QCD energy momentum tensor at finite temperature using gradient flow

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for

WHOT QCD collaboration

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Introduction

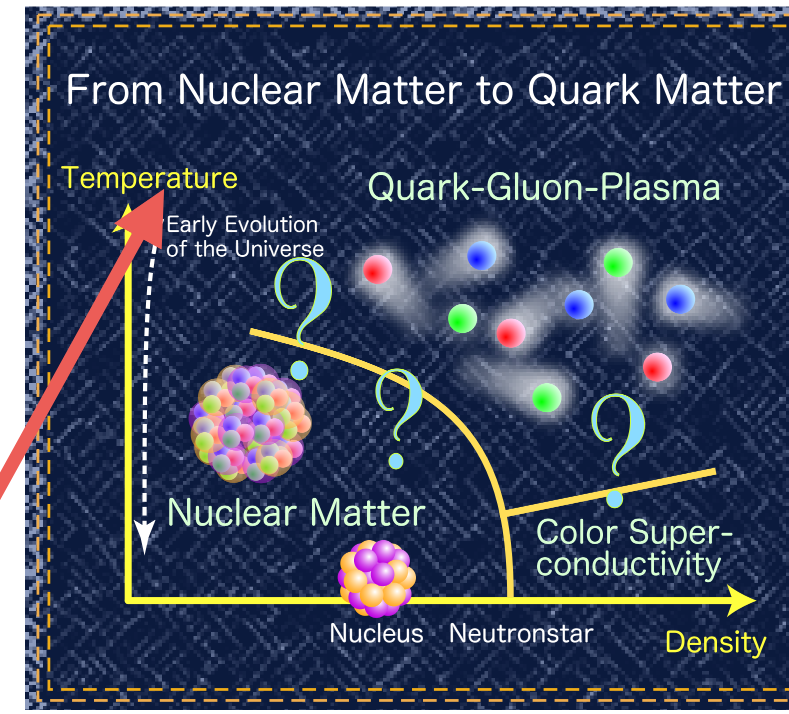
Energy momentum tensor ⟺ Poincare symmetry

$T_{\mu\nu}$

energy momentum

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

stress pressure



hot topics in QGP

- If we have $T_{\mu\nu}$
  - ➡ direct measurement of thermodynamic quantity
- Fluctuations and correlations of $T_{\mu\nu}$
  - ➡ specific heat, viscosity, $\cdots$

How to calculate $T_{\mu\nu}$ on lattice?

- Measure expectation values on lattice

  terms in QCD Lagrangian

  $\delta_{\mu\nu} F^a_{\rho\sigma}(x) F^a_{\rho\sigma}(x)$ $\delta_{\mu\nu} \bar{\psi}(x) \overleftrightarrow{\not{D}} \psi(x)$ $\delta_{\mu\nu} \bar{\psi}(x) \psi(x)$

  terms in QCD Lagrangian when trace is taken

  $F^a_{\mu\rho}(x) F^a_{\nu\rho}(x)$ $\bar{\psi}(x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \psi(x)$

- Renormalization

  Well established for E and P

  Karsch coefficients

  problems

  - non universal (No Poincare symmetry)
    - depends on: lattice action, operator
  - additive correction for $\delta_{\mu\nu} \bar{\psi}(x) \psi(x)$

How to calculate $T_{\mu\nu}$ on lattice?

Easier method for renormalization?

Gradient Flow

Narayanan-Neuberger(2006)
Lüscher(2009–)

Flow the gauge field

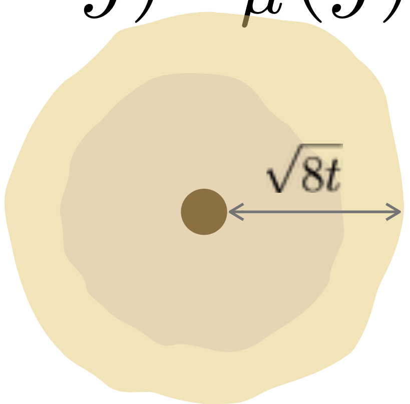
$$\partial_t A_\mu(t,x) = -\frac{\delta S_{\rm YM}}{\delta A_\mu} \qquad A_\mu(t=0,x) = A_\mu(x)$$

t: flow time, dim=[length^2]

A kind of diffusion equation $\partial_t A_\mu(t,x) = D_\nu G_{\nu\mu}$

Solution $A_\mu(t,x) = \int d^4y K_t(x-y) A_\mu(y) + \text{interactions}$

heat kernel $K_t(x) = \frac{e^{-x^2/4t}}{(4\pi t)^{D/2}}$



smear field within $\sqrt{8t}$

How to calculate $T_{\mu\nu}$ on lattice?

A great view point:

Gradient Flow as a renormalization scheme

Narayanan-Neuberger(2006), Lüscher(2010), Lüscher-Weisz(2011)

Gauge operators with flowed field $A_\mu(t,x)$

- does not have UV divergence
- does not have contact term singularity
- operators are renormalized

scale: $\sqrt{8t}$

absorbed

NP renormalized operator

lattice operator

flow + a→0

universal

finite ren.

$F^a_{\mu\nu}F^a_{\mu\nu}(x,t)$

$\mathrm{Re}\langle 1 - \square \rangle$

$\overline{\mathrm{MS}}$ scheme

differences in lattice action / lattice operator

How to calculate $T_{\mu\nu}$ on lattice?

From flowed operator to proper operator

NP renormalized in flow scheme

operator which satisfies WT identity

Need a matching factor like Wilson coefficient

scale matching $\mu = \frac{1}{\sqrt{8t}}$

Matching coefficients are calculable perturbatively at small t region

H.Suzuki, PTEP 2013, 083B03 (2013)

$$\{T_{\mu\nu}\}_{\mathrm{WT}}(x) = \lim_{t\to 0}\left\{c_1(t)\left[\tilde{\mathcal{O}}_{1\mu\nu}(t,x) - \frac{1}{4}\tilde{\mathcal{O}}_{2\mu\nu}(t,x)\right] + c_2(t)\left[\tilde{\mathcal{O}}_{2\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}_{2\mu\nu}(t,x)\right\rangle_{T=0}\right]\right\}$$

$\mathcal{O}_{1\mu\nu}(t,x) = F^a_{\mu\rho}F^a_{\nu\rho}(t,x)$ $\mathcal{O}_{2\mu\nu}(t,x) = \delta_{\mu\nu}F^a_{\rho\sigma}F^a_{\rho\sigma}(t,x)$

How to calculate $T_{\mu\nu}$ on lattice?

From flowed operator to proper operator

- Matching coefficients at one loop

$$c_1(t) = \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left(9\gamma - 18\ln 2 + \frac{19}{4}\right)$$

$$c_2(t) = \frac{1}{(4\pi)^2}\frac{33}{16}$$

Matching coefficients are calculable perturbatively at small t region

H.Suzuki, PTEP 2013, 083B03 (2013)

$$\{T_{\mu\nu}\}_{\mathrm{WT}}(x) = \lim_{t\to 0}\left\{c_1(t)\left[\tilde{\mathcal{O}}_{1\mu\nu}(t,x) - \frac{1}{4}\tilde{\mathcal{O}}_{2\mu\nu}(t,x)\right] + c_2(t)\left[\tilde{\mathcal{O}}_{2\mu\nu}(t,x) - \left\langle\tilde{\mathcal{O}}_{2\mu\nu}(t,x)\right\rangle_{T=0}\right]\right\}$$

$$\mathcal{O}_{1\mu\nu}(t,x) = F^a_{\mu\rho}F^a_{\nu\rho}(t,x) \qquad \mathcal{O}_{2\mu\nu}(t,x) = \delta_{\mu\nu}F^a_{\rho\sigma}F^a_{\rho\sigma}(t,x)$$

How to calculate $T_{\mu\nu}$ on lattice?

