16/8/3@ xQCD-2016

Topological susceptibility in finite-temperature (2+1)-flavor QCD with gradient flow

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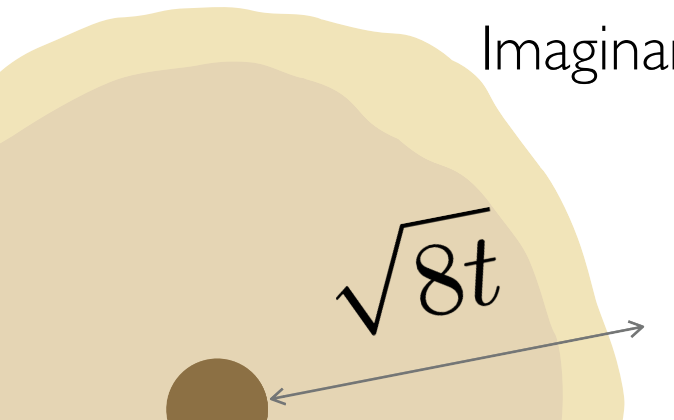
(WHOT-QCD Collaboration)

QCD Thermodynamics with Gradient Flow

Gradient flow

Lüscher(2009–), Narayanan-Neuberger(2006)



Imaginary evolution of the system into a fictitious "time" t preserving gauge sym. etc.:

(ex) pure gauge theory $\dot{B}_\mu = D_\nu G_{\nu\mu}$, $\quad B_\mu\big|_{t=0} = A_\mu \leftarrow$ original gauge field

We may view the flowed field $B\mu$ as a smeared $A\mu$ over a physical range of $\sqrt{(8t)}$.

It was shown that operators of flowed fields have no UV divergences nor short-dist. singularities at $t > 0$.

Lüscher-Weisz(2011)

GF provides us with a new physical (i.e. non-perturbative) renormalization scheme, which is directly calculable on the lattice in the $a \to 0$ limit.

This opened many possibilities to drastically simplify lattice evaluation of physical observables.

Our project: Application of GF to thermodyn. of (2+1)-flavor QCD

EMT by GF : H. Suzuki (2013), Makino-Suzuki (2014)

Chiral condensate by GF : Hieda-Suzuki (2016)

=> *Talk by Taniguchi (Aug. 1)*

Topological charge / susceptibility by GF : this talk

Gauge vs. Fermion Definitions for Topological Susceptibility

$$\chi_T = \frac{1}{V_4}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$

Gauge definition

Lüscher('10), Consonni-Engel-Giusti('15)

$$Q = \frac{1}{64\pi^2}\int d^4x \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

Equivalence shown with GW quarks on the lattice.
Large discrepancy found with non-chiral quarks.
E.g. Petreczky et al.(1606.03145): factor $\approx 2^4$ different χ at Nt=12 with HISQ.

Fermion definition

Bochicchio et al.('84), Giusti-Rossi-Testa('04)

$$\langle Q^2\rangle = \frac{m^2}{N_f^2}\left(\langle P^0 P^0\rangle - N_f\langle P^a P^a\rangle\right) = \frac{m^2}{N_f^2}\langle P^0 P^0\rangle_{\rm disc}$$

$$P^0 = \int d^4x \bar{\psi}(x)\gamma_5\psi(x)$$

$$P^a = \int d^4x \bar{\psi}(x) T^a \gamma_5 \psi(x) \qquad \psi = (\psi_1, \cdots, \psi_{N_f})$$

Note on the fermion definition

$$\langle Q^2\rangle = \frac{m^2}{N_f^2}\left(\langle P^0P^0\rangle - N_f\langle P^aP^a\rangle\right) = \frac{m^2}{N_f^2}\langle P^0P^0\rangle_{\rm disc}$$

Integrated Ward-Takahashi identities

Giusti-Rossi-Testa('04)

Abelian $\quad -2m\langle P^0\mathcal{O}\rangle + 2N_f\langle Q\mathcal{O}\rangle = i\langle\delta^0\mathcal{O}\rangle$

$\mathcal{O}=Q$ $\xrightarrow{\text{Abelian}}$ $N_f\langle Q^2\rangle = m\langle P^0Q\rangle$

singlet scalar

$\mathcal{O}=P^0$ $\xrightarrow{\text{Abelian}}$ $N_f\langle QP^0\rangle = m\langle P^0P^0\rangle - \langle S^0\rangle$

non-Abelian $\quad -2m\langle P^a\mathcal{O}\rangle = i\langle\delta^a\mathcal{O}\rangle$

non-singlet

$\mathcal{O}=P^b$ $\xrightarrow{\text{NA}}$

$$-2m\langle P^aP^b\rangle = i\langle\delta^aP^b\rangle = -\left(\delta^{ab}\frac{2}{N_f}\langle S^0\rangle + d_{abc}\underbrace{\langle S^c\rangle}_{0}\right)$$

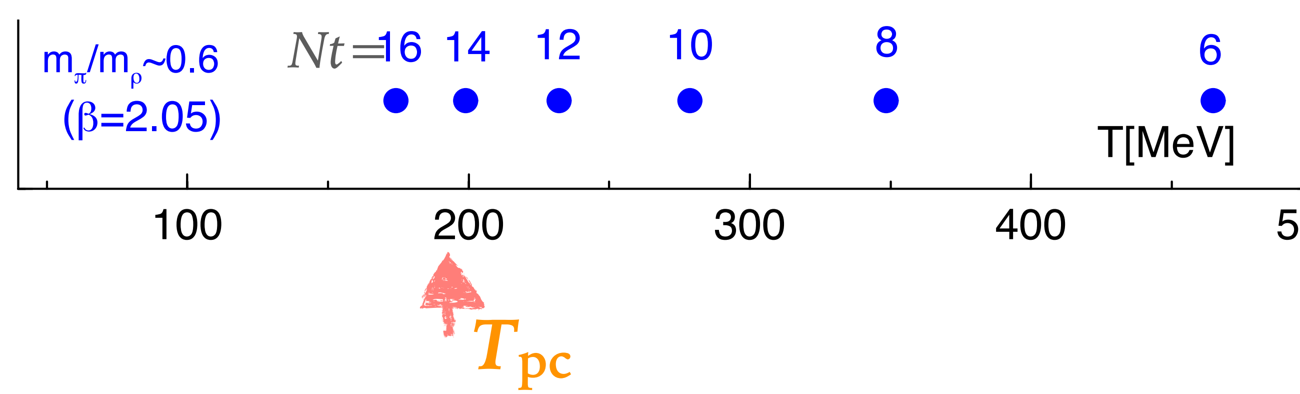
Simulation Parameters

WHOT-QCD, Phys.Rev.D85, 094508 (2012)

- ☑ Nf=2+1 QCD, Iwasaki gauge + NP-clover // fine lattice, physical s & heavy ud
- ☑ CP-PACS+JLQCD's $T = 0$ config. ($\beta = 2.05$, 28^3x56, $a \approx 0.07$fm, $m_{\rm PS}/m_{\rm V} \approx 0.63$) available on ILDG/JLDG
- ☑ $T > 0$ by fixed-scale approach, WHOT-QCD config.(32^3xNt, $Nt = 4, 6, 8, 10, 12, 14, 16$)
- ☑ gauge measurements at every config.
- ☑ quark measurements every 10/5 config's (T=/>0), using a noisy estimator method.
- ☐ continuum extrapolation => next step study

T (MeV)	$T/T_{\rm pc}$	N_t	$t_{1/2}$	gauge confs.
0	0	56	24.5	650
174	0.92	16	8	1440
199	1.05	14	6.125	1270
232	1.22	12	4.5	1290
279	1.47	10	3.125	780
348	1.83	8	2	510
464	2.44	6	1.125	500
697	3.67	4	0.5	700

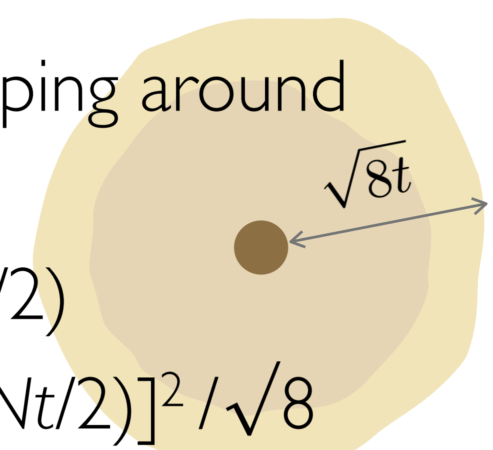
$T_{\rm pc}$ = 190 MeV assumed



To avoid oversmearing, wrapping around the lattice,

$$\sqrt{(8t/a^2)} \leq \min(Ns/2, Nt/2)$$

i.e., $\boldsymbol{t/a^2 \leq t_{1/2}} = [\min(Ns/2, Nt/2)]^2/\sqrt{8}$



Gauge and Quark Flows

Lüscher, JHEP 1008, 071 ('10); 1304, 123 ('13)

We adopt the simplest one suggested by Lüscher.

