Charm quark diffusion coefficient and relaxation time on the quenched lattice

Atsuro Ikeda, Masayuki Asakawa, Masakiyo Kitazawa

Osaka University

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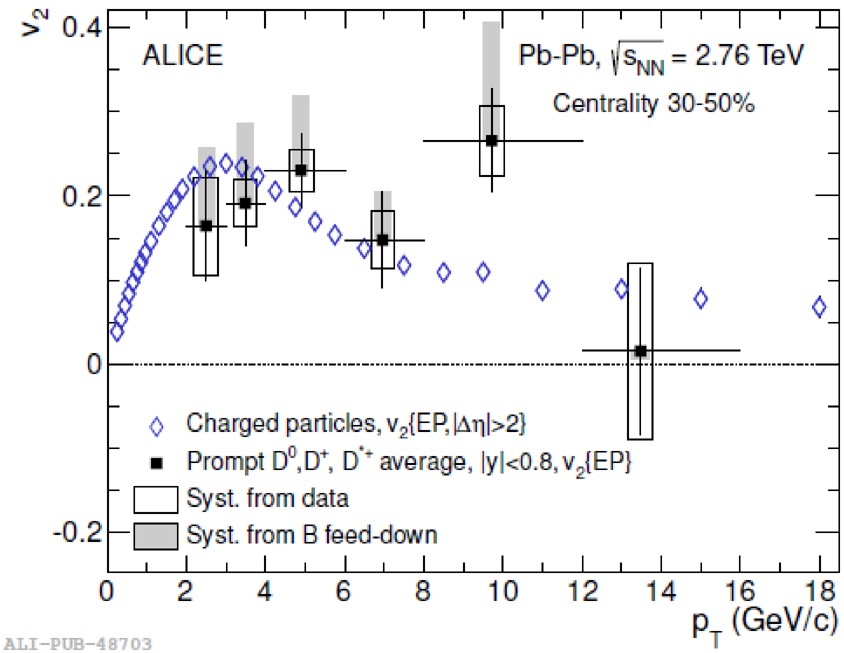
Anisotropic flow of open charm

- Large elliptic flow of open charm
  →charm flow ~ medium flow
- Rapid thermalization of charm quarks?

- **Diffusion coefficient** is an important quantity



Transport coefficient on the lattice

- Shear viscosity

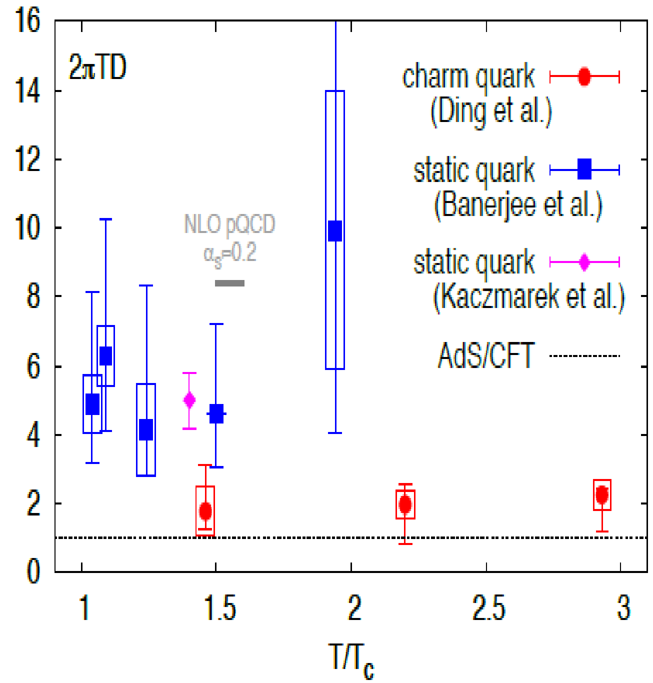
Karsch and Wyld 1987, Nakamura and Sakai 2005, Meyer 07, Haas 2013, Borsanyi et al. 2014, etc…

- Electric conductivity

Gupta 2004, Aarts et al. 2014, etc…

- Quark diffusion coefficient

Ding et al. 2011, Banerjee et al. 2012, Aarts et al. 2015, Francis et al. 2015 etc…



Ding et al. arXiv:1504.05274

There is a numerical difficulty, called ill-posed problem, and analyses still have uncertainty.

Measurement of Diffusion coefficient

Kubo formula

$$D = \frac{\pi}{3}\frac{1}{\chi}\lim_{\omega\to 0}\lim_{\vec{p}\to 0}\frac{\rho_{ii}(\omega,\vec{p})}{\omega}$$

$$G^E_{\mu\mu}(\tau,p) = \int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle j_\mu(\tau,\vec{x}) j^\dagger_\mu(0,\vec{0})\right\rangle$$

$$= \int_0^\infty d\omega \frac{\cosh((1/2T-\tau)\omega)}{\sinh(\omega/2T)}\rho_{\mu\mu}(\omega,\vec{p})$$

for $\mu = 0,1,2,3$

ill-posed problem

1. Ansatz for spectral function
   - Depend on ansatz
   - Lattice Euclidean correlator has a lattice artifact
2. Maximum entropy method
   - Reconstructed spectral function has the strong correlation in whole ω-space
   - Not sensitive to low energy structure

Our strategy

3. Structure of $G_{00}(\tau,p^2)$ *(new)*
   - $\omega^2\rho_{00}(\omega,p) = p_i p_j \rho_{ij}(\omega,p) = p^2\rho_L(\omega,p)$

