fine lattices (high T) with staggered fermions?

Related: How to get smaller errors at high T?

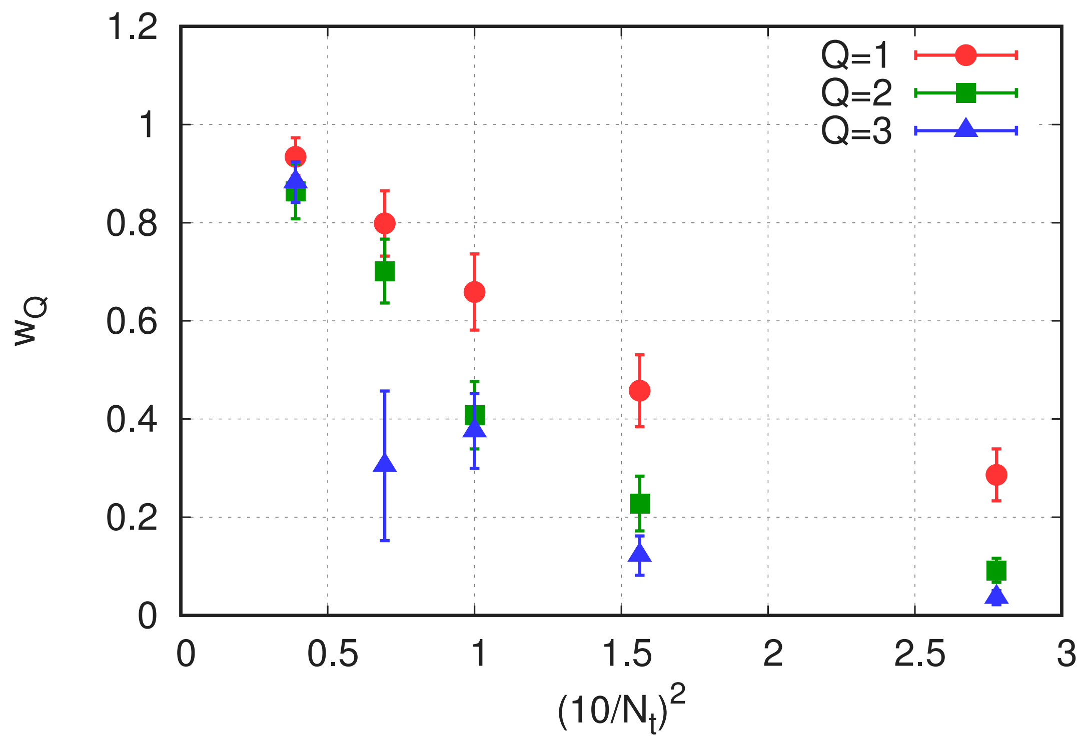
Borsanyi et al. 1606.07494

- Fix definition of Q (e.g. gradient flow)
- Look at lowest Q eigenvalues $\lambda_i(U) + m$, $\lambda_i(U) \neq 0$
- Should be (in continuum) m only
- Reweight for each flavor

$$w(U) = \prod_i \frac{m}{\lambda_i(U) + m}$$

Q1: Why are there so large cut-off effects on fine lattices (high T) with staggered fermions?

