Anderson localisation of Dirac eigenmodes in high temperature QCD

Analysis of background gauge fields

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August 1st 2016

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Outline

- Anderson localization basics (Hamiltonian & disorder)
- QCD spectrum at low and high temperature
- Random Matrix Theory (RMT), level spacing
- Finite size scaling studies, multifractals at criticality
- What is the disorder in QCD? **(*)**
- Dirac low-modes: analysis of background gauge fields
- Perspective and future works

Universality and the QCD Anderson Transition, **Giordano** et al., **PRL 112, 102002 (2014)**
An Ising-Anderson model of localisation in high-temperature QCD, **Giordano** et al., **JHEP 1504 (2015) 112**
Anderson transition and multifractals in the spectrum of the Dirac operator of Quantum Chromodynamics at high temperature, Ujfalusi et al., **1507.02162v1 [cond-mat.dis-nn]**
An Anderson-like model of the QCD chiral transition, **Giordano** et al. **JHEP 1606 (2016) 007**
…
(*) Anderson Localization in high temperature QCD: background configuration properties and Dirac eigenmodes, GC and **S. Hashimoto**, **JHEP 1606(2016) 056**

Anderson Localization

Metal-insulator transition (MIT)

P.W. Anderson, paper March 1958

Absence of diffusion in certain random lattices

Phys. Rev. 109 (5): 14921505

Spatial **localization** of the states of a system due to multiple quantum interference caused by **disorder**

Nobel prize 1977

Anderson Localization (AL)

Tight binding model Hamiltonian, non interacting electrons

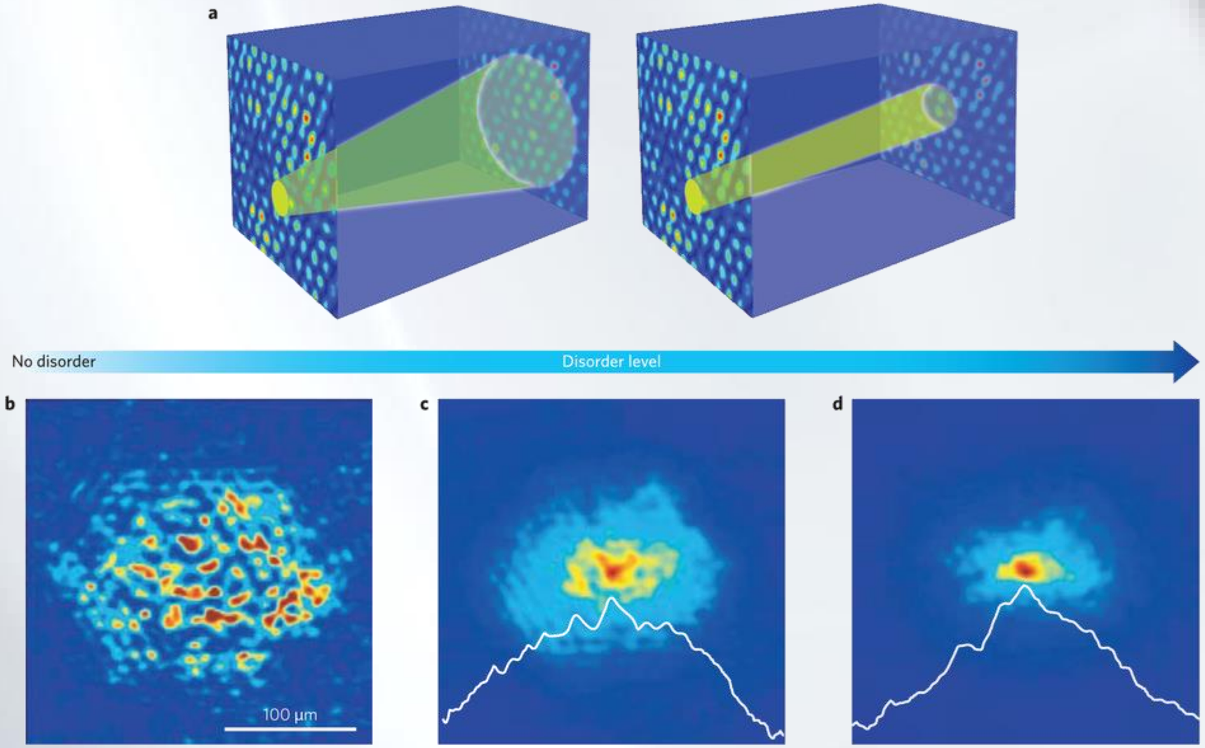
$$H = \sum_i \epsilon_i |i\rangle\langle i| - \sum_{ij} t_{ij} |i\rangle\langle j|$$

$$H\psi = E\psi$$

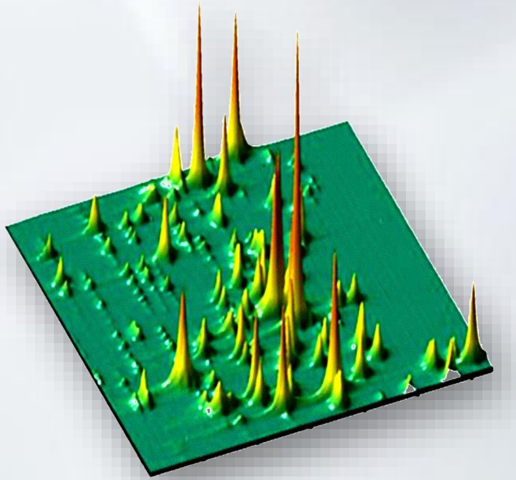
- Random noise ϵ_i (Uniform, Gaussian, Lorentz) (also hopping term)
- Second order quantum phase transition at E_c
- MIT occurs in 3D due to the scaling theory of localization, in 1 & 2D all electronic states are localized for any amount of disorder
- Interactions complicate the description (Coulomb potential)

AL, experiments

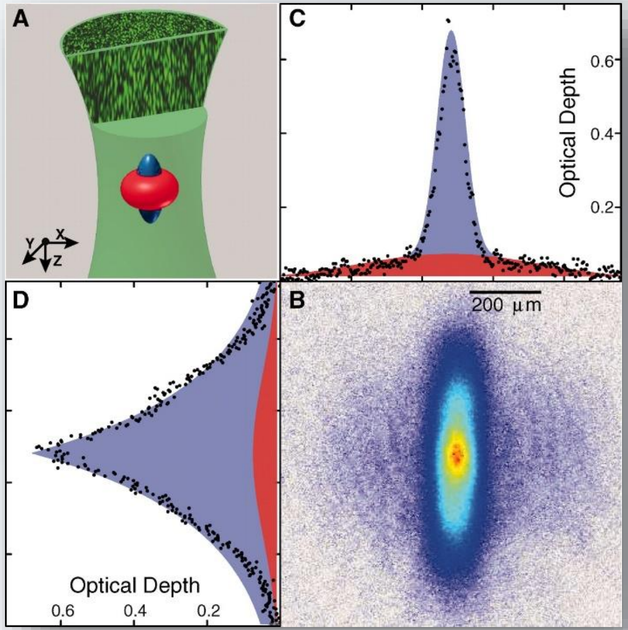
Electromagnetic, sound waves
Ultracold atoms



Anderson localization of light
Mordechai Segev, Yaron Silberberg , Demetrios N. Christodoulides
Nature Photonics 7, 197–204 (2013)



Cavity Quantum Electrodynamics with Anderson-Localized Modes
Luca Sapienza, Henri Thyrrestrup, Søren Stobbe, Pedro David Garcia, Stephan Smolka, Peter Lodahl
Science (2010) 327, 1352-1355



Three-Dimensional Anderson Localization of Ultracold Matter
S. S. Kondov, W. R. McGehee, J. J. Zirbel, B. DeMarco*
Science 7 October 2011: Vol. 334 no. 6052 pp. 66-68

AL, Random Schrödinger operator

$$\Big(-\Delta + V(x)\Big)\psi = E\psi$$

- $V(x)$ random potential, **disorder**
- Analogous to the Anderson tight-binding model
- Exponential spectral localisation above a critical disorder

AL, Random Schrödinger operator

$$\Big(-\Delta + V(x)\Big)\psi = E\psi$$

- Above a critical energy E_c, mobility edge → delocalisation
- (Second order) *quantum* phase transition at E_c

Two regions

- Localised states: Poisson distributed (classical dynamics)
- Delocalised states: Random Matrix Theory (chaotic dynamics)

Bohigas, Giannoni, Schmit conjecture 1984

QCD spectral density

QCD Dirac operator

$$D\psi = \lambda\psi$$

Chiral condensate vs low modes spectral density

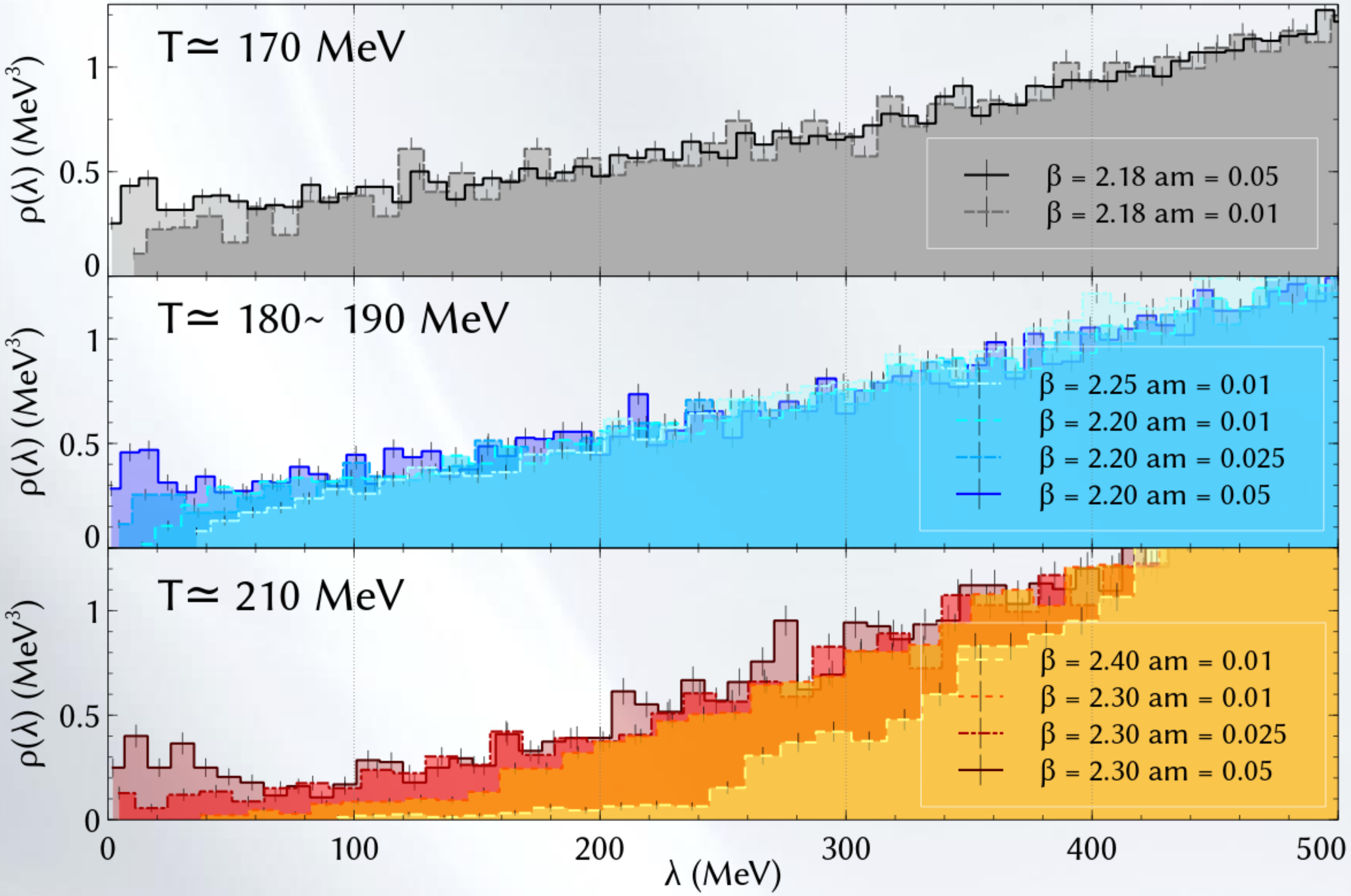
Banks-Casher relation:

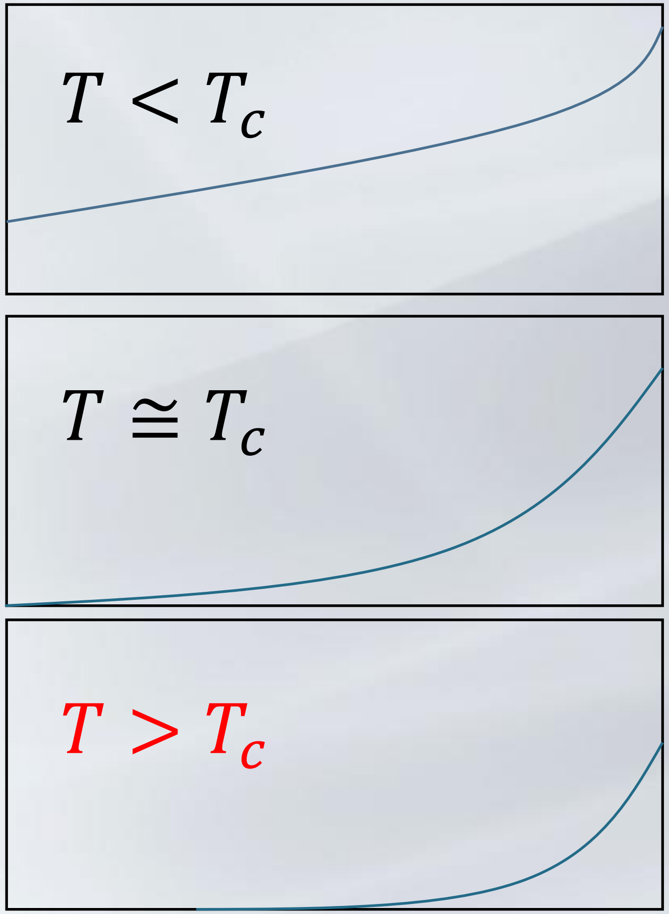
$$-\langle\bar{\psi}\psi\rangle = \Sigma = \pi \lim_{\lambda\to 0} \lim_{m\to 0} \lim_{V\to\infty} \rho(\lambda)$$