Gauge flow: standard Wilson flow

original gauge field at t = 0

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x)$$

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + [B_\mu(t,x), B_\nu(t,x)],$$

$$D_\nu G_{\nu\mu}(t,x) = \partial_\nu G_{\nu\mu}(t,x) + [B_\nu(t,x), G_{\nu\mu}(t,x)],$$

Quark flow: as suggested by Lüscher

original quark field at t = 0

$$\partial_t \chi_f(t,x) = \Delta\chi_f(t,x), \qquad \chi_f(t=0,x) = \psi_f(x),$$

$$\partial_t \bar{\chi}_f(t,x) = \bar{\chi}_f(t,x)\overleftarrow{\Delta}, \qquad \bar{\chi}_f(t=0,x) = \bar{\psi}_f(x),$$

$$\Delta\chi_f(t,x) \equiv D_\mu D_\mu \chi_f(t,x), \qquad D_\mu\chi_f(t,x) \equiv [\partial_\mu + B_\mu(t,x)]\,\chi_f(t,x),$$

$$\bar{\chi}_f(t,x)\overleftarrow{\Delta} \equiv \bar{\chi}_f(t,x)\overleftarrow{D}_\mu\overleftarrow{D}_\mu, \qquad \bar{\chi}_f(t,x)\overleftarrow{D}_\mu \equiv \bar{\chi}_f(t,x)\left[\overleftarrow{\partial}_\mu - B_\mu(t,x)\right]$$

only gauge fields involved

Quark field renormalizattion

$$\chi_R(t,x) = Z_\chi \chi_0(t,x) \qquad Z_\chi = \sqrt{\varphi(t)} \qquad \varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

Makino-Suzuki ('14)

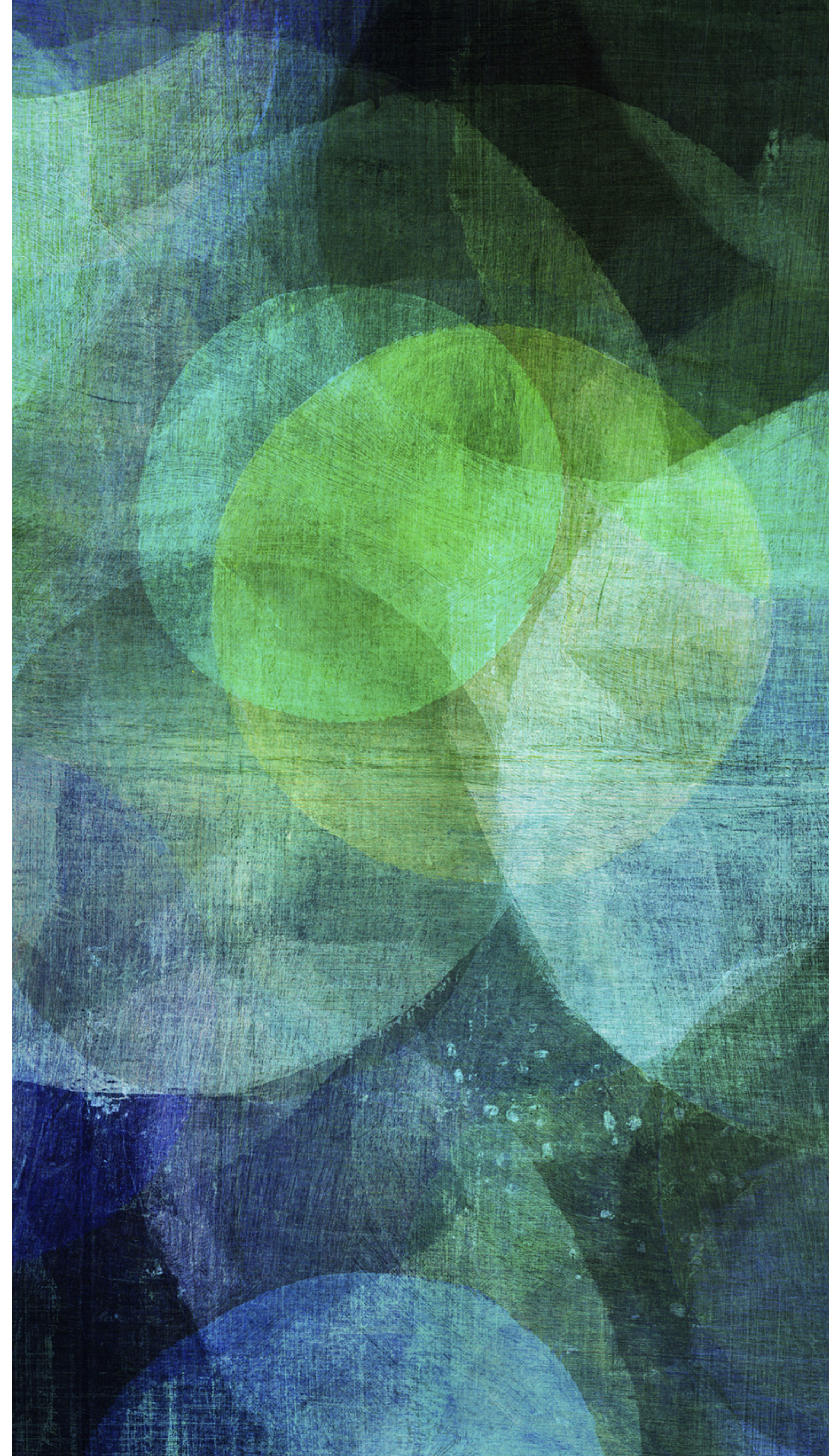
No more renormalization needed for any composite op's.

VEV(T = 0)

GAUGE DEFINITION

$$Q = \frac{1}{64\pi^2}\int d^4x\,\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}(x)F^a_{\rho\sigma}(x)$$

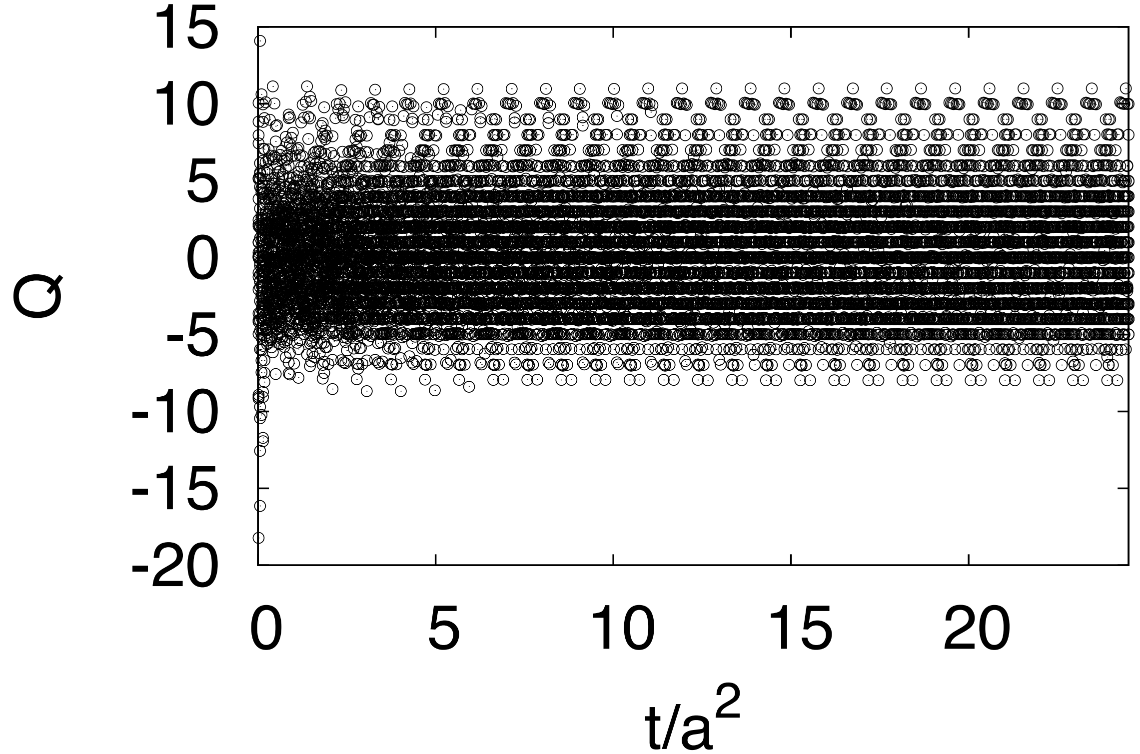
Use GF just as cooling. Extract info at large *t*.