Three steps to calculate $T_{\mu\nu}$

1. Flow the link variable $\partial_t U_\mu(t,x) U_\mu^\dagger(t,x) = -g_0^2 \partial_{x,\mu} S_{\text{lat}}(U)$

2. Calculate expectation value of flowed operators

$$\mathcal{O}_{1\mu\nu}(t,x) = F^a_{\mu\rho} F^a_{\nu\rho}(t,x) \quad \mathcal{O}_{2\mu\nu}(t,x) = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}(t,x)$$

appropriately defined on lattice

3. Multiply the coefficients and take t→0 limit

$$\{T_{\mu\nu}\}_{\text{WT}}(x) = \lim_{t\to 0}\left\{ c_1(t) \left[\tilde{\mathcal{O}}_{1\mu\nu}(t,x) - \frac{1}{4} \tilde{\mathcal{O}}_{2\mu\nu}(t,x) \right] + c_2(t) \left[\tilde{\mathcal{O}}_{2\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}_{2\mu\nu}(t,x) \right\rangle_{T=0} \right] \right\}$$

How to calculate $T_{\mu\nu}$ on lattice?

Previous works and lessons

FlowQCD Collaboration (2014-)

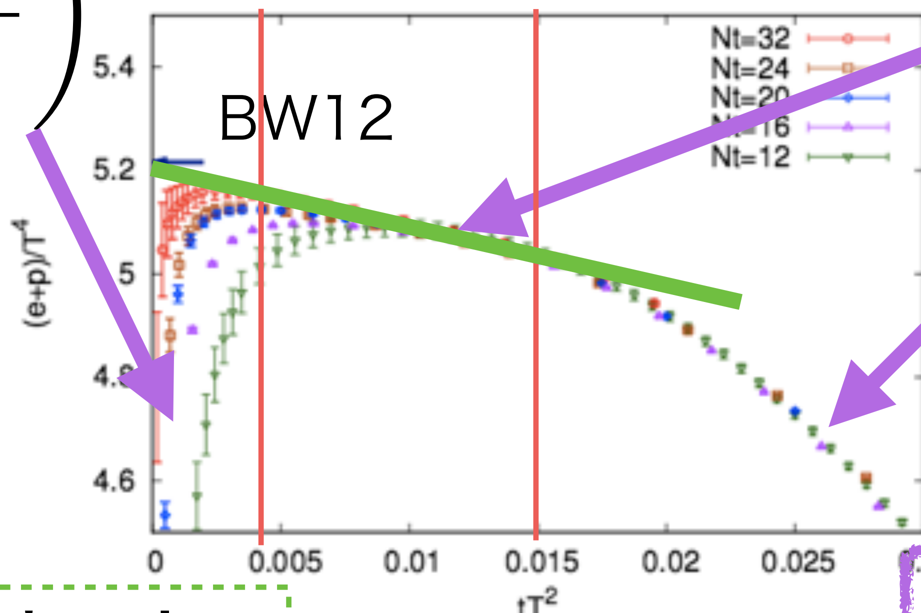
T=1.66Tc
quench

(E+P)/T^4

$O\left(\frac{a^2}{t}\right)$

$O(t)$

$O(t^2)$



why we need t→0

flowed operator on lattice

$\{T_{\mu\nu}\}(x,t,a) = \{T_{\mu\nu}\}_{\rm WT}(x)$ +t(dim6 operator)

the window region

$+\frac{a^2}{t}$ (dim4 operator) — tamed at large t

+t^2(dim8 operator) — tamed at small t

$+O(a^2T^2, a^2m^2, a^2\Lambda^2_{\rm QCD})$ → need to take a→0 limit

What's new? Quarks included!

Flow of quark field

Lüscher, JHEP 1304, 123 (2013)

$$\partial_t \chi(t,x) = D_\mu D_\mu \chi(t,x) \qquad \chi(t=0,x) = \psi(x)$$

$$\partial_t \bar{\chi}(t,x) = \bar{\chi}(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu \qquad \bar{\chi}(t=0,x) = \bar{\psi}(x)$$

flow the gauge field simultaneously

Renormalization is needed for quark field

$$\chi_R(t,x) = Z_\chi \chi_0(t,x)$$

No more renormalization is needed for composite op.

$$(\bar{\chi}(t,x)\chi(t,x))_R = Z_\chi^2 \, (\bar{\chi}(t,x)\chi(t,x))_0$$

How to calculate T$_{\mu\nu}$ on lattice?

Three steps to calculate T$_{\mu\nu}$

1. Flow the gauge and quark field

2. Calculate expectation value of flowed operators

3. Multiply the coefficients and take t→0 limit

$$\{T_{\mu\nu}\}^q_{\mathrm{WT}}(x) = \lim_{t\to 0}\Bigg\{c_3(t)\sum_{r=u,d,s}\left(\tilde{\mathcal{O}}^r_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}^r_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x)\right\rangle_{T=0}\right)$$
$$+c_4(t)\sum_{r=u,d,s}\left(\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}^r_{4\mu\nu}(t,x)\right\rangle_{T=0}\right) + \sum_{r=u,d,s} c^r_5(t)\left(\tilde{\mathcal{O}}^r_{5\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}^r_{5\mu\nu}(t,x)\right\rangle_{T=0}\right)\Bigg\}$$

How to calculate T$_{\mu\nu}$ on lattice?

3. Multiply the coefficients and take t→0 limit

$$\{T_{\mu\nu}\}^{q}_{\mathrm{WT}}(x)=\lim_{t\to 0}\Big\{c_3(t)\sum_{r=u,d,s}\left(\tilde{\mathcal{O}}^r_{3\mu\nu}(t,x)-2\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x)-\left\langle\tilde{\mathcal{O}}^r_{3\mu\nu}(t,x)-2\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x)\right\rangle_{T=0}\right)$$
$$+c_4(t)\sum_{r=u,d,s}\left(\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x)-\left\langle\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x)\right\rangle_{T=0}\right)+\sum_{r=u,d,s}c_5^r(t)\left(\tilde{\mathcal{O}}^r_{5\mu\nu}(t,x)-\left\langle\tilde{\mathcal{O}}^r_{5\mu\nu}(t,x)\right\rangle_{T=0}\right)\Big\}$$

$$\tilde{\mathcal{O}}^r_{3\mu\nu}(t,x)\equiv\varphi_r(t)\bar{\chi}_r(t,x)\left(\gamma_\mu\overleftrightarrow{D}_\nu+\gamma_\nu\overleftrightarrow{D}_\mu\right)\chi_r(t,x)$$

$$\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x)\equiv\varphi_r(t)\delta_{\mu\nu}\bar{\chi}_r(t,x)\overleftrightarrow{\not{D}}\chi_r(t,x)$$

$$\tilde{\mathcal{O}}^r_{5\mu\nu}(t,x)\equiv\varphi_r(t)\delta_{\mu\nu}\bar{\chi}_r(t,x)\chi_r(t,x)$$

wave function renormalization

$$\varphi_r(t)\equiv\frac{-6}{(4\pi)^2t^2\left\langle\bar{\chi}_r(t,x)\overleftrightarrow{\not{D}}\chi_r(t,x)\right\rangle_{T=0}}$$

VEV sub. $2\left\langle\tilde{\mathcal{O}}^r_{3\mu\nu}(t,x)\right\rangle_{T=0}=\left\langle\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x)\right\rangle_{T=0}=\frac{-6}{(4\pi)^2t^2}\delta_{\mu\nu}$

How to calculate $T_{\mu\nu}$ on lattice?