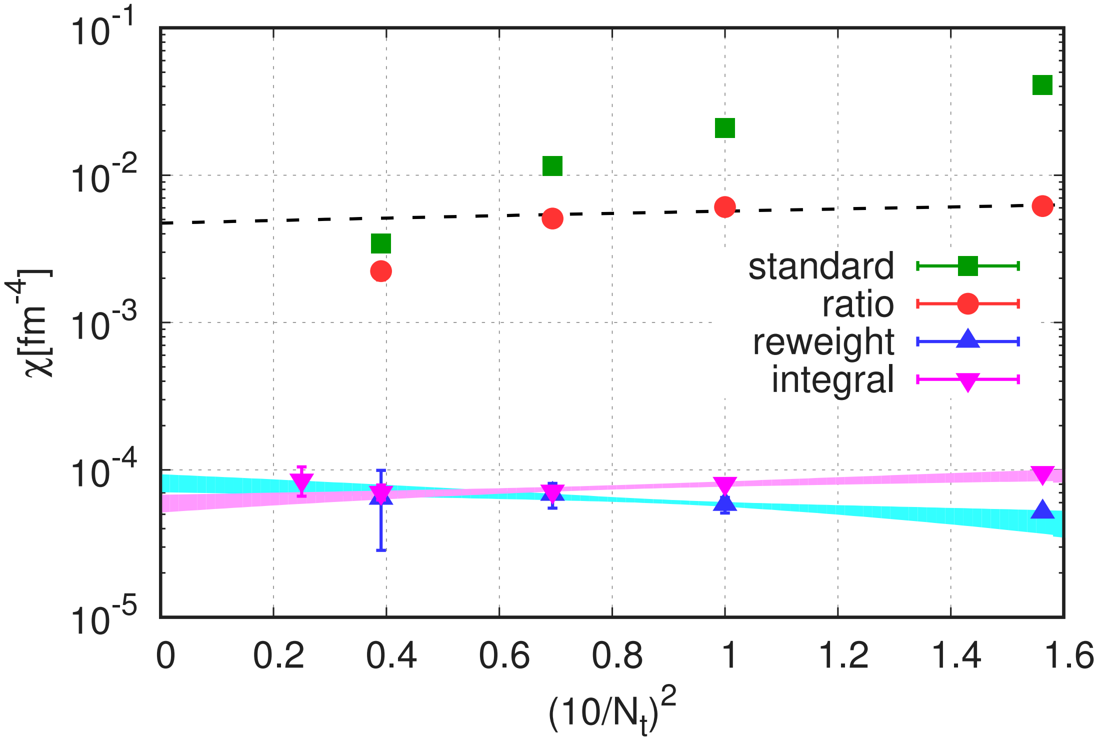
Reweighting: replaces the “wrong” low eigenvalues with the “correct” ones



Borsanyi et al. 1606.07494

Axion, topological susceptibility

Zero mode reweighting



Borsanyi et al. 1606.07494

$T = 300\ MeV$, Ratio: $\chi(T)/\chi(0)$

Axion, topological susceptibility

Q2: Is there a more efficient way of calculating χ at high T in general?

Fix definition of Q, Z_Q: partition function in sector Q

At high T only $|Q| = 1$ relevant, $Z_{-1} = Z_1$

$$\langle Q^2 \rangle = \frac{2Z_1}{Z_0}$$

Take derivatives: $b_1 = T \frac{d \log(Z_1/Z_0)}{dT}$

Get Z_1/Z_0 by integrating b_1 in T

Fix-Q integral method

pure gauge: Frison Kitano Matsufuru Mori Yamada 1606.07175

Julien's poster

dynamical: Borsanyi et al. 1606.07494

On the lattice in pure gauge:

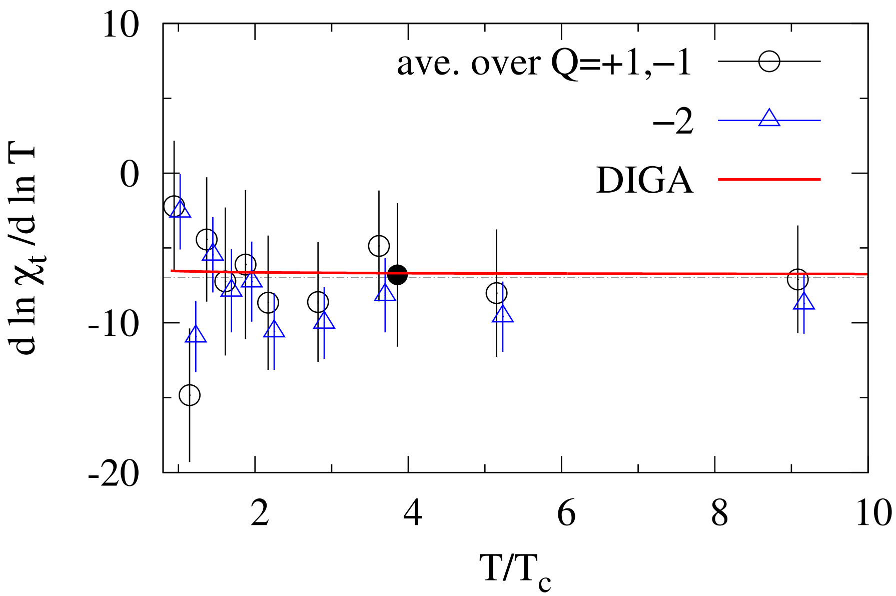
$$b_1 = a \frac{d\beta}{da} \langle S_g \rangle_{1-0}$$