➡ High energy component of $\rho_{00}(\omega,p)$ is suppressed by $1/\omega^2$ comparing with $\rho_{ii}(\omega,p)$

Linear response theory

- Consider the two relaxation process [Kadanoff and Martin 1963]

Classical source $h(r)$
$\delta\langle n(r)\rangle = \chi h(r)$
Turn off suddenly

compare

Perturbative Hamiltonian

$$H(r,z) = H_0(r) + \delta H(r,t)$$
$$\delta H(r,t) = e^{\epsilon t}\theta(-t)h(r)$$
$$\delta\langle n(r)\rangle = -i\int_{-\infty}^{t} dt'\langle[n(r,t),\delta H(r',t')]\rangle_{eq}$$

Relaxation process

Response lag caused by heavy quark mass

$$\left(\tau_{\mathrm{relax}}\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t}\right) j_0(x,t) = -D\nabla^2 j_0(x,t)$$

Low energy structure of the spectral function

$$\frac{\rho_{00}^{\mathrm{hydro}}(\omega,\vec{k})}{\omega} = \frac{1}{\pi}\frac{\chi D|\vec{k}|^2}{\omega^2 + \left(D|\vec{k}|^2 - \tau\omega^2\right)^2}$$

Kubo formula

$$\omega^2\rho_{00}(\omega,p) = p_i p_j \rho_{ij}(\omega,p)$$

$$D = \frac{\pi}{3}\frac{1}{\chi}\lim_{\omega\to 0}\lim_{\vec{k}\to 0}\frac{\rho_{ii}\left(\omega,\vec{k}\right)}{\omega}$$

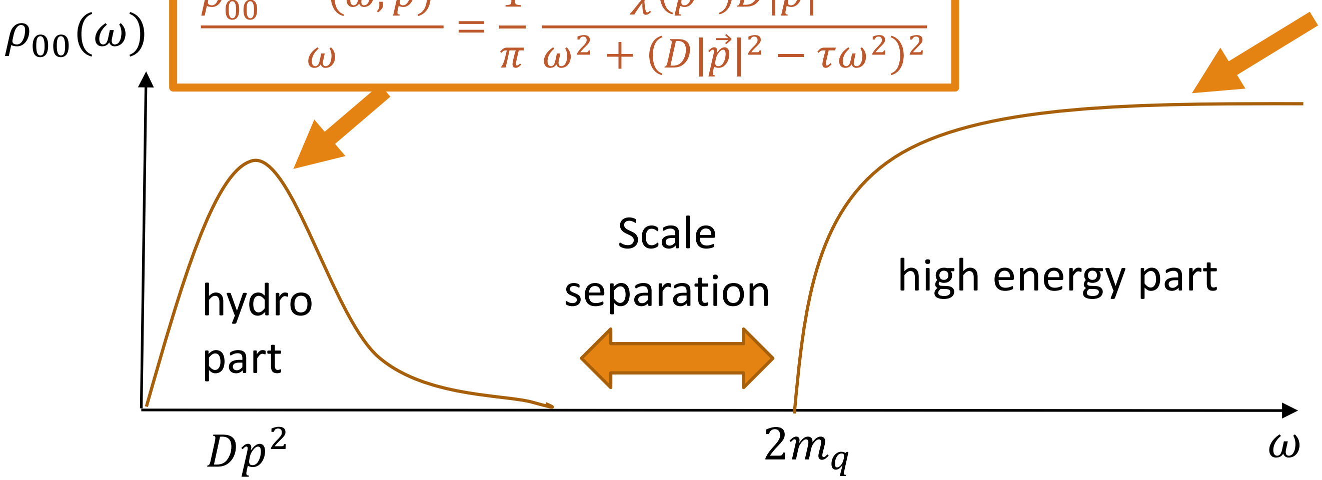
Assumptions

- Structure of the spectral function

$$\rho_{00}(\omega,\vec{p})=\rho_{00}^{hydro}(\omega,\vec{p})+\rho_{00}^{high}(\omega,\vec{p})$$

$$\frac{\rho_{00}^{\mathrm{hydro}}(\omega,\vec{p})}{\omega}=\frac{1}{\pi}\frac{\chi(p^2)D|\vec{p}|^2}{\omega^2+(D|\vec{p}|^2-\tau\omega^2)^2}$$

$$\rho_{\mu\mu}^{high}(\omega,\vec{p})\geq 0$$



- Quark number susceptibility

$$\chi(p^2)=\chi+\chi'\left(\frac{p}{T}\right)^2+\cdots$$

$$\chi'\ll\chi$$

$$G_{00}(\tau,0)=\chi T$$

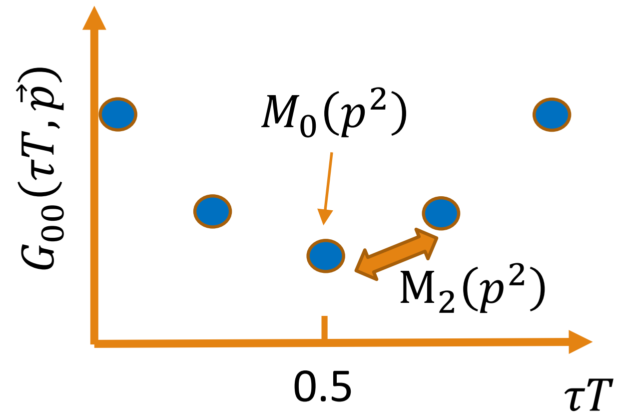
Mid-point expansion of $G_{00}(\tau, \mathrm{p})$

$$G_{00}(\tau,\vec{p}) = \int_0^\infty d\omega \left(1 + \frac{1}{2}\left(\frac{1}{2} - T\tau\right)^2 T^2\omega^2\right) \frac{\rho_{00}(\omega,\vec{p})}{\sinh\left(\frac{\omega}{2T}\right)} + O\left(\left(\frac{1}{2} - T\tau\right)^4\right)$$

$$\equiv M_0(p^2) + \frac{1}{2}\left(\frac{1}{2} - T\tau\right)^2 M_2(p^2) + \cdots$$