3. Multiply the coefficients and take t→0 limit

$$\{T_{\mu\nu}\}^{q}_{\mathrm{WT}}(x) = \lim_{t\to 0}\Bigg\{ c_3(t)\sum_{r=u,d,s}\left(\tilde{\mathcal{O}}^{r}_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}^{r}_{4\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}^{r}_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}^{r}_{4\mu\nu}(t,x)\right\rangle_{T=0}\right)$$
$$+c_4(t)\sum_{r=u,d,s}\left(\tilde{\mathcal{O}}^{r}_{4\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}^{r}_{4\mu\nu}(t,x)\right\rangle_{T=0}\right) + \sum_{r=u,d,s} c^{r}_{5}(t)\left(\tilde{\mathcal{O}}^{r}_{5\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}^{r}_{5\mu\nu}(t,x)\right\rangle_{T=0}\right)\Bigg\}$$

$$c_3(t) = \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left(2 + \frac{4}{3}\ln(432)\right)\right\}$$

$$c_4(t) = \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2$$

$$c^{r}_{5}(t) = -\bar{m}_r(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left(4\gamma - 8\ln 2 + \frac{14}{3} + \frac{4}{3}\ln(432)\right)\right\}$$

Makino-Suzuki, PTEP 2014, 063B02 (2014)