GF as Cooling

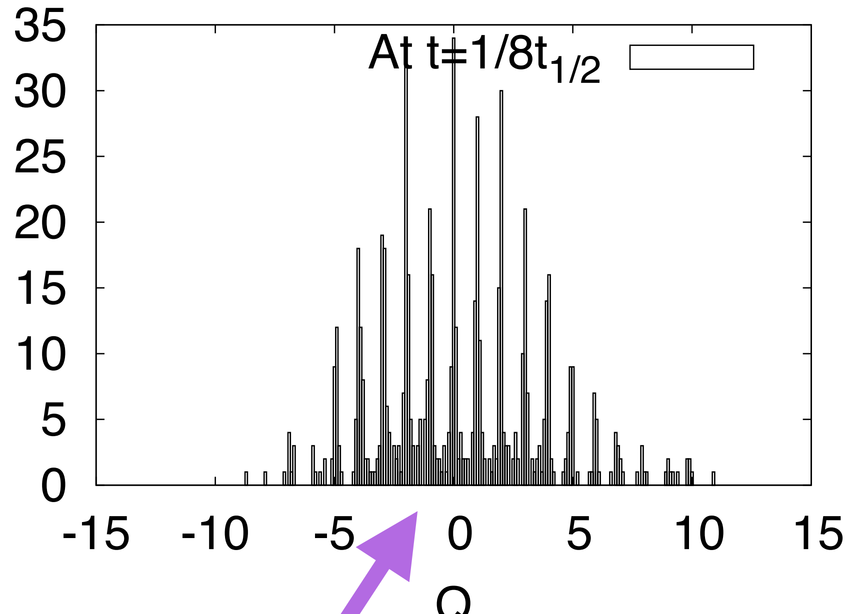
$T/T_c = 0$

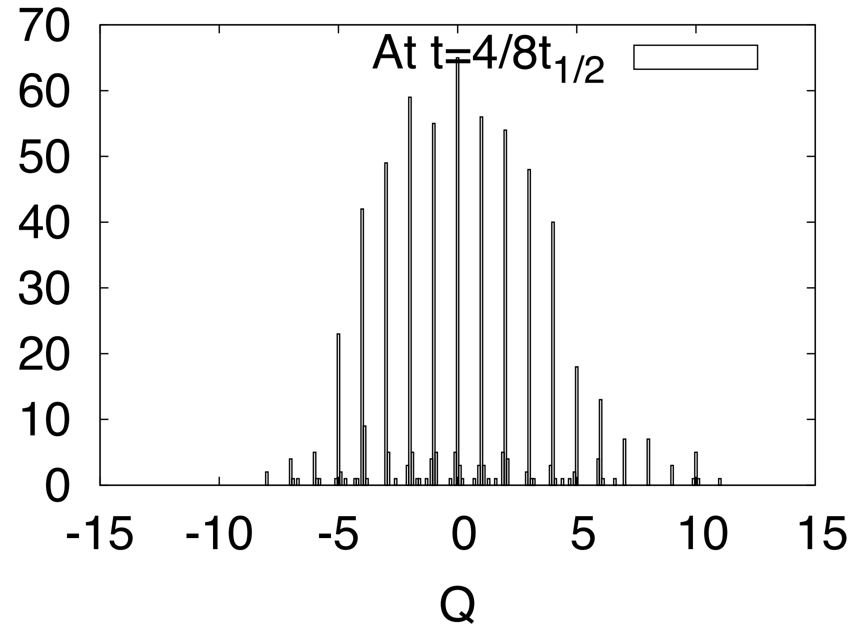
28^3x56

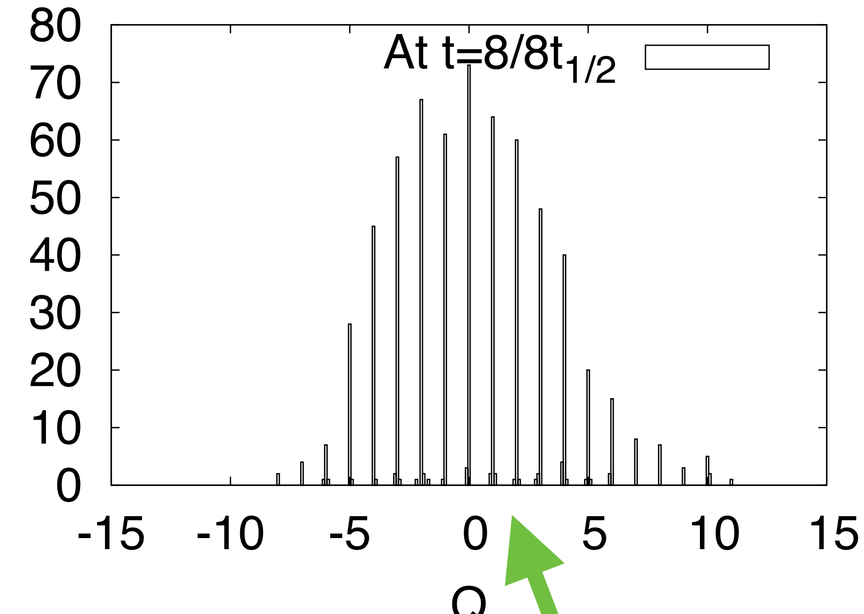


$t_{1/2} = 24.5$

GF works well as a cooling.