Non zero chiral condensate at low temperature

QCD spectral density

Banks-Casher relation

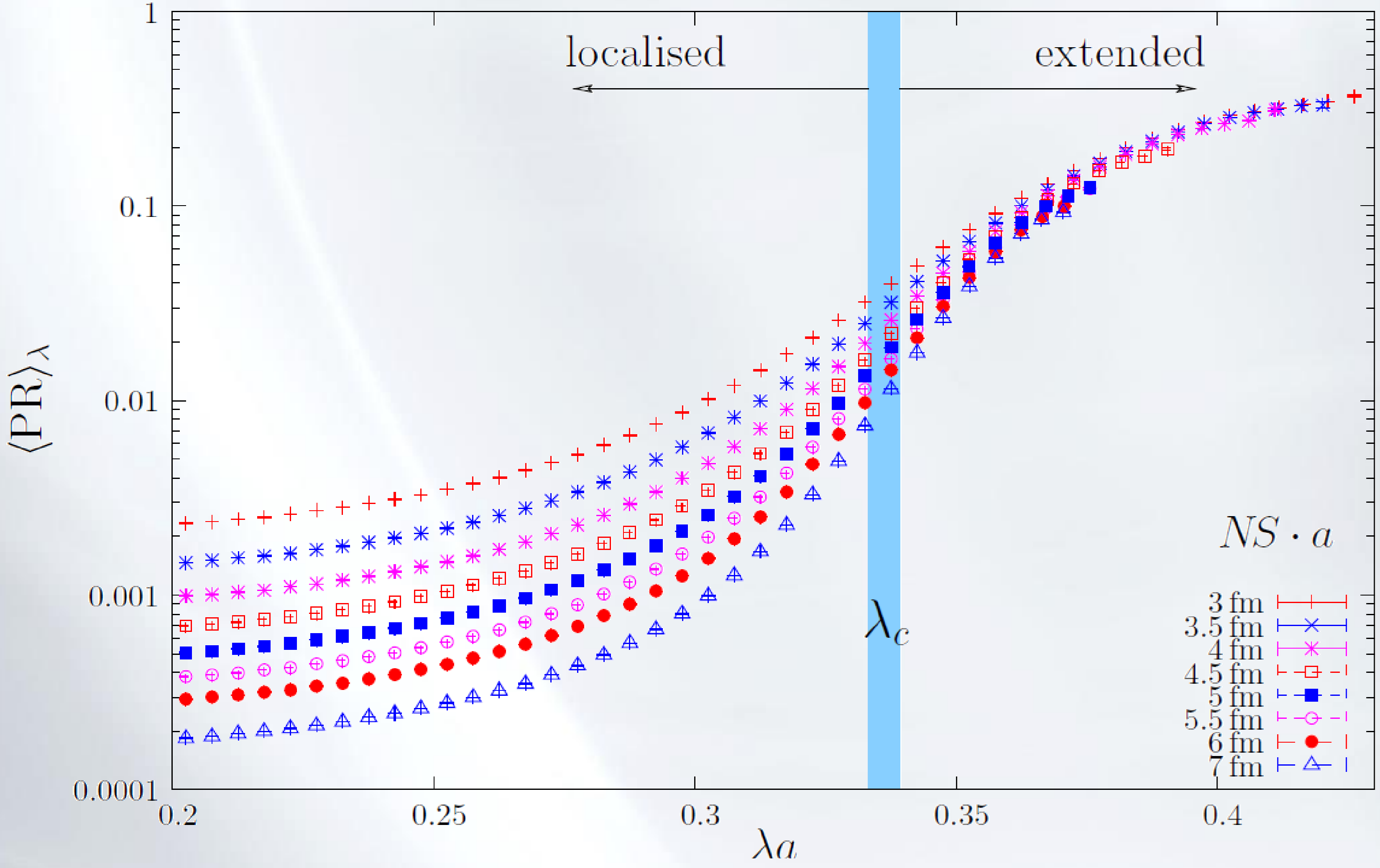




Cossu et al., PRD87 (2013) 11, 114514

Participation ratio

$$\langle PR_\lambda \rangle = (V \sum |\psi_\lambda|^4)^{-1}$$



Courtesy of Giordano, Kovács, Pittler

$$T \simeq 2.6 T_c$$
$$N_f = 2+1$$

$\langle PR \rangle$~ fraction of volume occupied by the mode

Generalized momenta:

$$\langle PR_\lambda^q \rangle = (V \sum |\psi_\lambda|^{2q})^{-1}$$

Random matrix theory (RMT)

Anderson model and random matrix theory

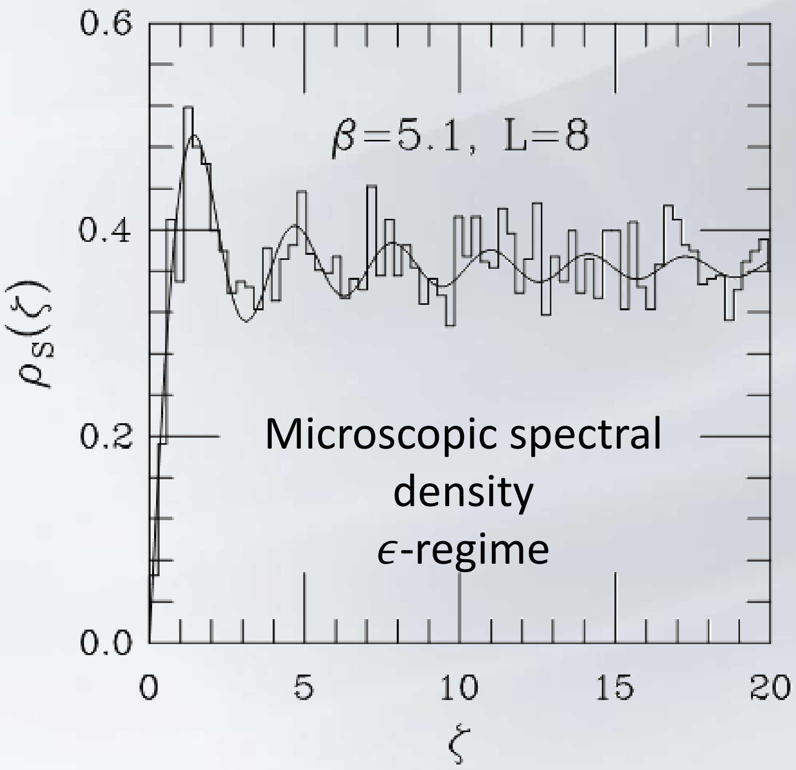
Wigner-Dyson classes: Unitary (GUE), Orthogonal (GOE), Symplectic (GSE)

Chiral classes (chGxE) can be defined too, same bulk statistical properties

QCD: Low temperature chRMT description successful (e.g. Verbaarschot, Wittig, Damgaard, Nishigaki, …)

High T?

chRMT description not valid anymore



Level spacing statistics

Unfolded level spacing distribution (ULSD), ***P(s)***

Level spacing, normalized by the local average spacing

$$s = \frac{\lambda_{i+1} - \lambda_i}{\langle \lambda_{i+1} - \lambda_i \rangle}$$

Poisson

$$P(s) = \exp(-s)$$

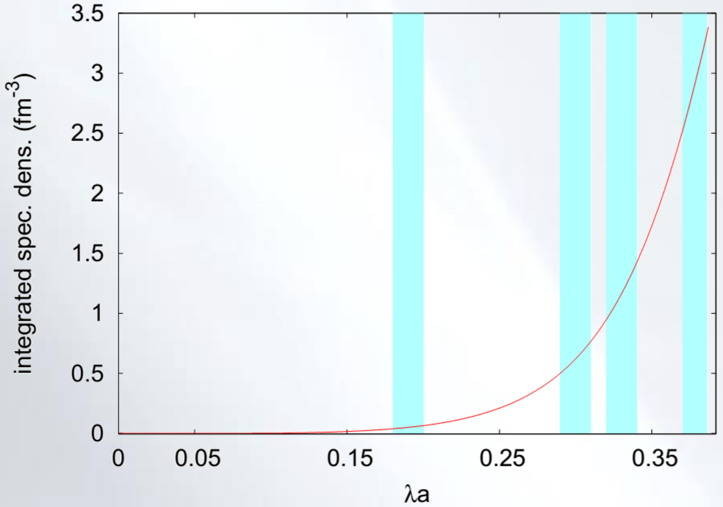
GUE (Wigner surmise)

$$P(s) = \frac{32}{\pi^2} s^2 \exp\left(-\frac{4}{\pi} s^2\right)$$

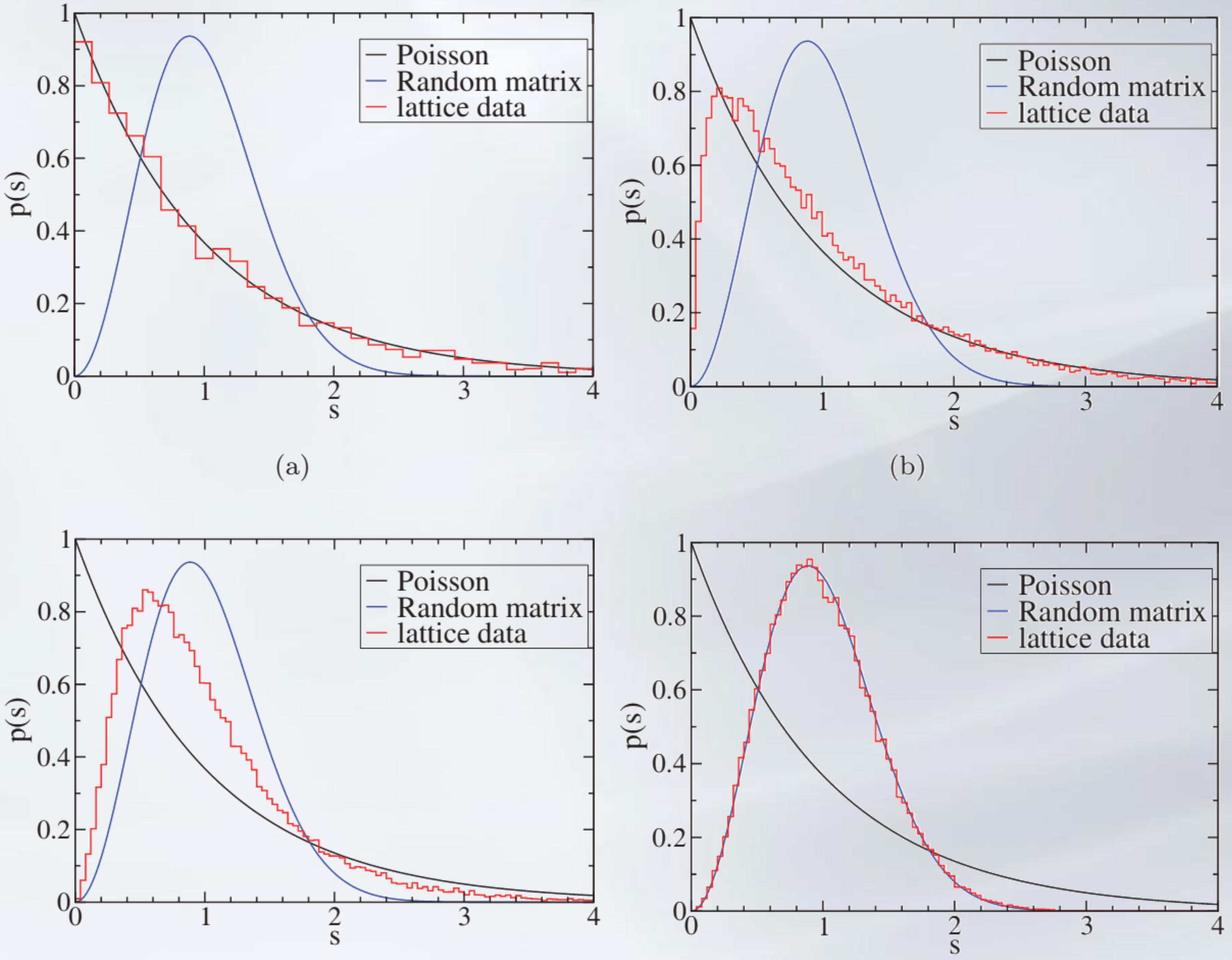
No free parameters!