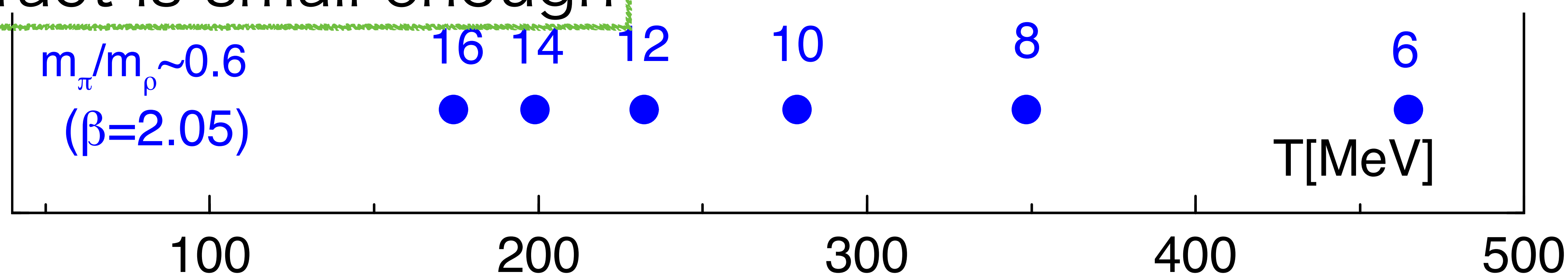
Numerical setups

- Iwasaki gauge action
  - β=2.05 : a∼0.07 [fm]
- Fixed scale method

aT~1/N_t artifact may be severe

  - T=1/(aNt), Nt=16, 14, 12, 10, 8, 6, 4

aT~1/N_t artifact is small enough



32^3xNt for T≠0

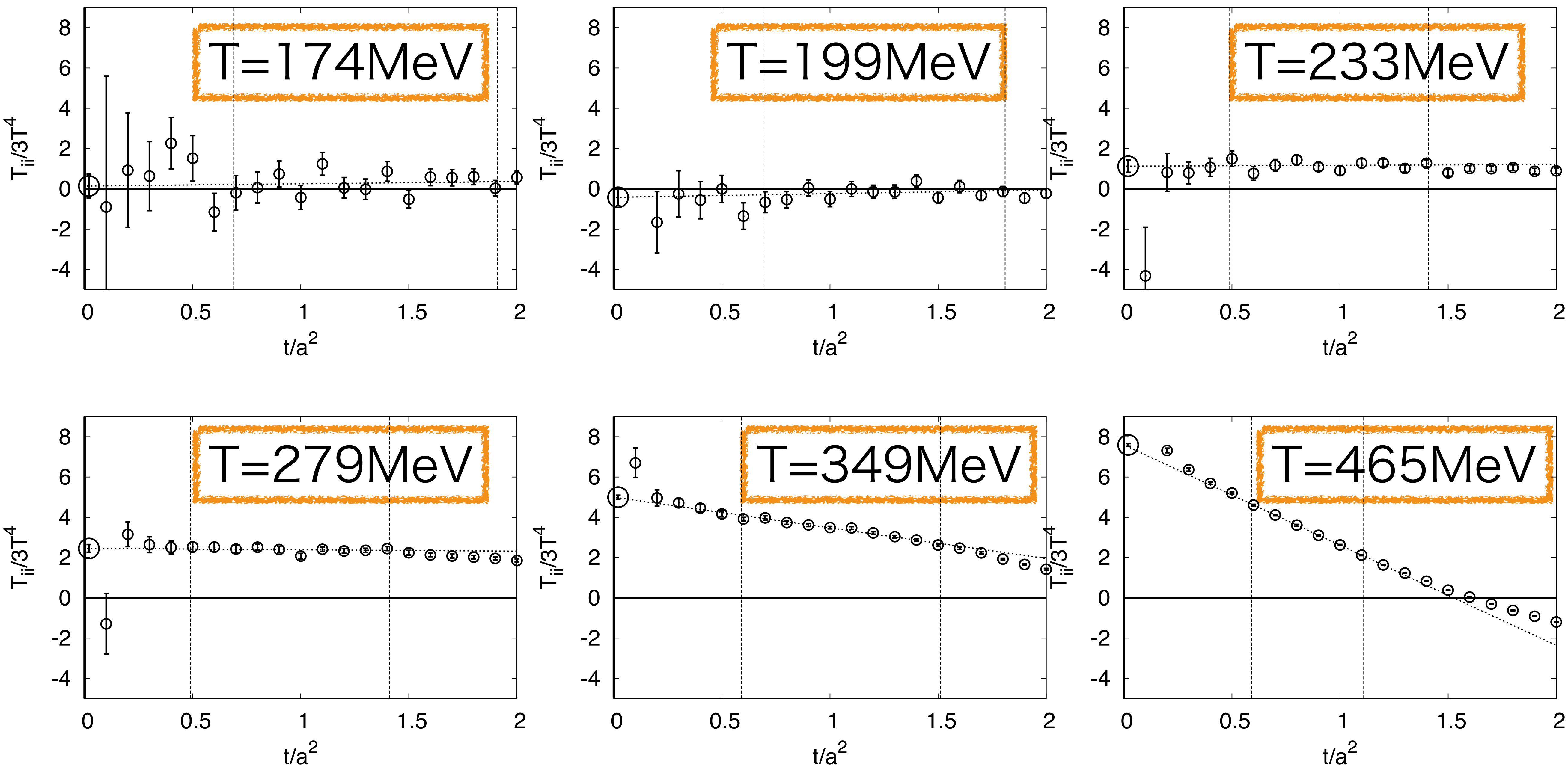
28^3x56 for T=0

- Nf=2+1
- NP improved Wilson fermion
- On an equal quark mass line

$$\frac{m_\pi}{m_\rho} \sim 0.6 \quad \frac{m_{\eta_{ss}}}{m_\phi} \sim 0.74$$

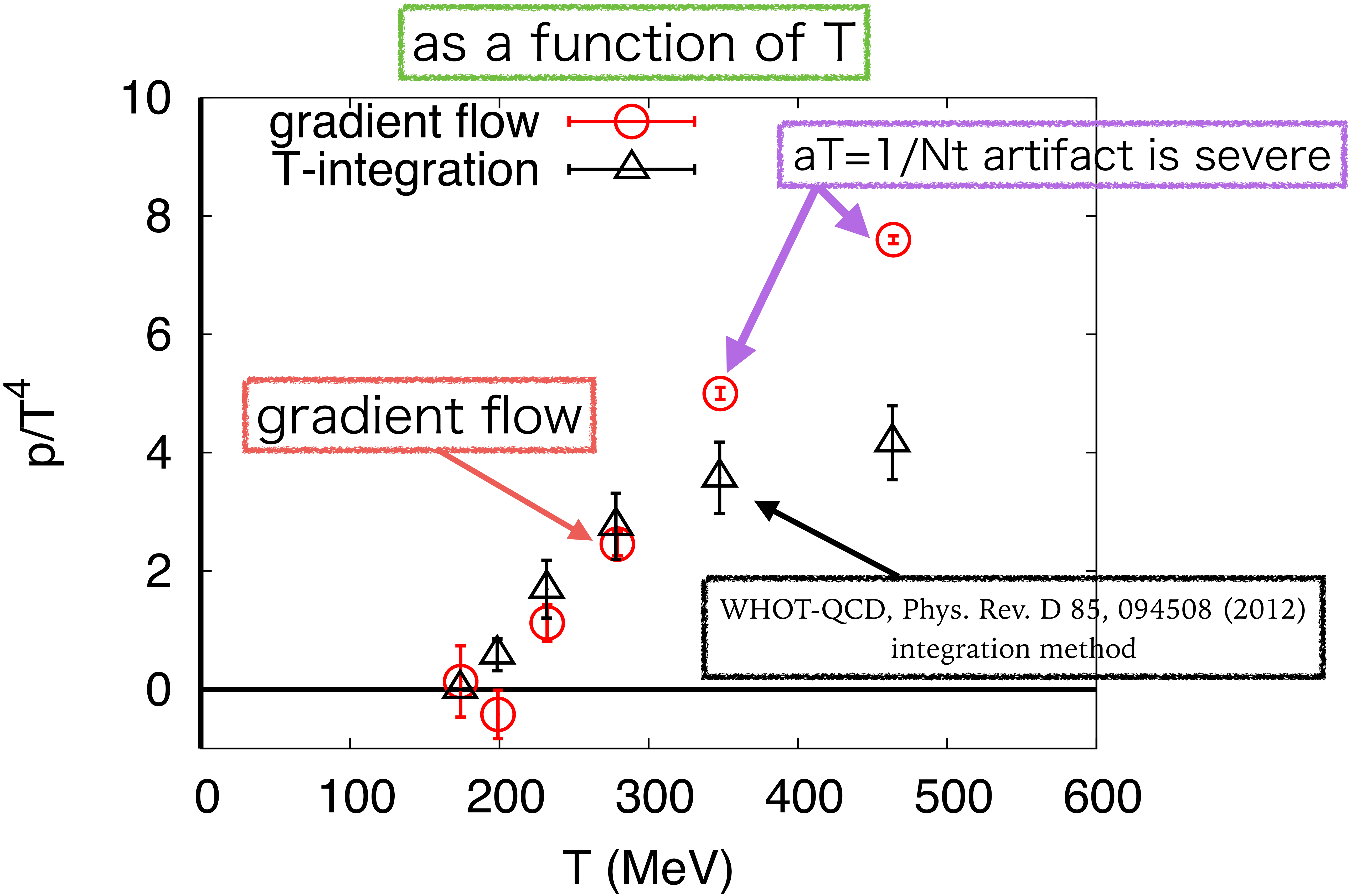
p/T^4

$t \to 0$ limit by linear extrapolation



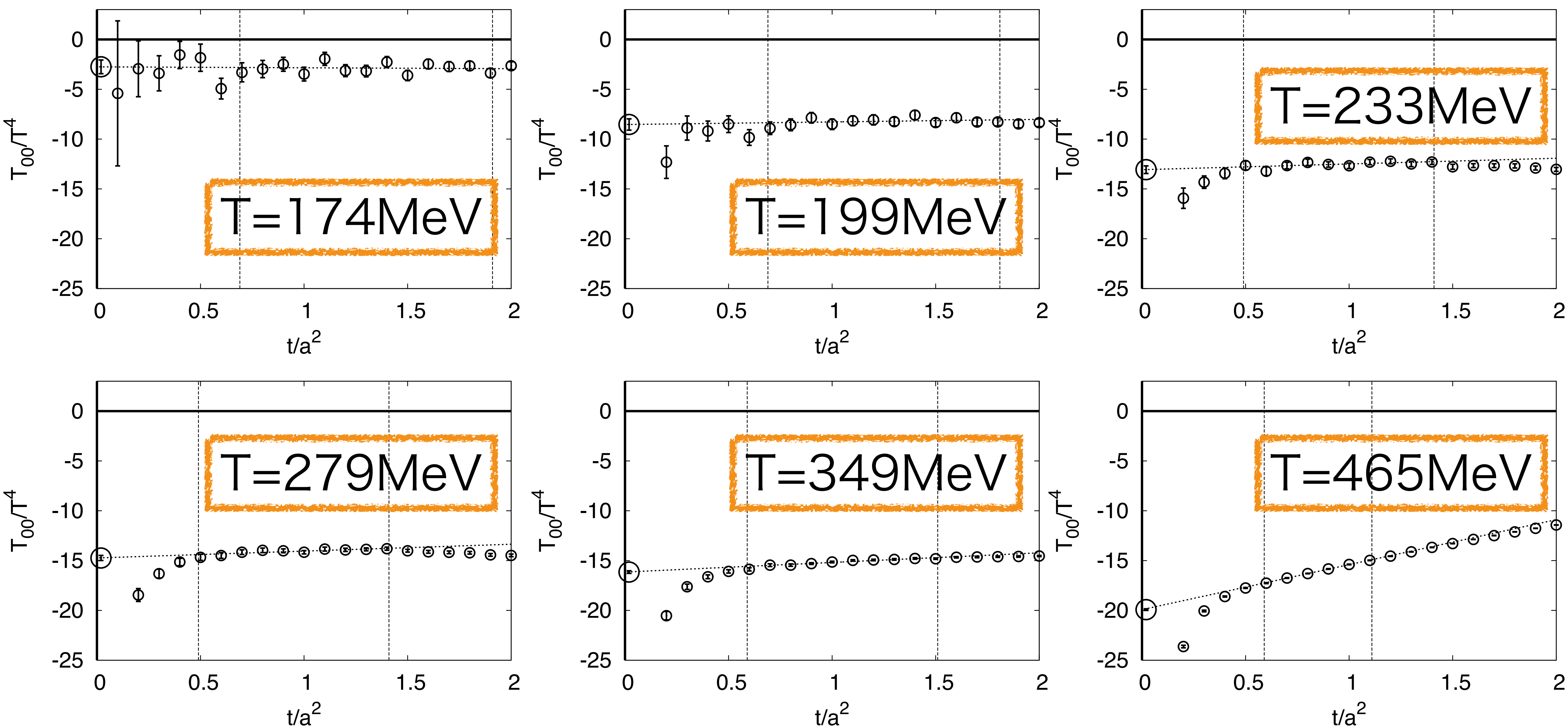
t/a^2 t/a^2 t/a^2

p/T^4


as a function of T
gradient flow
T-integration
aT=1/Nt artifact is severe
gradient flow
WHOT-QCD, Phys. Rev. D 85, 094508 (2012)
integration method
p/T4
T (MeV)
10
8
6
4
2
0
0
100
200
300
400
500
600