$G_{00}(\tau, \mathrm{p})$ around mid-point is the most sensitive to the low energy structure of the spectral function $\rho_{00}(\omega, p)$

But $M_0(0) = T\chi$ and $M_2(0) = 0$



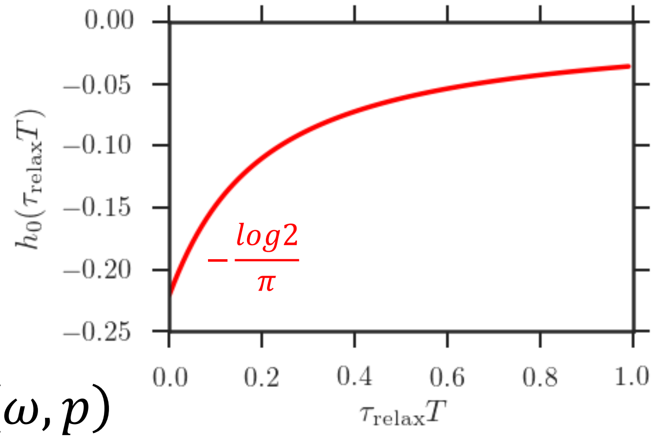
Study $\dfrac{\partial M_0(p^2)}{\partial \tilde{p}^2}$ *and* $\dfrac{\partial M_2(p^2)}{\partial \tilde{p}^2}$ at $\tilde{p} \to 0$

$$M_n(p^2) = M_n^{low}(p^2) + M_n^{high}(p^2)$$

$\tilde{p} \equiv p/T$

$\partial M_0(p^2)/\partial \tilde{p}^2$

$$\left.\frac{\partial M_0^{low}(p^2)}{\partial \tilde{p}^2}\right|_{\tilde{p}^2=0} = h_0(\tau_{\mathrm{relax}}T)\chi D T^2 + \chi' T$$



$$h_0(\tau_{\mathrm{relax}}T) \equiv \lim_{\tilde{p}^2 \to 0} \frac{\partial}{\partial(Dp^2)} \int_0^\infty d\omega \frac{1}{\sinh\left(\frac{\omega}{2T}\right)} \rho_{00}^{hydro}(\omega, p)$$

< 0

with $M_0(p^2) = M_0^{low}(p^2) + M_0^{high}(p^2)$ and $\frac{\partial}{\partial \tilde{p}^2} M_0^{high}(p^2) > 0$

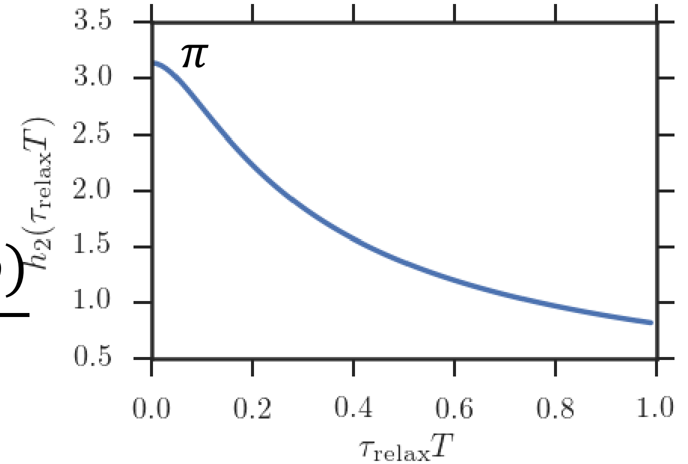
$$D_L T \equiv \frac{1}{h_0(\tau_{\mathrm{relax}}T)} \frac{T^2}{\chi} \left[\frac{\partial}{\partial \tilde{p}^2} \frac{M_0(p^2)}{T^3} - \frac{\chi'}{T^2} \right]\Bigg|_{p^2=0} < DT$$

$\partial M_2(p^2)/\partial \tilde{p}^2$

$$\left.\frac{\partial M_2^{low}(p^2)}{\partial \tilde{p}^2}\right|_{\tilde{p}^2=0} = h_2(T\tau_{\text{relax}})\chi D$$

$$h_2(T\tau_{\text{relax}}) \equiv \lim_{\tilde{p}^2\to 0}\int_0^\infty d\omega \frac{T^2}{\sinh\left(\frac{\omega}{2T}\right)} \frac{\partial \omega^2 \rho_{00}(\omega,p)}{\partial(Dp^2)}$$