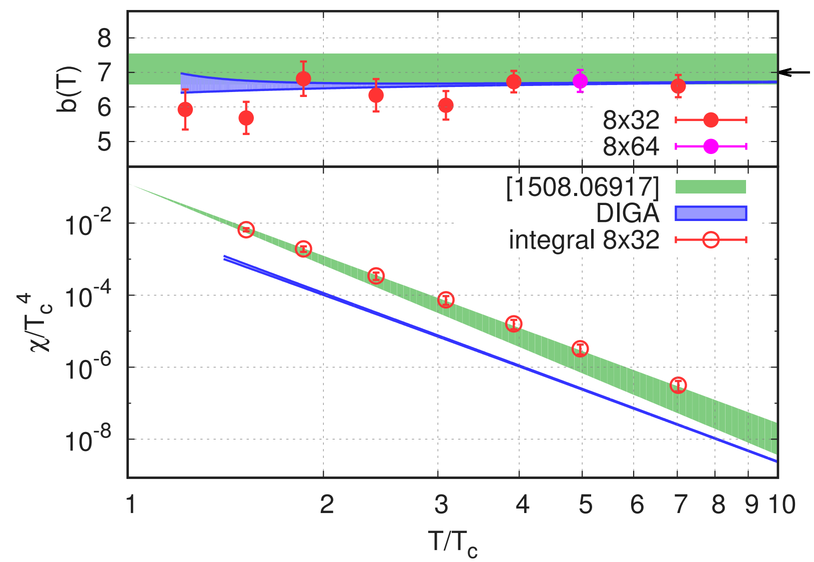
Need to measure $\langle S_g \rangle_{1-0} = \langle S_g \rangle_1 - \langle S_g \rangle_0$ in fixed-Q simulations and need scale $\beta(a)$

Important: difficulty does not grow with T

Axion, topological susceptibility

Fix-Q integral method in pure gauge





Left: Frison Kitano Matsufuru Mori Yamada 1606.07175
Right: Borsanyi et al. 1606.07494, pink: $Q = 8$ (less statistics needed)

Note: over-all factor mismatch with DIGA, as before

Fix-Q integral method with staggered dynamical fermions
Borsanyi et al. 1606.07494

$$b_1 = a\frac{d\beta}{da}\langle S_g\rangle_{1-0} + \sum_f \frac{d\log m_f}{d\log a} m_f \langle\bar{\psi}_f\psi_f\rangle_{1-0}$$