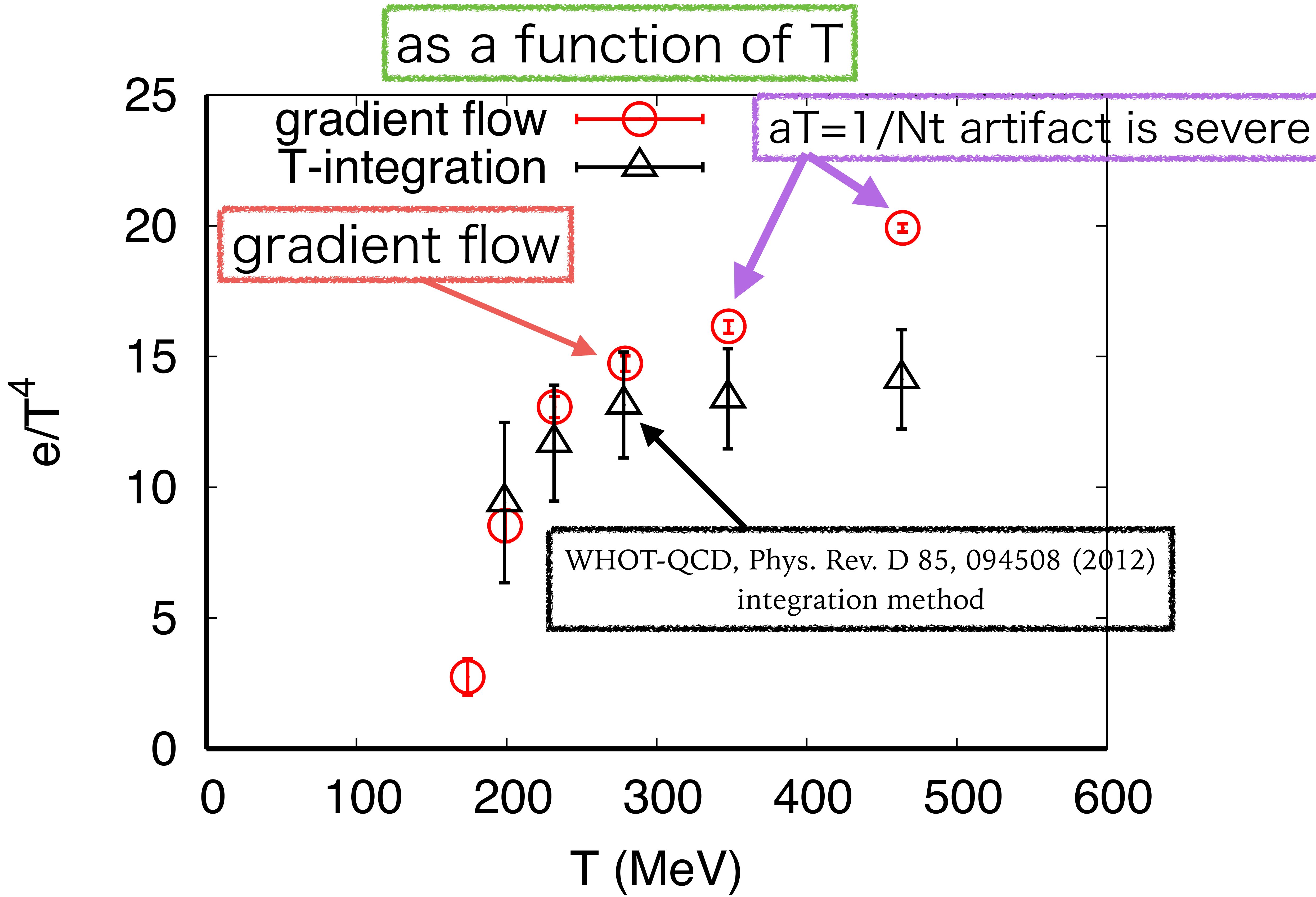
e/T^4

t→0 limit by linear extrapolation



t/a² t/a² t/a²

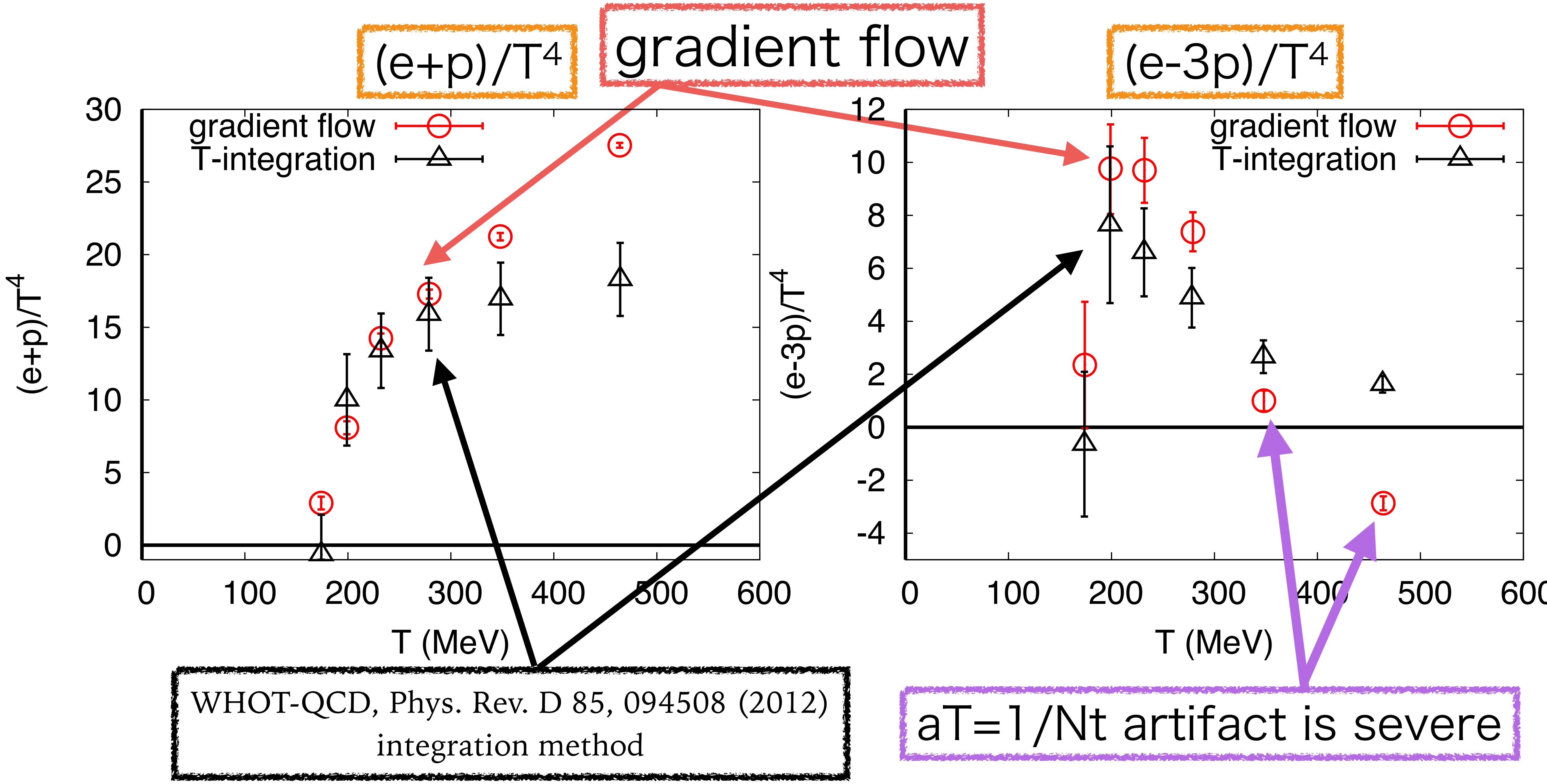
e/T^4



as a function of T
gradient flow
T-integration
aT=1/Nt artifact is severe
gradient flow
WHOT-QCD, Phys. Rev. D 85, 094508 (2012)
integration method
e/T4
T (MeV)
0
5
10
15
20
25
0
100
200
300
400
500
600

(e+p)/T^4 and (e-3p)/T^4

as a function of T



Measurement of chiral condensate

Only three steps!

1. Flow the gauge and quark field

2. Calculate VEV of flowed operators

3. Multiply the coefficients and take t→0 limit

$$(\overline{\psi}\psi)_{\overline{\rm MS}}(2{\rm GeV}) = \lim_{t\to 0} c_S(t) \frac{m_{\overline{\rm MS}}(1/\sqrt{8t})}{m_{\overline{\rm MS}}(2{\rm GeV})} \varphi(t) \bar{\chi}(t,x)\chi(t,x)$$