Level spacing statistics

- ULSD
- One parameter family of curves? (Nishigaki)



Courtesy of Giordano, Kovács, Pittler



(a)

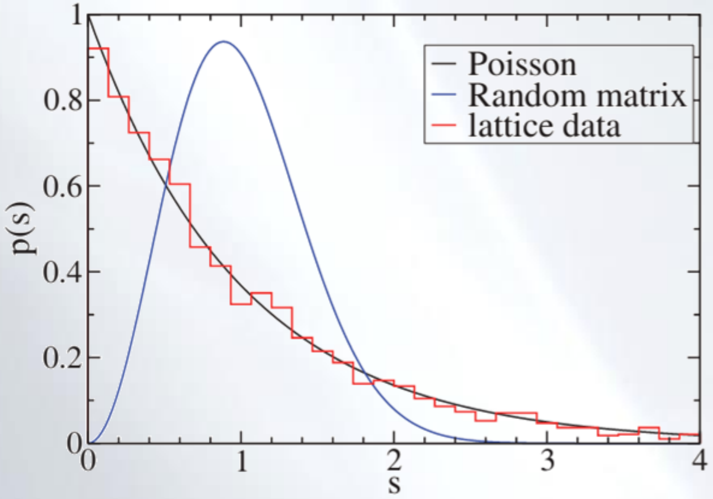
(b)

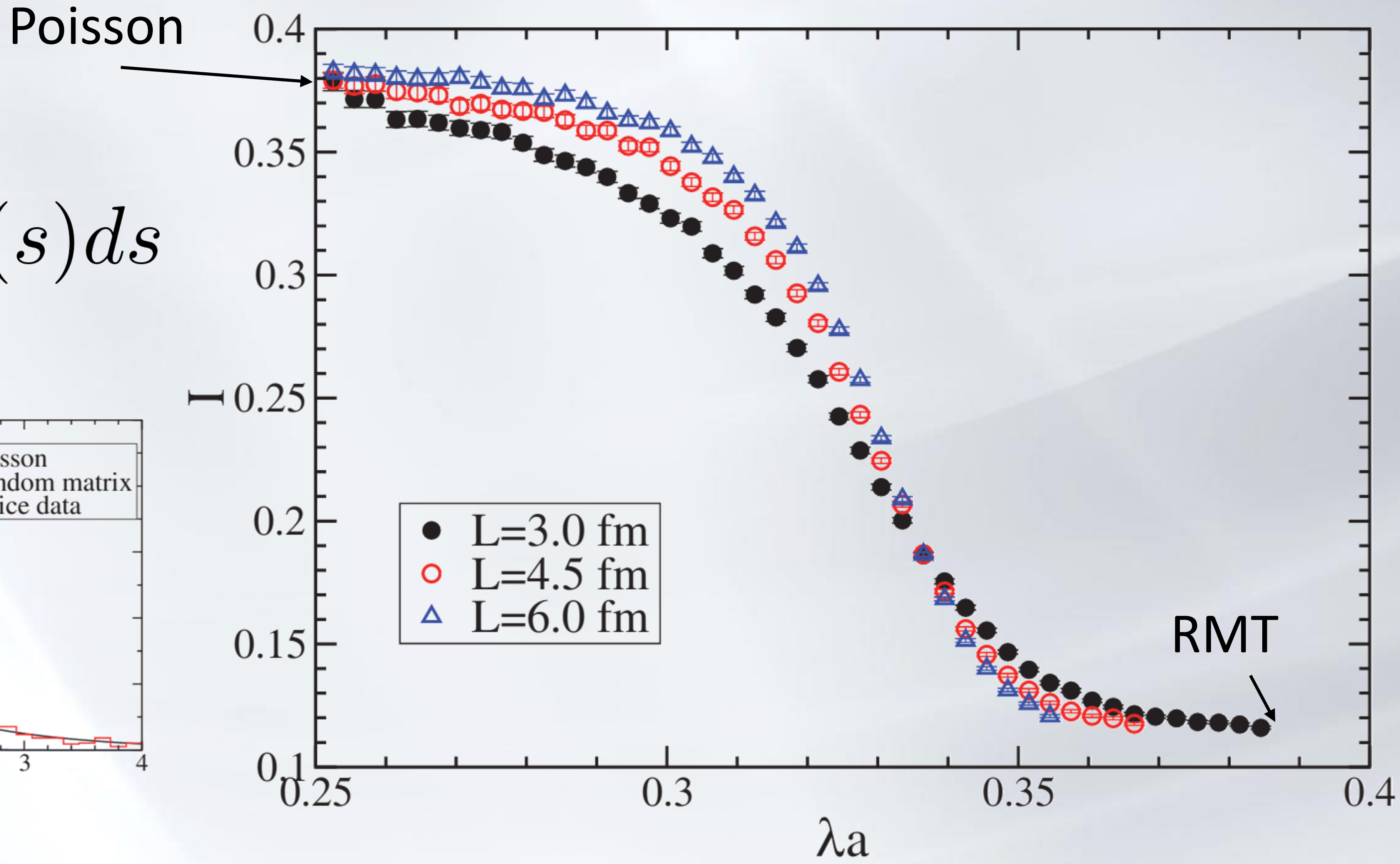
(c)

(d)

Finite size scaling

$$I = \int_0^{s_0} P(s)ds$$





Courtesy of Giordano, Kovács, Pittler

Finite size scaling

$$I(\lambda, L, \mu) = f((\lambda - \lambda_c)L^{1/\nu}, L^{y_h}\mu)$$

I is RG invariant

μ irrelevant direction

Best fit:

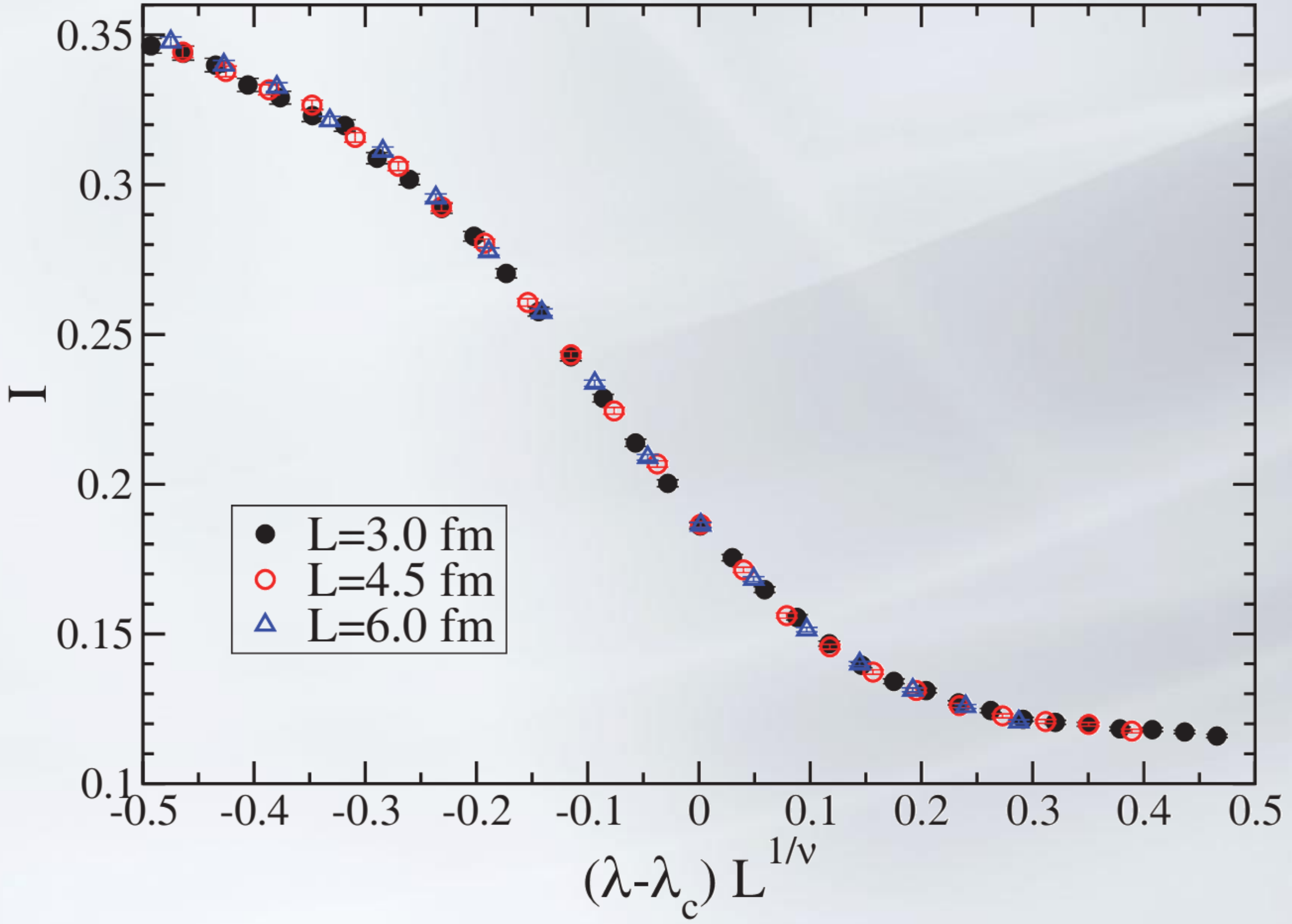
ν = 1.43(6)

Giordano et al. 2014

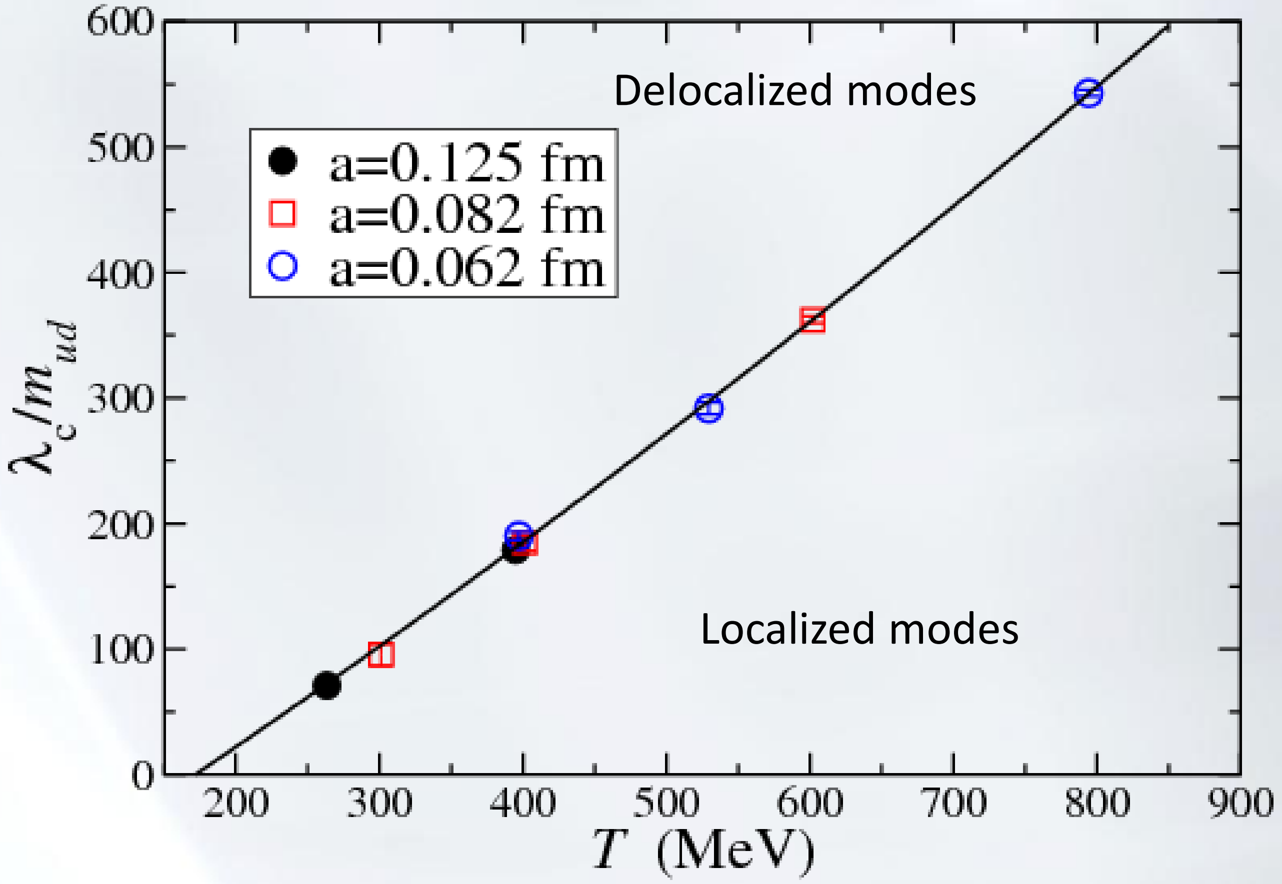
ν = 1.43(4)

Unitary 3D Anderson model

(Slevin, Ohtsuki 1999)