> 0



with $M_2(p^2) = M_2^{low}(p^2) + M_2^{high}(p^2)$, $\frac{\partial}{\partial \tilde{p}^2} M_2^{high}(p^2) > 0$

$$DT < D_U T \equiv \frac{1}{h_2(T\tau_{relax})} \frac{\partial}{\partial \tilde{p}^2} \left.\frac{M_2(p^2)}{\chi T}\right|_{p^2=0}$$

$$D_L T < DT < D_U T$$

Opposite sign of $h_0 < 0$ and $h_2 > 0$

Lattice set up

- Quenched lattice
- Wilson Fermion and standard Wilson gauge action
$\beta = 7.0, \gamma_F = 3.476$

[Asakawa, Hatsuda 2004]

- Anisotropic lattice with

$\xi = \frac{a_\sigma}{a_\tau} = 4$ and $N_\sigma = 128$

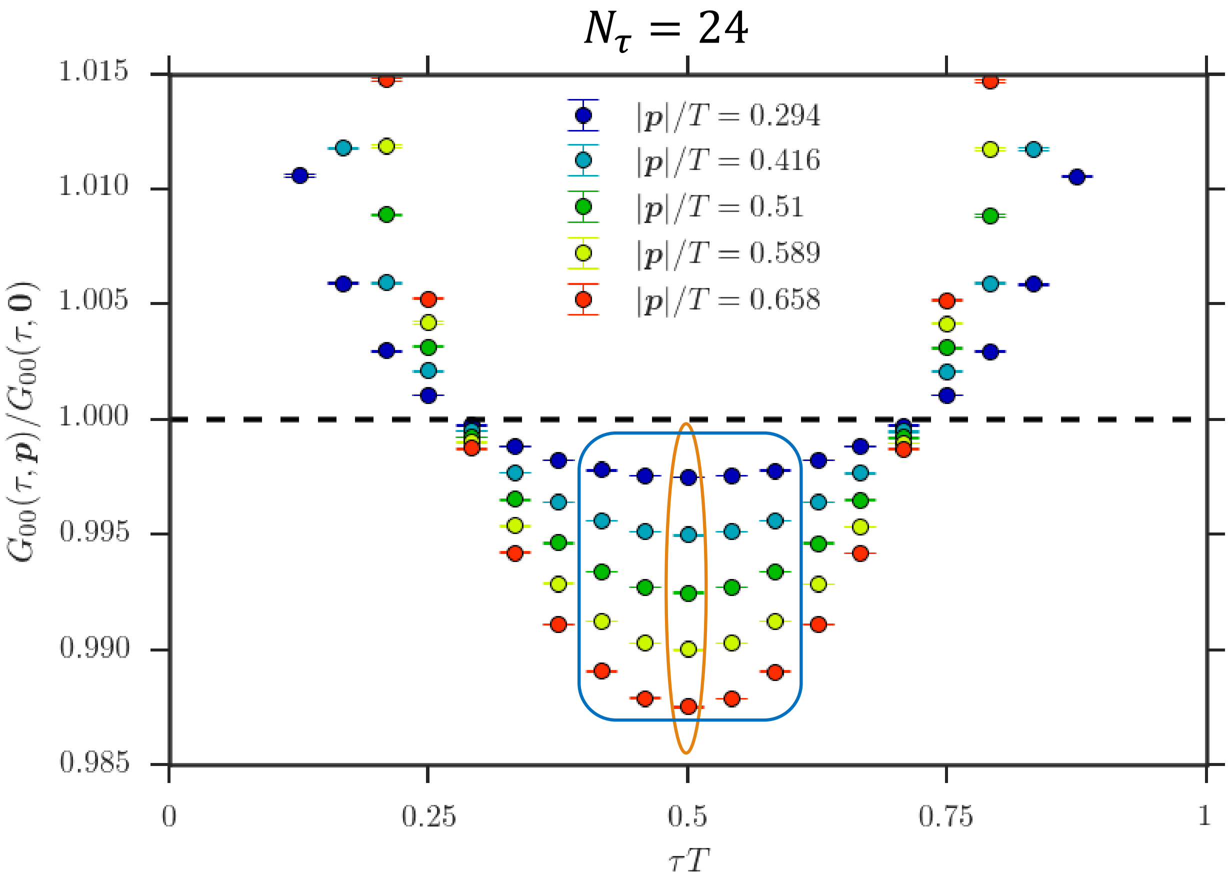
for high momentum resolution

$L_\sigma / L_\tau = 11.5 \sim 32$

N_τ	T/T_c	N_σ	$\Delta p/T$	Nconf
16	4.68	128	0.196	361
20	3.74	128	0.245	229
24	3.12	128	0.294	240
28	2.67	128	0.344	91
32	2.34	128	0.397	100
32	2.34	64	0.794	304
36	2.08	128	0.442	100
40	1.87	128	0.491	100
44	1.7	128	0.54	89