flowed operator

wave function renormalization

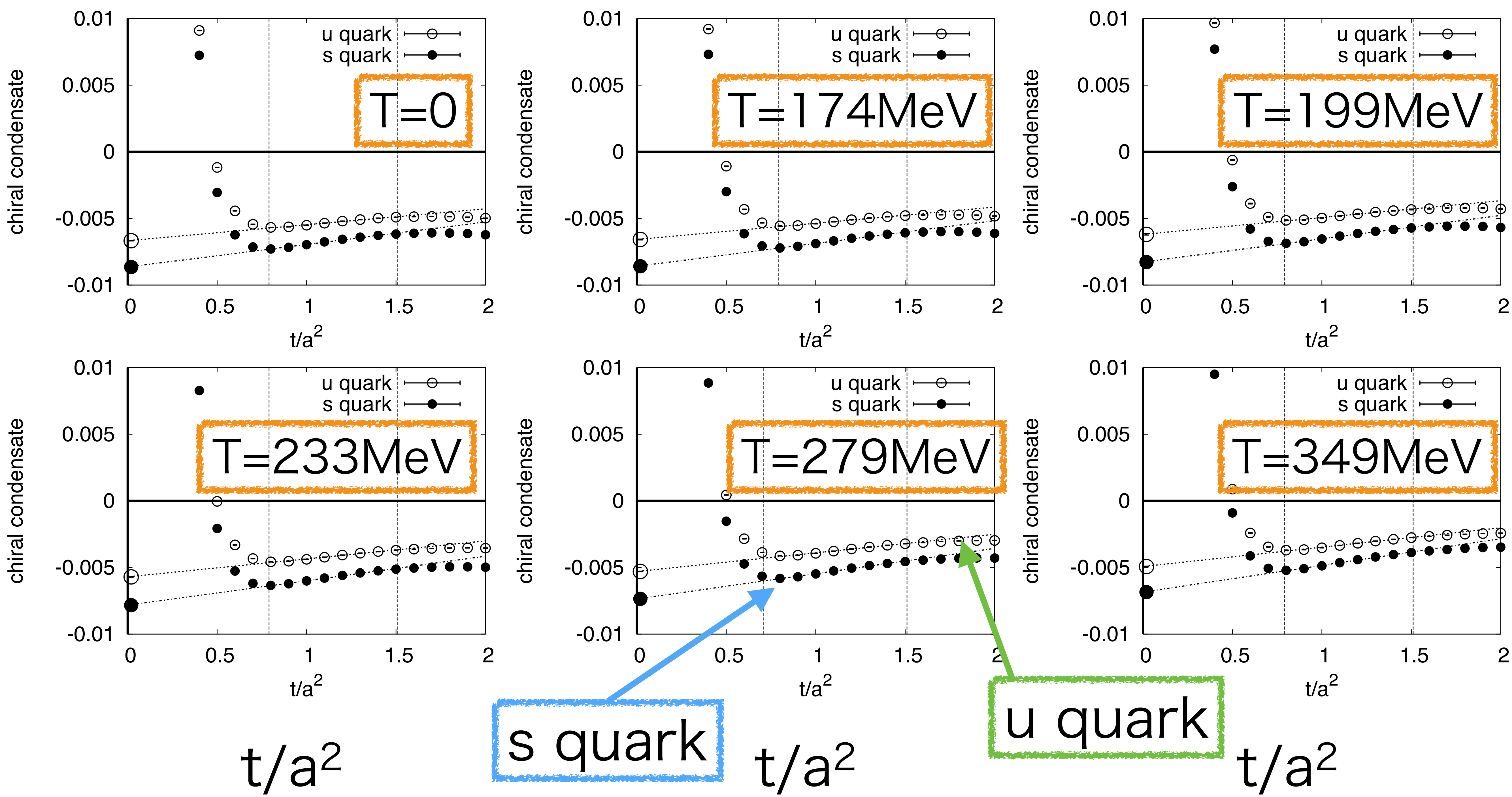
$$\varphi(t) = \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}(t,x) \overleftrightarrow{\not{D}} \chi(t,x) \right\rangle_{T=0}}$$

matching coefficient

$$c_S(t) = \left\{ 1 + \frac{\bar{g}_{\overline{\rm MS}}(1/\sqrt{8t})^2}{(4\pi)^2} \left(4\gamma - 8\ln 2 + 8 + \frac{4}{3}\ln(432) \right) \right\}$$