Mobility edge

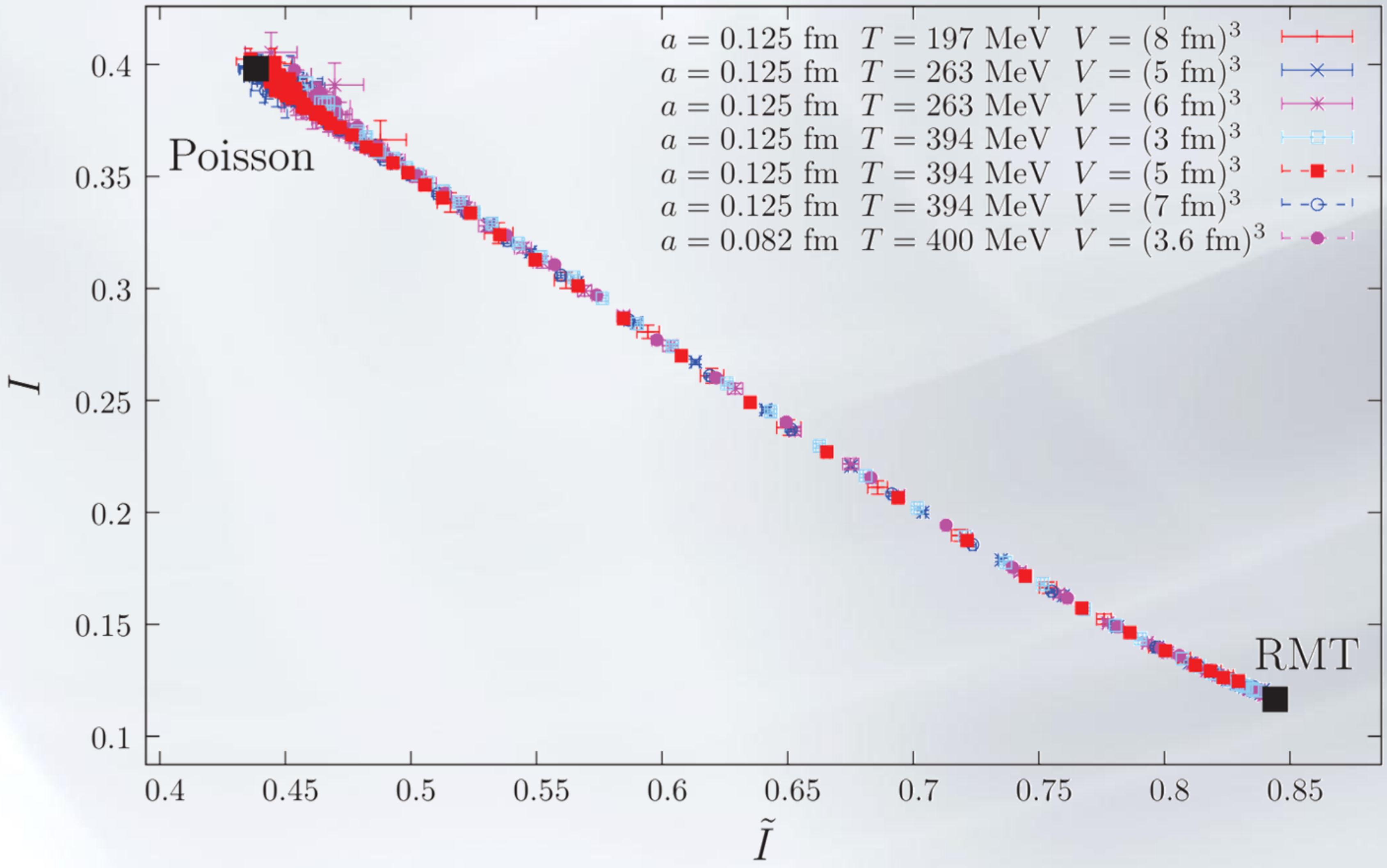


Kovacs, Pittler (2012)

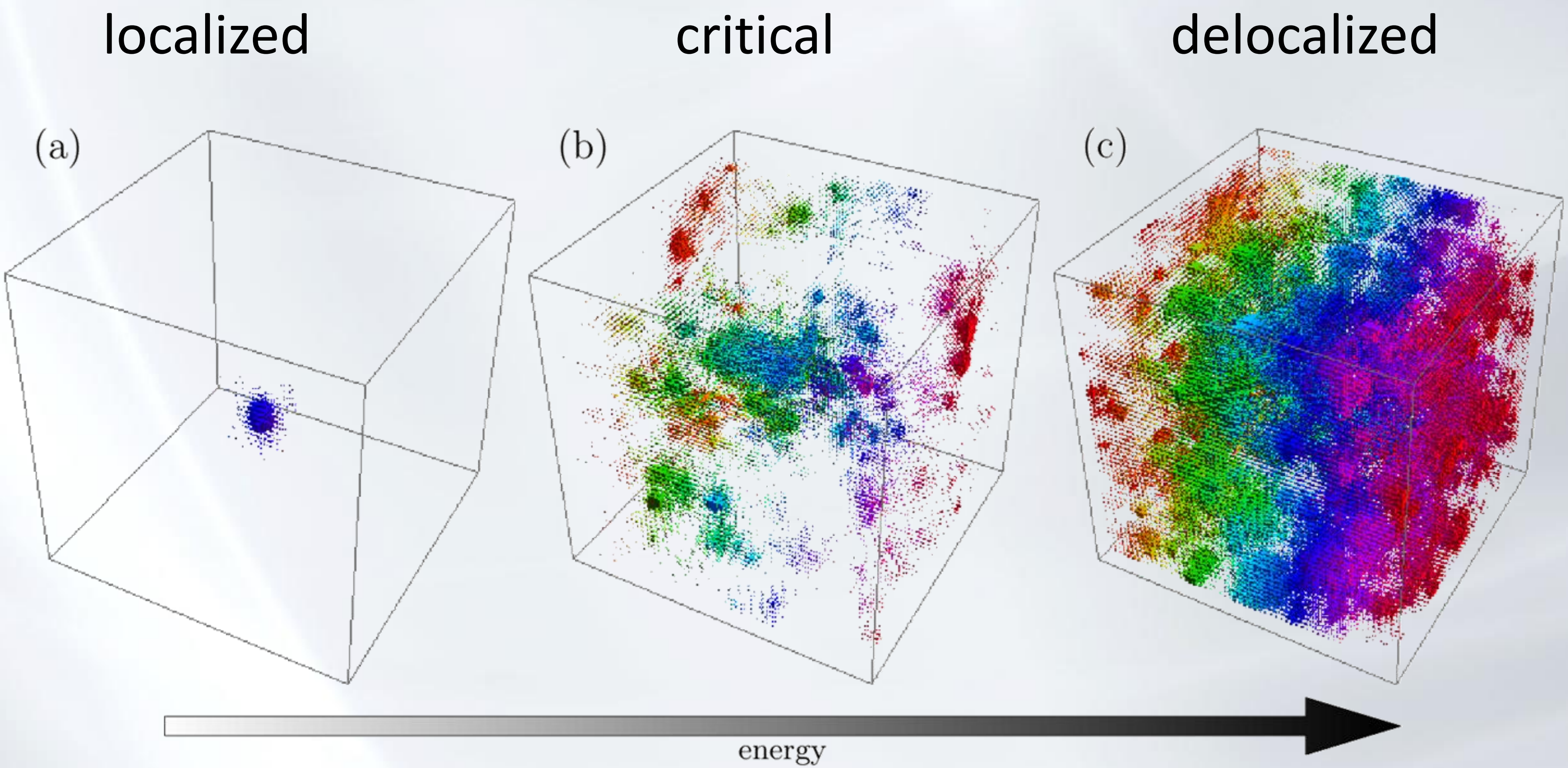
More and more localized modes by increasing temperature

Shape analysis

If one family of curves describes the transition, correlation of different statistics for different parameter sets will coalesce



Multifractals at criticality



QCD data, T ~ 2.6T_c , staggered fermions (Ujfalusi et al.)

Multifractals at criticality

Multifractals: fluctuations at all length scales

Scaling of wave function norm in a neighborhood of a point *x*, size ℓ

- Smooth $\sim\ell^d$
- Fractal $\sim\ell^\alpha$
- Multifractal $\sim\ell^{\alpha(x)}$, $\alpha(x)$ local dimension

Multifractals are characterized by families of fractal exponents

Multifractal exponents of 3D unitary Anderson model and QCD at high temperature agree (Ujfalusi et al.)

Understanding localisation in QCD

- Unitary Anderson Model: 3D, diagonal noise, V(x)
- QCD: 4D, off-diagonal noise (parallel transport)

Why the same universality class?

Polyakov line provides effective 3D diagonal (spatial) noise

$$\psi(N_T, x) = -P(x)\psi(0, x)$$

Bruckmann et al. (2011), Giordano et al. (2015-2016)

Understanding localisation in QCD

Change of basis of staggered operator to temporal momentum basis

- Polyakov line phases $\phi_a(x)$ explicitly provide diagonal noise
- Spatial hopping → Fourier transform of spatial links wrt time
  - mix temporal momentum components

Above T_c, P_L gets ordered ~ 1, effective gap in spectrum (lowest Matsubara frequency)

Fluctuations ($P_L \neq 1$) provide "trap" for eigenmodes by allowing smaller eigenvalues

Bruckmann et al. (2011), Giordano et al. (2015-2016)

Understanding localisation in QCD

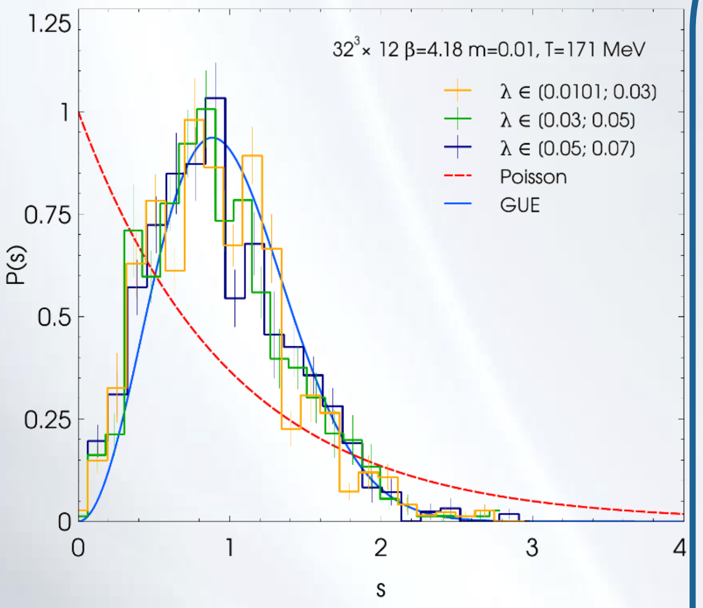
- Ordering of PL induces correlation across time-slices
  - reduced mixing of temporal momentum components
- Strong mixing of t-mom components → 4D system
- Reduced mixing of t-mom components → N_T 3D systems

- Decoupling seems important for localisation (Giordano et al. 2016)
  - Quenched toy model: Polyakov loop as Ising spins
  - retain only temporal couplings for spatial links
- Model: PL ordering suppresses χSB by opening a “gap” in the spectrum
- Reduced mixing of t-mom (larger coupling of temporal slices) necessary condition for suppression of spectral density

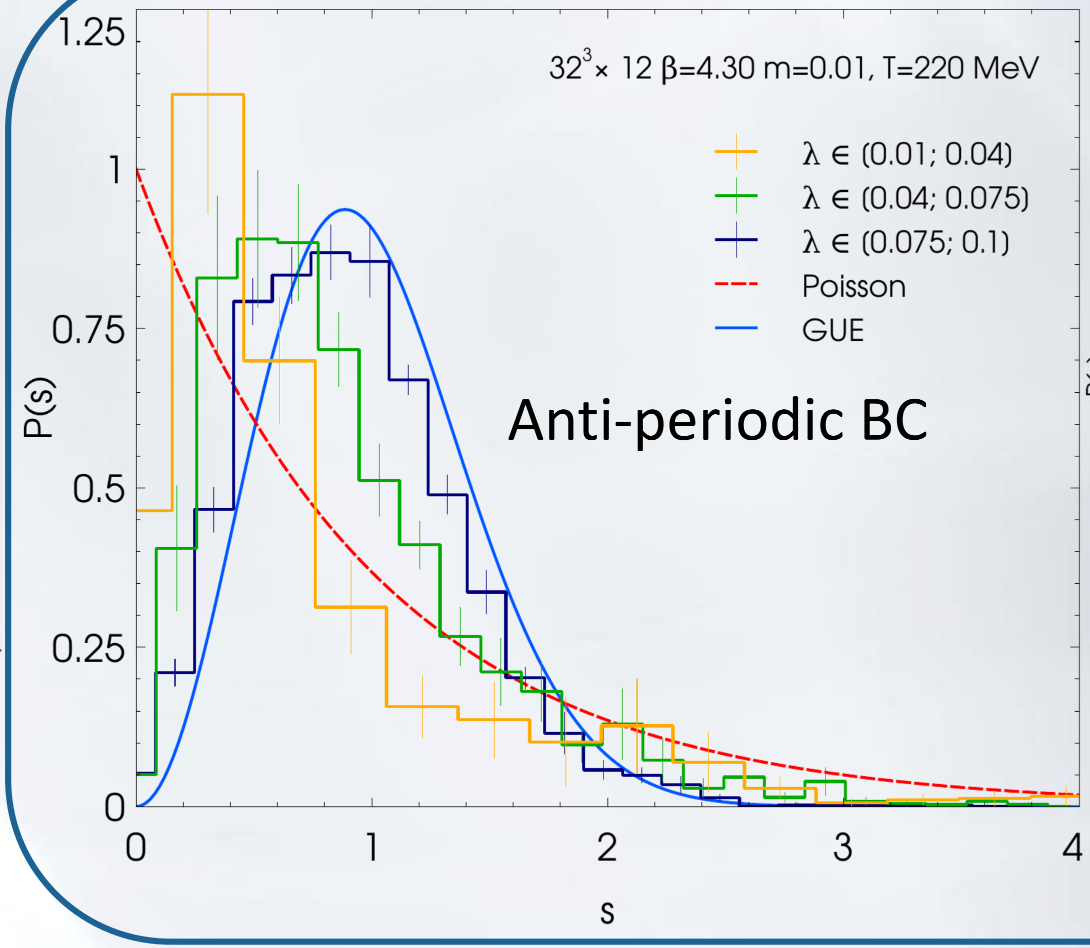
QCD with chiral fermions

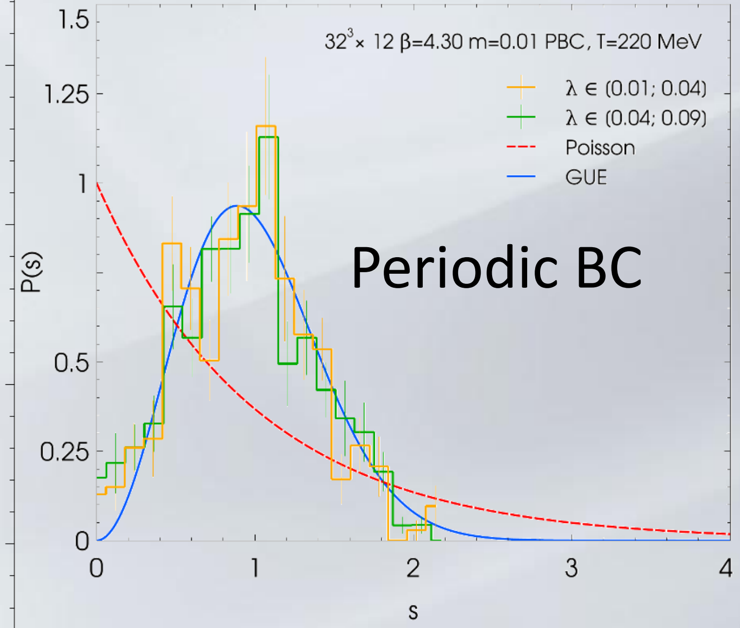
- Chiral fermions, domain-wall (GC and Hashimoto)
- Main question: what is the source of disorder?
- Let's investigate the gauge field background configurations
- Gauge invariant observables:
  - Polyakov loop
  - Local action
  - Local topology
- Chiral properties (left- and right-handed projections)
- Main conclusion:
  - Disorder → monopole-instantons (dyons)