Line of constant physics enters in fermionic contribution

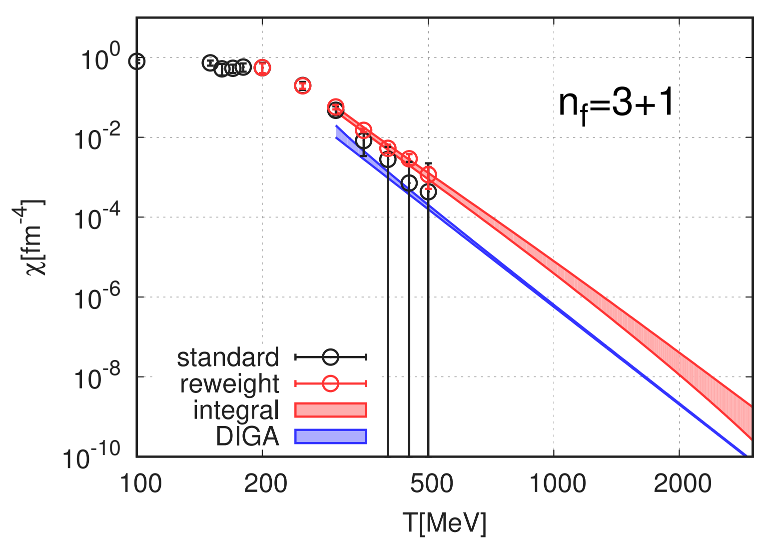
Integration in both β and m

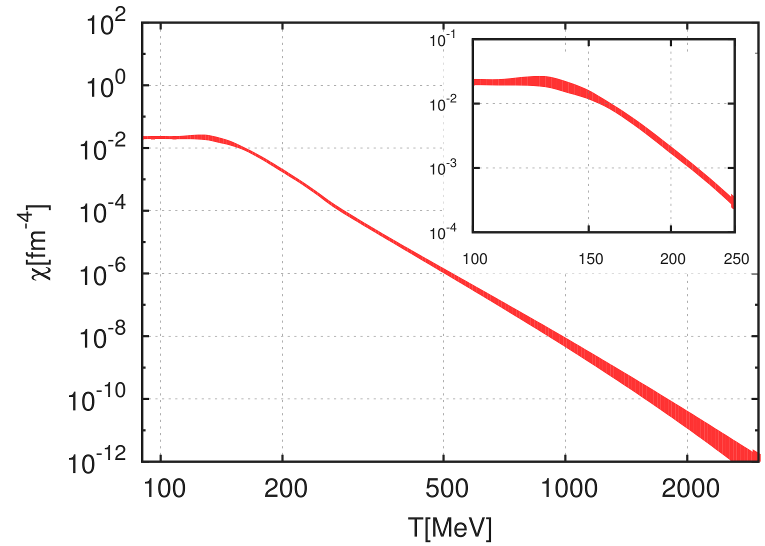
Note 1: $m_f\langle\bar{\psi}_f\psi_f\rangle_{1-0}$ finite $\rightarrow$ can integrate in m separately, using staggered or overlap fermions

Note 2: zero mode reweighting $w(U)$ has m-dependence

Axion, topological susceptibility

Fix-Q integral method with staggered/overlap dynamical fermions
Borsanyi et al. 1606.07494





Left: $3 + 1$
Right: $2 + 1 + 1$ integrating with overlap in mass

Continuum, m_{phys}

Axion cosmology

Once $\chi(T) = f_A^2 m_A^2(T)$, $\varepsilon(T)$, $s(T)$ available:

Axion equations of motion + Einstein equations

$$\frac{d^2\theta}{dt^2} + 3H(T)\frac{d\theta}{dt} + m_A^2(T)\frac{d}{d\theta}(1-\cos\theta) = 0$$