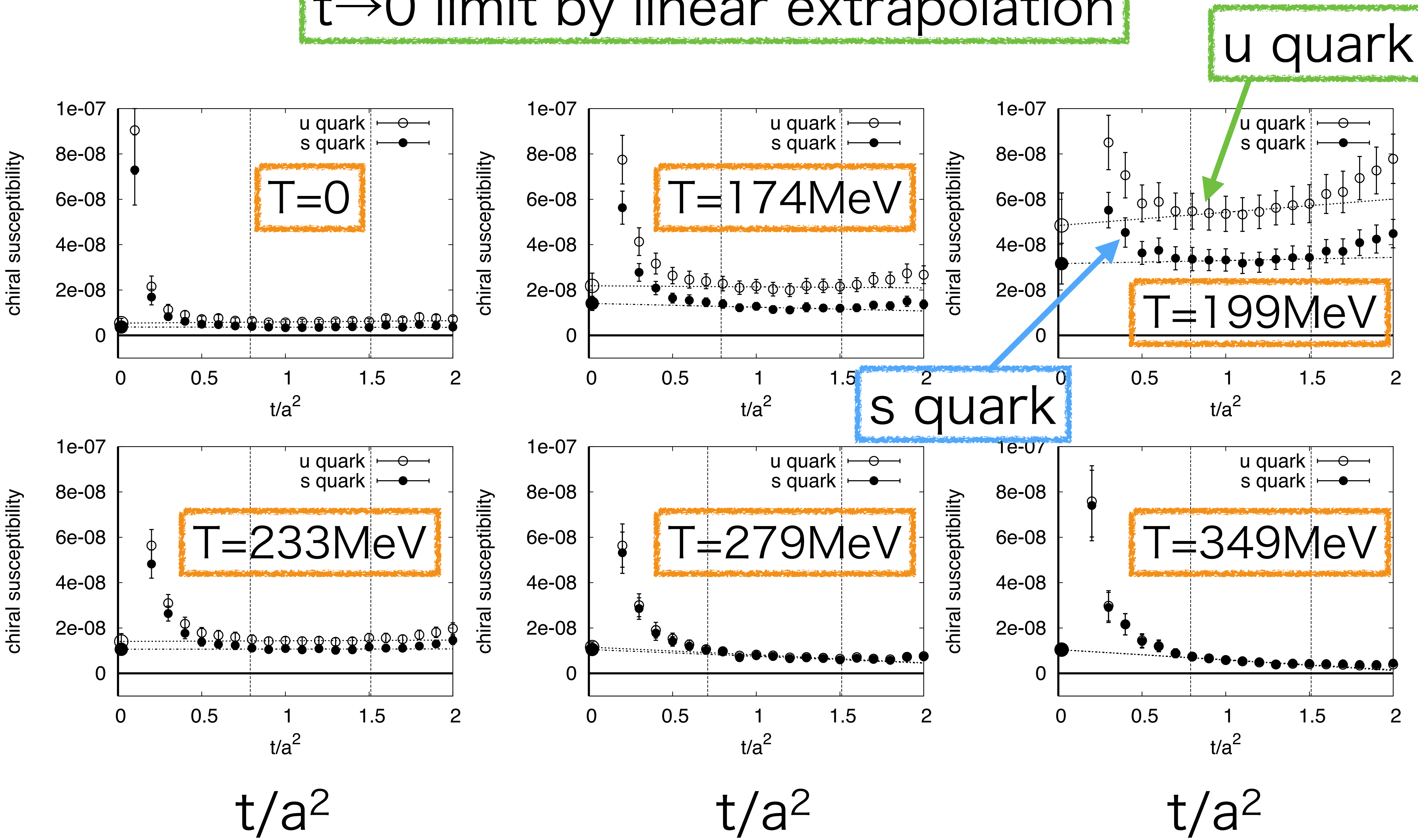
Chiral condensate

t→0 limit by linear extrapolation



disconnected chiral susceptibility

t→0 limit by linear extrapolation

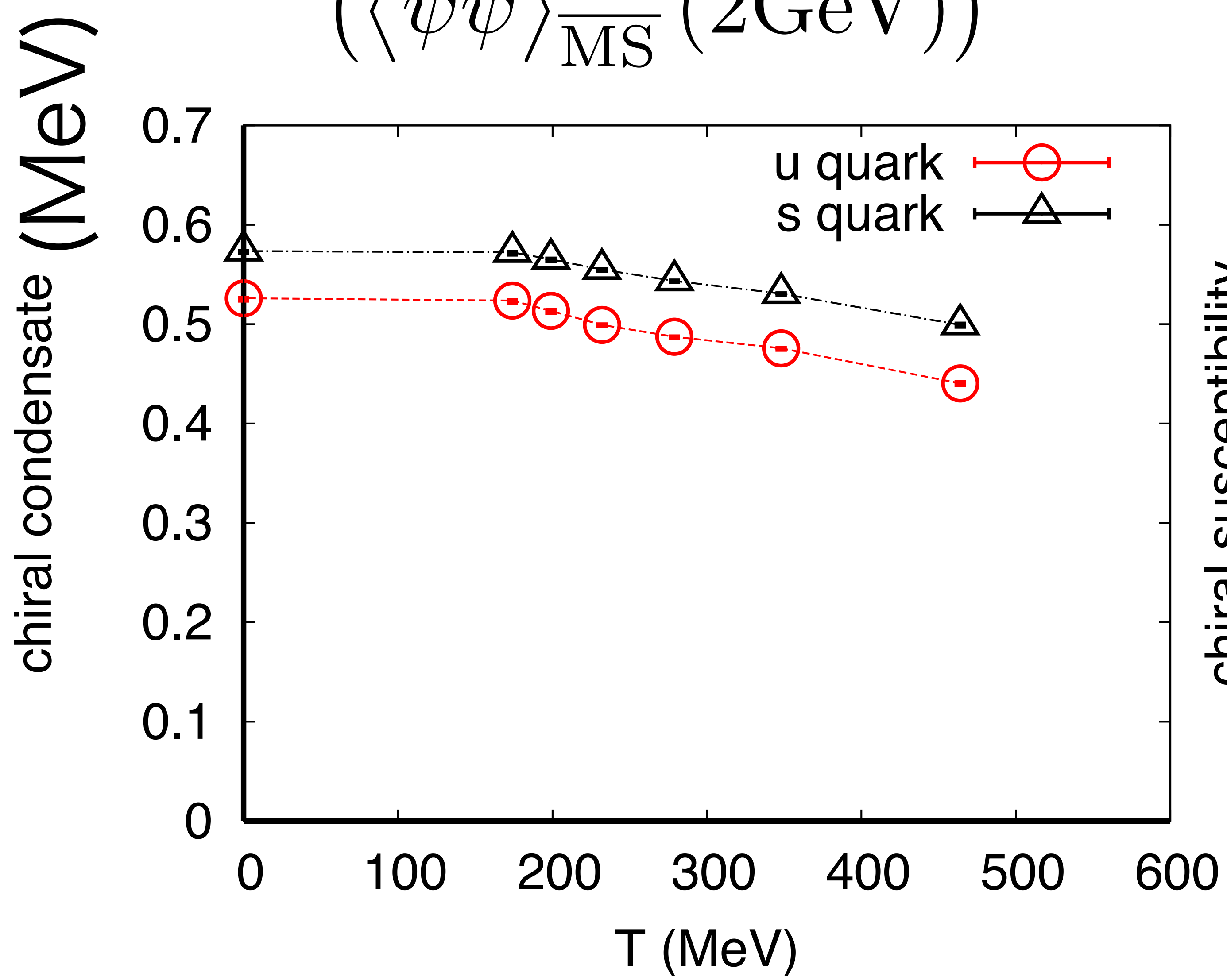


Chiral condensate

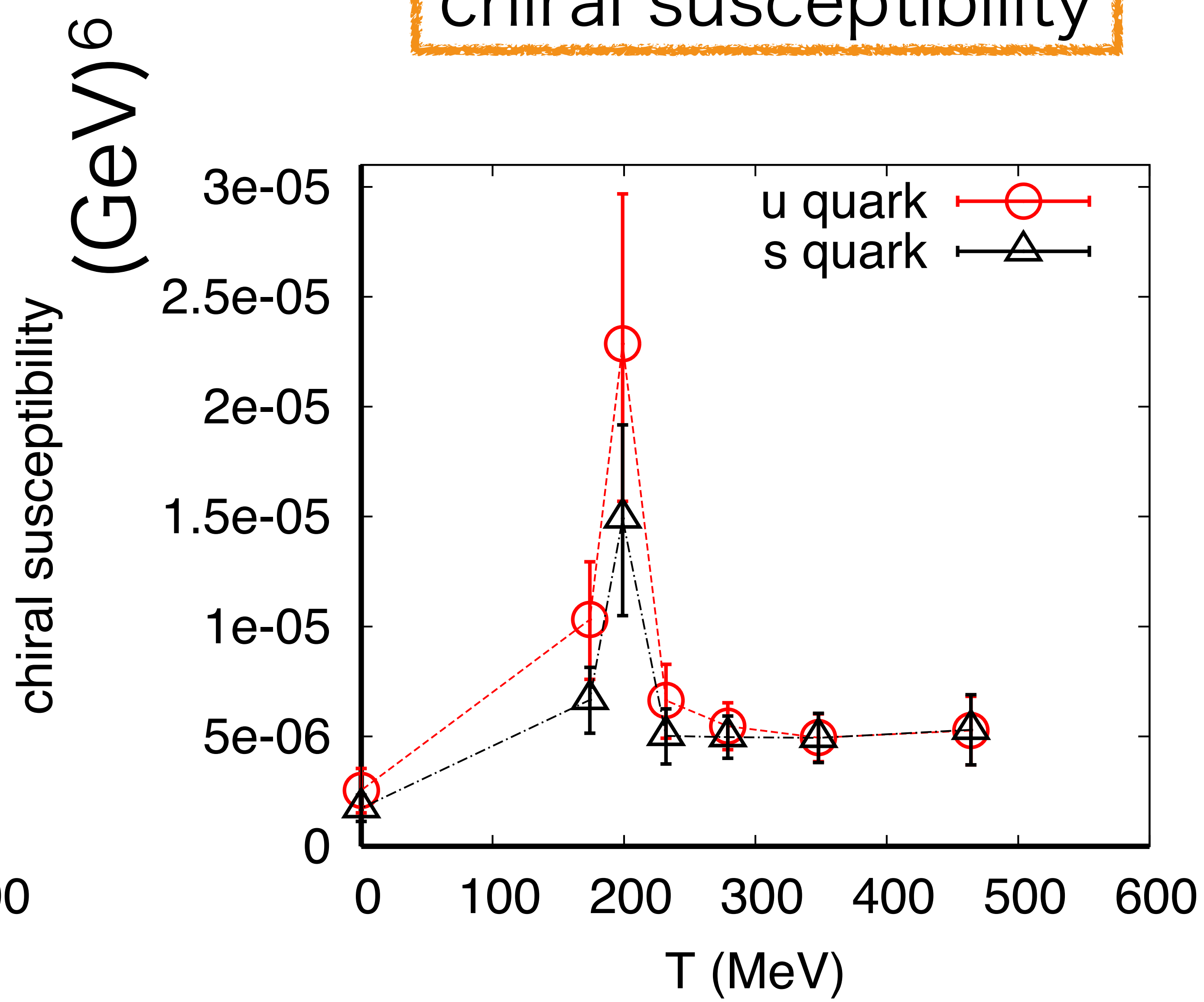
as a function of T

chiral condensate

$$\left(\langle\overline{\psi}\psi\rangle_{\overline{\rm MS}}(2{\rm GeV})\right)^{1/4}$$



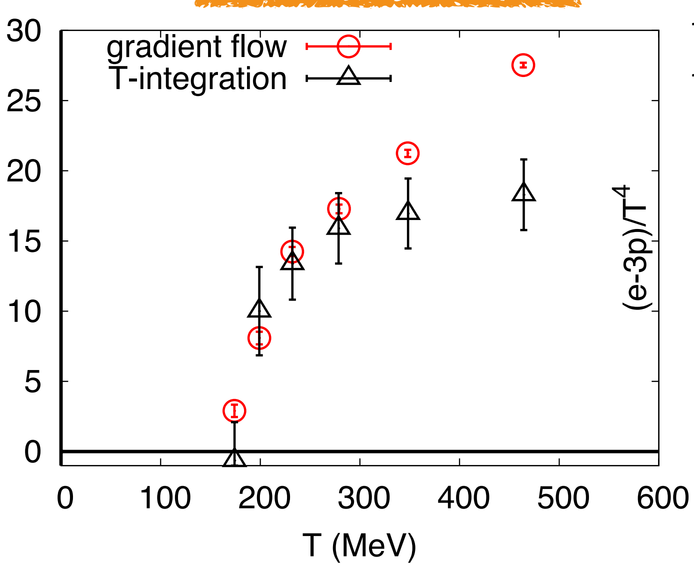
disconnected
chiral susceptibility



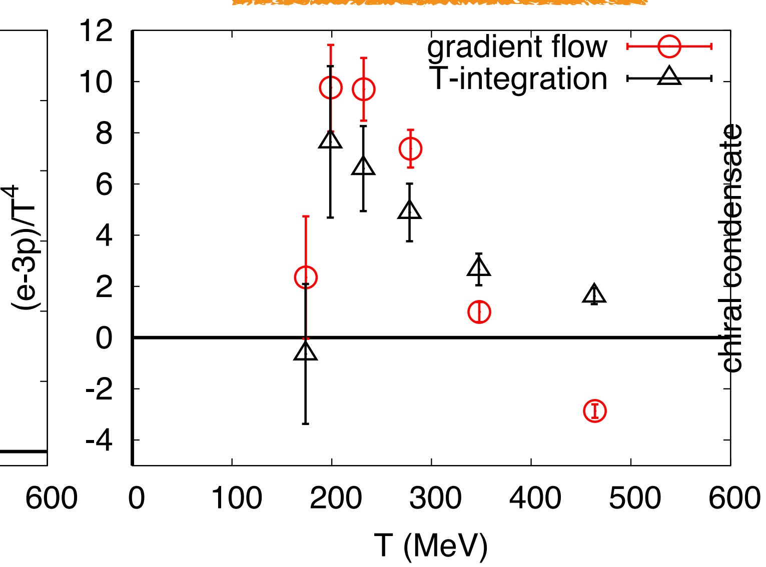
Summary

- Flow method works well for EM tensor!
  - as powerful as the derivative method.
- More suitable for Wilson fermion.
- We have exciting results:

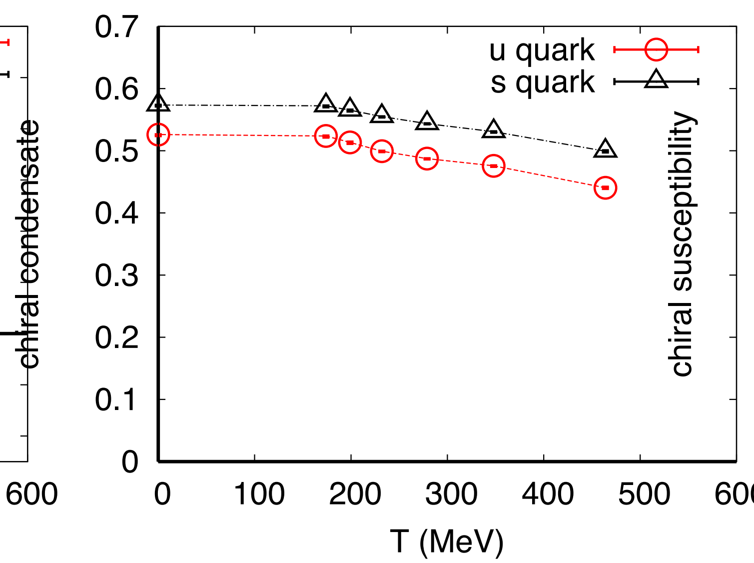
$(e+p)/T^4$



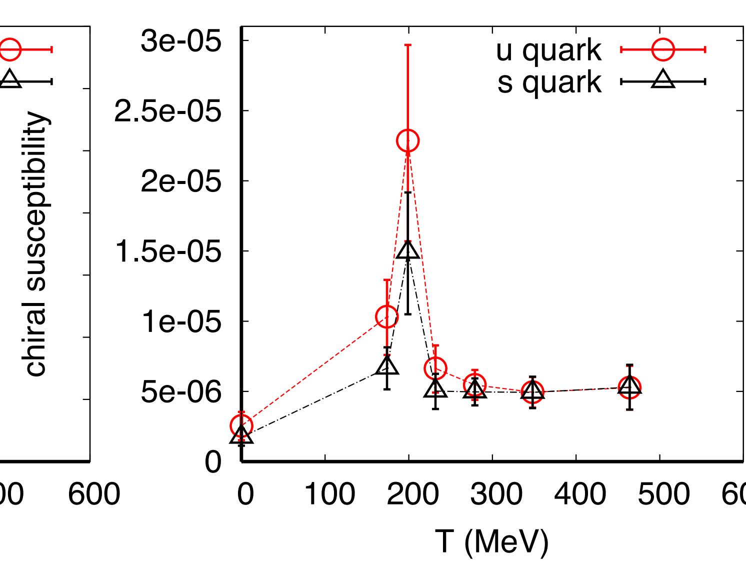
$(e-3p)/T^4$



chiral condensate



chiral susceptibility



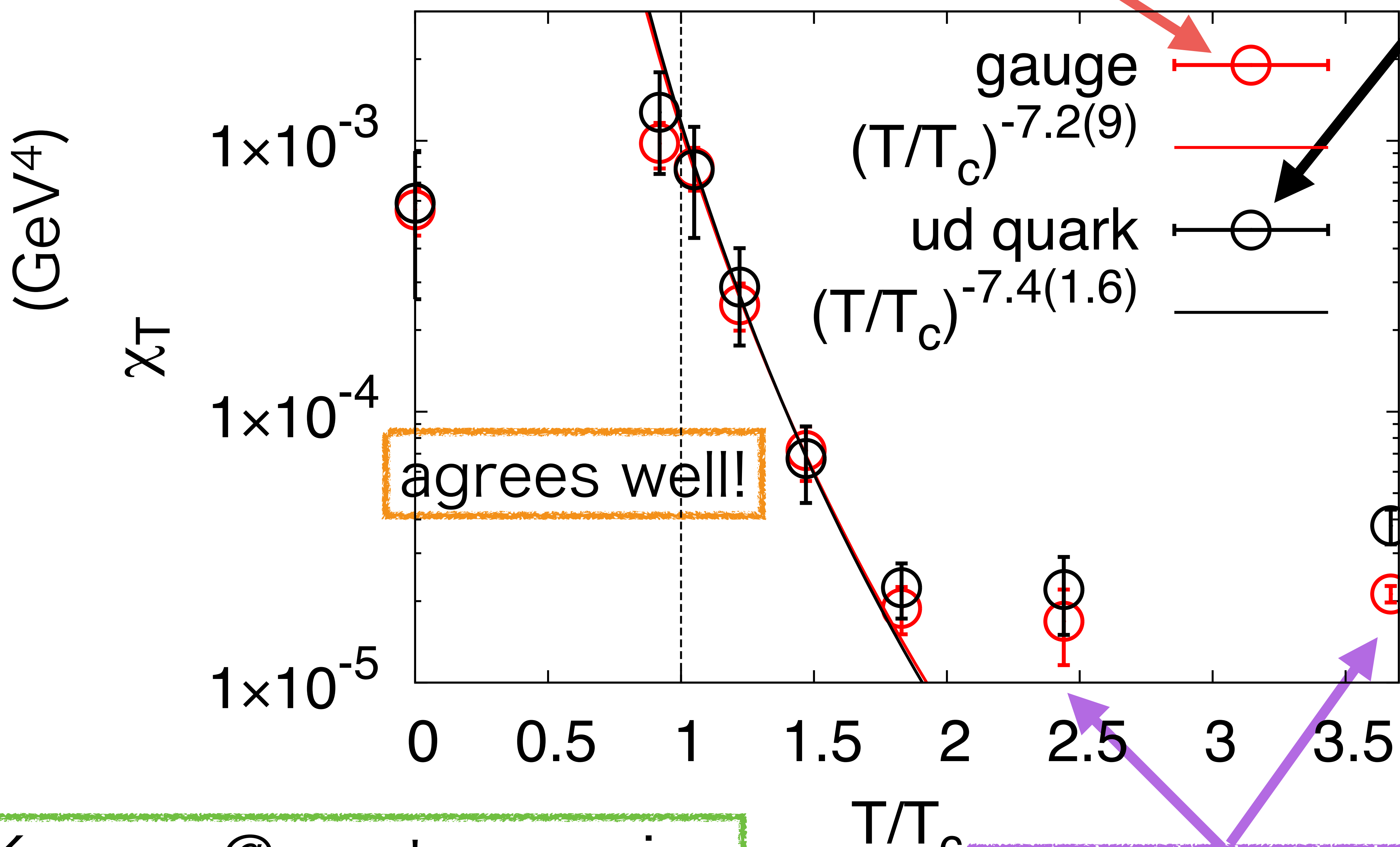
- Lattice artifact is severe for Nt=4, 6, 8
- We want work with fluctuation and correlation function using the flow!

topological susceptibility

$$\chi_T = \frac{1}{V_4}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$

$$Q = \frac{1}{64\pi^2}\int d^4x \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

$$\langle Q^2\rangle = \frac{m^2}{N_f^2}\langle P^0 P^0\rangle_{\rm disc}$$



Kanaya @ poster session

aT=1/Nt artifact is severe

Energy and Pressure

T=279MeV

contributions from gauge and quarks