Level spacing distribution



$T < T_c$

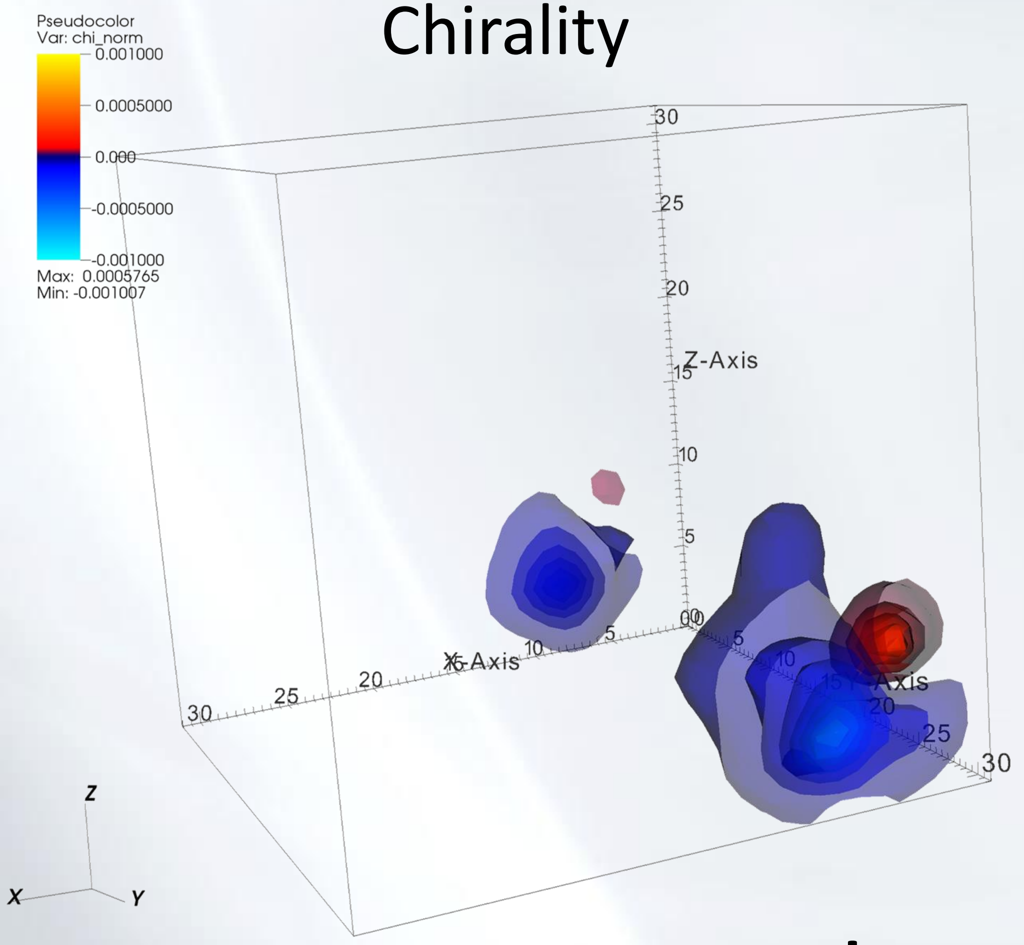




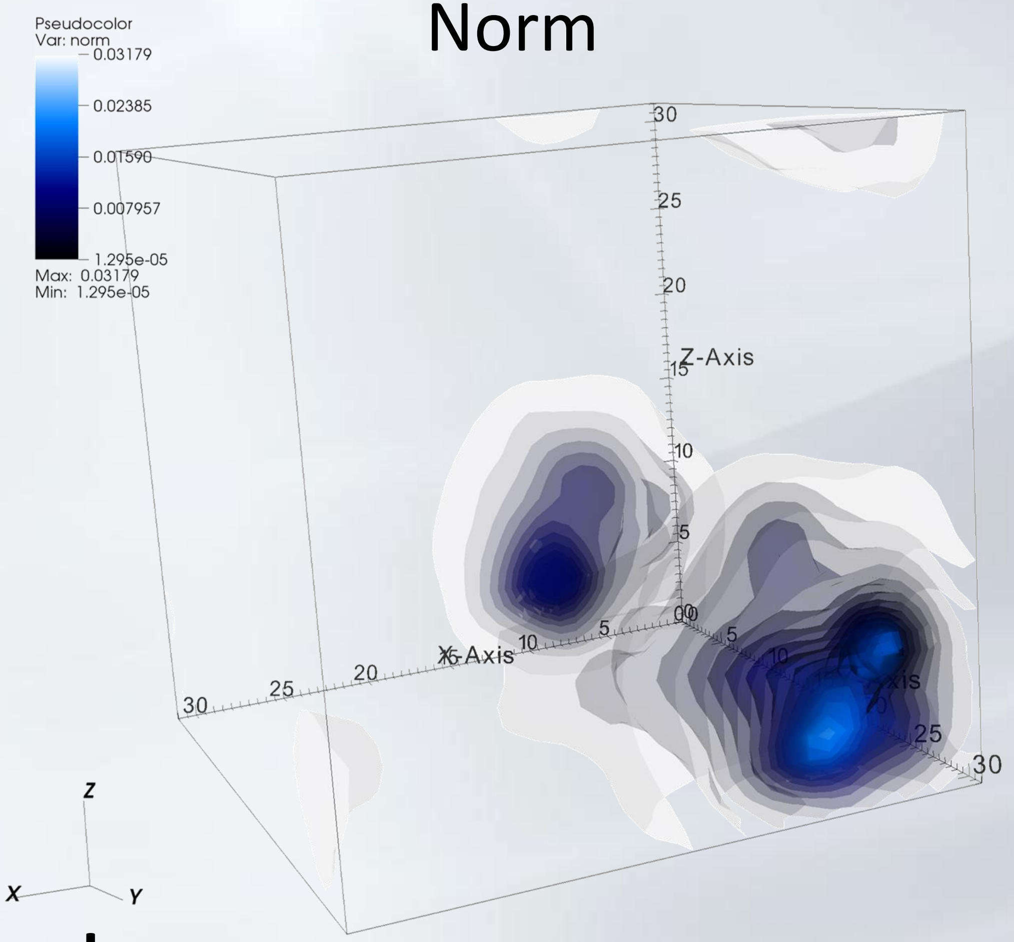
$T > T_c$

Investigating the disorder

Chirality



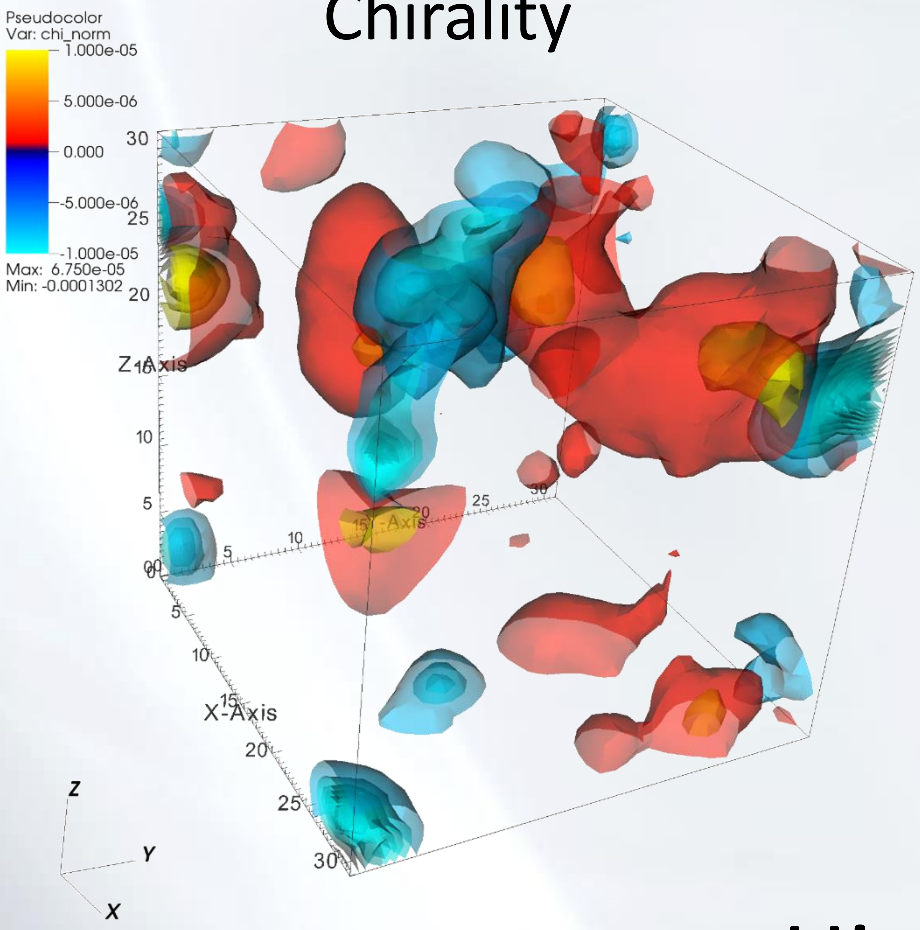
Norm



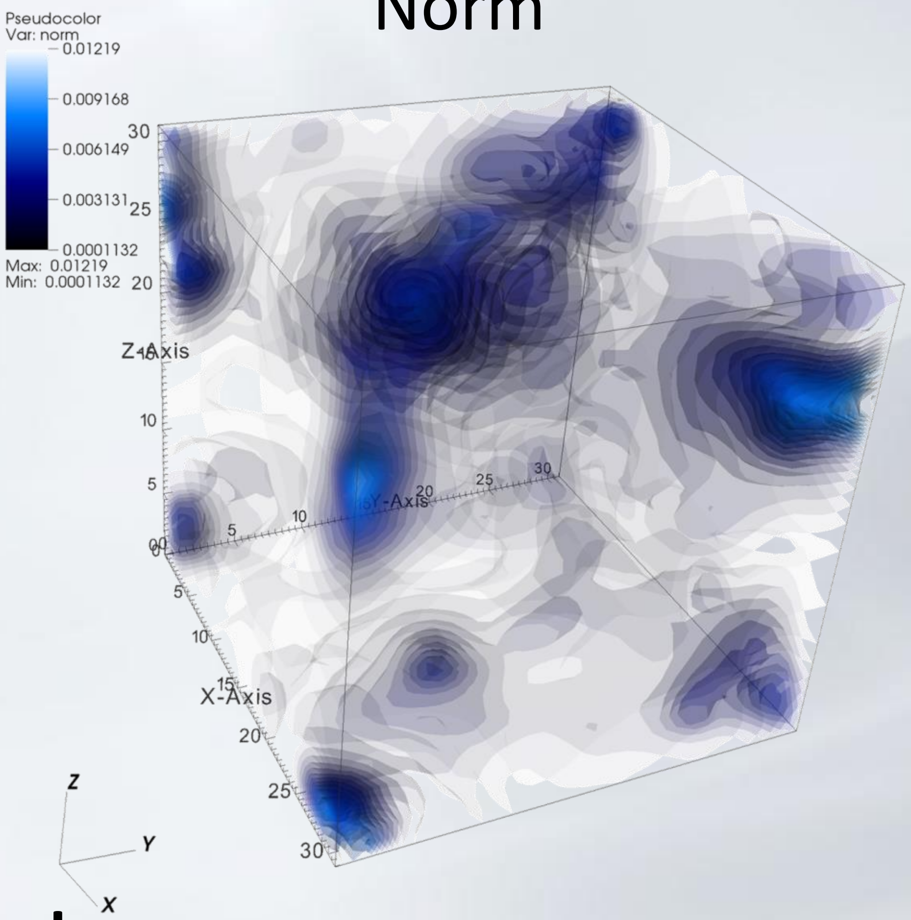
Low-mode

Investigating the disorder

Chirality



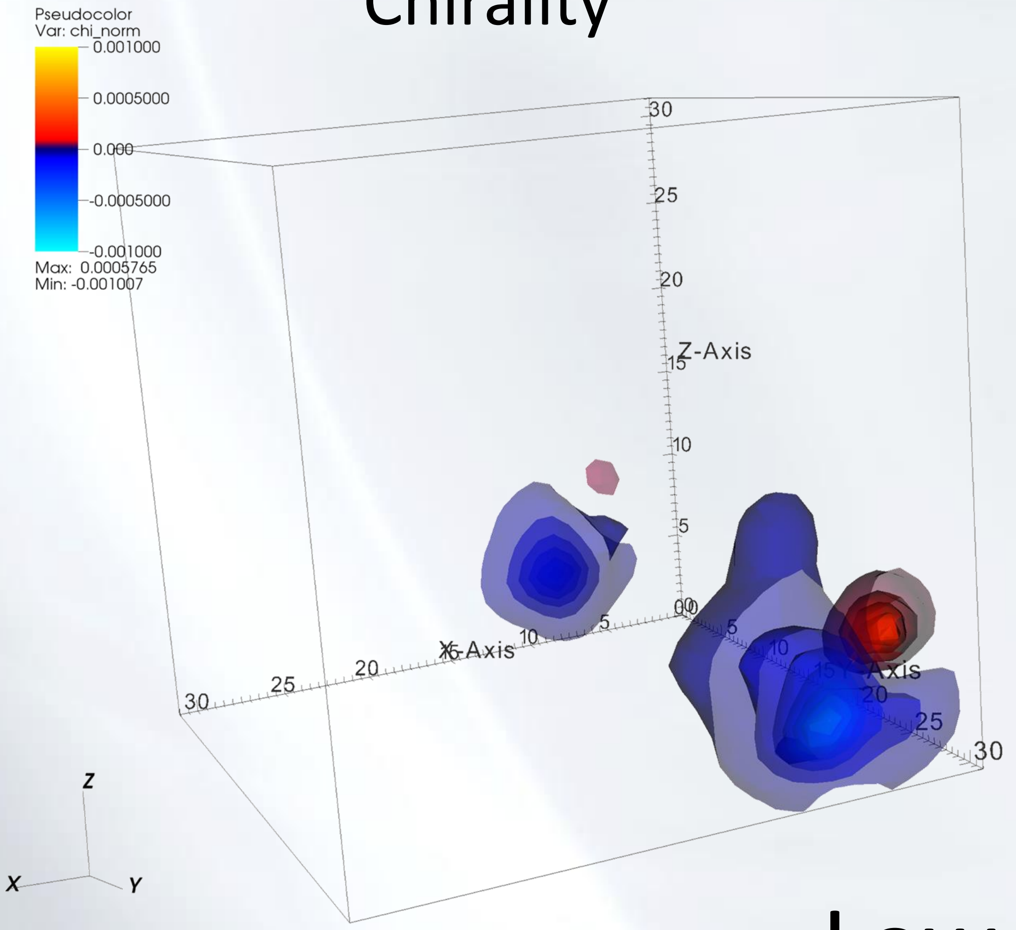
Norm



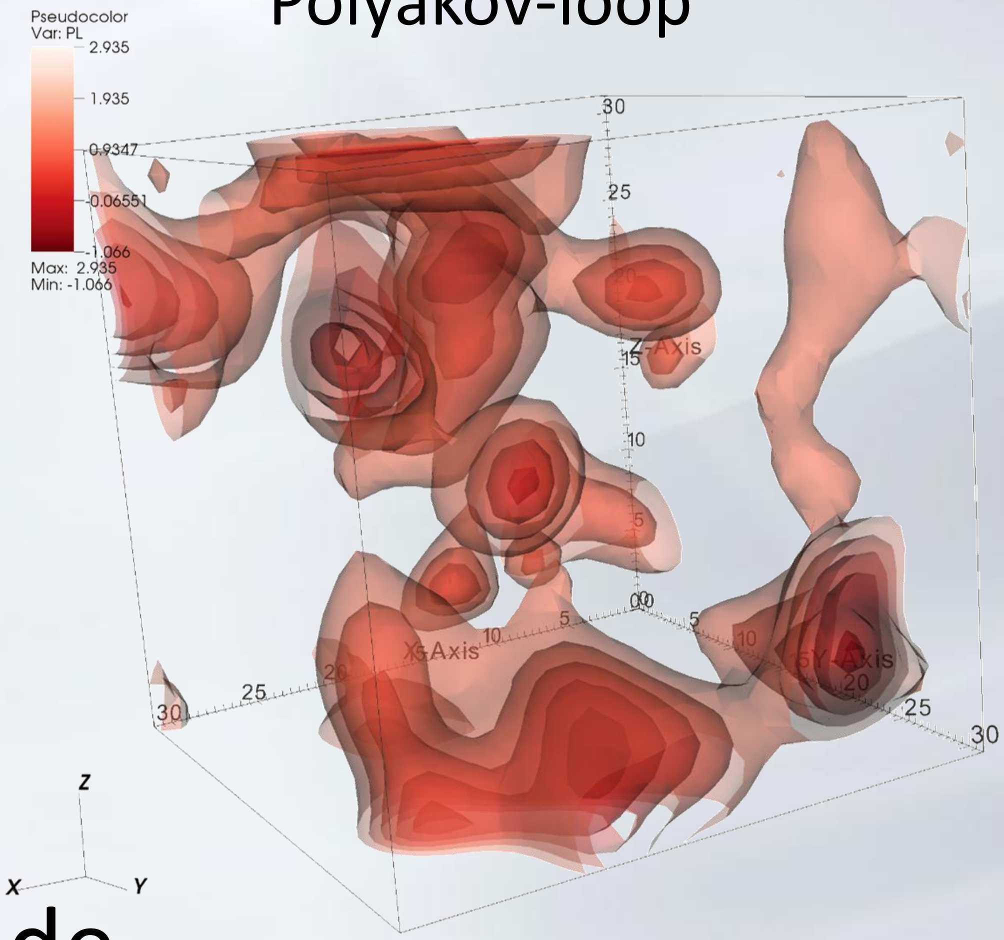
High-mode