$$H^2(T) = \frac{8\pi}{3M_{pl}^2}\varepsilon(T)$$

$$\frac{d\varepsilon}{dt} = -3H(T)s(T)T$$

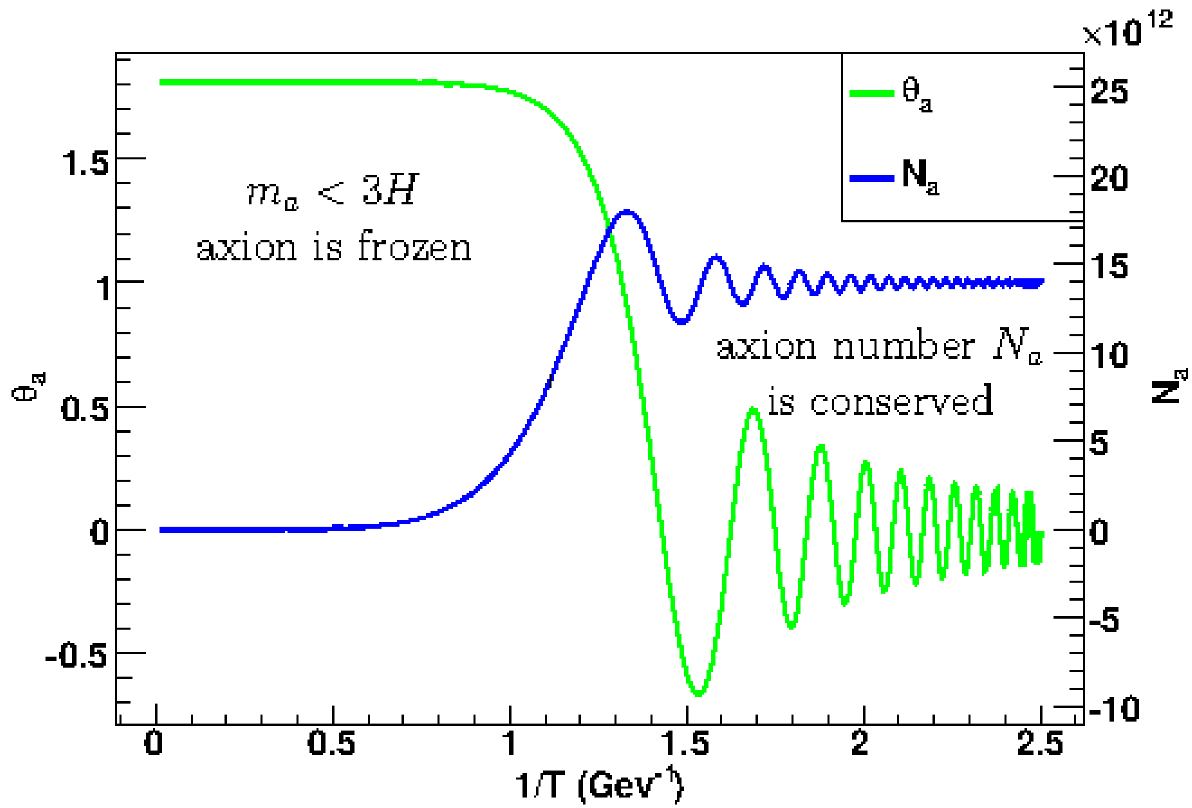
Key assumption: $\theta(x) = \theta(t)$ spatially constant $\rightarrow$
no strings, domain walls, etc

Numerical integration straightforward from initial $\theta_0 \to \theta(T, \theta_0, m_A)$

Qualitatively:

- Initially, $3H(T) \gg m_A(T)$, axion massless, number density $n_A = 0$
- Later, $m_A(T)$ increases, $H(T)$ decreases, θ goes towards zero
- At some point $3H(T) = m_A(T)$, oscillations, n_A jumps to non-zero value

Axion cosmology



Wantz Shellard 0910.1066

Axion cosmology

- Decaying oscillations, θ settles

- N_A conserved afterwards, S also (adiabatic) $\rightarrow n_A/s$ also

- $n_A(today) = n_A(T)/s(T)s(today)$

- $s(today) = \frac{2\pi^2}{45}\frac{43}{11}T_{CMB}^3$, $T_{CMB} = 2.725K$ (neutrinos, photons)

- $\rightarrow \varepsilon_A(today) = m_A n_A(today)$ energy density of axions today

From the numerical integration: $\rightarrow \varepsilon_A(today, m_A, \theta_0)$

From the numerical integration: $\rightarrow \varepsilon_A(today, m_A, \theta_0)$

Interpretation depends on whether symmetry breaking (scale f_A) before or after inflation

- pre-inflation scenario: single θ_0 (we are in one domain)
- post-inflation scenario: all θ_0 visible $\rightarrow$ average over them