Non-integer Q's

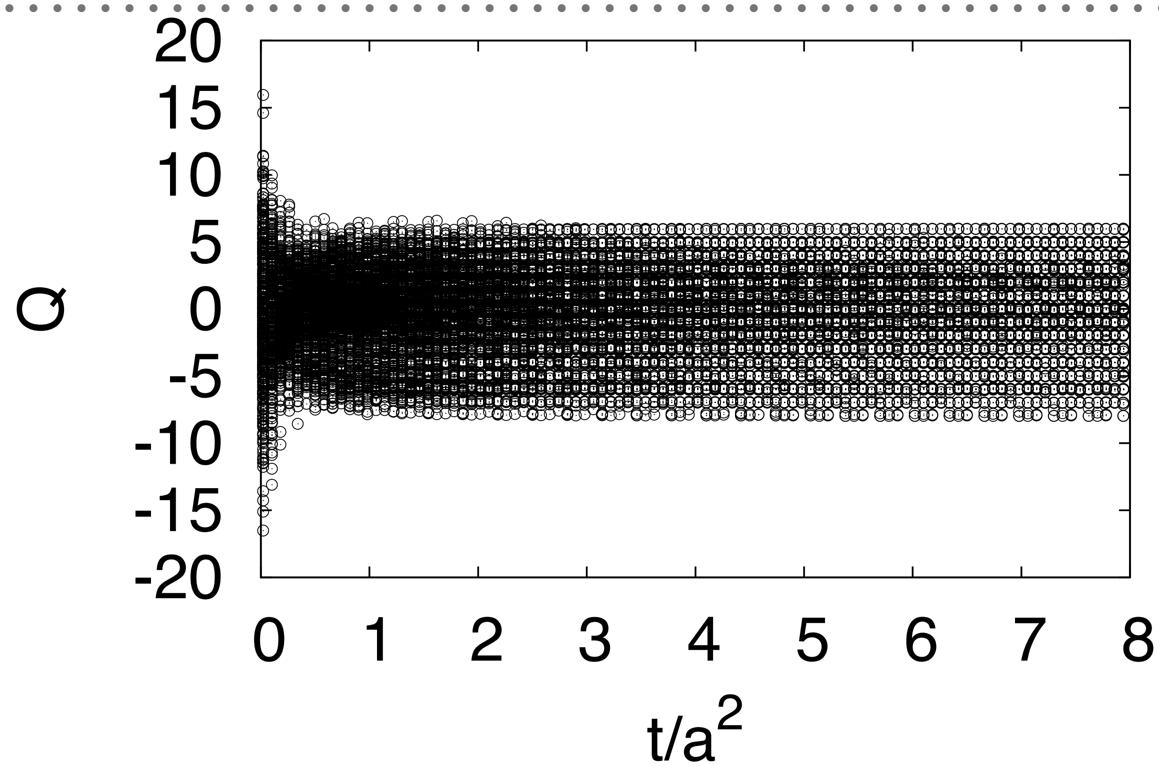
Accumulate to integer Q's

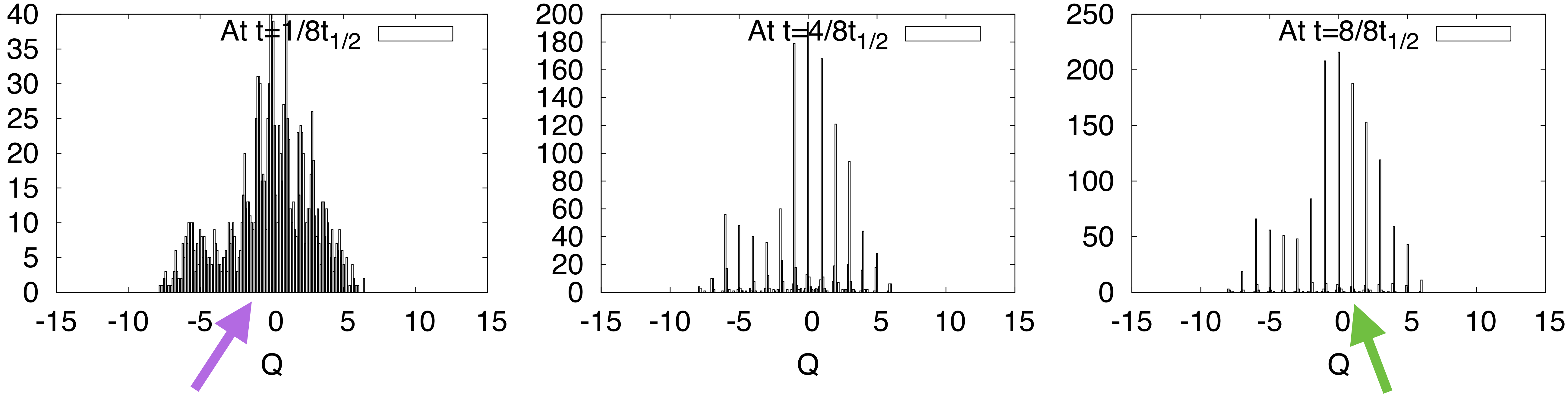
GF as Cooling

$T/T_c = 0.92$

32^3x16

$t_{1/2} = 8$





Non-integer Q's

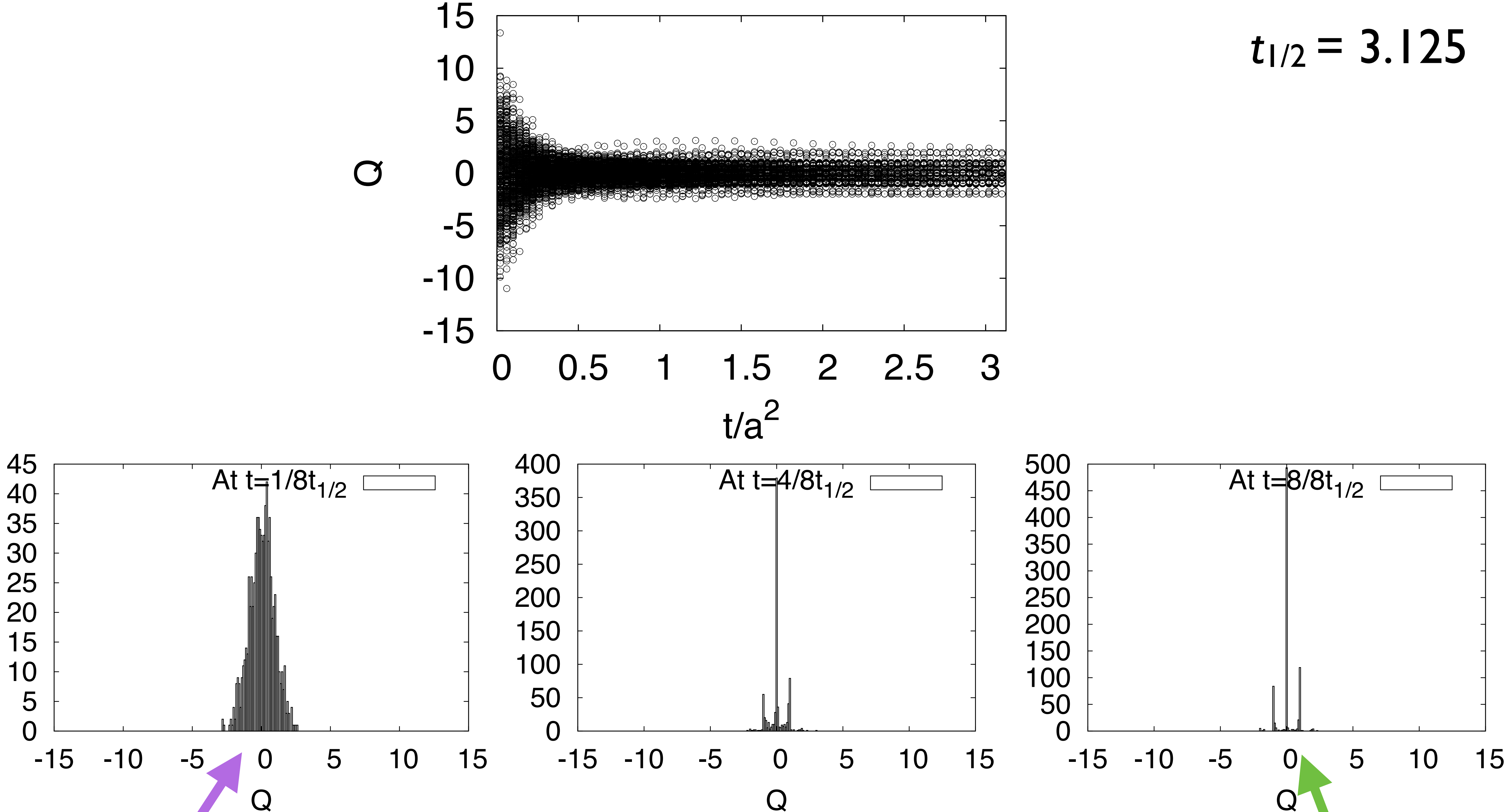
Accumulate to integer Q's

GF as Cooling