Investigating the disorder – Polyakov loop

Chirality



Polyakov-loop



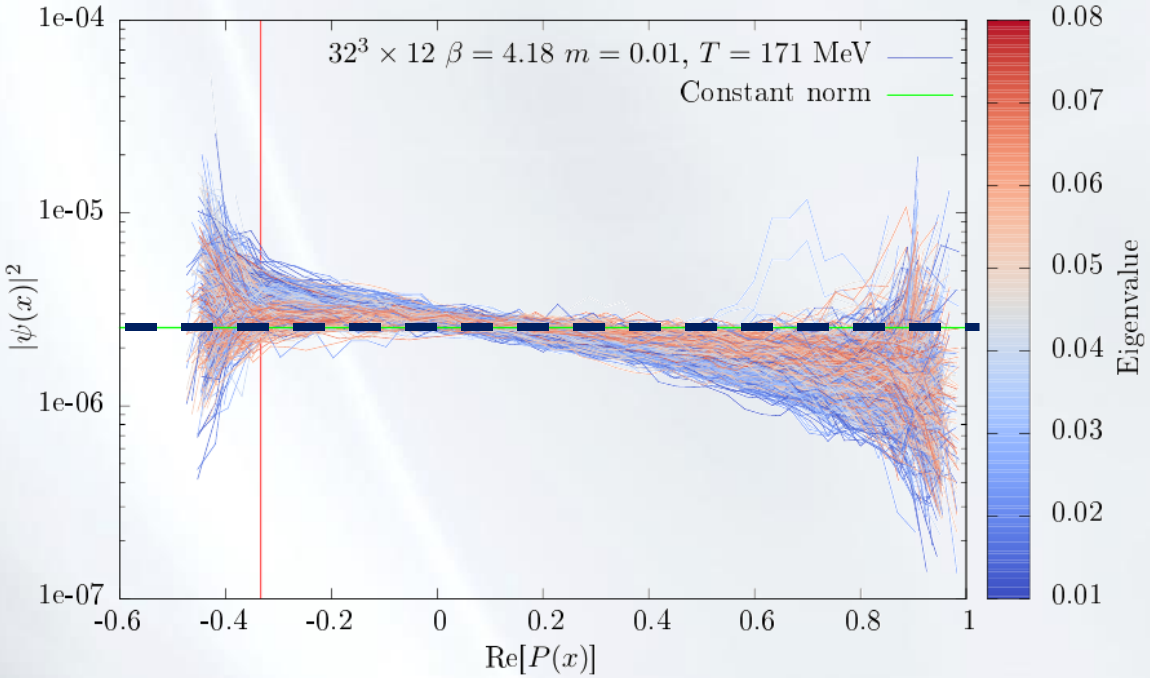
Low-mode

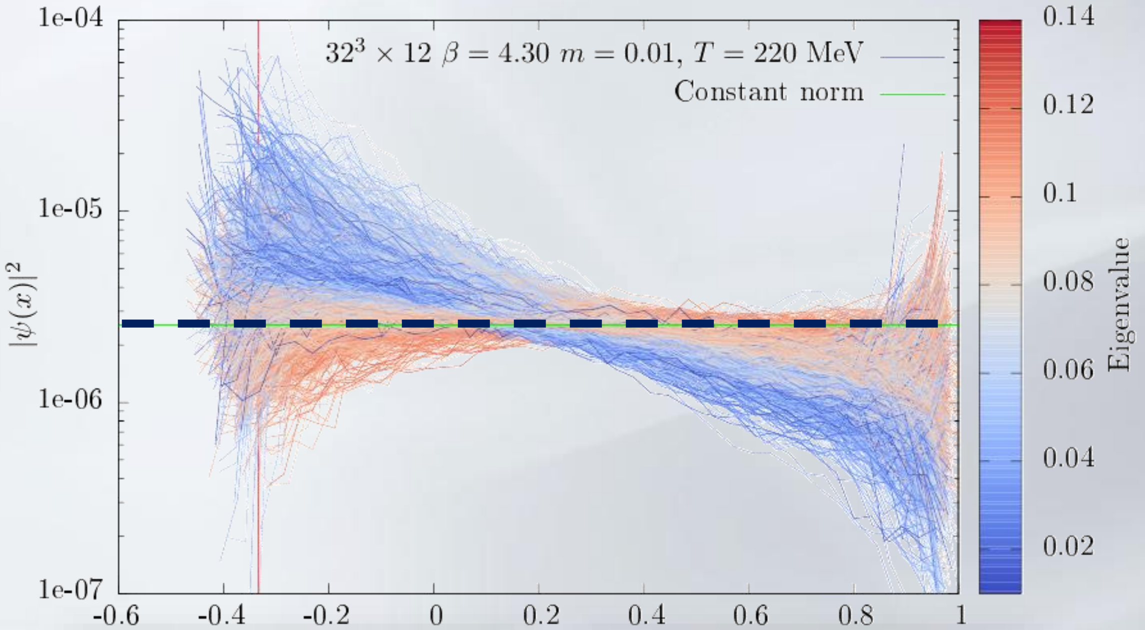
Correlations with the Polyakov loop

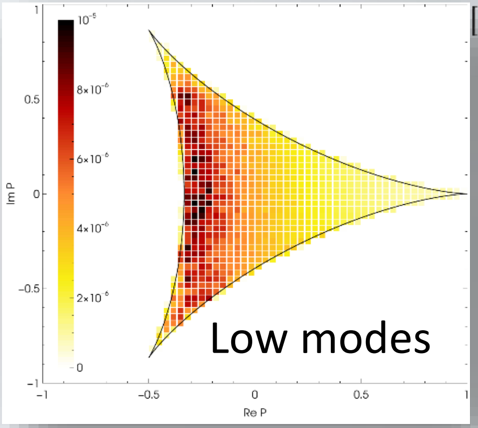
Below the phase transition

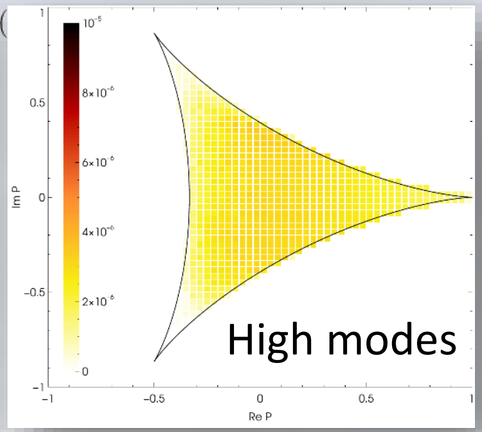
Above the phase transition

Eigenmode local norm







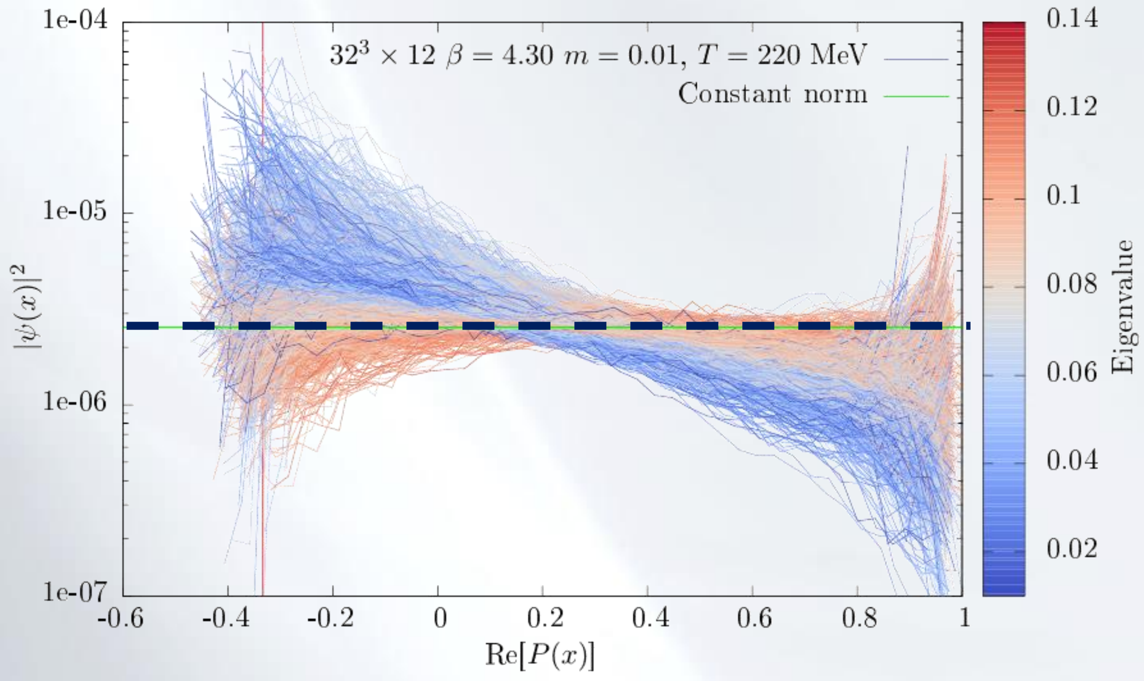


Correlations with the Polyakov loop