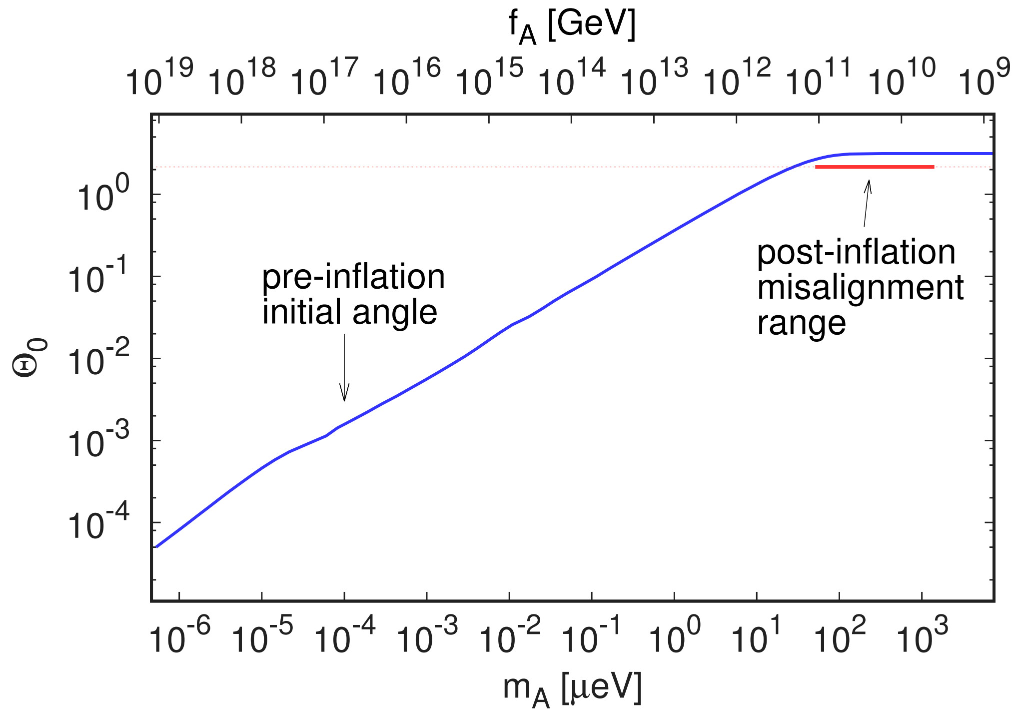
R_A: axion component of dark matter

$$\Omega_A = \frac{\varepsilon_A(today)}{\varepsilon_{crit}} \qquad R_A = \frac{\Omega_A}{\Omega_{DM}}$$

Pre-inflation scenario: $R = R(\theta_0, m_A)$, assuming a fix R value gives an implicit relation between θ_0 and m_A, for example $R = 1$. Measuring m_A in the future $\rightarrow$ initial condition of the Universe (θ_0), defects inflated away, less important

Post-inflation scenario: $\bar{R} = \bar{R}(m_A)$, assuming a fix $\bar{R}$ gives m_A directly. For example varying $\bar{R}$ between 1% and 50% $\rightarrow$ most plausible m_A-range, defects may be important

Axion cosmology, mass bounds



Borsanyi et al. 1606.07494

Only realignment: $m_A = 28(2)\mu eV$

50% realignment: $m_A = 50(4)\mu eV$

1% realignment: $m_A \sim 1500\mu eV$

Rest from defects (strings, domain walls, etc) within axion picture, potentially more types of DM also

Summary, continuum m_{phys} results

- T_c very mature from lattice QCD
- $2+1$ EoS also
- $2+1+1$ EoS available with stout staggered
- $\chi(T)$ available up to more than $10T_c$
- Solid axion mass bounds from QCD

Haven't talked about lots of interesting developments

- Renormalization of energy momentum tensor $\longrightarrow$ gradient flow (Monday 15:15 Yusuke Taniguchi)
- Correlators, transport coefficients (Monday 14:50 Victor Braguta)
- Finite density (Tuesday)
- Fluctuations of conserved charges at $\mu = 0$ (today Christian Schmidt)
- QCD on non-orientable manifolds, χ
- Shifted boundary conditions (pure gauge EoS high precision)
- etc.

Thank you for your attention!