Blue Gene/Q@KEK
Iroiro++

$G^{E}_{00}(\tau, p)$



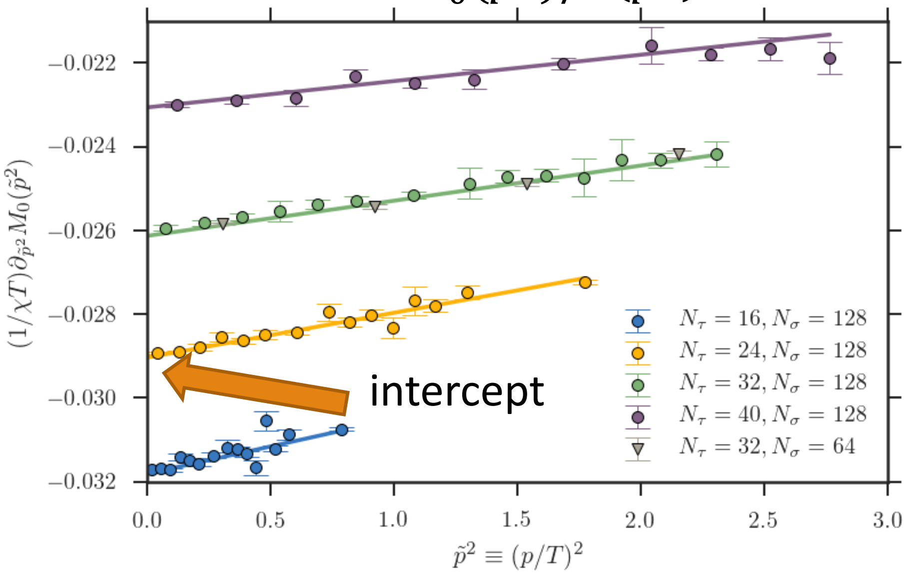
Momentum dependence of

1. Mid-point correlator
2. Curvature

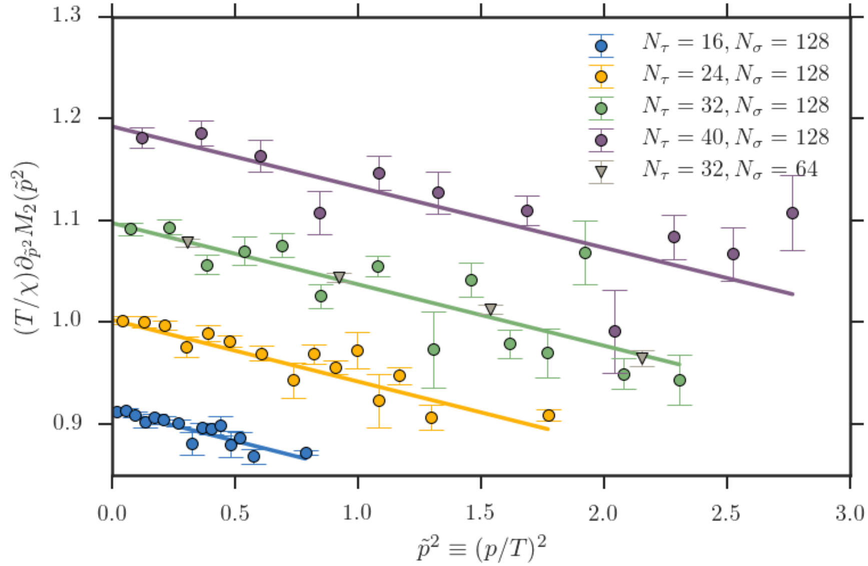
at $p \to 0$

$\partial M_0(p^2)/\partial(\tilde{p}^2)$ and $\partial M_2(p^2)/\partial(\tilde{p}^2)$

$\partial M_0(p^2)/\partial(\tilde{p}^2)$



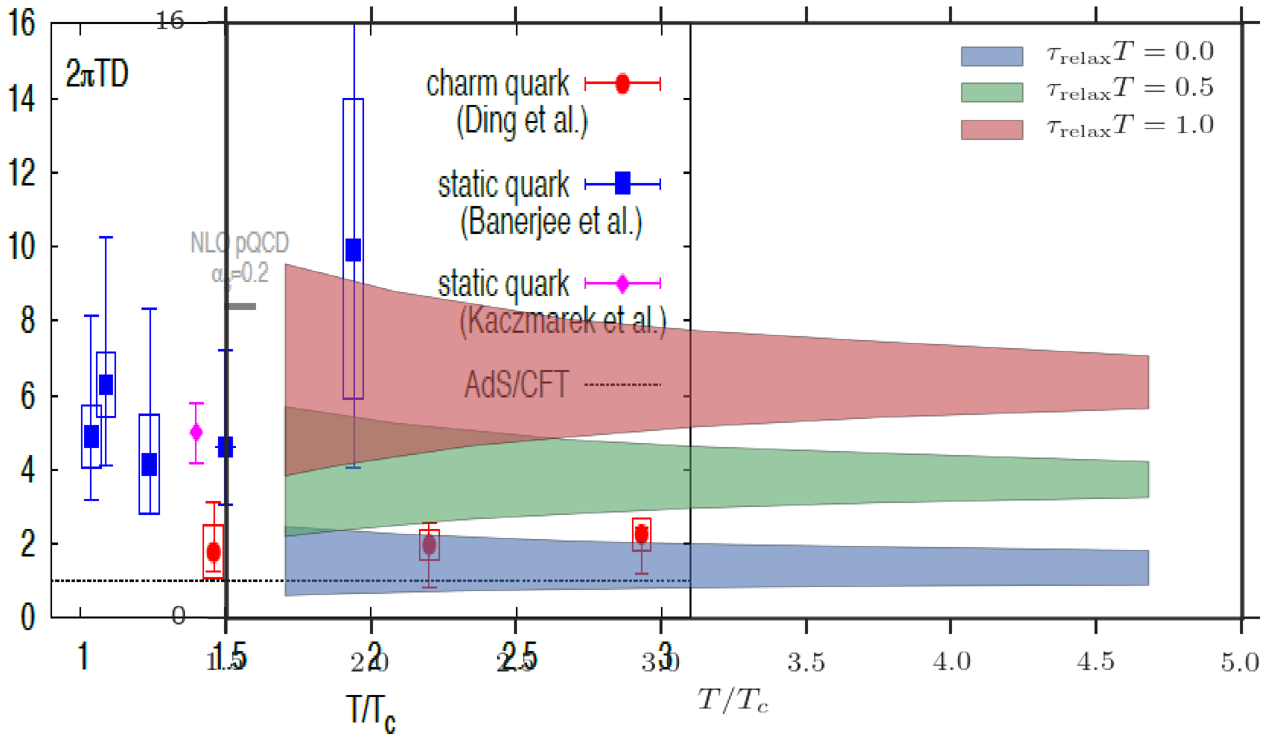
$\partial M_2(p^2)/\partial(\tilde{p}^2)$