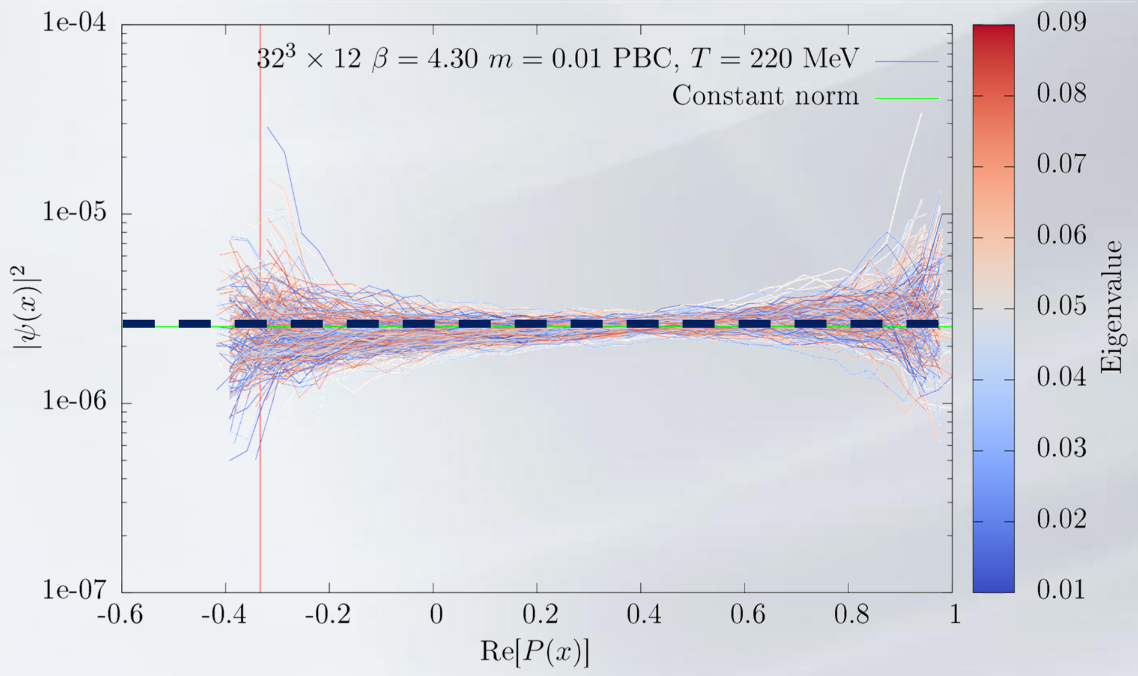
Now change the boundary conditions (BC) of the Dirac operator for the measurements

SAME background configurations on both panels

Anti-periodic BC



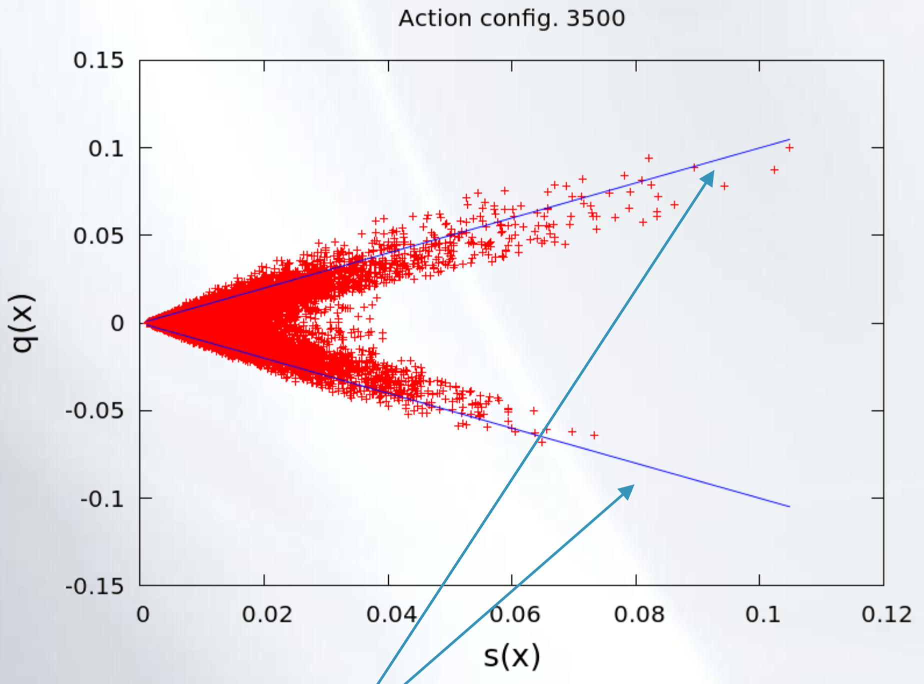
Periodic BC



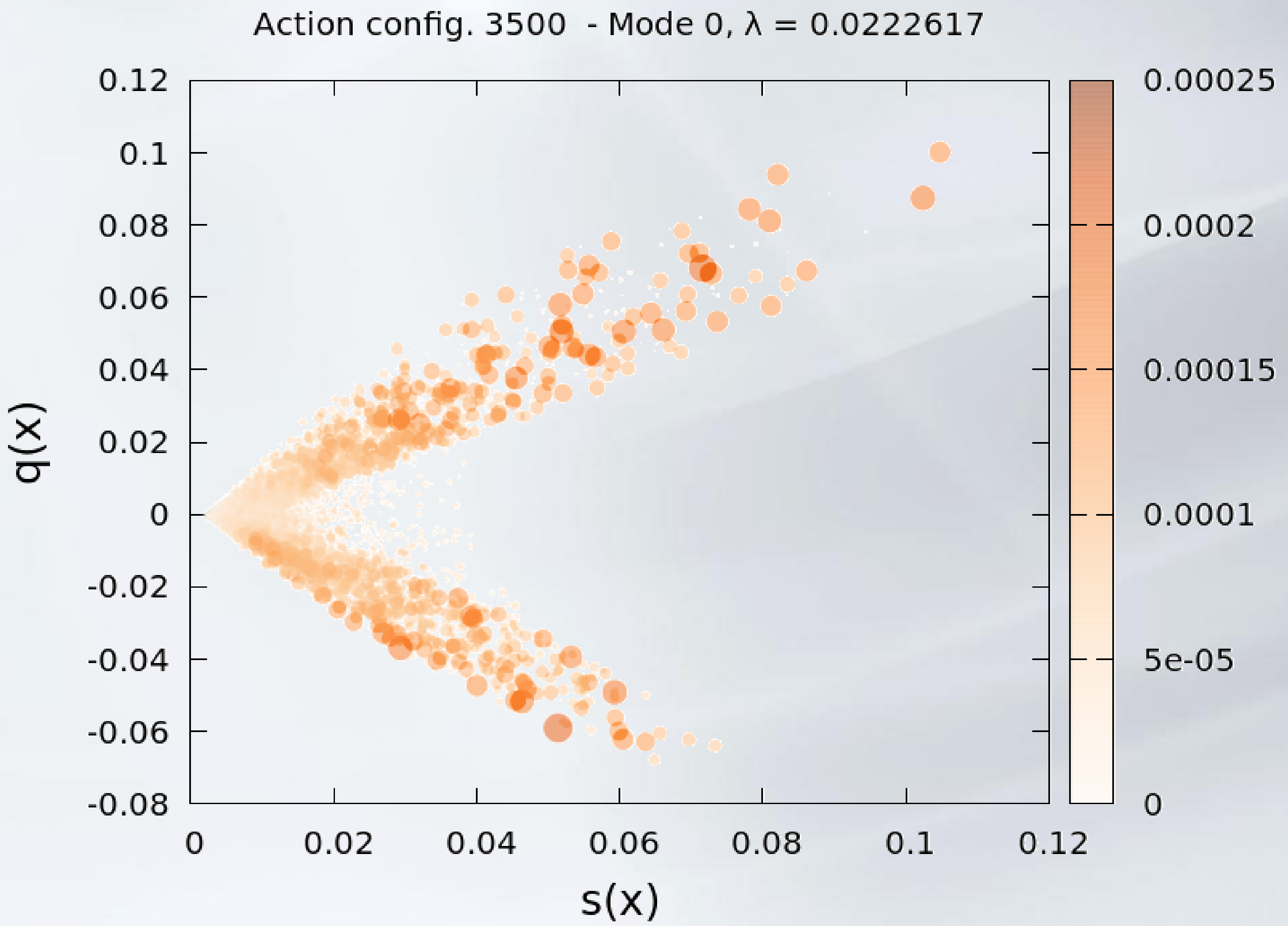
Action and topology

$$s(x) = F_{\mu\nu}F_{\mu\nu}(x)$$

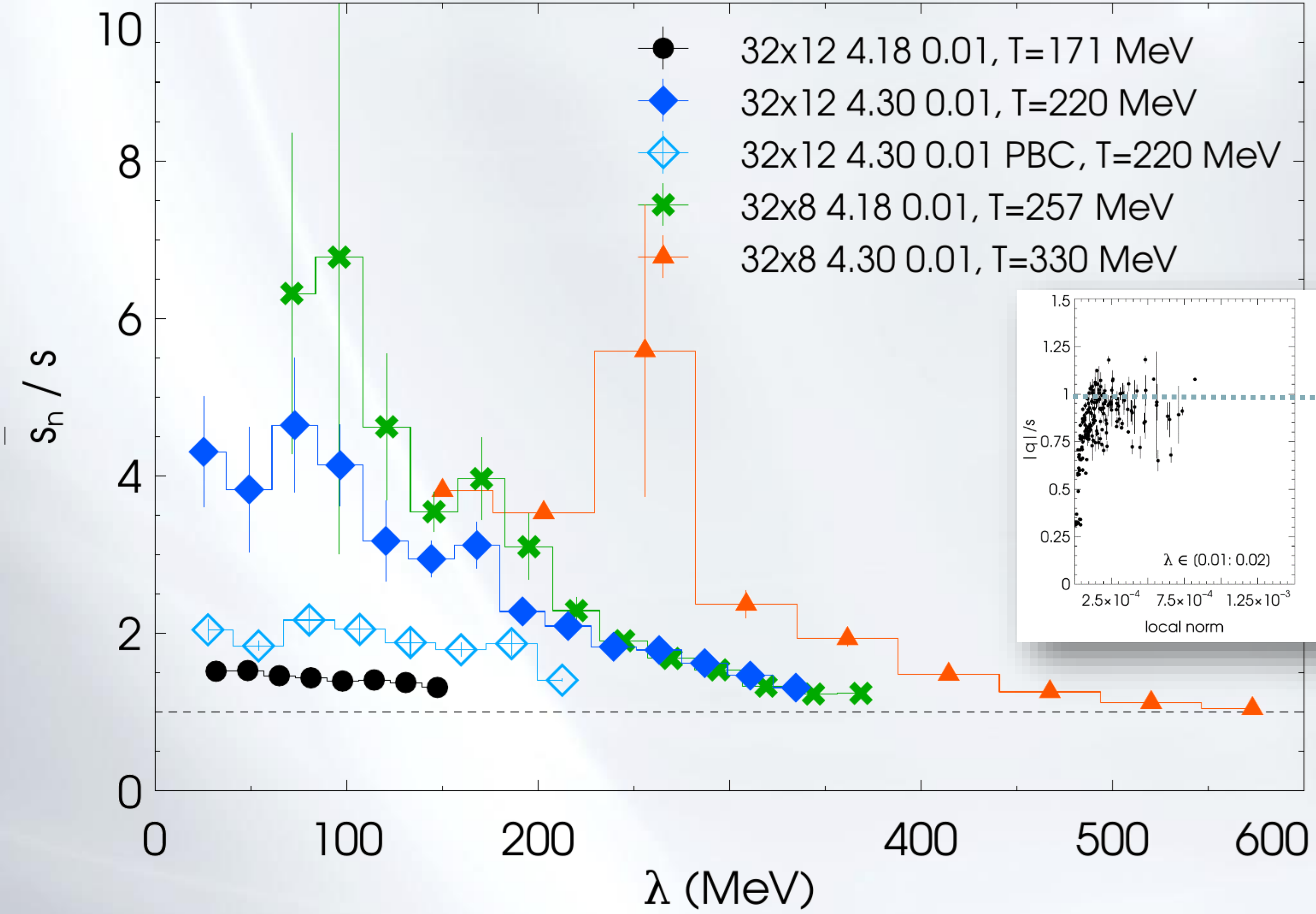
$$q(x) = F_{\mu\nu}\tilde{F}_{\mu\nu}(x)$$