$T/T_c = 1.47$

32^3x10

$t_{1/2} = 3.125$



Non-integer Q's

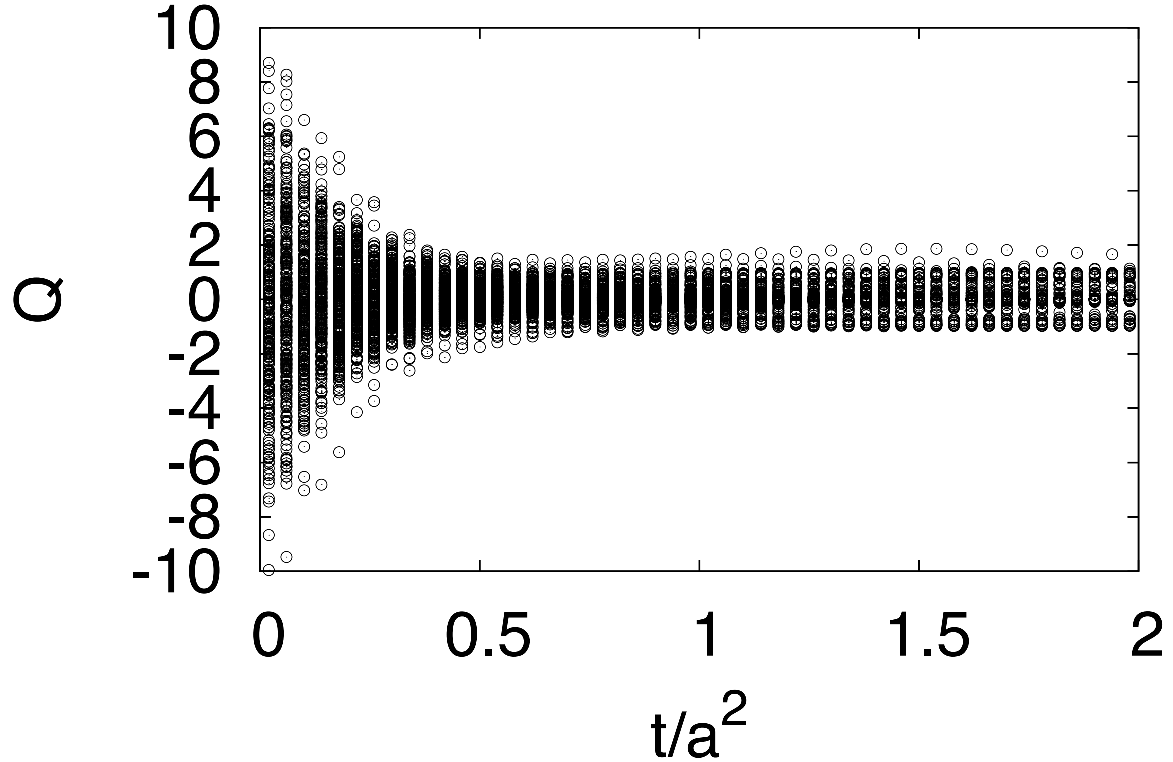
Accumulate to integer Q's

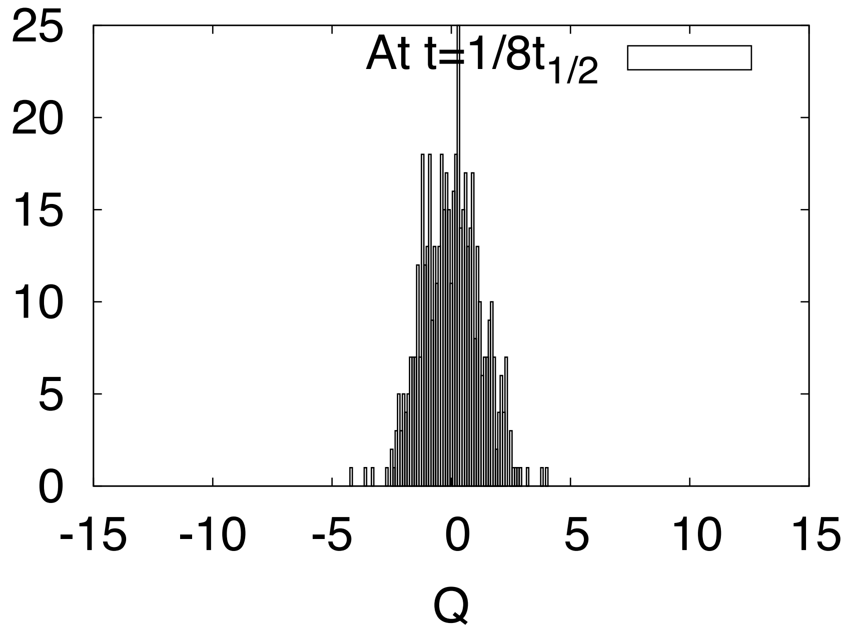
GF as Cooling

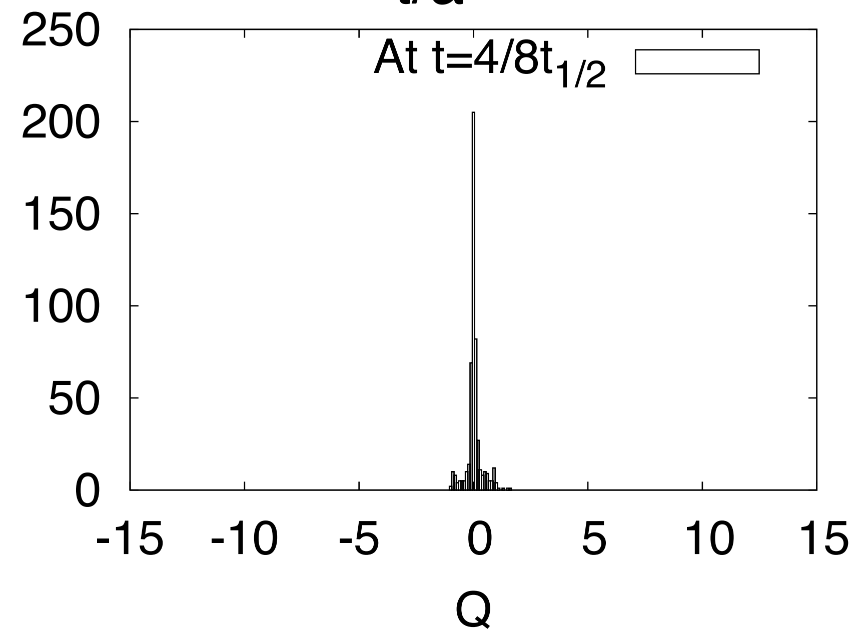
$T/T_c = 1.83$

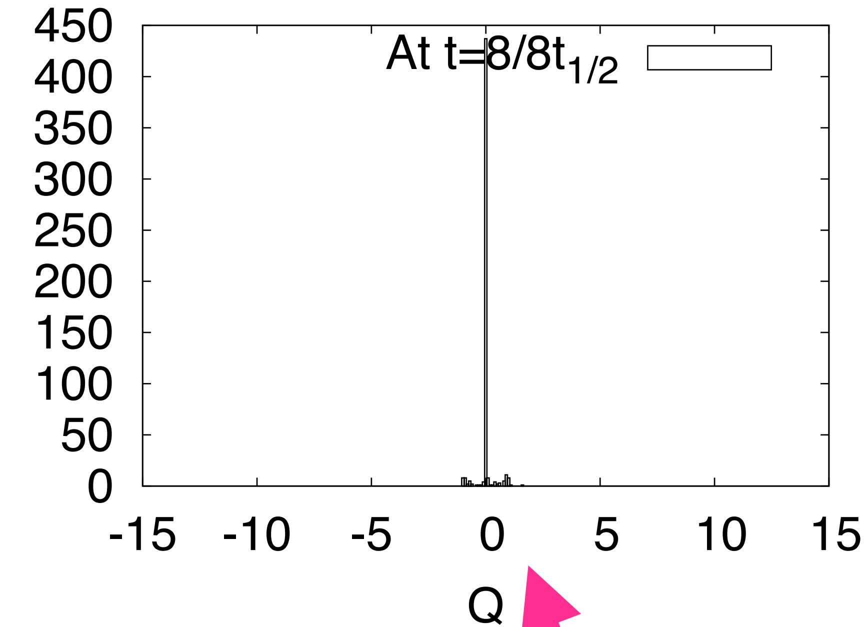