- Fit with linear function where $\tilde{p}^2 < 1$
- From $N_\tau = 32$, finite volume dependence is well suppressed

Result: $D_L(\tau_{relax})T < DT < D_U(\tau_{relax})T$

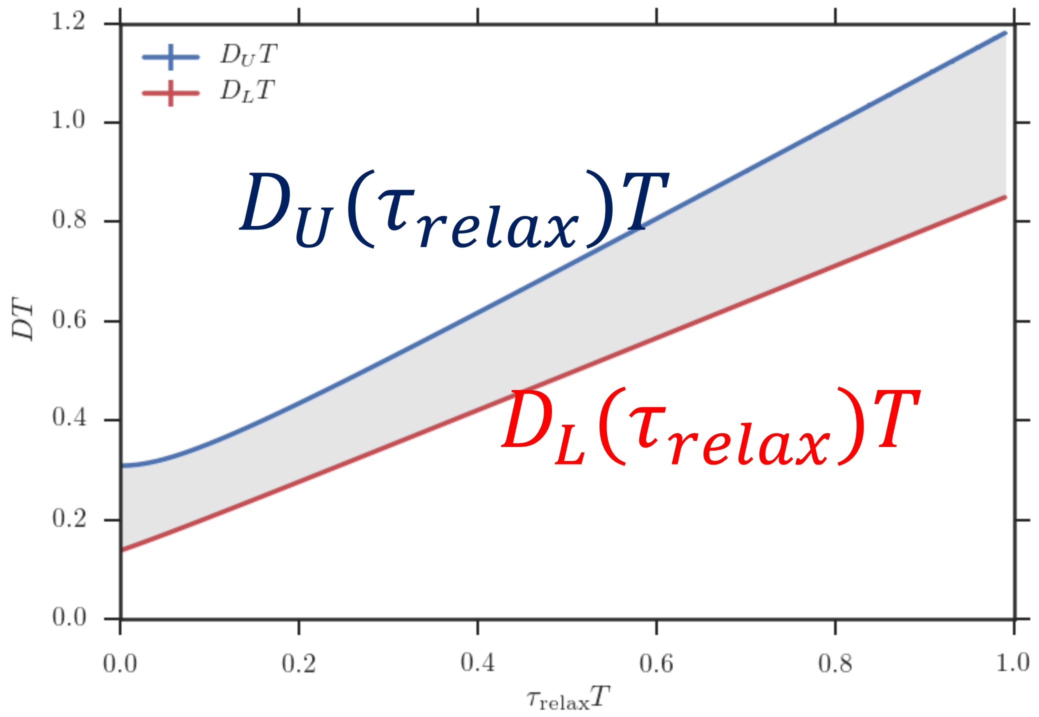
Ding et al. arXiv:1504.05274



- Consistent with previous works
- High energy contribution become larger for lower T
- Information on τ_{relax} is needed to determine D

Constraint on D and τ_{relax}

$N_\tau = 20,\ T/T_c = 3.74$



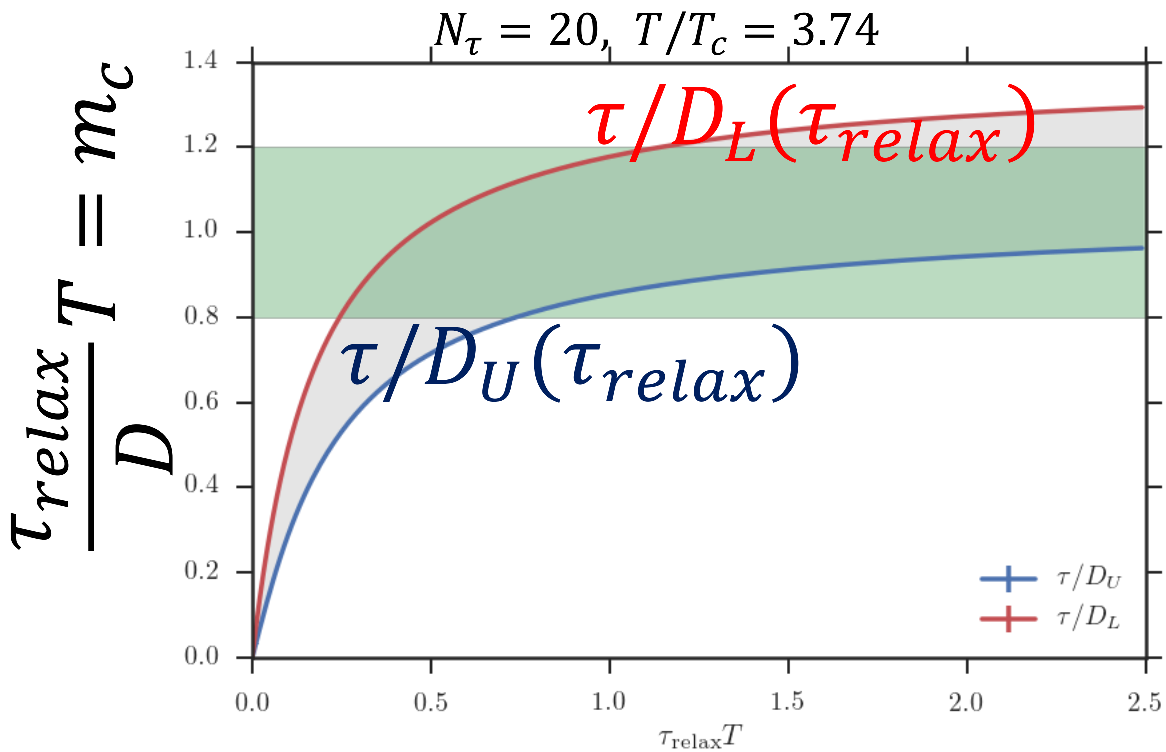
$D_L T$ at $\tau_{relax} = 0$ is still lower limit.