Self-dual fields

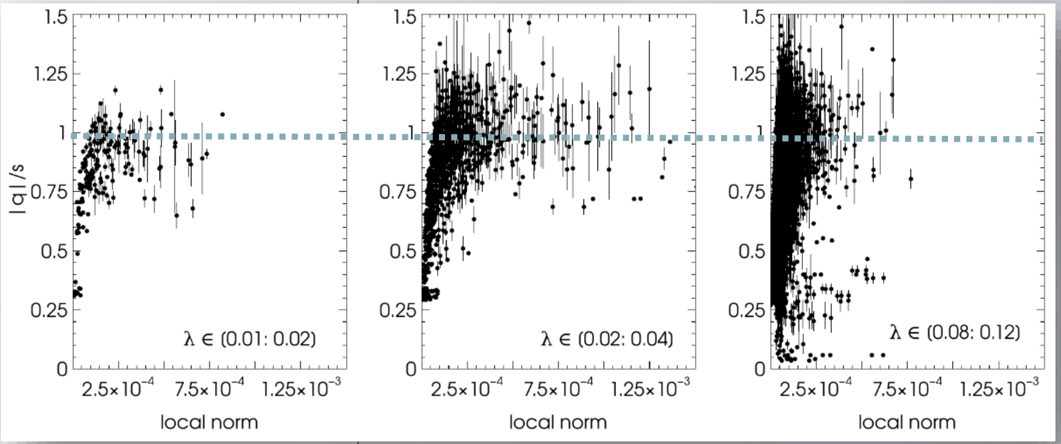


Action and topology



$$\bar{s}_n = \frac{\int \rho_n(x)^2 s(x) d^4x}{\int \rho_n(x)^2 d^4x}$$

$$\rho_n(x) = |\psi_n(x)|^2$$



Interpretation in terms of topological fluctuations

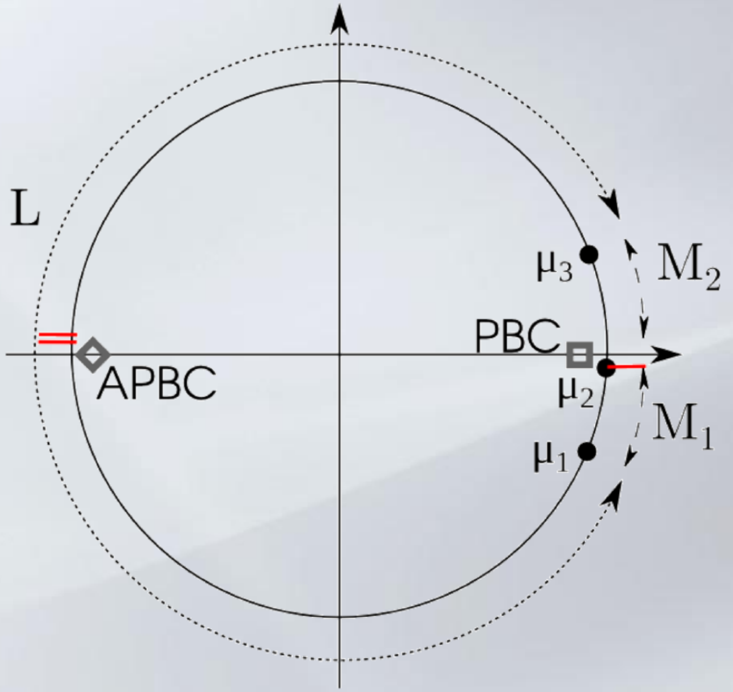
Class of solutions of the YM equation of motion in a non trivial Polyakov loop background (Van Baal et al.): monopole-instantons

$$P(\infty) = \exp[2\pi i \ \mathrm{diag}(\mu_1, \mu_2, \mu_3)], \qquad \nu_m \equiv \mu_{m+1} - \mu_m$$

- ✓ Self dual solutions, charged in each Cartan subgroup
- ✓ SU(N) N-1 species of BPS monopoles, 1 Kaluza-Klein (KK) from the compact dimension
- ✓ Finite temperature calorons are composite objects
  - ✓ N-1 BPS + 1 KK, electrically and magnetically neutral

Interpretation in terms of topological fluctuations

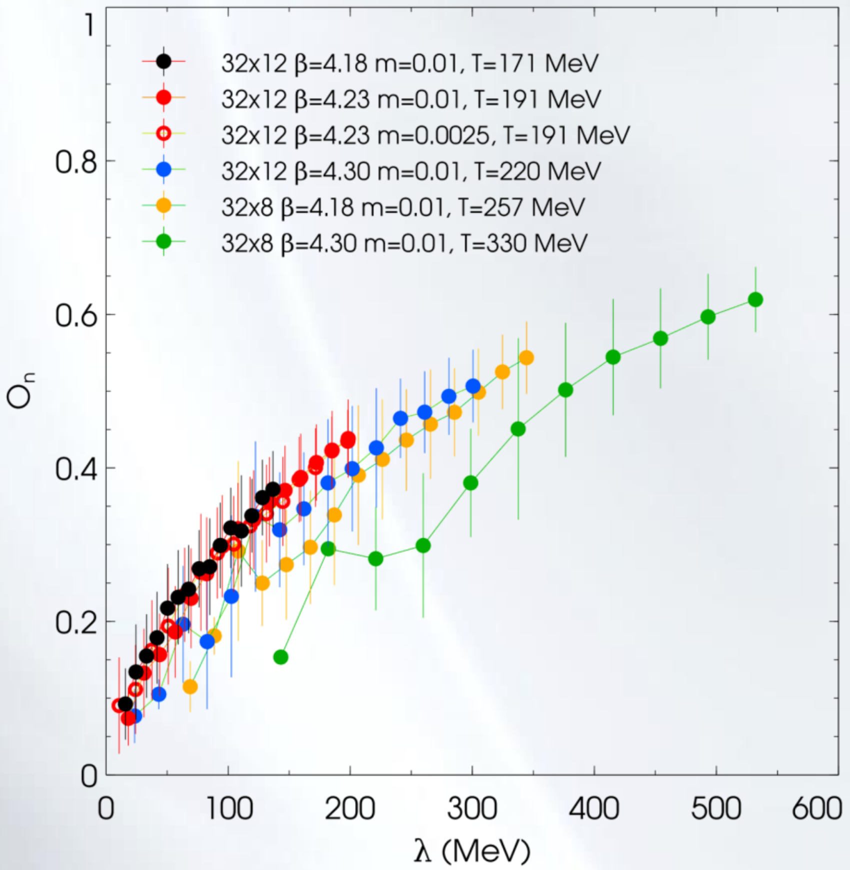
$$P(\infty) = \exp[2\pi i \ \mathrm{diag}(\mu_1, \mu_2, \mu_3)], \qquad \nu_m \equiv \mu_{m+1} - \mu_m$$



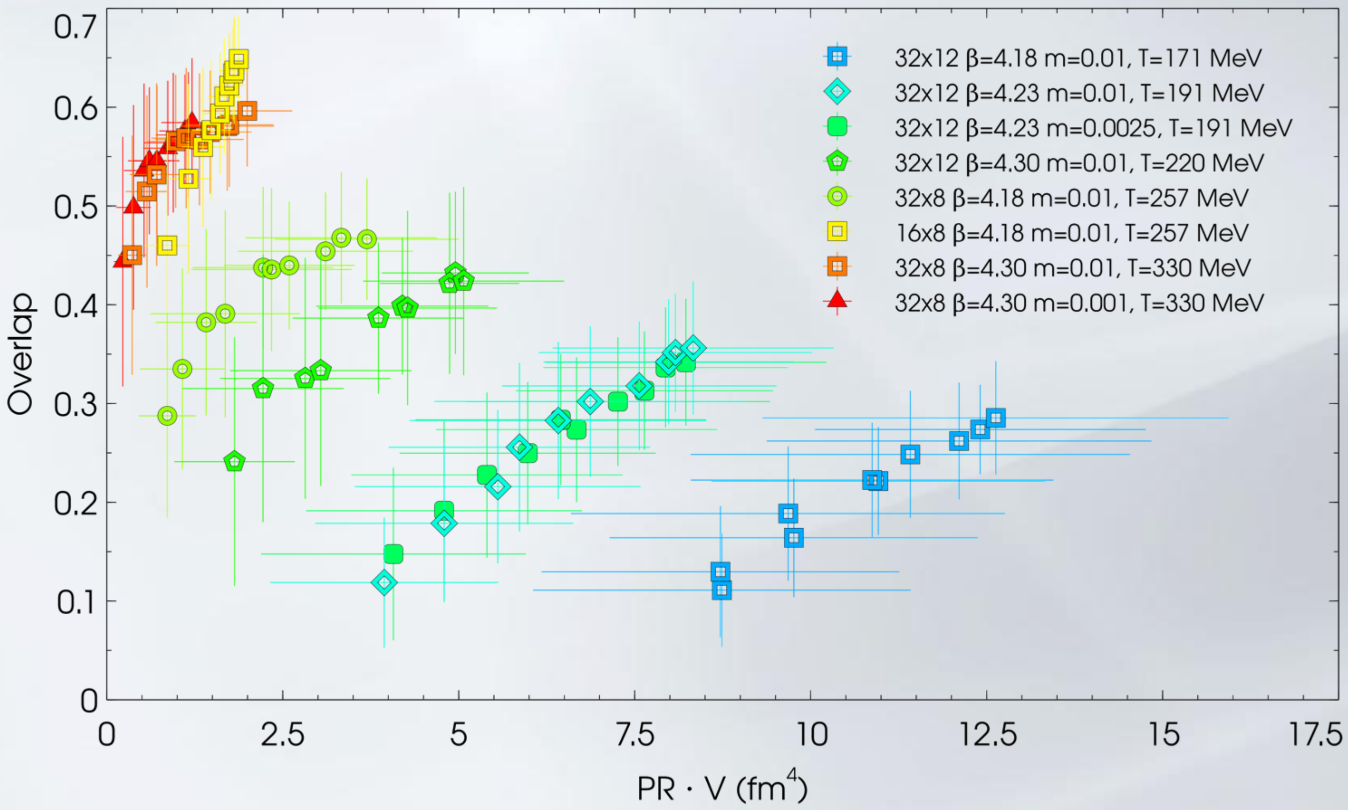
- ✓ Supported action: $S = \frac{8\pi^2}{g^2}\nu_m$
- ✓ Topological charge fractional in general
- ✓ KK monopoles at high T:
  - ✓ Large action support (“heavy”), suppressed
  - ✓ Polyakov loop at their centre = -1/3
- ✓ Boundary condition dependence of zero modes localisation Garcia-Perez et al.
- ✓ Low temperature all monopoles have the same action on average
  - ✓ No change of the spectrum with the boundary conditions below T_c

The properties measured on the lattice agree with the characteristics of molecules (pairs) of Kaluza-Klein monopole-instantons in SU(N)

Overlap of left-right eigenmode projections



Overlap monotonic with the eigenvalue



Overlap increases for more localized states

Conjecture: localisation triggers chiral symmetry restoration.

Conclusions

- ✓We measured the properties of gauge invariant observables in correlation with each one of the Dirac low eigenmodes
- ✓Boundary condition dependence of the localisation mechanism
- ✓Properties agree with the characteristics of KK monopole-instantons pairs in *SU(N)*

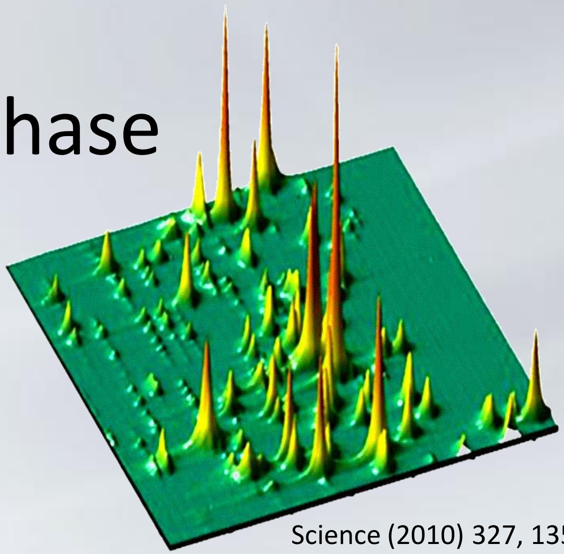
- ✓Conjectured mechanism relating restoration of chiral symmetry at high temperature to localisation
  - ✓Increased localisation → larger overlap → larger eigenvalues → chiral condensate suppressed
  - ✓Polyakov loop transition → localisation → chiral transition
- ✓Compatible results with staggered fermions analysis **model**
  - ✓Polyakov loop transition → localisation and chiral transition

Outlook

- Non perturbative effective potential for localisation
- Study interactions
- QCD with external magnetic field
  - Inverse catalysis (i.e. reduced chiral condensate with B around Tc)

Some ideas

- Fermions in different representations (different phase diagram)
- Imaginary chemical potential



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