32^3x8

$t_{1/2} = 2$



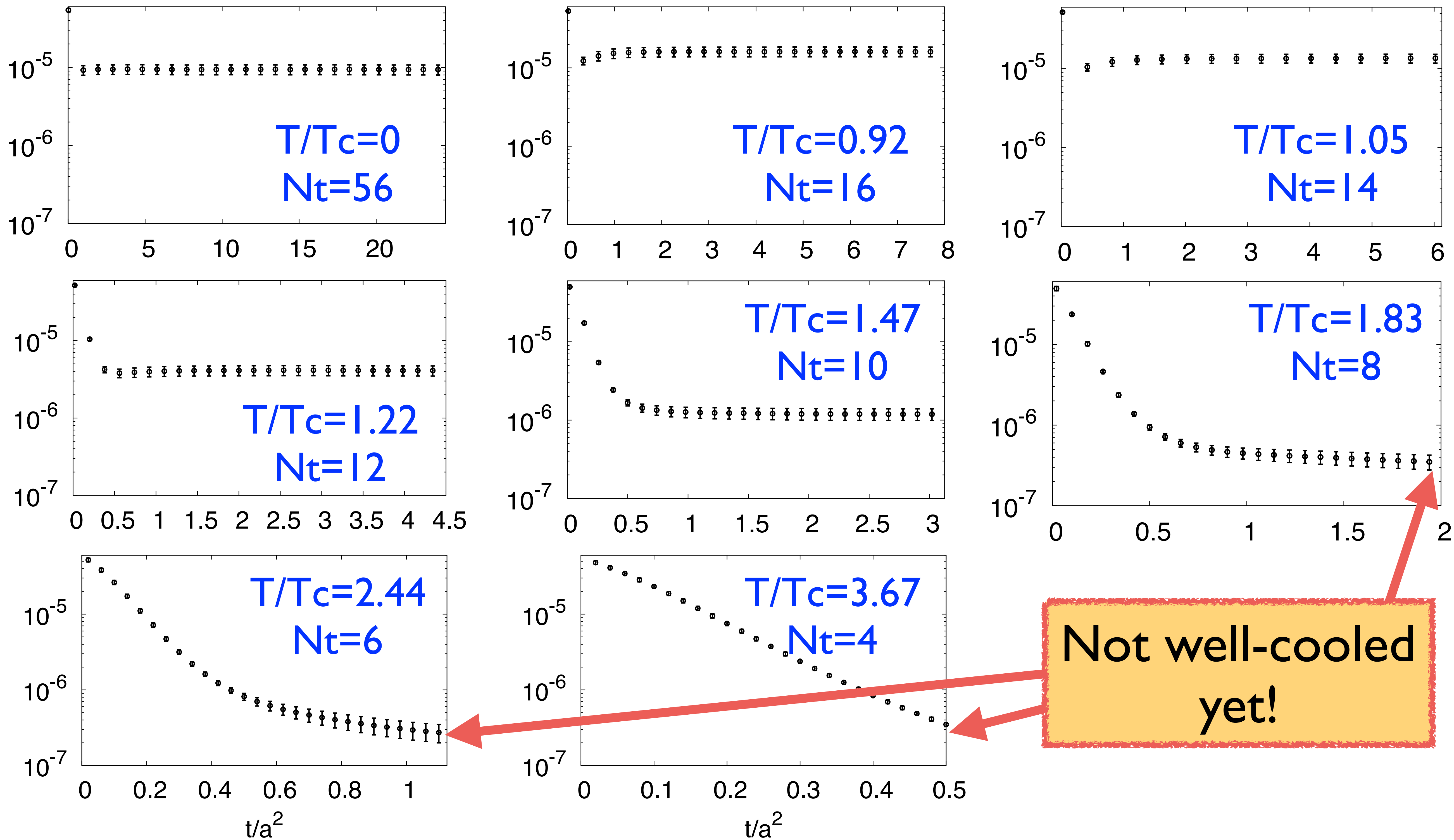






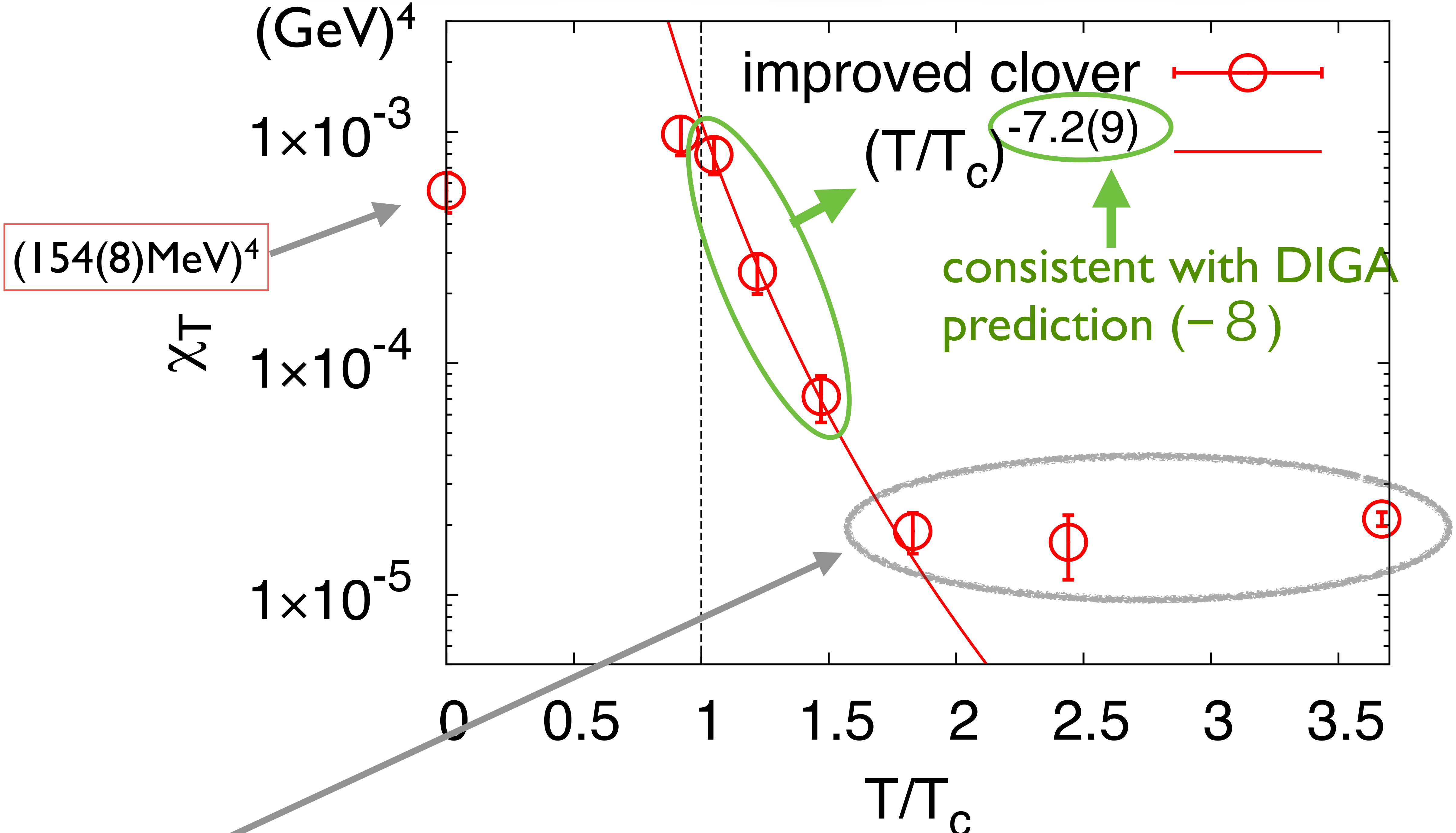
Starting to freeze to Q = 0.

GF-Cooled Topological Susceptibility (Gauge-Definition)



We have stopped GF at $t = t_{1/2}$, but we may go further as cooling.