τ_{relax} from Langevin dynamics

$$\frac{\tau_{relax}}{D} T = m_c$$

kinetic m_c on the lattice?

from Langevin dynamics or heavy quark limit [Petreczky, Teany 2006 Caron-Huot et al. 2009]



We need τ_{relax} or τ_{relax}/D on the lattice

Conclusion

- Constraint on D and τ_{relax} in (D, τ_{relax})-plane from the p-dependence of the mid-point correlator $G_{00}^{E}\left(\frac{1}{2T}, p\right)$ on the lattice with basic assumptions for the spectral function $\rho_{00}(\omega, \vec{p})$.
- We obtain $\partial M_0(p^2)/\partial(\tilde{p}^2)$ and $\partial M_2(p^2)/\partial(\tilde{p}^2)$ with good statistics.
- Spatial volume dependence was well suppressed even with $\frac{L_\sigma}{L_\tau} = 8.$

Future work

- Can we measure $\chi' = \left.\frac{\partial \chi(p^2)}{\partial(p^2)}\right|_{p^2=0}$ on the lattice?
- Other information on D and τ_{relax}?
- Estimate of high energy contribution of $\rho_{\mu\mu}(\omega, \vec{p})$ (MEM, ansatz for spectral function): $D_L(\tau_{relax})T < DT < D_U(\tau_{relax})T \Rightarrow$ equality

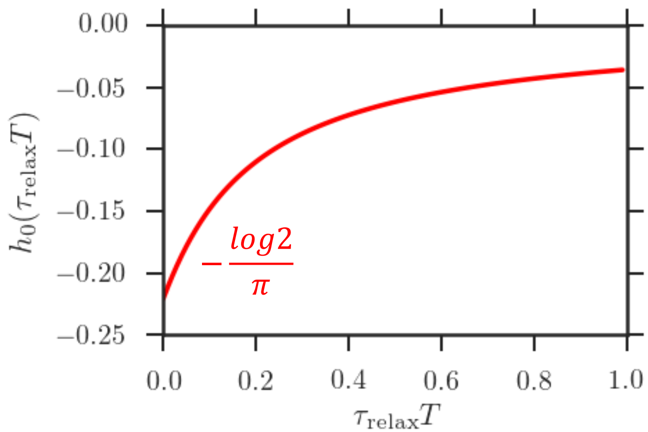
$h_0(\tau T)$ and $h_2(\tau T)$

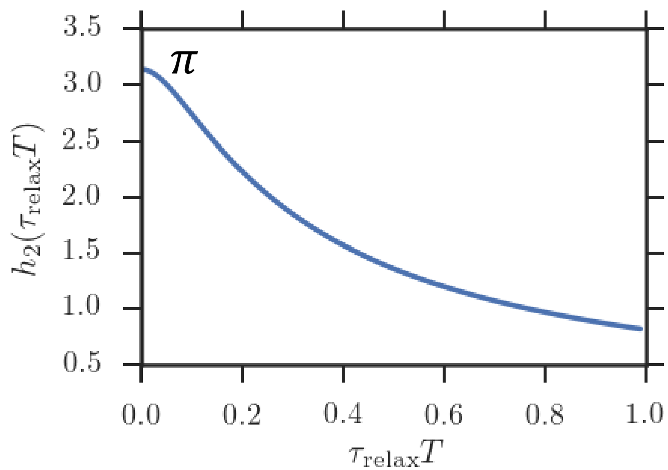
$$h_0(\tau_{\rm relax}T) = -\frac{log2}{\pi} + T\tau_{\rm relax}\left\{1 - F\left(\frac{1}{T\tau_{\rm relax}}\right)\right\} < 0$$

$$h_2(T\tau_{\rm relax}) = \frac{1}{T\tau_{\rm relax}} F\left(\frac{1}{T\tau_{\rm relax}}\right) > 0$$

$$F(a) \equiv \frac{a}{\pi}\int_0^\infty \frac{x}{x^2+1}\frac{1}{\sinh\frac{a}{2}x} = -1 + \frac{\mathrm{alog2}}{\pi} - \frac{\mathrm{a}}{\pi}\left[\Psi\left(\frac{a}{4\pi}\right) - \Psi\left(\frac{1}{2\pi}\right)\right]$$

$$\Psi(z) \equiv \frac{d}{dz} log\Gamma(z)$$





Low energy structure of $\rho_{00}(\omega, \vec{p})$

[Kadanoff and Martin 1963]