Results: Gauge definition

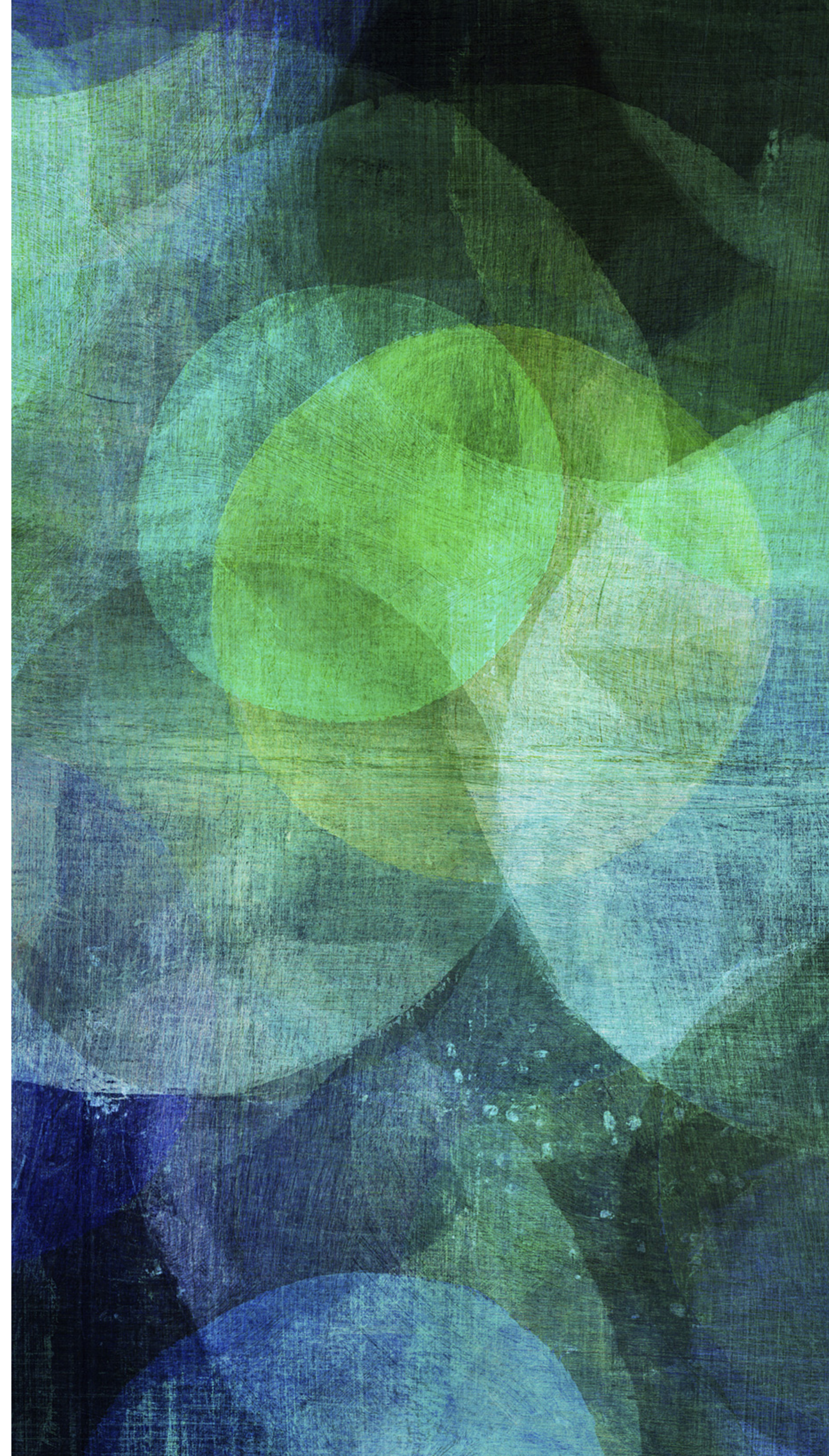


- GF up to $t_{1/2}$ insufficient, giving upper bounds.
- Freezing to Q=0 at T/Tc > 2.
- Small-Nt error severe for $Nt \leq 8$ (from EMT experience).

FERMION DEFINITION

$$\langle Q^2 \rangle = \frac{m^2}{N_f^2} \langle P^0 P^0 \rangle_{\text{disc}}$$

Use GF as a renormalization scheme.
Extract info in the $t \to 0$ limit.



Suzuki's Strategy

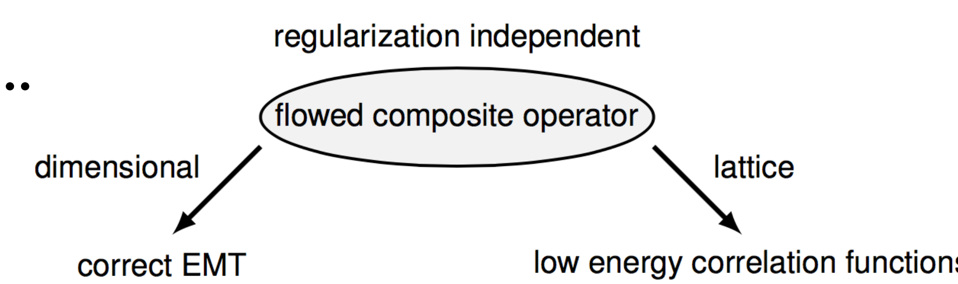
Evaluate $P^0 = \int d^4x \bar{\psi}(x)\gamma_5\psi(x)$ by GF following the H. Suzuki's strategy:

GF provides us with a physical (i.e. non-perturbative) renormalization scheme, which is directly calculable on the lattice in the *a* → 0 limit, and also with other continuum regularizations, such as the dimensional regularization, too.

=> Use the finite quantities to relate lattice and continuum regularizations.

Observables related to a symmetry which is violated on lattice:

O = chiral cond. / EMT / quark scalar densities / P_0 / ...



1) Define P_0 by a chiral W-T identity in a continuum scheme.
2) Relate it with a lattice operator through finite observable at $t > 0$ in the $a \to 0$ limit.
   By the GF evolution, however, unwanted operators can mix at $t > 0$.
3) Remove unwanted contributions by $t \to 0$.
   Matching with conventional re. scheme can be calculated by PT near the $t \to 0$ limit.

Fermion Definition by Suzuki's Strategy

Hieda-Suzuki, arXiv:1606.04193

P^0 in $\overline{\text{MS}}$ scheme at $\mu=1/\sqrt{(8t)}$ by GF

Quark field renormalization.

$$m_R(\bar\psi\gamma_5\psi)_R\Big|_{\overline{\text{MS}}}^{\mu=1/\sqrt{8t}} = \lim_{t\to 0}\Big[c_S(t)\, m_{\overline{\text{MS}}}(1/\sqrt{8t})\,\varphi(t)\,\bar\chi(t,x)\,\gamma_5\,\chi(t,x) + t\,\mathcal{O}_{D=6} + \cdots\Big]$$

Physics extracted by
$t \to 0$ extrapolation.

Matching with $\overline{\text{MS}}$ scheme.

On lattice with small but finite a:

We encounter additional mixings like $$+\frac{a^2}{t}\mathcal{O}_{D=4} + O(a^4) + \cdots$$

Note: lattice artifacts of our NP-clover are O(a^2).

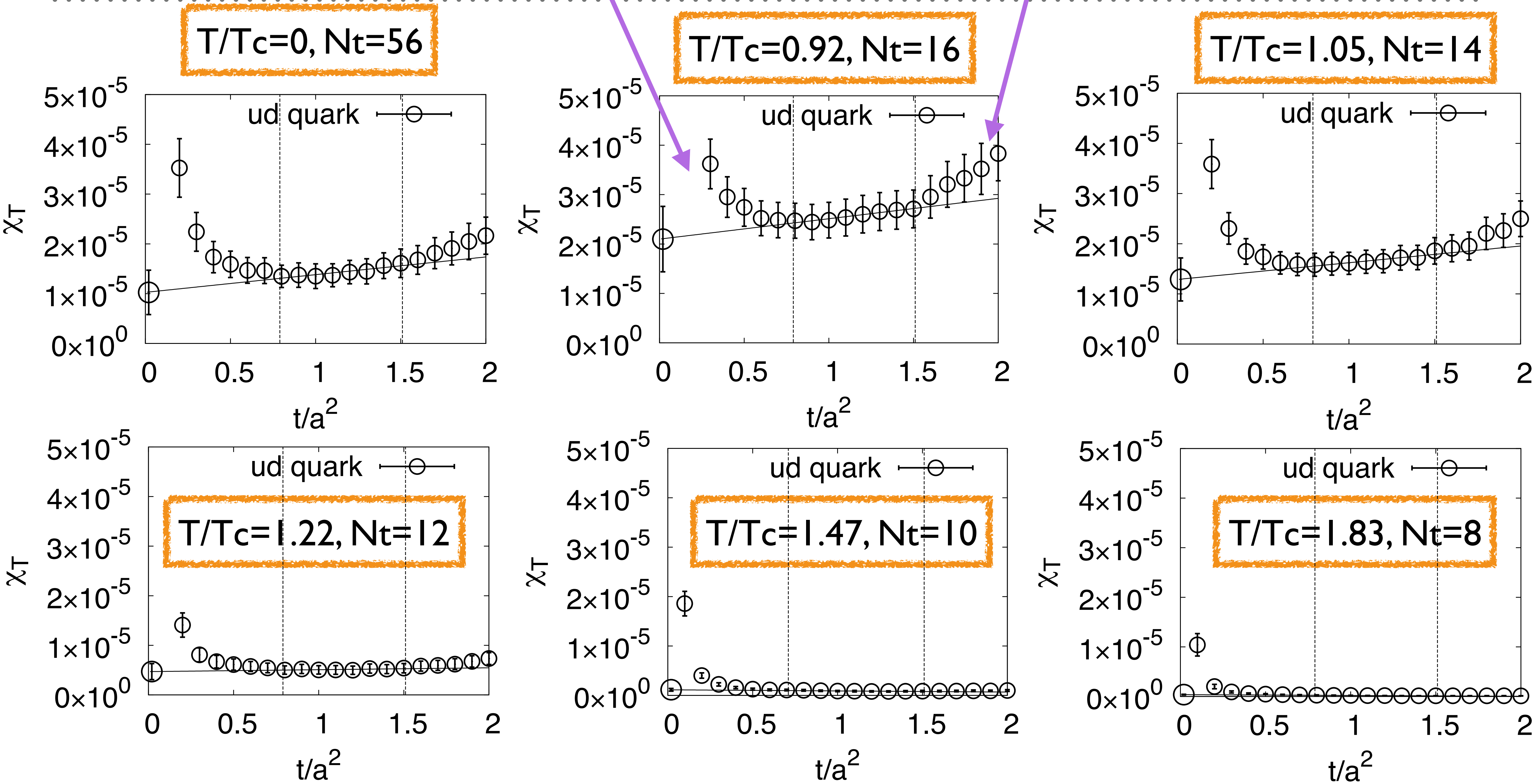
Ideally, we remove such singularity by taking $a \to 0$ before $t \to 0$.

We instead exchange the order of $a \to 0$ and $t \to 0$ extrapolations,
by removing such singularity by hand.

Our Data

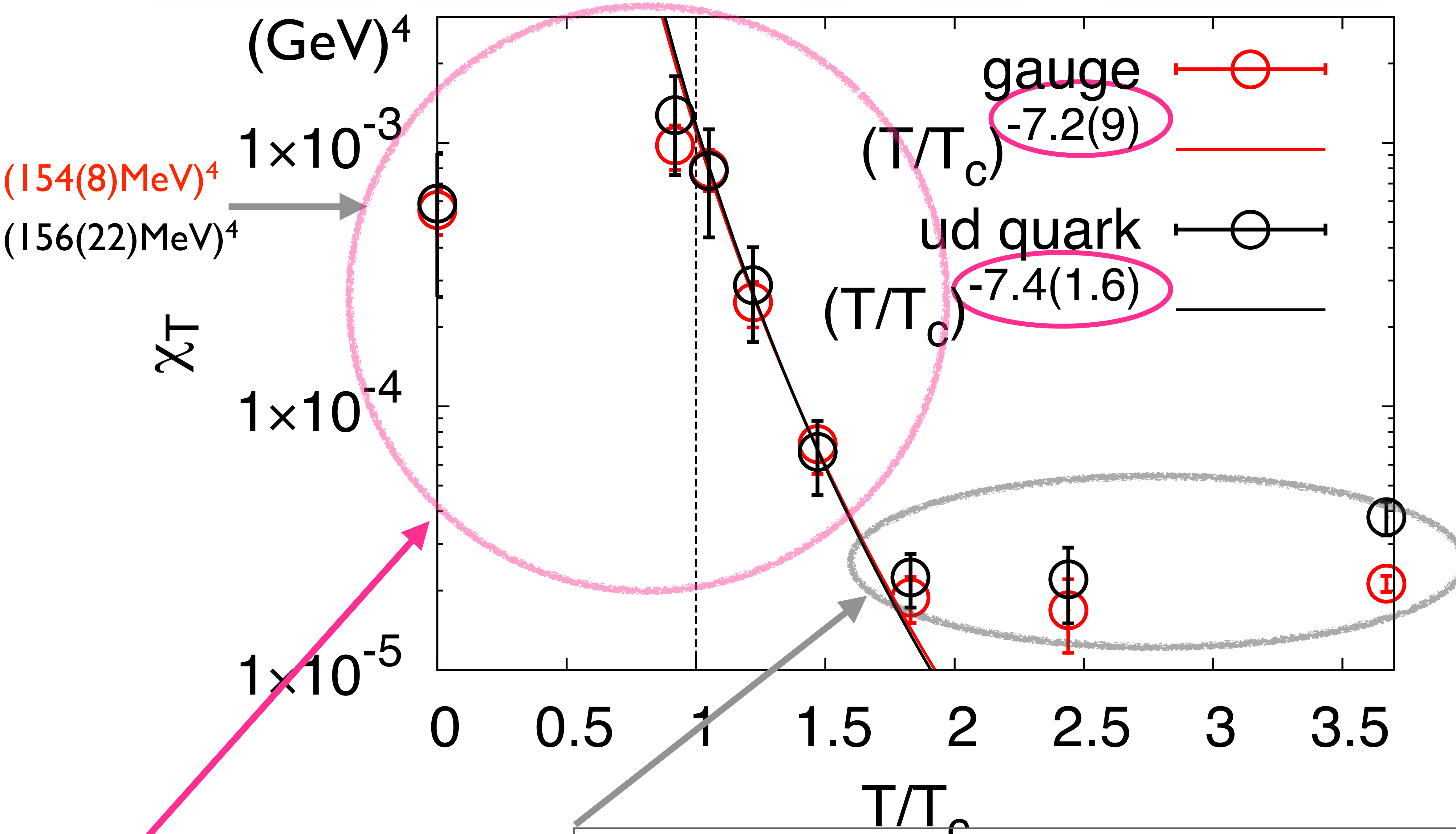
$$O\left(\frac{a^2}{t}\right) \qquad O\left(t^2\right)$$

using ud quark sector



We see clear linear windows avoiding singular terms as well as higher order terms within the meaning full range of below $t_{1/2}$.

Results: Gauge vs fermion definition

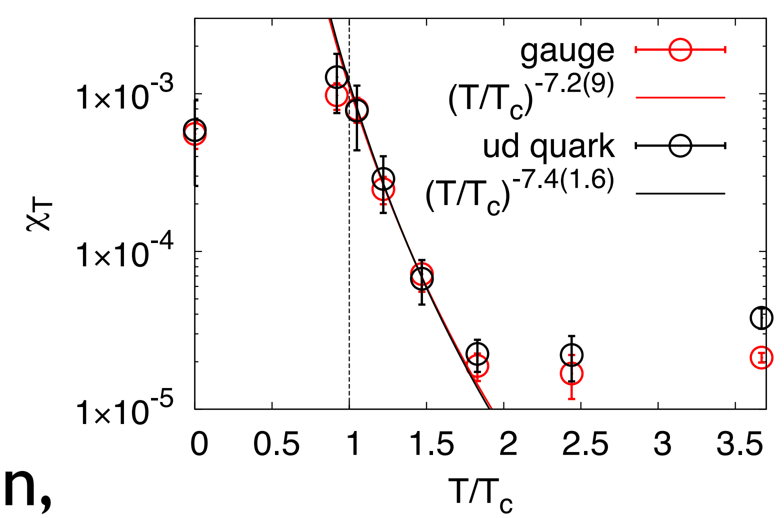


- Small-*Nt* error severe for *Nt*≤8 (from EMT)
- Freezing to Q=0 at T/Tc > 2.

- ☆ Both definitions agrees well at T/Tc < 1.5.
- ☆ Both exponents consistent with a DIGA prediction of –8.

SUMMARY

- We apply gradient flow ideas to investigate thermodynamics of (2+1)-flavor QCD. As the first test, we choose heavy ud quarks with physical s quark, on a fine lattice ($a \approx 0.07$fm, $m_{PS}/m_V \approx 0.63$), and adopt the fixed-scale approach.
- EMT / EoS / chiral cond. results presented by Y. Taniguchi on Monday.
- Topol. suscept. by gauge definition: GF as a cooling (t: large)
- Topol. suscept. by fermion definition: GF as a renormalization scheme ($t \to 0$)
- Completely different two estimations beautifully agree with each other,
- and reproduce T-dep. predicted by DIGA.
- But, note that our $m_\pi \sim 400$ MeV and finite a.



- Further study needed to complete the cont. extrapolation,
- and at lighter m_{ud}.