Consider the diffusion eq. as the relaxation process

$$\left(\tau_{\mathrm{relax}} \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t}\right) j_0(x,t) = D \nabla^2 j_0(x,t)$$

Response lag caused by heavy quark mass

Low energy structure of the spectral function

$$\frac{\rho_{00}^{\mathrm{hydro}}(\omega, \vec{p})}{\omega} = \frac{1}{\pi} \frac{\chi(\vec{p}) D |\vec{p}|^2}{\omega^2 + (D|\vec{p}|^2 - \tau\omega^2)^2}$$

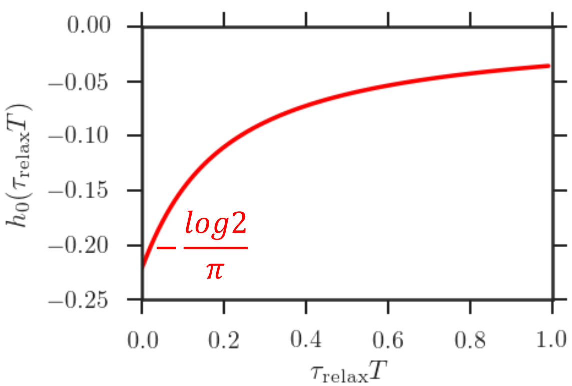
$$\omega^2 \rho_{00}(\omega, p) = p_i p_j \rho_{ij}(\omega, p)$$

Kubo formula

$$D = \frac{\pi}{3} \frac{1}{\chi} \lim_{\omega \to 0} \lim_{\vec{p} \to 0} \frac{\rho_{ii}(\omega, \vec{p})}{\omega}$$

$\partial M_0(p^2)/\partial\tilde{p}^2$

$$\frac{\partial M_0^{low}(p^2)}{\partial\tilde{p}^2} = \int_0^\infty d\omega \frac{1}{\sinh\left(\frac{\omega}{2T}\right)} \frac{\partial}{\partial(Dp^2)} \rho_{00}^{hydro}(\omega,p)\chi DT^2 + \chi' T$$



$$\xrightarrow{p^2\to 0} h_0(\tau_{\rm relax}T) = -\frac{log2}{\pi} + T\tau_{\rm relax}\left\{1 - F\left(\frac{1}{T\tau_{\rm relax}}\right)\right\} < 0$$

$$F(a) \equiv \frac{a}{\pi}\int_0^\infty \frac{x}{x^2+1}\frac{1}{\sinh\frac{a}{2}x} = -1 + \frac{a\log 2}{\pi} - \frac{a}{\pi}\left[\Psi\left(\frac{a}{4\pi}\right) - \Psi\left(\frac{1}{2\pi}\right)\right]$$

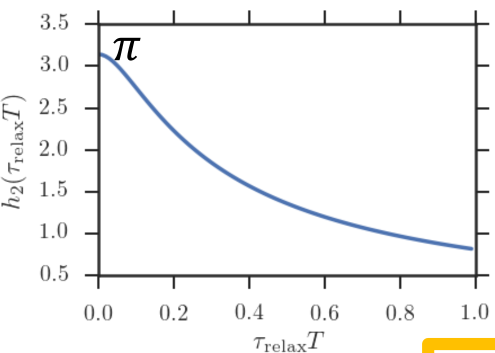
$$\Psi(z) \equiv \frac{d}{dz}log\Gamma(z)$$

with $M_0(p^2) = M_0^{low}(p^2) + M_0^{high}(p^2)$ and $\frac{\partial}{\partial\tilde{p}^2}M_0^{high}(p^2) > 0$

$$D_L T \equiv \frac{1}{h_0(\tau_{\rm relax}T)}\frac{T^2}{\chi}\left[\frac{\partial}{\partial\tilde{p}^2}\frac{M_0(p^2)}{T^3} - \frac{\chi'}{T^2}\right]\Bigg|_{p^2=0} < DT$$

$M_2(p^2)$: Momentum dependence

$$\frac{\partial}{\partial \tilde{p}^2} M_2^{low}(p^2) = \int_0^\infty d\omega \frac{1}{\sinh\left(\frac{\omega}{2T}\right)} \frac{\partial}{\partial (Dp^2)} \omega^2 T^2 \rho_{00}(\omega, p) \chi D + \frac{\chi'}{\chi} M_2^{low}(p^2)$$



$$\xrightarrow{p^2 \to 0} h_2(T\tau_{\rm relax}) = \frac{1}{T\tau_{\rm relax}} F\left(\frac{1}{T\tau_{\rm relax}}\right) > 0$$

with $M_2(p^2) = M_2^{low}(p^2) + M_2^{high}(p^2)$, $\frac{\partial}{\partial \tilde{p}^2} M_2^{high}(p^2) > 0$

$$DT < D_U T \equiv \frac{1}{h_2(T\tau_{relax})} \frac{\partial}{\partial \tilde{p}^2} \frac{M_2(p^2)}{\chi T}\bigg|_{p^2=0}$$

$$D_L T < DT < D_U T$$

Opposite sign of $\mathrm{h}_0 < 0$ and $h_2 > 0$